The Multi-period Petrol Station Replenishment Problem
Formulation and Solution Methods

Boers, Luke; Atasoy, Bilge; Homem de Almeida Correia, G.; Negenborn, Rudy R.

DOI
10.1007/978-3-030-59747-4_39

Publication date
2020

Document Version
Final published version

Published in
Computational Logistics

Citation (APA)
Boers, L., Atasoy, B., Homem de Almeida Correia, G., & Negenborn, R. R. (2020). The Multi-period Petrol Station Replenishment Problem: Formulation and Solution Methods. In E. Lalla-Ruiz, M. Mes, & S. Voß (Eds.), *Computational Logistics : Proceedings of the 11th International Conference, ICCL 2020* (pp. 600-615). (Lecture Notes in Computer Science; Vol. 12433). Springer. https://doi.org/10.1007/978-3-030-59747-4_39

The Multi-period Petrol Station Replenishment Problem: Formulation and Solution Methods

Luke Boers[1], Bilge Atasoy[1(✉)], Gonçalo Correia[2], and Rudy R. Negenborn[1]

[1] Department of Maritime and Transport Technology, Delft University of Technology, Delft, The Netherlands
lukeboers@gmail.com, {b.atasoy,r.r.negenborn}@tudelft.nl
[2] Department of Transport and Planning, Delft University of Technology, Delft, The Netherlands
g.correia@tudelft.nl

Abstract. We present a "rich" Petrol Station Replenishment Problem (PSRP) with real-life characteristics that represents the complexities involved in actual operations. The planning is optimised over multiple days and therefore, the new variant can be classified as the Multi-Period Petrol Station Replenishment Problem (MP-PSRP). A Mixed Integer Linear Programming (MILP) formulation is developed and a decomposition heuristic is proposed as a solution algorithm, which is evaluated with a case study from a real-life petrol distributor in Denmark. To determine delivery quantities, the heuristic uses the newly introduced *simultaneous dry run* inventory policy. A procedure is applied to improve the initial solution. A commercial solver is able to find feasible solutions only for instances with up to 20 stations and 7 days for the MILP model where optimality is guaranteed for instances up to 10 stations and 5 days. The heuristic on the other hand provides feasible solutions for the full case study of 59 stations and 14 days, within a time limit of 2 h.

Keywords: Petrol Station Replenishment · Inventory routing · Simultaneous dry run inventory policy · Decomposition heuristic

1 Introduction

In this paper, we develop a variant of the Petrol Station Replenishment Problem (PSRP), that addresses the optimisation of the distribution of several petroleum products to a set of petrol stations over a given planning horizon. The products, stored in underground tanks, are delivered to petrol stations using a heterogeneous fleet of vehicles with multiple compartments. The vehicles are assumed to be equipped with flow meters, that allow to split the fuel in a compartment over multiple stations. Delivery amount is limited by the available capacity of a storage tank at a station and the capacity of vehicles. The aim is to determine the optimal route for the vehicles in order to minimize total travel distance. Figure 1 shows a schematic representation of the problem.

E. Lalla-Ruiz et al. (Eds.): ICCL 2020, LNCS 12433, pp. 600–615, 2020.
https://doi.org/10.1007/978-3-030-59747-4_39

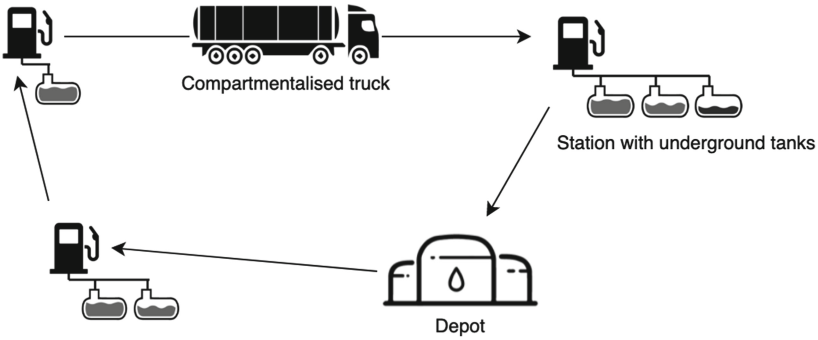

Fig. 1. A vehicle trip to multiple stations with different number of underground tanks

As the stations do not have to be replenished every day, the considered problem is solved over multiple days. The newly formulated problem can be classified as the Multi-Period Petrol Station Replenishment Problem (MP-PSRP). To decide when a station needs to be replenished the inventory levels at the stations are assumed to be known by the supplier. The supplier makes then, per time period, the following decisions:

– When stations are visited and which stations are combined into a trip (i.e., the routing decision for vehicles)
– Which vehicles are used to perform the trips
– The quantities per product that are transported to each station
– Which compartments are used to deliver the fuel
– For each trip the time a vehicle leaves and returns to the depot

These decisions comprise both inventory and routing decisions. Handling them simultaneously with Inventory Routing Problems (IRP) leads to more efficient operations since inventory and routing decisions are interrelated and these two factors are the main cost drivers of petroleum supply chain costs [9, 23]. For a comprehensive review of IRP we refer to [7]. Moreover, the problem in this paper relates to vehicle routing problems with multiple compartments due to the nature of the problem and we refer to [13] for a review.

Delivery quantities are set sufficiently high to prevent stock out of a product at a station. The daily demand of a product at a station is given by a deterministic forecast. However, in the real-life situation, the fuel consumption is not known beforehand. In this research, a safety stock level set to at least the average daily demand is used to cope with this uncertainty. The daily planning is usually made over night, with the latest available inventory levels. The safety stock reduces the risk of running out of a product, regardless of the time of delivery.

In this paper, a Mixed Integer Linear Programming (MILP) model is proposed to formulate the problem. Due to the complexity of the problem, solutions for real-life size situations cannot be found by an exact approach. Therefore, a decomposition heuristic is proposed as a planning method for this problem. Both the exact solution of the MILP and heuristic approach are evaluated with a case study from a real-life petrol distributor in Denmark.

The remainder of the paper is organised as follows. An overview of related literature is presented in Sect. 2 and the problem is defined in Sect. 3. The MILP model is presented in Sect. 4 and the heuristic is provided in Sect. 5. The results are discussed in Sect. 6 and Sect. 7 concludes with the main findings and recommendations for further research.

2 Literature Review

The PSRP has received substantial attention in the literature over the last decades, after the problem was formulated for the first time by [5]. A comprehensive overview of the history of the PSRP is provided by [4,11]. The authors of [10] developed an exact algorithm for the single day PSRP, where the number of stops per trip was limited to two. The problem is extended to multiple days by [12], that presented a MILP model with the objective to minimise costs with a penalty for overtime use of vehicles. In [19], the authors defined a MILP model for the PSRP which minimises both inventory and routing costs over multiple days. The number of stations that can be visited in a trip is limited to three and a station cannot be visited by more than one vehicle during a time period. Traditionally, the PSRP has been solved over a single day time span mainly because of the complexity of the problem. Several researchers have, however, shown that solving the problem over multiple days leads to more efficient solutions [1].

In [20], a Variable Neighbourhood Search (VNS) heuristic is proposed with a shaking procedure based on shifting deliveries between days in the planning horizon with a homogeneous fleet of vehicles. [23] also used a homogeneous fleet of compartmentalised vehicles to distribute fuel and formulated the problem as a MILP with the objective to minimise both inventory and routing costs.

A variant of the PSRP with a homogeneous fleet is proposed in [16]. The stations are visited after a minimum delivery quantity can be delivered, which is set to improve vehicle utilisation. The problem is solved with a Tabu search algorithm that is proven to find near-optimal solutions. The lower bounds of a reasonable sized problem are determined with Lagrangian relaxation. A variant of the PSRP with multiple compartments and time windows is presented by [4]. The time windows represent the scheduling horizon for each vehicle and there are two types of vehicles with different compartment sizes. The problem is solved using commercial solvers for the MILP formulation and a heuristic.

A variant of the Vehicle Routing Problem with Time Windows (VRPTW) is the problem with multiple use of vehicles, where each vehicle can perform multiple trips per time period. This problem is studied in [3] with a homogeneous fleet of capacitated vehicles. The researchers presented a MILP model and solved the problem using a branch-and-price algorithm. Another MILP model for a single day VRPTW is presented by [17].

The contribution of this paper is twofold: (i) the design of a decomposition heuristic with a new inventory policy called *simultaneous dry run concept*, which aims to minimise the number of visits to a station considering the delivery quantities across all tanks at that station, and (ii) the consideration of a unique combination of real-life characteristics that makes the problem "rich" as later highlighted in Table 1. Therefore, in the remainder of this section, we present further literature related to decomposition heuristics and real-life characteristics in routing problems.

2.1 Decomposition Heuristics

The combination of characteristics makes the considered variant of the MP-PSRP NP-hard, since it is already shown to be an NP-hard problem even for simpler problems [1,12,16,23]. An effective method to solve such complex problems is decomposition heuristics that can be considered under the umbrella of matheuristics. Matheuristics is a category of heuristics that divides a problem into sub problems, of which at least one sub problem is a MILP. Another class of matheuristics are improvement heuristics, where MILP models are used to improve a solution [2]. The heuristic presented in this research is a decomposition heuristic, based on decomposing the decision process of the supplier.

In [22], a decomposition heuristic is proposed for the PSRP, in which inventory was not considered. The problem was decomposed into assignment, routing and improvement procedures. A local search technique based on switching any two stations between service days was used as the improvement procedure. The authors of [9] solved an IRP by decomposing the decision process of the vendor into a three-phase heuristic focusing on the decisions of replenishment plan, delivery sequence and which routes to drive, respectively. Inventory management has also been included by [10], who decomposed the problem in a Tank Truck Loading Problem (TTLP) and a routing problem. Similar models are used by [12], where the planning is constructed for each day of the planning horizon.

The distinction of our decomposition heuristic is the incorporated *simultaneous dry run* inventory policy to determine delivery quantities, which are set in a way that limits the number of visits to a station. Furthermore, petrol products are transported by compartmentalised vehicles. These vehicles are often assumed not to be equipped with flow meters [4,10,12,17,20,23]. The absence of flow meters limits the flexibility in utilisation of vehicles, since flow meters allow to split the load from one compartment over multiple stations [8]. The case studied in this research assumes the availability of flow meters, which allows splitting loads.

2.2 Real-Life Case Studies

Several researchers developed models to solve real-life cases. In [18], the distribution of petrol for a network in Hong Kong is improved by applying the Vendor-Managed Inventory (VMI) concept. The proposed approach helped the company to increase the delivery volume and decrease driver costs. The authors of [22] used real-life cases to evaluate the performance of the solution methods to solve the PSRP. Compared to a planning made by a human operator, a maximum saving of 17.7% was achieved. In [21], the PSRP is investigated for a distributor of petroleum products in Oman. A MILP model is presented that eventually can be used to prepare a bid for auctions of transportation procurement.

A large petroleum company in China is considered by [16]. The distribution of petroleum products in provinces was modelled as an IRP and a heuristic is developed that is shown to provide near-optimal solutions. In [15], the authors considered the petrol distribution in the United States. A MILP model was used to determine the optimal supply chain design, while considering multi-modal transportation methods when determining locations for the facilities. The case of distribution for an Algerian petroleum company has been evaluated by [4], with a model for the PSRP with compartmentalised vehicles and time windows. The method proposed by the authors outperformed the solution created by the company, in terms of number of vehicles and total travel distance.

The case studies mentioned underline the growing interest in "richer" models, what promises even more effective methods for the future. For the definition of "rich" routing problems we refer to the survey by [6] where they mention several characteristics of VRP variants that lead to this rich category. As we consider real-life constraints related to time and stations, dynamism, heterogeneous fleet, and have linkage between inventory and routing decisions our paper fits to this concept. An overview of the problem characteristics considered in this research in reference to the most relevant literature is shown by Table 1. With this positioning, it is clear that our paper addresses a problem with a unique combination of characteristics that is not done before to the best of authors' knowledge.

Table 1. Positioning of our work in the literature. TH: Time Horizon S: Single-period, M: Multi-period, ST: Single Trip, MT: Multiple Trip, MP: Multi-Product, VU: Vehicle Use, VF: Vehicle Fleet, Comp.: Compartments, SL: Split Loads, HO: Homogeneous, HE: Heterogeneous, SR: Station Restrictions, Time Con.: Time constraints

Paper	IRP	Stops/trip	TH	MP	VU	VF	Comp.	SL	Time Con.	SR
Al-Hinai and Triki [1]		Unlimited	M	✓	ST	HE	✓			
Azi et al. [3]		Unlimited	S		MT	HO		✓	✓	
Benantar et al. [4]		Unlimited	S	✓	ST	HE	✓		✓	✓
Coelho and Laporte [8]	✓	Unlimited	M	✓	ST	HE	✓	✓		
Cornillier et al. [12]	✓	Max. 2	M	✓	MT	HE	✓		✓	
Li et al. [16]	✓	Unlimited	S		SU	HO			✓	
Macedo et al. [17]		Unlimited	S		MT	HO		✓	✓	
Popović et al. [20]	✓	Max. 3	M	✓	ST	HO	✓			
Vidović [23]	✓	Max. 4	M	✓	ST	HO	✓			
This research	✓	Unlimited	M	✓	MT	HE	✓	✓	✓	✓

3 Problem Definition

Let $G = (V, A)$ be a complete directed graph, where $V = (0, 1, ..., n)$ is a set of nodes and $A = \{(i, j) : i, j \in V\}$ is the set of arcs. Each arc (i, j) is associated with a travel distance c_{ij} and a travel time t_{ij}. The stations are represented by nodes $N = (1, ..., n)$ and the depot is given as node 0 or $n + 1$, depending on whether it is the initial or final node in a trip. Each node has a fixed service time st_i. Set $T = (1, ..., n)$ defines the time periods (days) in the planning horizon.

Set R contains the trips that can be driven by vehicles, with $|R|$ chosen large enough to enable the maximum number of trips a vehicle can perform during a daily work schedule. The trip indices are used in increasing order, which means that $r^{'} > r$ if a vehicle performs trip $r^{'}$ after trip r.

The stations are directly associated with nodes and have one or multiple underground tanks that store one type of product p. Set P contains all products, which are stored at the depot. Each underground tank has a maximum capacity C_i^p a safety stock IS_i^p and an initial inventory level at the start of the planning horizon I_{i0}^p. D_{it}^p gives the demand for each product at each station per day t.

Fuel is transported by a heterogeneous fleet of vehicles. Each vehicle $k \in K$ has multiple fuel compartments $f \in F_k$ with maximum capacity Q^{kf}. Station restrictions are included in the model by variable δ_i^k, which is 1 if station i can be visited by vehicle k. Operating hours of vehicles are represented by a time window $[a_t^k, b_t^k]$, where a_t^k is the start and b_t^k the end time for vehicle k during day t. M is an arbitrary large constant that is used to ensure a linear formulation.

Furthermore, the following assumptions are made:

- There is enough inventory at the depot to fulfil the replenishment plan.
- Inventory levels at stations are known by the supplier.
- Demand is considered as deterministic, based on a forecast. To deal with stochasticity and potential stock out, a safety stock level is determined for each tank at each station.
- A station can have only one underground tank per product.
- To transport the petrol products, a heterogeneous fleet of vehicles (equipped with flow meters) is used with compartments of known size.
- All vehicles are assumed to drive with the same speed, which means that they have the same travel time, given the same distance.
- Some vehicles cannot visit each station, due to spatial restrictions.
- Stations can be visited multiple times a day and 24/7, because it is assumed that a truck can always deliver the petrol products at the station.
- Vehicles run within the operating hours, i.e., work schedules of drivers.

4 Mathematical Formulation

The problem is formulated as an MILP. Let binary variable x_{ijrt}^k be 1 if and only if vehicle k drives arc (i, j) in trip r during day t. Binary variable y_{rt}^k is 1 if vehicle k drives trip r during day t. The same holds for binary variable z_{irt}^k, where i is added to define that station i is visited by vehicle k. If binary variable w_{rt}^{kfp} is 1, product p is loaded into compartment f of vehicle k during trip r on day t. If trip $r^{'}$ is driven after trip r by vehicle k during day t, binary variable $u_{rr't}^k$ is set to 1. Variable q_{irt}^{kfp} is used for the delivery quantity of product p, loaded in compartment f of vehicle k and delivered to location i during trip r

on day t. S_{irt}^k is the time vehicle k can start the service at station i during trip r on day t and I_{it}^p represents the inventory level of product p at station i at the end of day t. The problem can then be formulated as follows:

$$\min \sum_{t \in T} \sum_{r \in R} \sum_{k \in K} \sum_{(i,j) \in A} c_{ij}\, x_{ijrt}^k \tag{1}$$

Subject to

$$I_{it}^p \geq IS_i^p \qquad \forall p \in P, \forall n \in N, \forall t \in T \tag{2}$$

$$I_{i,t-1}^p + \sum_{r \in R} \sum_{k \in K} \sum_{f \in F_k} q_{irt}^{kfp} \leq C_i^p \qquad \forall p \in P, \forall i \in N, \forall t \in T \tag{3}$$

$$I_{it}^p = I_{i,t-1}^p + \sum_{r \in R} \sum_{k \in K} \sum_{f \in F_k} q_{irt}^{kfp} - D_{it}^p \qquad \forall p \in P, \forall i \in N, \forall t \in T \tag{4}$$

$$z_{irt}^k \leq \delta_i^k \qquad \forall k \in K, \forall i \in N, \forall r \in R, \forall t \in T \tag{5}$$

$$\sum_{j \in V} x_{0jrt}^k = 1 \qquad \forall k \in K, \forall r \in R, \forall t \in T \tag{6}$$

$$\sum_{i \in V} x_{ijrt}^k - \sum_{i \in V} x_{jirt}^k = 0 \qquad \forall k \in K, \forall j \in V, \forall r \in R, \forall t \in T \tag{7}$$

$$z_{irt}^k = \sum_{j \in V} x_{ijrt}^k \qquad \forall k \in K, \forall i \in N, \forall r \in R, \forall t \in T \tag{8}$$

$$q_{irt}^{kfp} \leq M\, z_{irt}^k \qquad \forall k \in K, \forall p \in P, \forall f \in F_k, \forall i \in N, \forall r \in R, \forall t \in T \tag{9}$$

$$\sum_{i \in N} q_{irt}^{kfp} \leq w_{rt}^{kfp}\, Q^{kf} \qquad \forall k \in K, \forall f \in F_k, \forall p \in P, \forall r \in R, \forall t \in T \tag{10}$$

$$\sum_{p \in P} w_{rt}^{kfp} \leq 1 \qquad \forall k \in K, \forall f \in F_k, \forall r \in R, \forall t \in T \tag{11}$$

$$S_{irt}^k \geq a_t^k \qquad \forall k \in K, \forall i \in V, \forall r \in R, \forall t \in T \tag{12}$$

$$S_{irt}^k \leq b_t^k \qquad \forall k \in K, \forall i \in V, \forall r \in R, \forall t \in T \tag{13}$$

$$S_{irt}^k + t_{ij} + st_i - M(1 - x_{ijrt}^k) \leq S_{jrt}^k \qquad \forall k \in K, \forall (i,j) \in A, \forall r \in R, \forall t \in T \tag{14}$$

$$S_{0r't}^k + M(1 - u_{rr't}^k) \geq S_{n+1,rt}^k \qquad \forall r, r^{'} \in R, r < r^{'}, \forall k \in K, \forall t \in T \tag{15}$$

$$\sum_{r \in R} \sum_{r' \in R | r' > r} u_{rr't}^k \geq \sum_{r \in R} y_{rt}^k - 1 \qquad \forall k \in K, \forall t \in T \tag{16}$$

$$z_{irt}^k \leq y_{rt}^k \qquad \forall k \in K, \forall i \in N, \forall r \in R, \forall t \in T \tag{17}$$

$$x_{ijrt}^k \in \{0,1\} \qquad \forall k \in K, \forall i,j \in V, \forall r \in R, \forall t \in T \tag{18}$$

$$y_{rt}^k \in \{0,1\} \qquad \forall k \in K, \forall r \in R, \forall t \in T \tag{19}$$

$$z_{irt}^k \in \{0,1\} \qquad \forall k \in K, \forall i \in N, \forall r \in R, \forall t \in T \tag{20}$$

$$w_{rt}^{kfp} \in \{0,1\} \qquad \forall k \in K, \forall f \in F_k, \forall p \in P, \forall r \in R, \forall t \in T \tag{21}$$

$$u_{rr't}^k \in \{0,1\} \qquad \forall k \in K, \forall r, r^{'} \in R, r < r^{'}, r^{'} = r + 1, \forall t \in T \tag{22}$$

$$q_{irt}^{kfp} \geq 0 \qquad \forall k \in K, \forall f \in F_k, \forall p \in P, \forall i \in V, \forall r \in R, \forall t \in T \tag{23}$$

$$S_{irt}^k \geq 0 \qquad \forall k \in K, \forall i \in V, \forall r \in R, \forall t \in T \tag{24}$$

$$I_{it}^p \geq 0 \qquad \forall p \in P, \forall i \in N, \forall t \in T \tag{25}$$

The objective function (1) minimises the total number of kilometres driven by all vehicles during the entire time horizon. Constraints (2) and (3) ensure that inventory levels for fuel in underground tanks stay above the safety stock level and below the maximum capacity. Constraints (4) keep track of the daily inventory level, taking into account the demand and deliveries. Station restrictions are imposed by constraints (5), which means that some vehicles cannot access all stations. Constraints (6) ensure that all vehicles start and end at the depot. Constraints (7) represent the flow conservation constraints. Constraints (8) and (9) link x and q variables to z, such that a vehicle can only drive to a station and deliver fuel when it is visited by that vehicle on a certain day. Constraints (10) ensure that compartment capacities are respected and constraints (11) make sure that only a single product is loaded into a compartment, if any.

Constraints (12–13) ensure that operating hours of the vehicles are respected. Constraints (14) ensure the consistency of start times at different stations based on the driving and service times. Constraints (15–17) make sure that, in case a vehicle performs multiple trips during a time period, these trips are driven consecutively. Lastly, x, y, z, w and u are defined as binary variables by (18)–(22) and constraints (19) maintain nonnegativity.

5 Heuristic Solution Method

We investigated two solution methods: (i) using an exact solver, Gurobi Optimizer version 8.1.1 [14], for MILP that is implemented in Python, (ii) a decomposition heuristic. In this section we describe the decomposition heuristic.

5.1 Decomposition Heuristic

A decomposition heuristic, based on decomposing the decision process of the supplier, is considered to solve the rich variant of the PSRP for real-life size instances. To create the planning for a single day, the decision process is decomposed into five phases. Since the planning for a certain day affects the inventory levels of the next day, a dynamic programming approach is required to create the planning day by day. After each iteration, inventory levels are adjusted with the delivery quantities. An overview of the heuristic procedure is shown in Fig. 2.

To improve the initial solution created by the decomposition heuristic, an improvement procedure is proposed (discussed in Sect. 5.2). We now discuss the five phases considered for the decomposition, which are used to generate the planning for a single day. This process is repeated for each day in the time horizon, that is T_e days, and the day in the considered iteration is noted as t_c.

Phase 1 - Order Generation. An order is generated for each underground tank and the expected day of depletion is determined, by deducting the daily demand from the inventory level. The day of depletion is the time period the inventory level is expected to drop below the safety stock level. An order is identified by the station, product and depletion day, since a station is assumed to have maximum one tank per product. The resulting list contains all underground tanks that are expected to deplete during the remainder of the considered time horizon $[t_c, T_e]$.

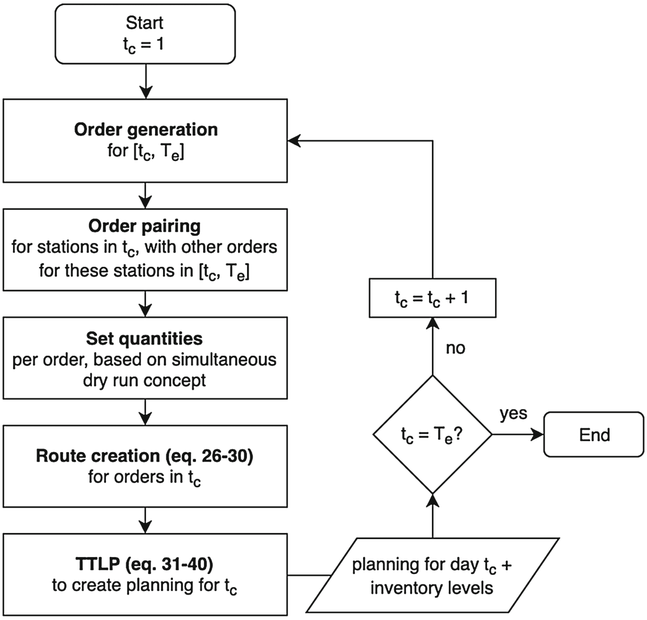

Fig. 2. Heuristic procedure, with T_e days in time horizon

Phase 2 - Order Pairing. The resulting list from phase 1 is filtered on underground tanks that deplete during the day for which the planning is generated t_c. Orders for the same stations as on this filtered list, with an expected depletion day in the future, are also added to the resulting list of phase 2. The output is a list of stations, with a set of tanks per station that are expected to deplete during t_c or a future time period.

Phase 3 - Set Quantities. Minimum and maximum delivery quantities are set according to the *simultaneous dry run* inventory policy, which we will now introduce. The maximum delivery quantity is set to the remaining tank's capacity. Minimum delivery quantities are set to a level that ensures the next delivery to a station to be postponed for as long as possible. When one of the tanks at a station depletes, the station needs to be visited. Postponing the next delivery is done by filling the tank with the highest consumption rate to the maximum, since this tank depletes the fastest. For the other tanks at the same station, the minimum delivery quantity is set to the expected sales during the same time that the tank with the highest consumption rate depletes again. This process, shown by Fig. 3, ensures that a station is visited as least as possible.

Phase 4 - Route Creation. This phase is about generating the routes based on the results of the previous phases. Binary variables y_r and z_{ir} are introduced to determine if route r is used. With the minimum delivery quantity $q^p_{ir,min}$ following from the previous phase, the route creation model can be stated as:

$$\min \sum_{r \in R} y_r \, c_r \tag{26}$$

Subject to

$$z_{ir} \leq x_{ir} \qquad \forall i \in N, \forall r \in R \tag{27}$$

$$z_{ir} \leq y_r \qquad \forall i \in N, \forall r \in R \tag{28}$$

$$\sum_{r \in R} z_{ir} = 1 \qquad \forall i \in N \tag{29}$$

$$\sum_{i \in N} (q^p_{ir,min} z_{ir}) \leq 0.9\, C_{k,min} \qquad \forall r \in R \tag{30}$$

The objective function (26) minimises the total travel distance for the considered time period. Constraints (27) make the choice to visit station i during route r only possible when the station is included in the predetermined route. Constraints (28) link variables y and z. Constraints (29) are used to ensure that each station is visited once. Constraints (30) limit the sum of the minimum delivery quantities to 90% of the total capacity of the largest vehicle. 90% of the capacity is used as maximum based on practical experience since different products need to be loaded into different compartments and a higher value may hinder feasibility, i.e., the next phase of the heuristic may not be able to find feasible solutions.

Phase 5 - Tank Truck Loading. In the final phase, routes are assigned to trucks while maximising vehicle utilisation for an efficient use of the vehicles. A variant of the Tank Truck Loading Problem (TTLP) by Cornillier et al. [10], is developed to execute this assignment procedure. Binary variable y^k_r is introduced to determine which vehicle k drives a route r and w^{kfp}_r is introduced to set in which compartment product p is loaded. The model is given as:

$$\max \sum_{r \in R} \sum_{i \in N} \sum_{k \in K} \sum_{f \in F} \sum_{p \in P} q^{kfp}_{ir} \tag{31}$$

Subject to

$$\sum_{k \in K} \sum_{f \in F} q_{ir}^{kfp} \geq q_{ir,min}^{p} \qquad \forall p \in P, \forall i \in N, \forall r \in R \tag{32}$$

$$\sum_{k \in K} \sum_{f \in F} q_{ir}^{kfp} \leq q_{ir,max}^{p} \qquad \forall p \in P, \forall i \in N, \forall r \in R \tag{33}$$

$$\sum_{p \in P} \sum_{f \in F} q_{ir}^{kfp} \leq M\, \delta_i^k \qquad \forall i \in N, \forall k \in K \forall r \in R \tag{34}$$

$$\sum_{i \in N} q_{ir}^{kfp} \leq w_r^{kfp}\, Q^{kf} \qquad \forall k \in K, \forall f \in F, \forall p \in P, \forall r \in R \tag{35}$$

$$\sum_{p \in P} w_r^{kfp} \leq 1 \qquad \forall k \in K, \forall f \in F, \forall r \in R \tag{36}$$

$$\sum_{k \in K} y_r^k = 1 \qquad \forall r \in R \tag{37}$$

$$\sum_{r \in R} (y_r^k\, t_r) \leq b^k - a^k \qquad \forall k \in K \tag{38}$$

$$w_r^{kfp} \leq y_r^k \qquad \forall k \in K, \forall f \in F, \forall p \in P, \forall r \in R \tag{39}$$

$$q_{ir}^{kfp} \geq 0 \qquad \forall k \in K, \forall f \in F, \forall p \in P, \forall r \in R \tag{40}$$

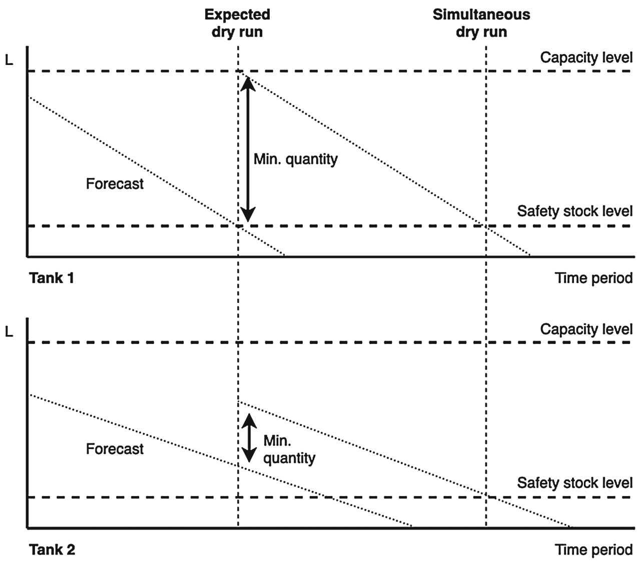

Fig. 3. Visualisation of the simultaneous dry run inventory policy. Shown graphs represent the inventory levels for two tanks at the same station. Minimum delivery quantities are set to ensure simultaneous dry run in the future.

The objective function (31) maximises the total delivered quantity, to ensure efficient vehicle utilisation. Constraints (32–33) set the delivery quantity to be within the range previously determined by the minimum and maximum delivery quantities. Constraints (34) implement station restrictions and constraints (35) limit the delivery quantity per compartment to its capacity. The fact that a compartment can only be filled with one type of product is implemented with constraints (36). Constraints (37) ensure that each route is driven by a vehicle exactly once and constraints (38) limit the duration of all routes driven by a vehicle by the driver schedule. Lastly, constraints (39) link variables w and y.

5.2 Improvement Procedure

For the initial solution, the decision to visit a station is based on the depletion day. This means that routing is not optimised over the entire time horizon and that the stations can not be visited later. To improve the initial solution, an improvement procedure is developed with solution candidates that are based on visiting stations one time period earlier. To evaluate if the solution improves, the planning is recreated for the remaining days. Since this is an extensive process, the number of candidates is limited to the number of station visits.

6 Computational Results

The performance of the decomposition heuristic is evaluated in reference to the benchmark which is the exact solution of the MILP. A real data set from a petrol distributor is used that consists of 1 depot, 59 gas stations and 4 vehicles. The experiments are executed on a computer with a 3.50 GHz 4-core processor and 32 GB of RAM. Shown results are averages of multiple experiments with subsets of the case study data set and S4-D5 stands for 4 stations over 5 days.

Table 2 shows the distance and computation time for MILP and the heuristic. The results are the average of performed experiments and the computation time is limited to7200 s. Optimal solutions for MILP can be found for instances with up to 8 stations and 5 days. For instances larger than 20 stations and 7 days, there is no available feasible solution. In terms of computation time, the heuristic provides advantages. However, the total travel increases with 19.1% compared to MILP solutions. If only experiments with 7 days are considered (representing practice), the average increase in travel distance is 15.6%. This can be explained by the fact that the heuristic maximises delivery amounts per vehicle and the MILP only ensures that the inventory level stays above the safety stock level.

Table 3 shows the heuristic results for real-life size instances. The initial solution found by the decomposition heuristic is improved during the improvement procedure by 4.6% on average. For the full data set, a solution is found within 2 h, which proves that the heuristic is an effective method for creating solutions for real-life instances.

As a validation of the heuristic, we also compared the results for 59 stations across 14 days to the results from the actual planning at the petrol distributor.

Table 2. Benchmark comparison. CT: computation time, Δ: difference in distance relative to MILP, *: not optimal, **: no feasible solution

Experiment	MILP		Heuristic		
	Distance (km)	CT (s)	Distance (km)	Δ	CT (s)
S4-D5	267	2	304	13.8%	2
S4-D7	480	469	550	14.6%	4
S6-D5	321	544	413	28.8%	3
S6-D7	582*	7200	690	18.5%	6
S8-D5	399	1955	501	25.5%	5
S8-D7	753*	7200	849	12.7%	8
S10-D5	454*	7200	581	28.1%	6
S10-D7	873*	7200	984	12.7%	14
S12-D5	478*	7200	606	26.8%	8
S12-D7	972*	7200	1063	9.4%	18
S20-D7	1376*	7200	1642	19.3%	39
S30-D7	–**	–	2281	–	79
Average				**19.1%**	

Table 3. Computational results for larger instances. CT: computation time, InSol, FinSol: initial and final solution.

Experiment	InSol (km)	FinSol (km)	Δ	CT InSol (s)	CT FinSol (s)
S20-D14	3931	3670	−6.6%	2	162
S30-D14	5588	5365	−4.0%	4	589
S40-D14	6965	6735	−3.3%	12	727
S50-D14	10267	9813	−4.4%	29	3747
S59-D14	11426	10877	−4.8%	14	5103

As this was not straightforward to analyse in detail due to differences in the algorithmic approach we can only mention indications for the reduction in travel distance (around 12%) and average number of stops per trip (around 26%). This shows that the proposed approach has a potential to be used in practice.

7 Conclusions and Future Research

We presented planning methods for the MP-PSRP with real-life characteristics and increased complexity. In order to deal with the increased computational complexity, a heuristic is developed, which is a combination of a decomposition heuristic and an improvement procedure. The decomposition heuristic uses the newly introduced *simultaneous dry run* inventory policy to determine maximum and minimum delivery quantities. The models have been tested with a data set based on the real-life situation for a petrol distribution company. It is found that no feasible solution is available for MILP within the two hour time limit for instances larger than 20 stations and 7 days. The heuristic can create the planning for the full case study with 59 stations over 14 days and compared to the actual planning used at the company shows a potential of reducing the travel distance. Nevertheless, the performance of the heuristic algorithm needs improvements as small size instances show gaps compared to the MILP solution.

Future research in this area is encouraged especially in the efficient solution algorithm development. The value of the *simultaneous dry run* inventory policy could be further explored by considering the exact time of delivery instead of the day of delivery. This would allow to also replenish the quantity of fuel that has already been sold during the day till the moment of delivery. Furthermore, the decomposition of the problem into days might be hindering the potential of our heuristic for a multi-period setting. Therefore, further research into different decomposition approaches might be promising. Another direction for further research could be to simulate deliveries with unexpected events, to determine the effect of uncertainty on the distribution performance.

Acknowledgements. We are grateful for the support of AMCS B.V. by providing the case study data, especially Jelmer Brandt and Kristian Milo Hauge with valuable contributions.

References

1. Al-Hinai, N., Triki, C.: A two-level evolutionary algorithm for solving the petrol station replenishment problem with periodicity constraints and service choice. Ann. Oper. Res. **286**(1), 325–350 (2018)
2. Archetti, C., Speranza, M.G.: A survey on matheuristics for routing problems. EURO J. Comput. Optim. **2**(4), 223–246 (2014)
3. Azi, N., Gendreau, M., Potvin, J.Y.: An exact algorithm for a vehicle routing problem with time windows and multiple use of vehicles. Eur. J. Oper. Res. **202**(3), 756–763 (2010)
4. Benantar, A., Ouafi, R., Boukachour, J.: A petrol station replenishment problem: new variant and formulation. Logist. Res. **9**(1), 1–18 (2016)
5. Brown, G.G., Graves, G.W.: Real-time dispatch of petroleum tank trucks. Manag. Sci. **27**(1), 19–32 (1981)
6. Caceres-Cruz, J., Arias, P., Guimarans, D., Riera, D., Juan, A.: Rich vehicle routing problem: survey. ACM Comput. Surv. **47**(2), 1–28 (2014)
7. Coelho, L.C., Cordeau, J.F., Laporte, G.: Thirty years of inventory routing. Transp. Sci. **48**(1), 1–19 (2014)
8. Coelho, L.C., Laporte, G.: Classification, models and exact algorithms for multi-compartment delivery problems. Eur. J. Oper. Res. **242**(3), 854–864 (2015)
9. Cordeau, J.F., Laganà, D., Musmanno, R., Vocaturo, F.: A decomposition-based heuristic for the multiple-product inventory-routing problem. Comput. Oper. Res. **55**, 153–166 (2015)
10. Cornillier, F., Boctor, F.F., Laporte, G., Renaud, J.: An exact algorithm for the petrol station replenishment problem. J. Oper. Res. Soc. **59**(5), 607–615 (2007)

11. Cornillier, F., Boctor, F., Renaud, J.: Heuristics for the multi-depot petrol station replenishment problem with time windows. Eur. J. Oper. Res. **220**(2), 361–369 (2012)
12. Cornillier, F., Boctor, F.F., Laporte, G., Renaud, J.: A heuristic for the multi-period petrol station replenishment problem. Eur. J. Oper. Res. **191**(2), 295–305 (2008)
13. Derigs, U., Gottlieb, J., Kalkoff, J., Piesche, M., Rothlauf, F.: Vehicle routing with compartment: applications, modeling and heuristics. OR Spectr. **33**(4), 885–914 (2011)
14. LLC Gurobi Optimization: Gurobi Optimizer Reference Manual (2019). http://www.gurobi.com
15. Kazemi, Y., Szmerekovsky, J.: Modeling downstream petroleum supply chain: the importance of multi-mode transportation to strategic planning strategic planning. Transp. Res. Part E: Logist. Transp. Rev. **83**, 111–125 (2015)
16. Li, K., Chen, B., Sivakumar, A.I., Wu, Y.: An inventory-routing problem with the objective of travel time minimization. Eur. J. Oper. Res. **236**(3), 936–945 (2014)
17. Macedo, R., Alves, C., De Carvalho, J.M., Clautiaux, F., Hanafi, S.: Solving the vehicle routing problem with time windows and multiple routes exactly using a pseudo-polynomial model. Eur. J. Oper. Res. **214**(3), 536–545 (2011)
18. Ng, W.L., Leung, S.C., Lam, J.K., Pan, S.W.: Petrol delivery tanker assignment and routing: a case study in Hong Kong. J. Oper. Res. Soc. **59**(9), 1191–1200 (2008)
19. Popović, D., Bjelić, N., Radivojević, G.: Simulation approach to analyse deterministic IRP solution of the stochastic fuel delivery problem. Proc. Soc. Behav. Sci. **20**, 273–282 (2011)
20. Popović, D., Vidović, M., Radivojević, G.: Variable neighborhood search heuristic for the inventory routing problem in fuel delivery. Expert Syst. Appl. **39**(18), 13390–13398 (2012)
21. Triki, C., Al-hinai, N., Kaabachi, I., Krichen, S.: An optimization framework for combining the petroleum replenishment problem with the optimal bidding in combinatorial auctions. Int. J. Supply Oper. Manag. **3**(2), 1318–1331 (2016)
22. Triki, C.: Solution methods for the periodic petrol station. J. Eng. Res. **10**(2), 69–77 (2013)
23. Vidović, M., Popović, D., Ratković, B.: Mixed integer and heuristics model for the inventory routing problem in fuel delivery. Int. J. Prod. Econ. Part C **147**, 593–604 (2014)