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DOI
10.1016/j.omega.2019.08.008

Publication date
2020

Document Version
Final published version

Published in
Omega (United Kingdom)

Omega 97 (2020) 102101

Contents lists available at ScienceDirect

Omega

journal homepage: www.elsevier.com/locate/omega

Portfolio-based airline fleet planning under stochastic demand☆

Constantijn A.A. Sa[a], Bruno F. Santos[a,*], John-Paul B. Clarke[b]

[a] Air Transport and Operations, Faculty of Aerospace Engineering, Delft University of Technology, Kluyverweg 1, Delft 2629 HS, the Netherlands
[b] The Daniel Guggenheim School of Aerospace Engineering, Georgia Institute of Technology, Atlanta, GA 30332, USA

ARTICLE INFO

Article history:
Received 1 May 2018
Accepted 12 August 2019
Available online 13 August 2019

Keywords:
Airline fleet planning
Robust planning
Portofolio-based planning
Ornstein–Uhlenbeck process

ABSTRACT

Airlines operate their fleet of aircraft over a relatively long time horizon during which the realized stochastic demand has the potential to profoundly impact the airlines' financial performance. This makes the investment in a fleet of aircraft a highly capital-intensive long-term commitment, associated with inherent risks. We propose an innovative three-step airline fleet planning methodology with the primary objective of identifying fleets that are robust to stochastic demand realizations. The methodology presents two main innovation aspects. The first one is the use of the mean reverting Ornstein–Uhlenbeck process to model the long-term travel demand, which is then combined with discrete-time Markov chain transitions to generate demand scenarios. The second innovative aspect is the adoption of a portfolio-based fleet planning perspective that allows for an explicit comparison of different fleets, in size and composition. Ultimately, the methodology yields for each fleet in the portfolio a distribution of net present values of operating profit across the planning horizon and a list of key financial and operational metrics per year. The robustest fleet can be selected based on the operating profit generating capability across different realizations of stochastic demand. An illustrative case study is presented as a proof of concept. The case study is used to demonstrate the type of results obtained and to discuss the usefulness of the methodology proposed.

1. Introduction

1.1. Airlines' poor financial performance

Airlines have low profit margins and consequently are among the poorest performers when it comes to providing return on invested capital (ROIC). For example, between 2004 and 2011, North American airlines annually returned 4.1% to their investors, which is lower than the average weighted cost of capital (WACC) of 7.4% [1].

There are a plethora of reasons underlying this poor profitability, which can be partially explained via Porter's five forces model [2]: the bargaining power of suppliers (i.e. aircraft and engine manufacturers, labor unions); bargaining power of buyers (i.e. passengers); relatively easy market entrance conditions; regulation; fierce price competition due to the commoditization of air transportation; a fragmented industry structure; and problems with the air transport value chain [1].

Two other factors can be added to these five. The first is the volatile nature of airlines operating profitability. The cyclical demand and the inconstant fuel prices can significantly impact the evolution of year-to-year operating profits and could contribute to a critical financial state of the airline or even bankruptcy [3]. The second factor to add is the fact that airline orders for new aircraft tend to be synchronized with years of high profit. Due to the lead time between the order and delivery of aircraft, these aircraft are often delivered in periods of downturn of the business cycle which causes overcapacity [4–6].

The combination of all these factors results in a persistently low profit margin, an inability to meet return requirements (i.e. ROIC lower than WACC), and a high risk of bankruptcy due to year-to-year volatility in demand and fuel prices.

1.2. The airline fleet planning problem

Fleet planning is the most strategic long-term consideration in airline planning and can profoundly impact the financial performance and operational flexibility of an airline. The fleet planning problem involves the management of the fleet size and composition over time by deciding on matters such as: how many aircraft to acquire, which aircraft types to acquire, when to acquire them, when to dispose them and decisions regarding leasing or buying. These decisions are commonly addressed separately or sequentially, to produce a fleet planning plan [7]. Often, the fleet

☆ This manuscript was processed by Associate Editor I. Ljubic.

* Corresponding author.
E-mail address: b.f.santos@tudelft.nl (B.F. Santos).

https://doi.org/10.1016/j.omega.2019.08.008
0305-0483/

Nomenclature

B	Set of scenarios generated
D	Set of Monte Carlo simulations
F	Set of fleets in portfolio
H	Set of hub airports
K	Set of aircraft types
M	Set of OD demand matrices per year
N	Set of airports
S	Set of sample values
Y	Set of years in planning horizon
Z	Set of OD pairs

planning decisions are closely tied to decisions on network development [8,9], which deals with the question on which markets (i.e. origin-destination pairs) to serve and which routing network to employ (e.g. hub-and-spoke or point-to-point). Investing in an aircraft fleet is a highly capital-intensive long-term commitment which bears inherent risk because the fleet is deployed across a long-term planning horizon over which uncertainty will materialize, both on the revenue side (e.g. stochastic demand) as well as on the cost side (e.g. fuel price volatility). Consequently there is a need for airlines to have a robust fleet that is resilient and flexible to this uncertainty in terms of profit generating capability.

Numerous measures can be taken to achieve robustness to uncertainty; revenue management and pricing models can be used to favorably influence demand patterns; hedge contracts can reduce the exposure to fuel price volatility; and there is an increasing trend towards aircraft leasing because of the flexibility benefits and reduced up-front investment cost. Leasing comes at an operational cost for the airline however, due to a compensation for the incurred risk that is transferred to the leasing company. This cost of purchasing flexibility from another entity could potentially be avoided by focusing on robustness during the fleet investment process by having the robustness built into the fleet composition itself. This research presents an innovative methodology that aims to identify fleets that are robust to stochastic demand realizations.

1.3. Literature review

1.3.1. Buying versus leasing

A major consideration in fleet planning is whether to buy or lease the aircraft in the fleet. In [10], the authors address this aspect of the fleet composition problem by focusing on the optimal lease/own mix for airlines that experience cyclical and stochastic demand. Specifically, they propose a formulation for the cost trade-off between owning an aircraft, which yields reduced capital cost and increased expected cost of overcapacity, as opposed to leasing an aircraft. Through a case study on 23 airlines in the period 1986–1993, they show that the optimal portion of leased aircraft with respect to all the aircraft in the fleet lies between 40% and 60%. The authors concluded by noting that aircraft lease contracts act as a means for risk sharing between airlines, which have reduced risk through increased flexibility in capacity management, and leasing companies that require a risk premium for their incurred risk.

In a more recent work [11], the authors approach the fleet planning problem from the same perspective and proposes a binary-integer linear programming model for aircraft replacement strategy. The objective function minimizes the total discounted cost of buying, leasing, operating and maintaining aircraft over a planning horizon of 10 years. Moreover, it includes two other cost terms. One that represents additional costs associated with owning aircraft, such as spare parts, hangars and crew training; An another term that accounts for the sale of aircraft. Five observations are made from the results, that apply to both of the case studies that were performed: new aircraft are favored over old aircraft irrespective of buying/leasing decisions; solutions with short-term leases are favored; old aircraft are to be sold; fleet diversity is discouraged; and leasing is preferred over buying. The latter observation is consistent with [10,12]. Although a method is proposed that incorporates a considerable number of terms in the objective function and constraints, the contribution fails to account for uncertainty in demand. Rather a sensitivity analysis is performed on lease and buy prices (i.e. plus or minus 50%). When analyzing the magnitude of these different cost terms it is observed that operation and maintenance cost are the major cost drivers when evaluated over the long term. The results of a case study indicate a strategy towards leasing new aircraft of common aircraft types over the short term and moreover shows that aircraft with a higher purchase price and a higher operating efficiency are preferred over aircraft that are less expensive to acquire but more costly to operate.

1.3.2. Dynamic capacity allocation

In an effort to account for stochastic demand in the fleet composition problem, the authors of Listes and Dekker [13] propose a two-stage stochastic programming model for fleet composition optimization where robustness is added to the fleet planning decision by including stochastic demand and using the concept of demand driven dispatch, as introduced in [14]. The latter concept acknowledges the existence of uncertainty in future demands when decisions about fleet compositions or initial fleet assignments are made, and tries to accommodate that uncertainty by having fleets that consist of aircraft of different sizes but within the same crew-compatible family so that they can be swapped when more information about the actual demand becomes available close to the day of operation. Moreover, it is noted that when the stochastic model is solved with integrality constraints the optimality gap is smaller than 0.5%, which is comparable to the order of magnitude of the optimality gaps that result from linear relaxation in deterministic models. Although Listes and Dekker [13] makes a great step forward when it comes to considering stochastic demand in the fleet planning decision, the approach is limited due to a sole focus on short cycle variations in stochastic demand that are to be solved using re-assignment. The approach fails to account for the longer term uncertainty in demand that is characteristic to fleet planning. A scenario aggregation solution algorithm is used to solve the fleet composition problem in the first stage. The assumption is made that demand is independent and follows a normal distribution, which is discretized into a set of scenarios using descriptive sampling.

1.3.3. Multi-period fleet planning

Initial studies on the multi-period fleet planning problem date from the ’80s [15]. These initial models were deterministic and simplified representations of the problem, producing less efficient results. The current trend is to consider demand uncertainty when planning the fleet for multiple years. For instance, in [12] the authors propose an optimal replacement schedule for airline fleets using a stochastic dynamic programming model which is solved using backward computing. A grey topological forecasting method combined with Markov-chain is used to model the stochastic demand on a market level. Subsequently, a frequency dependent market share estimation is used to calculate demand at the airline level. From the results it is observed that high volatility in demand drives fleet planning decisions to favor leasing over buying. The objective function minimizes three cost terms per period over a multi-period planning horizon. These cost terms are operating cost, aircraft replacement cost and a penalty cost which arises from the potential difference between forecasted and actually realized demand. Operating cost are assumed to be dependent on aircraft *sta-*

tus, which is defined as: aircraft age, type and mileage travelled. A sensitivity analysis is performed on the aircraft age and average lease cost. Although a profound step forward is made using the sophisticated demand forecasting method that accounts for the cyclical demand, the authors of Hsu et al. [12] note that this method still lacks the influence of non-cyclical (i.e. random) variations in demand as result of, for instance, terrorist attacks and aircraft accidents.

One drawback of earlier attempts in literature to include the non-cyclical nature of stochastic demand was noticed by Khoo and Teoh [16]. This is referred to as *"the possibility of unexpected events that could take place unexpectedly"*. In order to capture this, the authors propose the formulation a stochastic demand index (SDI). The SDI is developed in multiple steps by identifying a range of possible unexpected events such as disease outbreaks and natural disasters, as well as the probability distributions of these situations based on their historical occurrence. Then the occurrence of all these uncertain events is modeled using a Monte Carlo simulation, combined with a traditional demand forecast that does not account for uncertain events, in order to arrive at a single SDI for each operating period. The SDI is then used as an input to a fleet management optimization model.

More recently, Du et al. [17] proposed a multi-period schedule for a set of heterogeneous airport towing tractors, under demand uncertainty, flight schedule disruptions and different cost structures. The model optimizes fleet size and mix by determining the time of buying, overhauling and selling tractors. The authors propose a 4-step approach for demand aggregation and demonstrating the application of the model in a case study with a major European airport. In [18], a scenario tree approach to solve the multi-period airline fleet planning problem. The authors considered that the nodes of the tree represent the decisions points in different stages of the planning horizon and the branches represent the scenario paths with demand variations. A mixed-integer linear programming model is proposed to determine the optimal fleets. Given that some scenario paths share common nodes of the tree, the scenarios are modeled as interdependent and solved together. The probabilities associated with each scenario are used to compute fleet probability tables for each time stage in the scenario tree. The authors only consider cyclical and pre-defined demand variation cases. In a very recent paper, Wang et al. [19] addressed the multi-period fleet planning problem for the chartering problem in the shipping industry. The authors model the problem as a tactical fleet composition problem taking into account market uncertainties. A two-stage stochastic programming model is proposed, in which the planning period is divided in two periods to capture different confidences levels in the estimation of the market conditions (i.e., demand, fuel prices and spot rates).

1.4. Research objective

The motivation behind this fleet planning research work stems from three observations.

- First, in the long-term, air transportation demand is resilient to external shocks (Fig. 1). That is, the demand evolution is composed by cyclical and non-cyclical variations, however, in the long-term, it readjusts from non-cyclical variations and gravitates around a long-term trend.
- Second, in practice fleet planners make their decisions based on a set of different alternatives, which are explicitly compared. The fleet optimization models present in the literature usually focus on obtaining a single optimal fleet and do not capture all elements that influence the fleet planning process in practice.
- Finally, optimization models and models that explore the evolution of stochastic variables tend to be computationally demanding (as highlighted by, e.g., [16,18]). These properties make it challenging to combine these methodologies into one fleet planning modeling framework that provides meaningful results.

Following these observations, the goal of this research work is to develop an innovative fleet planning model that realistically considers the long-term stochastic nature of air travel demand and generates meaningful results.

Results are considered *meaningful* if they allow for the explicit comparison of both financial and operational performance metrics of different fleets, across the planning horizon for numerous realizations of stochastic demand. To achieve this objective the proposed methodology adopts a portfolio of fleets (each of different size or composition) and uses an optimization model that simultaneously considers network development and frequency planning. This allows for the explicit comparison of the profit generating capability of each fleet from the portfolio across a long-term planning horizon across numerous realizations of stochastic demand.

1.5. Contribution

The contribution of this paper is two-fold:

- A long-term (multi-year) consideration of stochastic demand per origin-destination pair is presented by modeling demand as a mean reverting Ornstein–Uhlenbeck process and using discrete-time Markov chain transition probability matrices to generate scenarios.
- The adoption of portfolio-based fleet planning perspective allows for explicit comparison of different fleets in terms of size and composition on both financial and operational performance metrics. Robust fleets can be selected based on their operating profit generating capability across different realizations of stochastic demand across the long-term planning horizon.

1.6. Outline

The remainder of the paper is outlined as follows. The methodology, which consists of a three-step modeling framework that deals with simulation, optimization and scenario generation, is presented in Section 2. Each of the three models is elaborated in detail. The case study definition and results are presented in Section 3. The conclusions from this work are drawn in Section 4.

2. Methodology

2.1. The overarching modeling framework

A three-step modeling framework is proposed and visualized in Fig. 2.

The modeling framework takes two inputs: the historical passenger demand of each origin-destination pair (OD pair) under consideration, and a portfolio of fleets where each fleet has a given size (i.e. total number of aircraft in the fleet) and composition (i.e. mix of aircraft types). Model 1 is used to identify the historical stochastic characteristics of each of the Z OD pairs under consideration and outputs a set of M OD demand matrices per year that represent the range of uncertainty within a year, for each of the Y years in the planning horizon. Model 2 takes one OD demand matrix as input as well as one fleet from the portfolio and returns the resulting annual operating profits by deploying the fleet based on the given demand in an optimal fashion. This optimization process is iterated for each combination of fleet from the portfolio (F) and each OD demand matrix within a year (M) and across years (Y). Consequently, a value matrix with size $F \cdot M \cdot Y$ is filled with annual operating profits per fleet, per OD demand matrix within the

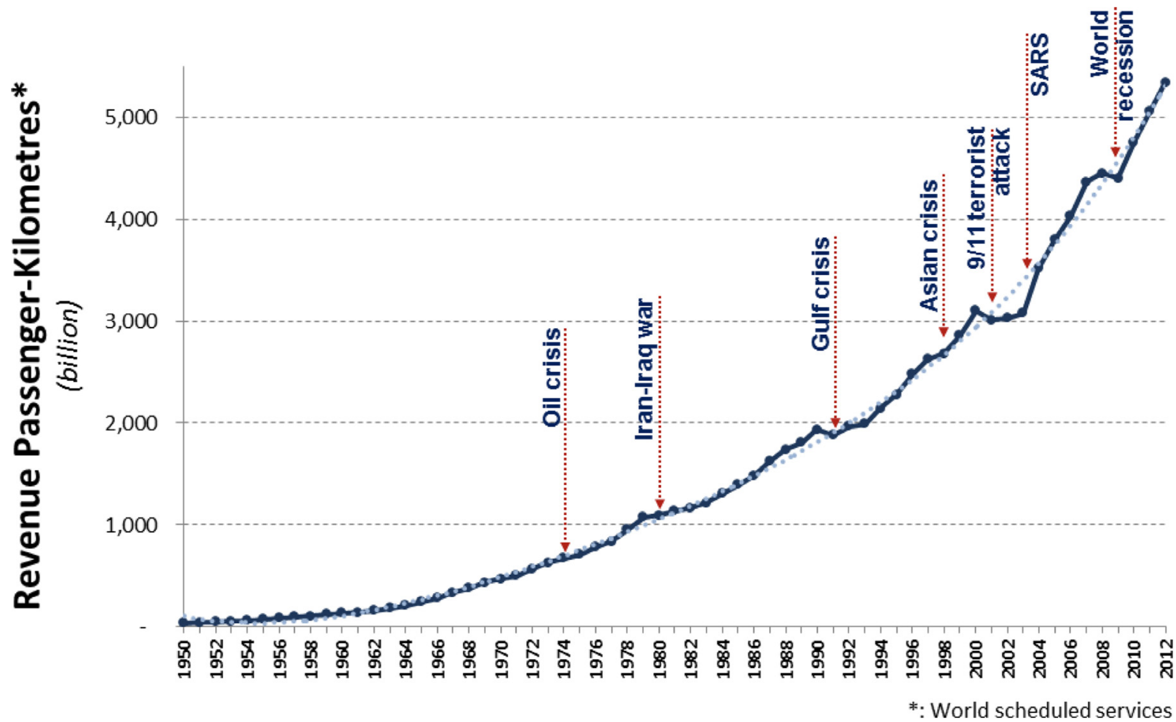

Fig. 1. Global air transport revenue passenger-kilometers, from 1950 to 2012 (source: ICAO).

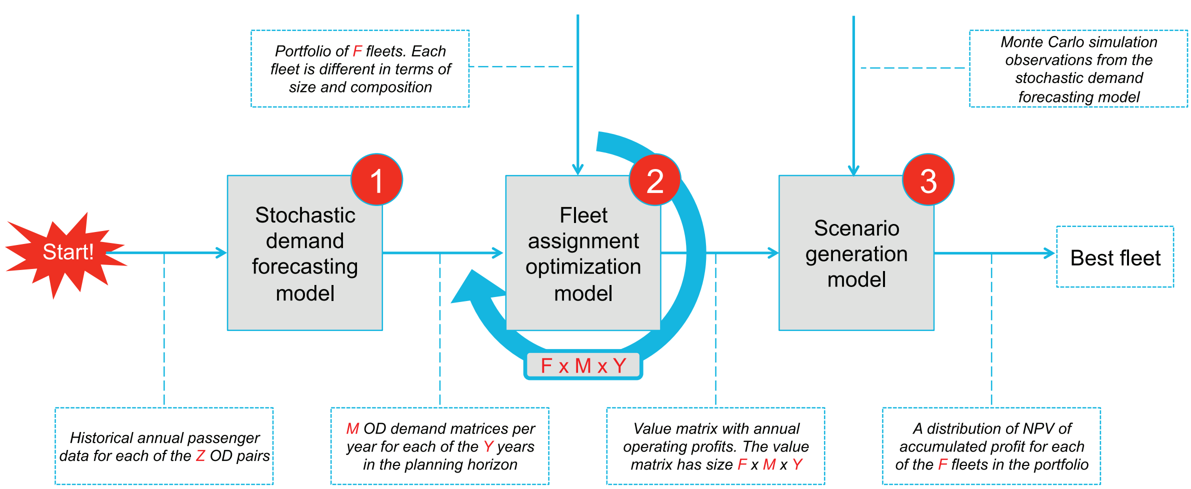

Fig. 2. The proposed solution methodology consists of three underlying models.

year (M), per year in the planning horizon (Y). As a third and final step, Model 3 generates paths (i.e. scenarios) through the value matrix across the planning horizon based on the underlying transition probability of demand sample values. The sequence of Y annual operating profits within each scenario is reduced to a single net present value (NPV). By iterating the scenario generation process numerous (B) times, a distribution of NPVs is obtained for each fleet in the portfolio. All three models are detailed in the next three sections.

2.2. Model 1: simulation of stochastic demand

In order to explore the evolution of stochastic demand into the future, the historical characteristics of the stochastic nature of demand need to be captured in a mathematical expression. In this research, the mean reverting Ornstein–Uhlenbeck process [20,21] is used to model the stochastic nature of air travel demand.

The mean reverting process has been successfully applied to model variables that tend to be cyclical. Prime examples include the modeling of stock, commodity and option prices [22,23]. Ultimately these variables tend to correlate to the cyclical behavior of gross domestic product (GDP). Another example is provided in [24], in which a Ornstein–Uhlenbeck process was proposed to model the retail demand uncertainty in a two-stage supply chain inventory planning problem. The authors assume that the demand process is driven by the market, but cyclically dominated by seasonal variations. In this research the mean reversion concept is applied to model the stochastic nature of air travel demand. Although modeling future stock prices or retail demand are a different activity than modeling future air travel demand, the underlying causes

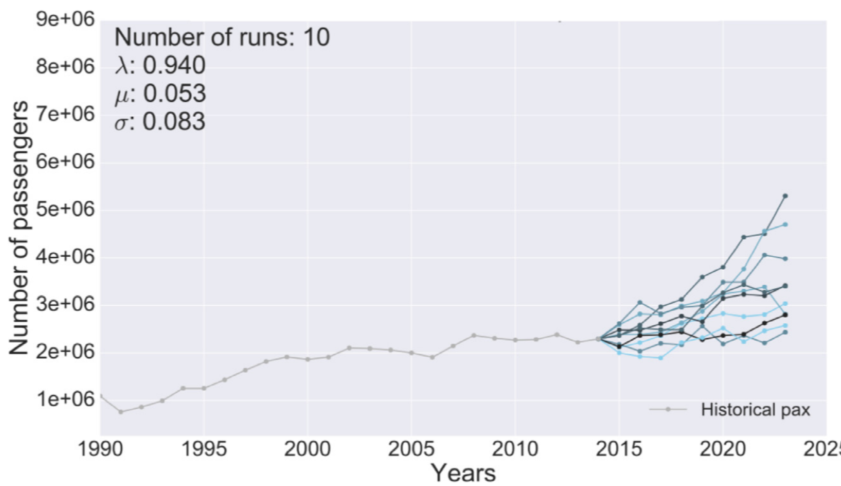

Fig. 3. Example of a Monte Carlo simulation of the mean reverting process, after 10 runs.

for the variation in these variables share a common dependence on GDP variations or seasonality effects.

2.2.1. Mathematical formulation

The mean reverting process is represented by the following equation;

$$X_{t+1} = X_t + \lambda(\mu - X_t) + \sigma dW_t \quad (1)$$

where X_{t+1} is the to be forecasted future air travel demand growth rate between time t and $t = t + 1$, X_t is the air travel demand growth rate between time $t - 1$ and t, λ is the speed of mean reversion, μ is the long-term mean growth rate, σ is the standard deviation of the historical estimation error and W_t is a random shock with N∼(0,1). As can be seen from the $\lambda(\mu - X_t)$ term, the expected corrective movement towards the long-term average growth rate at each point in time depends on the speed of mean reversion λ and the difference between the demand growth rate at time t, X_t, and the long-term average demand growth rate, μ.

In physical terms, the process displays similar behavior to a spring; the more a spring is stretched with respect to its equilibrium length, the higher the force with which the spring pushes back – i.e., the larger the difference between a passenger growth rate at a certain point in time and the average passenger growth rate, the higher the tendency to revert back to the mean passenger growth rate in the subsequent point in time.

Furthermore the randomness of future demand growth rates is captured in the last term of the equation, σdW_t, which resembles a random error shock with mean 0 and standard deviation equal to the standard deviation of the historical estimation error which is inherited from the estimation of the model parameters. The mean reverting model parameters λ, μ and σ are fixed over the time and are estimated by rewriting the mean reversion equation into a form that is suitable for linear least squares regression.

The concept of mean reversion is applied to forecast future demand growth rates. However, ultimately the goal is to forecast future demand levels, which are calculated using;

$$Dem_{t+1} = Dem_t \cdot (1 + X_{t+1}) \quad (2)$$

where Dem_{t+1} is the demand value at time $t + 1$, Dem_t is the demand value at time t and X_{t+1} is the growth rate of demand between t and $t + 1$.

2.2.2. Sampling strategy

Based on the mean reverting process, D Monte Carlo simulation runs are performed for each year in the planning horizon for each OD pair under consideration (see Fig. 3 for an example with $D = 10$). These D observations of realized stochastic demand are then sampled into a set of S sample values representative of the underlying probability distribution. This is done by grouping the D Monte Carlo simulation observations across S bins with each an equal number of observations. That is, the bins are defined such that each bin represents 1/10th of the D dataset, equal probability bin histograms with 10 bins are adopted which set the bin edges at the 0, 10th, 20th, ..., 100th percentiles. The average of all $\frac{D}{S}$ observations within each bin is then taken as a sample value.

The method yields S demand sample values per year per OD pair. However, ultimately this data should be stored in a set of S OD demand matrices per year, where each OD demand matrix contains demand sample values of all OD pairs while ensuring that each of those sample values corresponds to the same part of the distribution. This simplification greatly reduces the number of OD demand matrices per year (M) from $M = S^Z$ to $M = S$, thereby reducing the computation times from impracticable large (i.e., several years) to reasonable (i.e., several minutes or few hours). In the present work is also assumed that the demand growth for each OD pair is independent. However, correlated random shocks (W_t) could be easily incorporated in the current model by, e.g., adopting the Cholesky factorization method when generating these random values [25]. A variance-covariance matrix would have to be precomputed to express the correlation between OD pairs.

2.3. Model 2: fleet assignment optimization

The goal of the optimization model is to optimally allocate a fleet in terms of operating profit given one OD demand matrix. This is achieved by mathematically formulating the optimization problem as an Integer Linear Programming (ILP) optimization model that optimizes fleet assignment per aircraft type on a weekly flight frequency basis. The mathematical formulation consists of a profit maximizing objective function and a set of demand, capacity, physical and integrality constraints. The formulation is such that it allows for both point-to-point and hub-and-spoke network routing networks. Nevertheless, only two-leg itineraries are considered for the hub-and-spoke case (i.e., origin node to hub and hub to destination node).

In order to optimize for profit on an airline level, the weekly flight frequency per airport pair per aircraft type as well as the weekly passenger flow per OD pair (i.e. both nonstop and connecting flow) are determined. Therefore, three sets of decision variables are defined. The first referring to the weekly flight frequency per aircraft type per airport pair; the second representing the weekly passenger flow per OD pair for nonstop connections; and the third representing the weekly passenger flow per OD pair for passenger connecting in any of the hubs of the airline.

A number of inputs are used for this optimization process: a given OD demand matrix that contains weekly demand values for each OD pair, a given fleet which is characterized by the number of aircraft per aircraft type K, the specific aircraft characteristics of each aircraft type (seats, cruise speed, range, daily utilization, turnaround times, fixed cost, variable cost), yields and distances between airports. The optimization model returns weekly operating profit which is then multiplied by 52 to arrive at an estimated annual operating profit. That is, it is assumed that this average week is representative of the demand throughout the year and thereby neglects seasonality as well as trend growth throughout the year. Besides financial results, operational performance metrics such as the average network load factor and aircraft utilization can also be derived.

2.3.1. Nomenclature

Sets

N	Set of airports
K	Set of aircraft types
H	Set of hub airports (H $\subset$ N)

Parameters

$Q_{o,d}$	demand between airports o and d
$D_{o,d}$	distance between airports o and d
$yield_{o,d}$	yield per route for nonstop connections
$yield^h_{o,d}$	yield per route for connections through hub h
AC^k	number of aircraft of aircraft type k in the fleet
U^k	aircraft maximum utilization per week for aircraft type k
C^k_{fix}	aircraft ownership cost per aircraft type k
C^k_{var}	aircraft operating cost per aircraft type k (i.e. CASM)
s^k	number of seats per aircraft type k
vc^k	cruise speed per aircraft type k
T^{dep}_i	taxi time per departure
T^{arr}_j	taxi time per arrival airport
$range^k$	range per aircraft type k

Decision variables

$x_{o,d}$	Nonstop passenger flow between origin airport o and destination airport d
$w^h_{o,d}$	Connecting passenger flow for passengers from origin airport i and destination airport d via the hub airport h
$z^k_{i,j}$	Number of flights (i.e. flight frequency) between airport i and airport j operated by aircraft type k

2.3.2. Objective function formulation

The objective function aims to maximize operating profit and consists of four terms; operating revenue stemming from nonstop passenger flow, operating revenue stemming from connecting passenger flow, ownership cost and operating cost. The mathematical formulation is given by;

$$\begin{aligned}\text{Maximize profit} = &\sum_{o\in N}\sum_{d\in N}\left[yield_{o,d}\cdot D_{o,d}\cdot x_{o,d}\right]\\ &+\sum_{o\in N}\sum_{d\in N}\sum_{h\in H}\left[yield^h_{o,d}\cdot D_{o,d}\cdot w^h_{o,d}\right]\\ &-\sum_{k\in K}\left[AC^k\cdot C^k_{fix}\right]\\ &-\sum_{i\in N}\sum_{j\in N}\sum_{k\in K}\left[C^k_{var}\cdot D_{i,j}\cdot s^k\cdot z^k_{i,j}\right] \end{aligned} \quad (3)$$

2.3.3. Constraints formulation

$$x_{o,d}+\sum_{h\in H}w^h_{o,d}\le Q_{o,d} \qquad \forall\ o,d\in N, o\ne d \quad (4)$$

$$x_{i,j}+\sum_{m\in N}w^j_{i,m}\le\sum_{k\in K}z^k_{i,j}\cdot s^k \qquad \forall\ i\in N\backslash H, j\in H, i\ne j \quad (5a)$$

$$x_{i,j}+\sum_{m\in N}w^i_{m,j}\le\sum_{k\in K}z^k_{i,j}\cdot s^k \qquad \forall\ j\in N\backslash H, i\in H, i\ne j \quad (5b)$$

$$x_{i,j}\le\sum_{k\in K}z^k_{i,j}\cdot s^k \qquad \forall\ i,j\in N\backslash H, i\ne j \quad (5c)$$

$$\sum_{j\in N}z^k_{j,i}=\sum_{j\in N}z^k_{i,j} \qquad \forall\ i\in N, k\in K \quad (6)$$

$$\sum_{i\in N}\sum_{j\in N}z^k_{i,j}\cdot\left[\frac{D_{i,j}}{vc^k}+T^{dep}_i+T^{arr}_j+TAT^k\right]\le AC^k\cdot U^k \qquad \forall k\in K \quad (7)$$

$$z^k_{i,j}=0 \qquad \forall\ i,j\in N, i\ne j, k\in K \text{ if } range^k<D_{i,j} \quad (8)$$

$$x_{o,d}\in\mathbb{Z}^+,\ \ w^h_{o,d}\in\mathbb{Z}^+,\ \ z^k_{i,j}\in\mathbb{Z}^+ \quad (9)$$

The first set of constraints in the optimization model ensures that the assigned passenger flows cannot exceed the demand (Eq. (4)).

Eqs. (5a) –(5c) ensure that the passenger flow in a certain flight segment between airport i and airport j must be smaller than or equal to the capacity offered between airports i and j. The capacity in a flight segment is the total number of seats offered between the two airports, computed by multiplying the number of flights per aircraft type between these airports $z^k_{i,j}$ by the number of seats per aircraft type s^k. Focusing on a flight segment between two airports i and j, both of these airports can be either a regular airport or act as a hub. Therefore, the passenger flow between the two airports can be composed by only nonstop passengers (5c) or by a mix of nonstop and connecting passengers ((5a), (5b)).

The aircraft continuity at the airports is guaranteed by constraints 6. These constraints ensure that the total number of inbound flights per aircraft type k that arrive at airport i from all airports j must be equal to the total number of outbound flights per aircraft type k that depart from airport i to all airports j.

Eq. (7) ensures per aircraft type that the total weekly operational time does not exceed the weekly aircraft utilization. The aircraft utilization per aircraft type should not be based on a 24 h per day availability. It should rather reflect the average available hours to operation per week when considering the need for scheduled and unscheduled maintenance. The total weekly operational time is a function of the number of flights of each aircraft type between each airport pair, the flight time, the taxi time and turnaround time. The flight time is a function of the distance between two airports $D_{i,j}$ and the cruise speed of the aircraft type vc^k. The taxi times are airport dependent and depend on whether the flight is inbound or outbound. The turnaround times range from 30 min to one hour and are based on the assumption that larger aircraft have higher turnaround times.

Each aircraft type is characterized by its maximum range. Eq. (8) ensures that a flight between two particular airports i and j can only be operated by a particular aircraft type k if the range of the respective aircraft type is equal to or larger than the distance between two airports.

Finally, Eq. (9) defines the domain for the decision variables included in the model.

2.3.4. The size of the LP matrix

The LP-matrix contains the decision variables (i.e. columns) and constraints (i.e. rows) of the problem. It is considered interesting to know how the size of the LP-matrix of the optimization problem scales with increasing problem size. The problem size is defined by the number of airports, hubs and aircraft types under consideration. This explicit relationship between the problem size and LP-matrix is given by;

$$\text{Decision variables} = N^2 + N^2H - 2N + N^2K - NK \tag{10}$$

$$\text{Constraints} = 2(N^2 - N) + NK + K + D_k \tag{11}$$

with N the number of airports, H the number of hubs, K the number of aircraft types and D_k is the number of aircraft range constraints, which is specific to the characteristics of the aircraft types and the distances between the airports under consideration.

2.3.5. Iterating over the optimization model

The optimization process is solved for each fleet-OD demand matrix combination. Consequently, the optimization model is run $F \cdot M \cdot Y$ times which results in an equal amount of annual operating profits that are stored in a value matrix. The Gurobi Optimizer is used to find the optimal solution for each problem solved.

2.3.6. Model output

Each optimization model run returns the optimal decision variable values (i.e., weekly OD passenger flow and weekly airport-to-airport aircraft flow per aircraft type) and the optimal objective function value (i.e., weekly operating profit). These optimal values can be used to derive a range of financial and operational performance metrics.

Moreover, the annual operating profits returned by each optimization run are stored in a so-called value matrix (Fig. 4), which contains annual operating profit for each fleet F in the portfolio for each of the M OD demand matrices within the year, for each of the Y years in the planning horizon.

Both the vast amount of financial and operational performance metrics as well as the value matrix with annual operating profit data provide valuable information that can be used to compare fleets. However, one more step in the methodology allows for an even more profound analysis of the impact of the evolution of stochastic demand on the robustness of operating profit generating capability of the different fleets in the portfolio: the scenario generation model.

2.4. Model 3: scenario generation

The goal of the scenario generation model is to generate numerous paths through the value matrix across the planning horizon by using the underlying stochastic nature of demand.

A single path across the planning horizon is basically a sequence of elements in the value matrix. This sequence of annual operating profits is driven by the underlying sequence of OD demand matrices.

Each OD demand matrix contains demand sample values of all OD pairs, which are drawn from the same part (i.e. the same bin number) of their distribution of Monte Carlo simulation observations. Therefore, a sequence of year-to-year annual operating profit values essentially is driven by the year-to-year transition behavior of individual Monte Carlo simulation observations from one bin number to another. Please refer to Lohndorf [26] for a recent discussion on scenario generation methods and on the error that arises from using a small set of scenarios.

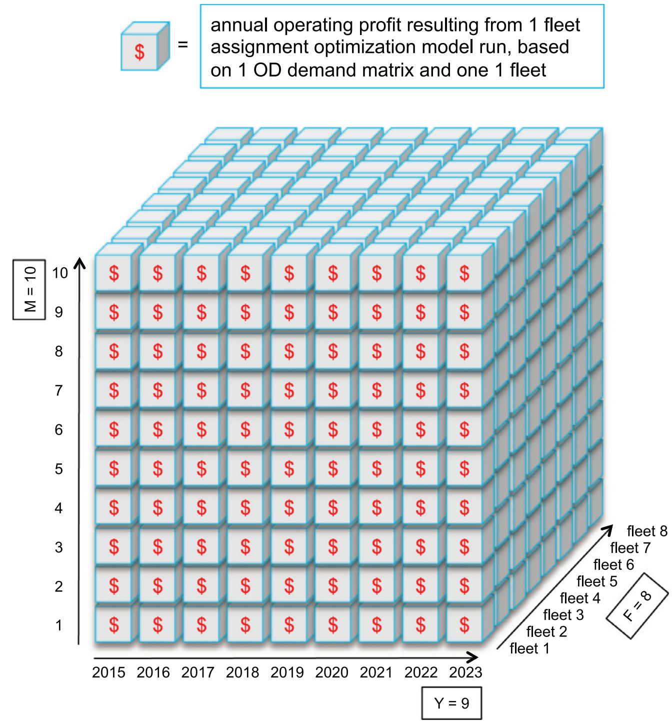

Fig. 4. An example value matrix encompassing 8 fleets ($F = 8$) in the portfolio, 9 planning years ($Y = 2015, \ldots, 2023$) and 10 OD demand matrices per year ($M = 10$),resulting in $FYM = 8910 = 720$ profit values.

2.4.1. Discrete-time Markov chain

A discrete-time Markov Chain (DTMC) is used to describe the stochastic process of the transition behavior of Monte Carlo simulation observations from one bin number in year t to another bin number in year $t + 1$. The Markov property describes the memorylessness of the stochastic process: the probability of arriving in a future state only depends on the present state. A transition probability matrix contains the transition probabilities of transitioning from state i at time t to state j at time $t + 1$. Based on the Markov property the transition probability should be a square matrix (i.e. the number of states must remain constant over time) and each row should add up to one (i.e. the total probability of arriving in any of the states must be 1).

Translated to the context of this research, a DTMC can be used to model the stochastic process of the evolution of Monte Carlo simulation observations that are outputted by the stochastic demand forecasting model. D Monte Carlo simulation observations per year per OD pair are equally distributed across S bins. These S bins are the discrete states in this research context. Subsequently the transition probability matrix has size $S \times S$.

2.4.2. OD demand matrix based transition probability matrices

Fig. 5 serves as an illustration of the transition process when considering 5000 Monte Carlo simulation observations ($D = 5000$) and 4 bins ($S = 4$). An observation can transition from any of the 4 states at time t to any of the 4 states at time $t + 1$ resulting in S^2 possible transitions. Because each bin contains the same number of observations ($\frac{D}{S}$), each row of observed transitions (i.e. counts) is converted to probabilities by multiplying each element in the row with:

$$\frac{1}{\frac{D}{S}} \tag{12}$$

In a similar fashion, such a transition probability matrix can be constructed for each consecutive year combination ($Y - 1$) in the Y

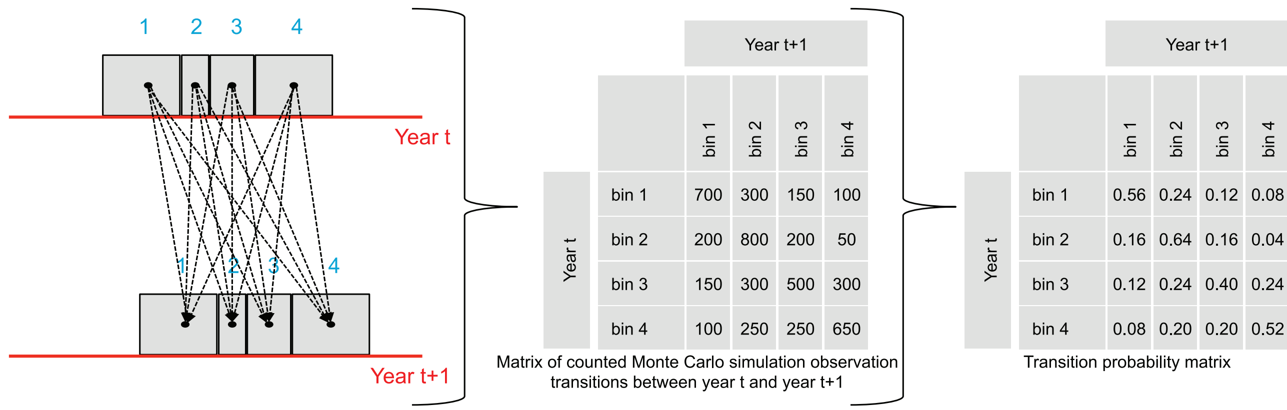

Fig. 5. An example of the transition process.

years of the planning horizon and for each OD pair Z under consideration, resulting in $Z \cdot (Y-1)$ transition probability matrices.

For the construction of OD demand matrix based transition probability matrices the same underlying principles of counting observations and transforming these to probabilities can be used. However, the counting process is iterated for all Z OD pairs and therefore each row contains $Z \cdot \frac{D}{S}$ observations. Consequently, rows are normalized by multiplying each element with;

$$\frac{1}{\frac{D}{S} \cdot Z} \tag{13}$$

It is noted that this aggregation is made possible by the decision presented in Section 2.2 to set the number of unique OD demand matrices per year equal to the number of sample values ($M = S$). As result of that decision, each OD demand matrix contains demand sample values of different OD pairs that are based on the same bin number.

2.4.3. Path generation

Due to the memoryless property of the DTMC, a scenario can be generated throughout the planning horizon of Y years by utilizing the $Y-1$ OD demand matrix based transition probability matrices. The process of one scenario generation resembles a roulette process which is executed $Y-1$ times in sequence using the known probabilities from the $Y-1$ transition probabilities and acknowledging that the first roulette is defined by a $1/S$ probability for each state (i.e. bin number).

A scenario is essentially a sequence of Y bin numbers; i.e. one bin number for each year in the planning horizon. The sequence of OD demand matrices can in turn be related to a sequence of annual operating profit values using the value matrix. Ultimately, the generation of B scenarios results in B sequences of annual operating profits each of length Y.

2.4.4. A distribution of NPVs

One scenario corresponds to a sequence of Y annual operating profit values. These Y values can be reduced to a single monetary value; the so called net present value (NPV) in the following fashion;

$$NPV = \sum_{t=1}^{Y} \frac{\text{annual profit}_t}{(1+r)^t} \tag{14}$$

where r is the discount rate and t is the year, with t = 1, ..., Y. When B scenarios are generated the resulting B NPVs can be used to construct a distribution of NPVs. Moreover, this procedure is executed for each fleet F in the portfolio so that ultimately F distributions of NPVs are outputted that can be used to compare the profit generating capabilities of the different fleets in the portfolio based on the underlying evolution of stochastic demand across the planning horizon. It is noted that each fleet is subject to the same set of scenarios which is required to ensure fair comparison.

Table 1
Case study specific variable values.

Notation	Definition	Case study value
F	# Fleets in portfolio	8
Y	# Years in planning horizon	9
D	# Monte Carlo simulations	5000
S	# Sample values	10
M	# OD demand matrices per year	10
N	# Airports under consideration	10
Z	# OD pairs under consideration	100
H	# Hubs under consideration	1
K	# Aircraft types under consideration	3
B	# Scenarios generated	5000

3. Case study

3.1. Context

A small real-world based case study serves as proof of concept of the proposed methodology. The purpose of this case study is to illustrate the applicability of the proposed methodology, evaluating the type of results that can be generated. It does not represent any specific airline operating in the market. Table 1 presents the size of the sets considered for the case study.

The forecasting period consists of 9 years ($Y = 9$) with 2014 as the last historical year and the following forecasting years: 2015, 2016, . . ., 2023. This time span roughly coincides with one business cycle [1].

Based on Eqs. (10) and (11), and noting that the number of aircraft range constraints is 38, it can be derived that the number of decision variables and constraints are 450 and 251, respectively.

3.2. Model parameter data

The data used in this case-study is explained in the next subsections. The data can be downloaded from https://doi.org/10.4121/uuid:90abf0a2-369e-4a52-9a2c-518ab9f66478.

3.2.1. Demand data

The historical passenger data is extracted from the TranStats database of the Bureau of Transportation Statistics (BTS), which is part of the United States Department of Transportation (US DOT). The underlying dataset that was used is the T-100 Domestic Market (U.S. Carriers) data table that contains monthly scheduled US

Table 2
Aircraft characteristics per aircraft type: number of seats (s^k), cruise speed (vc^k), range ($range^k$), daily utilization (U^k), turnaround time (TAT^k), ownership cost (C^k_{fix}), variable cost (C^k_{var}) and purchase price (IC^k).

Attributes	s^k	vc^k	$range^k$	U^k	TAT^k	C^k_{fix}	C^k_{var}	PP^k
Units	#	miles/hour	miles	hours	hours	USD	USD	USD
A	75	514	1401	11	0.75	1.23E+06	0.11	2.45E+07
B	162	543	3582	12	1.00	3.95E+06	0.09	7.90E+07
C	295	555	8510	14	1.50	1.10E+07	0.08	2.19E+08

domestic passenger data based on a 10 percent ticket sale information dataset, aggregated for all airlines for the period 1990–2014.

A group of 10 different airports ($N = 10$) was selected for this case study. They include the 10 most high-density OD connections in the US domestic economic passenger market, according to 2014 air travel data. The OD demand matrix has then the size 10×10. A 20% market share is assumed for each OD pair when reducing total market demand to airline specific demand.

3.2.2. Yield data

Yield is defined as revenue per revenue-passenger-mile in 2014 US Dollar cents. The yields are based on average fare data in 2014. This data stems from the BTS US DOT database and the underlying dataset that was used is Table 1a Domestic Airline Airfare Report (2011 - 2014) which contains average fare data per OD pair per quarter for the period 2011–2014 for a large set of OD pairs in the US. Yield data is calculated by dividing the fares by their origin-destination distance. It is noted that the average fares that form the basis for this dataset are not only averaged for the year 2014, but also reflect average fares as listed by all airlines in the marketplace, irrespective of the offered service (i.e. nonstop or connecting). For simplification, the ratio between yields for nonstop and connecting passengers is set at 1.0 in the case study. This results in the nonstop flow being more profitable because it generates the same level of revenue at a lower cost. In practice, these yield ratios are airline specific and highly depend on how an airline prioritizes nonstop or connecting flow per OD pair in their revenue management models, based on the competitive environment.

3.2.3. Aircraft characteristics

Three different aircraft types are considered – a regional jet (A), a narrow body (B) and a wide body (C) aircraft (Table 2). The aircraft types are differentiated by their characteristics, which are the number of seats, cruise speed, range, daily utilization, turnaround time, variable cost, ownership cost and purchases price. The number of seats s^k, cruise speed vc^k, range $range^k$ and purchase price (i.e. list price) are based on information provided on the internet [27]. Weekly utilization U^k, turnaround times TAT^k and operating cost C^k_{var} are based on previous studies done by the authors (e.g., [18]) and on the assumption that larger aircraft tend to have a higher daily utilization, higher turnaround time and lower unit operating cost [8]. The yearly ownership cost C^k_{fix} are based on the aircraft purchase price and assuming a 20-year linear depreciation period and residual value of 15% at the end of the depreciation period, which is based on an example depreciation scheme provided by Doganis [28]. This way, the purchase prices are not considered in the NPV calculations. They are replaced by the yearly ownership costs. For the sake of simplicity and following Repko and Santos [18], we assumed that the ownership costs reflect either the lease costs or the depreciation costs per period.

3.2.4. Fleet portfolio

The portfolio of fleets considered is presented in Table 3. Each of the 8 fleets in the portfolio ($F = 8$) is characterized by the number of aircraft per aircraft type. The fleets were created so they could represent different fleet configurations of similar fleet sizes.

Table 3
Portfolio of fleets.

	Aircraft types			
Fleet	A	B	C	Total
Fleet 1	4	4	4	12
Fleet 2	5	5	5	15
Fleet 3	10	2	2	14
Fleet 4	2	10	2	14
Fleet 5	2	2	10	14
Fleet 6	15	0	0	15
Fleet 7	0	15	0	15
Fleet 8	0	0	15	15

It was estimated that a fleet of 15 aircraft would be enough to transport most of the demand considered. Therefore, fleets of 12 to 15 aircraft were considered. Fleets 1–2 have compositions with the same amount of aircraft of each type, but Fleet 1 represents an option with less aircraft. Fleets 3–5 have a predominance of one aircraft type in the fleet. Fleets 6–8 represent options with just a single aircraft type in the fleet.

3.2.5. Airport characteristics

The taxi-in and taxi-out times stem from the BTS US DOT database. The underlying dataset that was used is the Airline On-Time Statistics - Origin and Destination Airport dataset that provides 2014 data on taxi-in and taxi-out times in minutes per OD pair averaged for all airlines.

3.2.6. Inflation, discount rate and tax rate

Inflation is considered across the planning horizon on the cost side (i.e., operating and ownership cost) and on the revenue side (i.e., nonstop and connecting yields). In the case study the inflation is assumed to be 1.5% per year for all the 9 years in the planning horizon. This number is calculated as the average inflation in the US between 2010 and 2014 which is based on data from the US Bureau of Labor Statistics (US BLS).

For the NPV calculation, the discount rate r is set at 7.4% which is the average historical WACC for US airlines (both legacy carriers and low cost carriers) between 2004 and 2011 [1]. The effective corporate tax rate in the US depends on a federal and state component and is assumed to be 39%. Using the tax rate, a simplified version of the return on invested capital (ROIC) is calculated as:

$$\mathrm{ROIC} = \frac{\text{Annual operating profit} - \text{tax}}{\text{Investment}} = \frac{\text{Annual operating profit} \cdot (1 - \text{tax rate})}{\text{Investment}}. \quad (15)$$

3.3. Results

The three models together produce a vast amount of results, the majority of which are used as intermediate results that are part of the methodology. The most important final results in terms of the overarching methodology are:

– The distribution of net present values of profit across the planning horizon across the range of stochastic demand, for each fleet in the portfolio

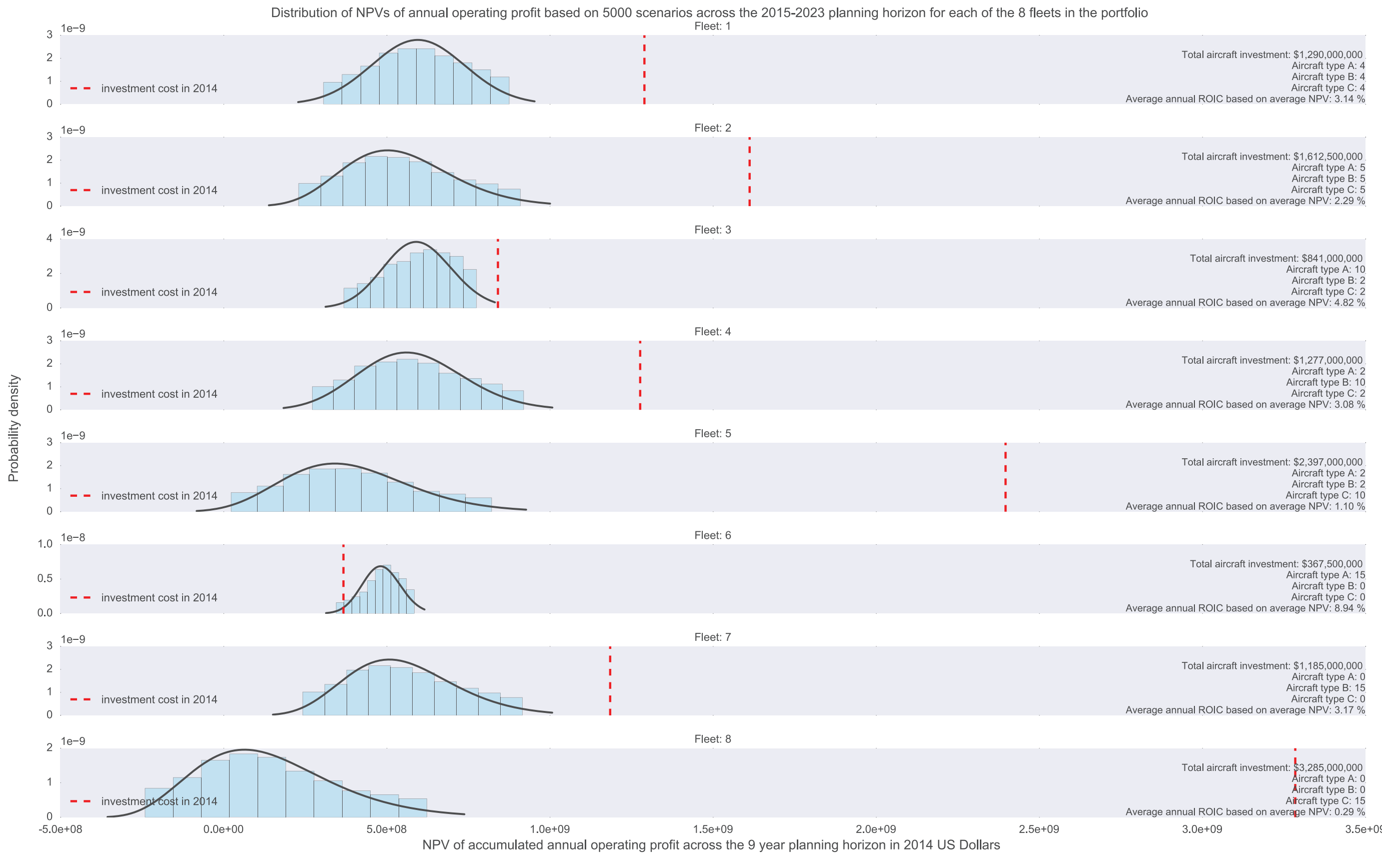

Fig. 6. A distribution of net present values based on annual operating profits across the planning horizon across numerous realizations of stochastic demand, for each fleet in the portfolio.

– Table with all financial and operational performance metrics per fleet (*F*), per year (*Y*), per OD demand matrix within the year (*M*)

3.3.1. *Distribution of net present profit values for each fleet in the portfolio*

Fig. 6 presents for each fleet in the portfolio the distribution of net present profit (NPVs) values based on the 5.000 scenarios across the planning horizon. Four key attributes can be evaluated in order to compare the different fleets from the portfolio.

– The mean of the distribution
  - This gives insight in the absolute operating profit generation capability of the fleet across the planning horizon across the range of stochastic demand.

– The spread of the distribution:
  - This provides insight in the robustness of the profit generating capability of a fleet to the range of stochastic of demand it is subject to across the planning horizon. A wide distribution indicates a lot of uncertainty, while a narrow distribution indicates little uncertainty.

– The location of the distribution with respect to the level of investment required
  - This observation relates the profit generating capability of a fleet to the magnitude of the investment cost that is required to purchase the fleet. Whereas operating profit is an indicator of how efficient the assets (i.e. the fleet) are deployed, the difference between profitability and investment can be used as an indicator of how efficient the investment is generating a return.

– The level of investment required
  - In the case of limited capital to invest in a fleet, the amount of investment required (i.e., the location of the 'investment cost in2014' line) could make some of the fleets unfeasible.

The third observation reveals a key insight in the difference between the profit generating capability of a fleet and its capability to generate returns on invested capital: there can be fleets with high operating margins and low returns. A fleet could be very profitable in operation in absolute terms (i.e. a distribution with a high mean) but at the same time can be a poor investment because of the disproportionate level of investment that is required to get to that level of absolute operating profits.

3.3.2. *Financial and operational performance metrics*

Each iteration returns the same type of financial and operational performance metrics as presented in Table 4, which are stored in one large data table with 720 cells. This allows for an explicit comparison of both financial and operational performance metrics between different fleets across different realizations of stochastic demand across the 9 years in the planning horizon.

3.3.3. *How should a fleet planner interpret the results?*

The information that stems from the two datasets can be used to explicitly compare fleets. First, the distribution of NPVs can be used by fleet planners to get a high level insight in the magnitude and uncertainty of the operating profits across a 9-year planning horizon and how these operating profits relate to the required fleet investment. Second, the vast amount of both financial and operational data can be used to unravel the underlying factors that drive the distribution of profitability; what are the aircraft utilizations of the different aircraft types in the fleet? What is the average network load factor? How many passengers are spilled? What is the

Table 4
Example of the financial and operational performance metrics for portfolio Fleet 6.

Financial performance metrics		Operational performance metrics	
Metric	Value	Metric	Value
Weekly revenue	$3,597,750	Weekly OD pax transported	42,900
Weekly operating cost	$1,986,880	Weekly seats offered	45,700
Weekly ownership cost	$300,360	Weekly seats-miles offered	48,477,034
Weekly operating profit	$1,310,509	Percentage nonstop flow	96%
Annual operating profit	$68,146,505	Percentage OD demand satisfied	21.4%
Operating profit margin	36.43%	Average network load factor	98%
Annual after-tax profit	$41,569,368	Number of OD pairs served	7
Total investment cost	$367,500,000	Utilization per aircraft type:	
Annual return on invested capital	11.31%	Type A	100%
Spilled revenue	80.49%		

Table 5
An overview of computation times per model.

Model	Total computation time (s)	Number of runs	Time per run (s)
Model 1	5400	$Y \cdot Z = 9 \cdot 100 = 900$	6
Model 2	520	$F \cdot Y \cdot M = 8 \cdot 9 \cdot 10 = 720$	0.72
Model 3	1100	$B \cdot F = 5.000 \cdot 8 = 40.000$	0.0275

spilled revenue? How many OD pairs are served? What percentage of the passengers is transported nonstop? What are the weekly operating cost and ownership cost? What is the routing network? This vast amount of detailed information can be used for subsequent detailed analysis.

A fleet planner from industry is likely to select the fleet with a distribution with a high mean, low uncertainty (i.e. high robustness) and a beneficial relation between NPVs and required investment in terms of ROIC, which in this illustrative case study corresponds to Fleet 6 in Fig. 6.

3.3.4. Results analysis

From the results it can be concluded that fleets with the same amount of aircraft of the same type (Fleets 1–2) do not produce very promising results. Both cases have a similar NPV distribution and an investment cost that is higher than the maximum expected NPV. The best results are obtained for the fleets with more type A aircraft (Fleets 3 and 6). These are the least cost options and provide narrow NPV distributions. Fleet 6 is the only fleet in the portfolio that has a high probability of having a NPV higher than the investment costs. However, this is also the fleet with lower maximum expected NPV. For a more risk prone decision maker, Fleets 3, 4 and 7 could be a good option. For these fleets, the consideration of aircraft lease options could result in a good alternative. If all or some of the aircraft in the fleet would be lease, the investment costs could be spread over time, resulting in investment costs in 2014 lower than the ones presented in Fig. 6.

The fleets with a predominance of wide body aircraft (Fleets 5 and 8) prove to be unsuitable to the network considered. They are associated with high investment costs and their NPV distributions are the most to the left. The latter is a result of, on one hand, the lower utilization of the aircraft due to higher turn-around times, and on the other hand, lower load factors combined with higher costs per flight.

3.4. Computation times

The computation times of the presented small case study for each of the three models is presented in Table 5. The three models are run sequentially. Thus, the total computation time is simply calculated as the sum of the computation times of the three models:

$$
\begin{aligned}
CT_{\text{TOTAL}} &= CT_{\text{model 1}} + CT_{\text{model 2}} + CT_{\text{model 3}} \\
&= Y \cdot Z \cdot 6 + F \cdot Y \cdot M \cdot 0.72 + B \cdot F \cdot 0.0275 \text{ s} \\
&= 9 \cdot 100 \cdot 6 + 8 \cdot 9 \cdot 10 \cdot 0.72 + 5000 \cdot 8 \cdot 0.0275 \text{ s} \\
&= 117 \text{ min.}
\end{aligned}
\tag{16}
$$

The computational times for each model and for the complete model framework, show how demanding is the stochastic multi-period fleet planning problem. For a larger real world case study, the computation would amount to several hours of computation. However, it is considered valuable to harvests insight into how the computation time scales with increasing problem size. Armed with that insight, fleet planning decision makers can make explicit trade-offs with regards to the level of detail they wish to consider (i.e. number of fleets in the portfolio, aircraft types, OD pairs under consideration, Monte-Carlo simulations) and the corresponding computation times to get to a solution.

4. Summary and conclusions

This paper proposes a three-step methodology that harvests insight into the operating profit generating capability of different fleets portfolios, in terms of size and composition, over a multi-year planning horizon under stochastic demand. The long-term stochastic nature of demand growth rates is modeled as a mean reverting Ornstein–Uhlenbeck process and explored using a Monte Carlo simulation, per origin-destination pair. Demand scenarios are generated by using discrete-time Markov chain transition probability matrices that are based on the transition behavior of the evolution of stochastic demand realizations.

An illustrative small case study is used in this paper to demonstrate the type of results obtained with the proposed methodology. The proposed airline fleet planning modeling framework has the potential to identify robust fleet plans through the detailed consideration of stochastic demand per origin-destination pair across a long term planning horizon, and being able to compare both financial and operational performance metrics of different fleets across a multi-year planning horizon across numerous realizations of stochastic demand. The methodology is generic and can be applied to any airline, irrespective of the business model, size, routing network and preference with regards to aircraft types or risk profile.

Besides the contribution of this work, with a innovative approach to solve the fleet planning problem, this paper opens the opportunity for further research. For instance, one of the limitations of this work is that no competition elements are considered.

Instead, a 20% market share is assumed for each OD pair when reducing total market demand to airline specific demand. The implication of the absence of a market share model on the results of this research is that the distribution of NPVs is likely to shift to the right for fleets with more aircraft, considering they can offer a higher flight frequency and thus capture a larger share of the market and vice versa for smaller fleets.

Another research opportunity is the consideration of portfolios of variable fleet size and composition over time. The consideration of aircraft replacement would be an interesting development in future studies. However, this development would, on one hand, require the analysis of more fleet portfolios and, on the other hand, make it more challenging the comparison of different portfolios across the planning horizon.

Supplementary material

Supplementary material associated with this article can be found, in the online version, at doi:10.1016/j.omega.2019.08.008.

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