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Piston-Driven Pneumatically-Actuated Soft Robots: Modeling and Backstepping Control

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***Abstract*—Actuators' dynamics have been so far mostly neglected when devising feedback controllers for continuum soft robots since the problem under the direct actuation hypothesis is already quite hard to solve. Directly considering actuation would have made the challenge too complex. However, these effects are, in practice, far from being negligible. The present work focuses on model-based control of piston-driven pneumatically-actuated soft robots. We propose a model of the relationship between the robot's state, the acting fluidic pressure, and the piston dynamics, which is agnostic to the chosen model for the soft system dynamics. We show that backstepping is applicable even if the feedback coupling of the outer on the inner subsystem is not linear. Thus, we introduce a general model-based control strategy based on backstepping for soft robots actuated by fluidic drive. As an example, we derive a specialized version for a robot with piecewise constant curvature.**

***Index Terms*—Control applications, PID control, robotics.**

I. Introduction

Continuum soft robots are systems entirely made of deformable materials, so to resemble the trunk of an elephant [1]. Controlling these systems is challenging because of the infinite amount of Degrees of Freedom (DoFs), the multi-body dynamics, nonlinear potentials, underactuation, and the high degree of uncertainties [2]. Combining feedback controllers and simplified dynamical models can help taming this complexity and achieve good experimental performance [3]–[5].

Accurate low-dimensional models of the continuum dynamics have been thoroughly investigated in recent years [6]–[8], serving as the base for model-based controllers [9], [10]. In comparison, researchers have devoted little or no attention to modeling the actuator dynamics, despite this being far from a negligible effect in practice, in particular for pneumatic actuation. The lack of models pairs with the scarcity of model-based dynamic controllers. Existing strategies only rarely reason on the actuators' dynamics, if not through simple heuristics. For example, [4], [11] use a combination of PID control and inversion of quasi-static linear approximations to compensate for the actuators' dynamics. This strategy may present clear limitations in terms of performance and stability assessment.

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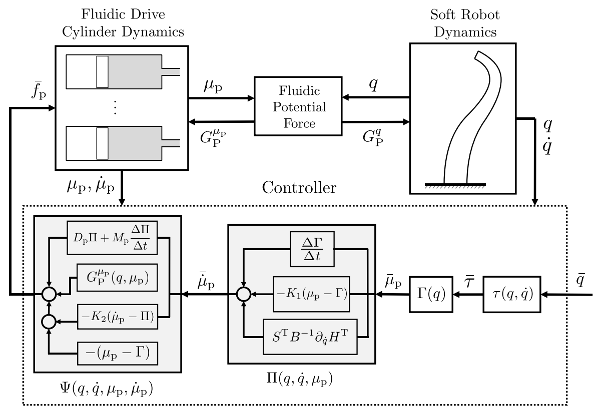


Fig. 1. Schematic block diagram of the proposed nonlinear backstepping controller for a pneumatically-actuated soft robot. The approach considers both the fluidic drive cylinder and the soft system dynamics. It is agnostic to the chosen soft system controller in configuration-space $\tau(q, \dot{q})$.

As model-based control of soft robots becomes a mature discipline, the need for general ways of dealing with actuators' dynamics becomes more pressing. In this letter we deal with this challenge by following a backstepping approach - which is an established strategy to deal with dynamical systems with triangular structure. A pneumatic model based on the ideal gas law is derived and the pneumatic actuation system is compensated in a quasi-static fashion in Falkenhahn *et al.* [12]. Recent work by Wang *et al.* [13] uses backstepping for control of a continuum soft bending arm. Although interesting, the work is limited because it targets a linear model of a single DoF. Similarily, Franco *et al.* [14] derive an energy-based control scheme for pneumatic manipulators while using a backstepping-based controller for comparison purposes. Both pieces of work focus on pneumatic actuation with valves and thus cannot be immediately applied to systems actuated with fluidic drive cylinders.

To conclude, this letter targets the dynamic control of piston-driven pneumatic-actuated soft robots (see for example Fig. 2, 3). We provide general strategies for (i) augmenting existing dynamic models of soft robots through a description of pneumatic actuation, (ii) controlling these systems via model-based feedback. As an example, we specialize the

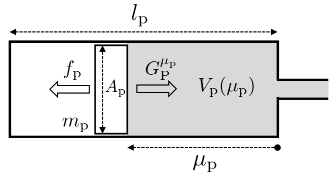


Fig. 2. Fluidic drive cylinder parameters for a piston of mass m_p, length l_p and cross-sectional area A_p: f_p describes the actuation force while $G_\mathrm{P}^{\mu_\mathrm{p}}$ is the conservative force applied by the compressed fluid on the cylinder. μ_p represents the actuators' state variable. These pneumatic pistons could be for example actuated by current-controlled DC motors or linear electric actuators [15].

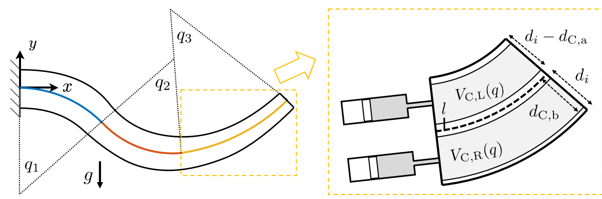


Fig. 3. Shape regulation under PCC approximation - **Left:** A planar soft robot consisting of three segments each modelled to have constant curvature **Right:** Model parameters for fluidic volume in soft segment chambers. Each chamber is actuated independently by a fluidic drive cylinder connected through tubing.

model to planar soft robots satisfying the Piecewise Constant Curvature (PCC) assumption [4] including the proposal of a kinematic model for the air volume in the chambers, and the controller to the set-point regulation of configuration. In this context, we also propose a simplified, potential coupling-aware PID-like controller. We provide simulations showing the effectiveness of both strategies.

II. Dynamic Model

We consider the robot made by a sequence of n_S segments. Each segment is described with n_D configuration variables by using one of the many modeling techniques developed in the state of the art [6]–[9]. We denote with $n_\mathrm{q} = n_\mathrm{S} n_\mathrm{D}$ the total number of configuration variables, which also represents the approximated DoFs of the soft arm. Although we show planar kinematic relations for the PCC-case in Figure 3 as an example, the dynamic model derived in this section is agnostic to the chosen kinematic approximation.

All of kinematic modelling techniques produce multi-body dynamics of the unactuated soft robot as follows [10]

$$B(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) + K(q) + D(q,\dot{q}) = 0, \tag{1}$$

where $q \in \mathbb{R}^{n_\mathrm{q}}$ describes the configuration of the robot in generalized coordinates, $B(q) \in \mathbb{R}^{n_\mathrm{q}\times n_\mathrm{q}}$ the inertial matrix, $C(q,\dot{q}) \in \mathbb{R}^{n_\mathrm{q}\times n_\mathrm{q}}$ contains the Coriolis and Centrifugal forces and $G(q) \in \mathbb{R}^{n_\mathrm{q}}$ compensates for the gravitational effects. The elastic (restoring) forces are captured in the matrix $K \in \mathbb{R}^{n_\mathrm{q}}$ and the natural damping is represented by $D(q,\dot{q}) \in \mathbb{R}^{n_\mathrm{q}}$.

Each segment is actuated through a set of n_C dedicated chambers. Adapting the pressure in a chamber will lead to to a different chamber volume and ultimately resulting in a changed configuration of the segment. Each chamber is connected to a dedicated piston as shown in Figure 3. If more than one chamber is connected to a same piston, it can be considered to be the same chamber for the sake of this letter. These hypotheses are not paramount, but they are instrumental to maintain the simplicity of notation. Accordingly, the total number of pistons is described with $n_{\mu_\mathrm{p}} = n_\mathrm{S} n_\mathrm{C}$. Please note that if $n_\mathrm{C} = n_\mathrm{D}$, the model of the soft robot is fully-actuated, if $n_\mathrm{C} > n_\mathrm{D}$ it is over-actuated, and with $n_\mathrm{C} < n_\mathrm{D}$ under-actuated respectively. The dynamics of the piston when not interacting with the fluid can be easily written as being

$$M_\mathrm{p}\ddot{\mu}_\mathrm{p} + D_\mathrm{p}\dot{\mu}_\mathrm{p} + G_\mathrm{P}^{\mu_p} = f_\mathrm{p}, \tag{2}$$

where $\mu_\mathrm{p} \in \mathbb{R}^{n_{\mu_\mathrm{p}}}$ denoting the displacement of every piston from the zero-volume configuration, $M_\mathrm{p} \in \mathbb{R}^{n_{\mu_\mathrm{p}}\times n_{\mu_\mathrm{p}}}$ the mass matrix of the piston system, $G_\mathrm{P}^{\mu_p} \in \mathbb{R}^{n_{\mu_\mathrm{p}}}$ describing the conservative force caused by the compressed fluid acting on the pistons, and $D_\mathrm{p} \in \mathbb{R}^{n_{\mu_\mathrm{p}}\times n_{\mu_\mathrm{p}}}$ the damping matrix of the piston system. As the piston system is fixed to the ground and connected via tubing to the robot chambers, note that the gravity force here is constant, so w.l.o.g. we consider it to be zero (or alternative as being compensated by a constant off-set in f_p).

In first approximation, we model the compressible fluid (typically air) as an ideal gas. Furthermore, we consider the process to be isothermal and that no exchange of fluid with the external world is happening. We neglect the volume of fluid in any tubes connecting the pistons with the chambers. The overall volume of the fluid can be evaluated as

$$V(q,\mu_\mathrm{p}) = V_\mathrm{C}(q) + V_\mathrm{p}(\mu_\mathrm{p}) = V_\mathrm{C}(q) + A_\mathrm{p}\mu_\mathrm{p}, \tag{3}$$

where $V(q,\mu_\mathrm{p}) \in \mathbb{R}^{n_{\mu_\mathrm{p}}}$ describes the total volume of fluid stored in the system, $V_\mathrm{C}(q) \in \mathbb{R}^{n_{\mu_\mathrm{p}}}$ the volume of fluid in each chamber and $V_\mathrm{p}(q) \in \mathbb{R}^{n_\mathrm{S} n_\mathrm{p}}$ the volume in the piston with $A_\mathrm{p} \in \mathbb{R}^{n_{\mu_\mathrm{p}}}$ the cross-sectional area of every piston. We will present an example of analytical derivation of $V_\mathrm{C}(q)$ in Section IV-A. For now we consider it known.

The total energy stored in the system due to fluid compression is

$$\begin{aligned}\mathcal{U}_\mathrm{fluid}(q,\mu_\mathrm{p}) &= \sum_{j=1}^{n_{\mu_\mathrm{p}}} \int_{V_{j,0}}^{V_j(q_i,\mu_{\mathrm{p},j})} -\big(p_j(\nu) - p_\mathrm{atm}\big)\mathrm{d}\nu \\ &= \sum_{j=1}^{n_{\mu_\mathrm{p}}} -\alpha_{\mathrm{air},j}\left(\ln\frac{V_j(q_i,\mu_{\mathrm{p},j})}{V_{j,0}} - \frac{V_j(q_i,\mu_{\mathrm{p},j})}{V_{j,0}} + 1\right),\end{aligned} \tag{4}$$

where $V_j(q_i,\mu_{\mathrm{p},j}) = \frac{\alpha_{\mathrm{air},j}}{p_j(q_i,\mu_{\mathrm{p},j})}$ represents the total fluidic volume in the system of chamber and piston j in segment i. We assume that this fluid system is filled with air at atmospheric pressure p_atm with an initial volume of $V_{j,0} = V_j(0,l_\mathrm{p})$ (e.g., straight robot configuration and with fully extended pistons). This lets us find an expression for α_air:

$$\alpha_{\mathrm{air},j} = n_j RT = p_\mathrm{atm}V_{j,0} = p_j(q,\mu_\mathrm{p})V_j(q,\mu_\mathrm{p}). \tag{5}$$

The force exerted on the ith segment of the robot by the fluid is

$$\begin{aligned}G_{\mathrm{P},i}^{\mathrm{q}}(q_i,\mu_{\mathrm{p},j}) &= \partial_{q_i}\mathcal{U}_\mathrm{fluid}(q,\mu_\mathrm{p}) \\ &= -\partial_{q_i}V_{C,j}\big(p_j(q_i,\mu_{\mathrm{p},j}) - p_\mathrm{atm}\big),\end{aligned} \tag{6}$$

Similarly, the force applied on jth piston by the fluid is

$$\begin{aligned}G_{\mathrm{P},j}^{\mu_\mathrm{p}}(q_i,\mu_{\mathrm{p},j}) &= \partial_{\mu_{\mathrm{p},j}}\mathcal{U}_\mathrm{fluid}(q,\mu_\mathrm{p}) \\ &= -A_{\mathrm{p},j}\big(p_j(q_i,\mu_{\mathrm{p},j}) - p_\mathrm{atm}\big),\end{aligned} \tag{7}$$

The overall dynamic model is

$$\begin{aligned}&B(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) + K(q) + D(q,\dot{q}) + G_\mathrm{P}^\mathrm{q}(q,\mu_\mathrm{p}) = 0, \\ &M_\mathrm{p}\ddot{\mu}_\mathrm{p} + D_\mathrm{p}\dot{\mu}_\mathrm{p} + G_\mathrm{P}^{\mu_\mathrm{p}}(q,\mu_\mathrm{p}) = f_\mathrm{p},\end{aligned} \tag{8}$$

which is always underactuated. Note that this structure is similar to the one of classic flexible joint robots under Spong's approximation [16] due to the fact that the fluidic drive cylinders are fixed to the ground. Two of the major differences making the control problem harder are that that $n_{\mu_\mathrm{p}} \neq n_\mathrm{q}$, and that G_P^q and $G_\mathrm{P}^{\mu_\mathrm{p}}$ are not linear. The latter renders the feedback coupling of the outer on the inner subsystems non-affine. In the rest of this letter, we will use the following definitions to simplify the notation

$$\begin{aligned} f(q,\dot{q}) &= -B^{-1}(q)(C(q,\dot{q})\dot{q} + G(q) + K(q) + D(q,\dot{q})),\\ g(q,\mu_\mathrm{p}) &= -B^{-1}(q)\big(G_\mathrm{P}^\mathrm{q}(q,\mu_\mathrm{p})\big). \end{aligned} \tag{9}$$

III. Backstepping Control of Piston-Driven Pneumatically-Actuated Soft Robots

This section discusses the main contribution of this letter, a backstepping-based approach to generalize controllers $\Gamma(q,\dot{q})$ designed in the directly actuated case, to systems that can be modeled through (8). We suppose that we have access to $\Gamma(q,\dot{q})$ controlling the piston position μ_p. Next, we perform backstepping twice to the controllers of the piston velocity $\dot{\mu}_\mathrm{p}$ and the piston actuation force f_p and prove the stability of each controller with Lyapunov arguments. The derived model-based control approach assumes that all model parameters are known, and all states are measurable (namely the configuration q, its time derivative $\dot{q}$, the piston position μ_p and the piston velocity $\dot{\mu}_\mathrm{p}$), and that there are no disturbances or model uncertainties. We first introduce a Lemma, which will be instrumental to the proof of the main theorem. It allows to relate an offset in the actuation-space to a change in acceleration in configuration-space that is proportional to the offset.

Lemma 1: The input field defined in (9) verifies

$$g(q,\mu_\mathrm{p,a}) - g(q,\mu_\mathrm{p,b}) = -B^{-1}(q)S(q,\mu_\mathrm{p,a},\mu_\mathrm{p,b})(\mu_\mathrm{p,a}-\mu_\mathrm{p,b}),$$

$\forall \mu_\mathrm{p,a}, \mu_\mathrm{p,b} \in \mathbb{R}^{n_{\mu_\mathrm{p}}}$ and $q \in \mathbb{R}^{n_\mathrm{q}}$ with $S \in \mathbb{R}^{n_\mathrm{q} \times n_{\mu_\mathrm{p}}}$ so defined

$$S_{i,j} = \frac{A_{\mathrm{p},j}\alpha_{\mathrm{air},j}\partial_{q_i}V_{\mathrm{C},j}}{(V_{\mathrm{C},j}(q_i) + A_{\mathrm{p},j}\mu_{\mathrm{p,a},j})(V_{\mathrm{C},j}(q_i) + A_{\mathrm{p},j}\mu_{\mathrm{p,b},j})}.$$

Proof: We express the left term of the equality using (9)

$$g(q,\mu_\mathrm{p,a}) - g(q,\mu_\mathrm{p,b}) = -B^{-1}(q)(G_\mathrm{P}^\mathrm{q}(q,\mu_\mathrm{p,a}) - G_\mathrm{P}^\mathrm{q}(q,\mu_\mathrm{p,b})),$$

where we recognize the term $B^{-1}(q)$ appearing in the Lemma. The term between brackets can be adjusted by using (6)

$$\begin{aligned} &G_{\mathrm{P},i}^\mathrm{q}(q,\mu_\mathrm{p,a}) - G_{\mathrm{P},i}^\mathrm{q}(q,\mu_\mathrm{p,b})\\ &= -\left(\sum_{j=1}^{n_{\mu_\mathrm{p}}} \frac{\alpha_{\mathrm{air},j}\partial_{q_i}V_{\mathrm{C},j}}{V_{\mathrm{C},j}(q_i) + A_{\mathrm{p},j}\mu_{\mathrm{p,a},j}} - \sum_{j=1}^{n_{\mu_\mathrm{p}}} \frac{\alpha_{\mathrm{air},j}\partial_{q_i}V_{\mathrm{C},j}}{V_{\mathrm{C},j}(q_i) + A_{\mathrm{p},j}\mu_{\mathrm{p,b},j}}\right)\\ &= \sum_{j=1}^{n_{\mu_\mathrm{p}}} \frac{(\mu_{\mathrm{p,a},j} - \mu_{\mathrm{p,b},j})A_{\mathrm{p},j}\alpha_{\mathrm{air},j}\partial_{q_i}V_{\mathrm{C},j}}{(V_{\mathrm{C},j}(q_i) + A_{\mathrm{p},j}\mu_{\mathrm{p,a},j})(V_{\mathrm{C},j}(q_i) + A_{\mathrm{p},j}\mu_{\mathrm{p,b},j})}. \end{aligned} \tag{10}$$

The Lemma follows by simple factorization of the latter term. ■

Thus, even if the robot side of the dynamics (8) is not affine in control when taking μ_p as input, still Lemma 1 provides some structure that we leverage in the next theorem.

Theorem 1: Suppose that a $\Gamma(q,\dot{q})$ exists s.t. the reduced system

$$\ddot{q} = f(q,\dot{q}) + g(q,\Gamma(q,\dot{q})) \tag{11}$$

converges to a desired trajectory $\bar{q}(t)$, $\forall (q(0),\dot{q}(0)) \in \mathbb{R}^{2n_\mathrm{q}}$. Suppose that the convergence is proven by Lyapunov arguments through the function $H(q,\dot{q})$. Then the closed loop of the full system (8) and the controller

$$\begin{aligned} f_\mathrm{p} &= \Psi = G_\mathrm{P}^{\mu_\mathrm{p}} + D_\mathrm{p}\Pi + M_\mathrm{p}\dot{\Pi} - K_2(\dot{\mu}_\mathrm{p} - \Pi) - (\mu_\mathrm{p} - \Gamma),\\ \Pi &= \dot{\Gamma} - K_1(\mu_\mathrm{p} - \Gamma) + S^\mathrm{T}(q,\mu_\mathrm{p},\Gamma)B^{-1}(q)\partial_{\dot{q}}H^\mathrm{T}, \end{aligned} \tag{12}$$

with $K_1, K_2 \succ 0$, is such that $q \to \bar{q}$ and $\mu_\mathrm{p} \to \Gamma(\bar{q},\dot{\bar{q}})$, $\forall (\mu_\mathrm{p}(0),\dot{\mu}_\mathrm{p}(0)) \in \mathbb{R}^{2n_{\mu_\mathrm{p}}}$, and $\forall (q(0),\dot{q}(0)) \in \mathbb{R}^{2n_\mathrm{q}}$.

Proof: We first consider the problem of deriving a controller under the assumption that the velocity of the piston v_p is set by a controller. This serves as a first step toward the general solution of the problem. System (8) is thus reduced into

$$\begin{aligned} &B(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) + K(q) + D(q,\dot{q}) + G_\mathrm{P}^\mathrm{q}(q,\mu_\mathrm{p}) = 0,\\ &\dot{\mu}_\mathrm{p} = v_\mathrm{p}. \end{aligned} \tag{13}$$

We introduce the following control Lyapunov candidate

$$W(q,\dot{q},\mu_\mathrm{p}) = H(q,\dot{q}) + \frac{1}{2}(\mu_\mathrm{p} - \Gamma)^\mathrm{T}(\mu_\mathrm{p} - \Gamma)\ , \tag{14}$$

which can thus be differentiated obtaining

$$\begin{aligned} \dot{W}(q,\dot{q},\mu_\mathrm{p}) &= \dot{H} + (\mu_\mathrm{p} - \Gamma)^\mathrm{T}(v_\mathrm{p} - \dot{\Gamma})\\ &= \partial_q H\dot{q} + \partial_{\dot{q}}H(f(q,\dot{q}) + g(q,\mu_\mathrm{p})) + (\mu_\mathrm{p} - \Gamma)^\mathrm{T}(v_\mathrm{p} - \dot{\Gamma})\\ &= \partial_q H\dot{q} + \partial_{\dot{q}}H(f(q,\dot{q}) + g(q,\Gamma(q,\dot{q})))\\ &\quad + \partial_{\dot{q}}H(g(q,\mu_\mathrm{p}) - g(q,\Gamma(q,\dot{q}))) + (\mu_\mathrm{p} - \Gamma)^\mathrm{T}(v_\mathrm{p} - \dot{\Gamma}), \end{aligned} \tag{15}$$

where we first used the chain rule on $\dot{H}$ and then we added and subtracted $\partial_{\dot{q}}Hg(q,\Gamma(q,\dot{q}))$. We now propose the controller $v_\mathrm{p} = \Pi(q,\dot{q},\mu_\mathrm{p})$ for stabilizing this system, with

$$\Pi(q,\dot{q},\mu_\mathrm{p}) = \dot{\Gamma} - K_1(\mu_\mathrm{p} - \Gamma) + S^\mathrm{T}(q,\mu_\mathrm{p},\Gamma)B^{-\mathrm{T}}(q)\partial_{\dot{q}}H^\mathrm{T}.$$

The derivative of the Lyapunov candidate for the closed loop system is thus

$$\begin{aligned} \dot{W}(q,\dot{q},\mu_\mathrm{p}) &= \partial_q H\dot{q} + \partial_{\dot{q}}H(f(q,\dot{q}) + g(q,\Gamma(q,\dot{q})))\\ &\quad + \partial_{\dot{q}}H(g(q,\mu_\mathrm{p}) - g(q,\Gamma(q,\dot{q}))) - (\mu_\mathrm{p} - \Gamma)^\mathrm{T}K_1(\mu_\mathrm{p} - \Gamma)\\ &\quad + \partial_{\dot{q}}HB^{-1}(q)S(q,\mu_\mathrm{p},\Gamma)(\mu_\mathrm{p} - \Gamma), \end{aligned}$$

where we exploited that all terms are scalar to extract the transpose of the last one. This equation can be simplified by invoking Lemma 1 into

$$\begin{aligned} \dot{W}(q,\dot{q},\mu_\mathrm{p}) &= \partial_q H\dot{q} + \partial_{\dot{q}}H(f(q,\dot{q}) + g(q,\Gamma(q,\dot{q})))\\ &\quad - (\mu_\mathrm{p} - \Gamma)^\mathrm{T}K_1(\mu_\mathrm{p} - \Gamma). \end{aligned} \tag{16}$$

Consider now that H is a Lyapunov function for (11) under the control action Γ. This assures that

$$0 > \dot{H} = \partial_q H\dot{q} + \partial_{\dot{q}}H(f(q,\dot{q}) + g(q,\Gamma(q,\dot{q}))). \tag{17}$$

Note that we are considering here the case of strict sign definiteness of $\dot{H}$. However, the same results can be achieved in the case of semi-definiteness. We can now conclude that $\dot{W} < 0$, thus proving that the controller Π stabilizes (13). This conclude the first step of the proof.

We now reiterate this sequence of operations, to generalize the controller Π to work on the actual system (8). The complete Lyapunov candidate that we propose is

$$Q = W + \frac{1}{2}(\dot{\mu}_\mathrm{p} - \Pi)^\mathrm{T}M_\mathrm{p}(\dot{\mu}_\mathrm{p} - \Pi), \tag{18}$$

with time derivative

$$\begin{aligned}\dot{Q} &= \dot{W} + (\dot{\mu}_\mathrm{p} - \Pi)^\mathrm{T}(\Psi - G_\mathrm{P}^{\mu_\mathrm{p}} - M_\mathrm{p}\dot{\Pi})\\ &= \partial_q W\dot{q} + \partial_{\dot{q}} W\ddot{q} + \partial_{\mu_\mathrm{p}} W\dot{\mu}_p\\ &\quad + (\dot{\mu}_\mathrm{p} - \Pi)^\mathrm{T}(\Psi - G_\mathrm{P}^{\mu_\mathrm{p}} - D_\mathrm{p}\dot{\mu}_\mathrm{p} - M_\mathrm{p}\dot{\Pi}).\end{aligned}$$

We therefore propose the controller

$$\Psi(q, \dot{q}, \mu_\mathrm{p}, \dot{\mu}_\mathrm{p}) = G_\mathrm{P}^{\mu_\mathrm{p}} + D_\mathrm{p}\Pi + M_\mathrm{p}\dot{\Pi} - K_2(\dot{\mu}_\mathrm{p} - \Pi) - \partial_{\mu_\mathrm{p}} W^\mathrm{T}, \tag{19}$$

which generates the following closed loop Lyapunov candidate

$$\begin{aligned}\dot{Q} &= \partial_q W\dot{q} + \partial_{\dot{q}} W\ddot{q} + \partial_{\mu_\mathrm{p}} W\Pi\\ &\quad - (\dot{\mu}_\mathrm{p} - \Pi)^\mathrm{T}(K_2 + D_\mathrm{p})(\dot{\mu}_\mathrm{p} - \Pi) < 0,\end{aligned} \tag{20}$$

where we exploit that W is a Lyapunov function for the previous system when $\dot{\mu}_\mathrm{p} \equiv \Pi$. This assures the asymptotic stability of the closed loop system, when (19) is used. The Theorem follows considering that $\partial_{\mu_\mathrm{p}} W^\mathrm{T} = (\mu_\mathrm{p} - \Gamma)$. ■

IV. Shape Regulation Under PCC Approximation

This section provides an example of application of the proposed model augmentation and model-based control strategy for the setpoint regulation of a pneumatically actuated planar soft robot, modeled through PCC approximation with acting gravity forces.

A. Model

Background - PCC dynamic model: We consider a planar soft robotic arm consisting of three segments analogue to [4] modelled using the PCC [17] assumption, but the formulation can be easily extended to the 3D case while neglecting the torsional deformations. Alternatively, a strain-based parameterization could be employed [9]. We assume a weight distribution of $m_i = \int_0^{l_i} \rho_i(s')ds'$ along the center line of the segment i. Gravity is acting along the vector $g \in \mathbb{R}^2$. We consider the following equations of motions with diagonal matrices K and D:

$$\begin{aligned}B(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) + Kq + D\dot{q} + G_\mathrm{P}^\mathrm{q}(q, \mu_\mathrm{p}) &= 0,\\ M_\mathrm{p}\ddot{\mu}_\mathrm{p} + D_\mathrm{p}\dot{\mu}_\mathrm{p} + G_\mathrm{P}^{\mu_\mathrm{p}}(q, \mu_\mathrm{p}) &= f_\mathrm{p}.\end{aligned} \tag{21}$$

Fluid Volume in Chamber: A model of the fluid volume in the chambers as a function of the configuration of the segment is required to evaluate the conservative forces by the fluid as specified in (6) and (7). In this section, we derive a simple analytical model based on Constant Curvature (CC) kinematics. It is assumed that the volume of the chamber is only dependent on the curvature of the segment as we model the segment length $l_{0,i}$ to stay constant and the chambers to be inextensible in radial direction of the curvature. We visualize the model and its parameters in Figure 3. Thus, the volume of chamber j part of segment i is defined as:

$$V_{\mathrm{C},j}(q_i) = \int_{d_{\mathrm{C,a},j}}^{d_{\mathrm{C,b},j}} b_\mathrm{C}\, l_i(d'_\mathrm{C})\, \mathrm{d}d'_\mathrm{C}, \tag{22}$$

where b_C describes the constant planar thickness of the chamber and $l_i(d'_\mathrm{C})$ the length of segment i at offset d'_C from the center-line. The function $l_i(d'_\mathrm{C})$ is derived by the properties of constant curvature of the segment

$$l_i(d'_\mathrm{C}) = l_{0,i} - q_i d'_\mathrm{C}. \tag{23}$$

The integration inherits an opposite sign for the change of volume with q_i for the left and right chamber respectively for an inner and outer chamber wall radius of $0 < d_\mathrm{C,a} < d_\mathrm{C,b} < d_i$ and a continuum segment of radius d_i:

$$V_{\mathrm{C},j}(q_i) = b_\mathrm{C}\Big(l_{0,i}(d_\mathrm{C,b} - d_\mathrm{C,a}) \mp \frac{q_i}{2}(d_\mathrm{C,b}^2 - d_\mathrm{C,a}^2)\Big). \tag{24}$$

The partial derivative $\partial_q V_\mathrm{C}$ is determined as

$$\partial_{q_i} V_{\mathrm{C},j} = \mp 0.5\, b_\mathrm{C}\, (d_\mathrm{C,b}^2 - d_\mathrm{C,a}^2). \tag{25}$$

B. Set Point Control

Configuration-space control: Consider the following regulator of desired configuration $\bar{q} \in \mathbb{R}^{n_\mathrm{q}}$,

$$\bar{\tau}(q, \bar{q}) = K\bar{q} + G(q), \tag{26}$$

where $\bar{\tau} \in \mathbb{R}^{n_\mathrm{q}}$ is the torque in configuration space. Asymptotic stability of the equilibrium $\bar{q}$ is proven through the Lyapunov function $H(q, \dot{q}) = \dot{q}^\mathrm{T}B(q)\dot{q}/2 + q^\mathrm{T}Kq/2$, which yields the time derivative $\dot{H} = -\dot{q}^\mathrm{T}D\dot{q}/2 \le 0$.

Mapping from configuration to actuation space with force balance: Our backstepping controller requires access to $\Gamma(q, \dot{q})$ which returns a desired piston $\bar{\mu}_\mathrm{p}$ given the desired torque $\bar{\tau}$ and the current state of the soft system $(q, \dot{q})$. In the planar case with inextensible segments, there exist a redundancy in actuating the pistons controlling the pressure in the left and right chambers of a segment to trigger $\bar{\tau}$ on the segment. Thus, we decide to solve this redundancy by equally attributing the desired torque to both pistons.

We assume that the system is calibrated at a straight configuration $q_\mathrm{t0} = 0$ with pistons preloaded at position $\mu_{p,t0} \in \mathbb{R}^{n_{\mu_\mathrm{p}}}$ leading to a fluidic volume of

$$V_{\mathrm{t0},j} = V_{\mathrm{C},j}(q_{\mathrm{t0},i}) + \mu_{\mathrm{p,t0},j} A_{\mathrm{p},j} \tag{27}$$

in the system. After preloading, the fluids in the left and right chambers each apply a preloaded torque of magnitude $G_\mathrm{P,t0}^q \in \mathbb{R}^{n_\mathrm{q}}$ on the soft system. It is implicitly assumed that the piston length l_p, piston area A_p, the preloaded piston position $\mu_\mathrm{p,t0}$ and the preloaded volume V_t0 are equal for the left and right chambers of segment i. We can write the conservative forces acting on the left chamber $G_\mathrm{P,L}^q(q, \mu_\mathrm{p}) \in \mathbb{R}^{n_\mathrm{q}}$ and right chamber $G_\mathrm{P,R}^q(q, \mu_\mathrm{p}) \in \mathbb{R}^{n_\mathrm{q}}$ as differences from the neutral conservative force $G_\mathrm{P,t0}^q$

$$G_\mathrm{P,L}^q = G_\mathrm{P,t0}^q + \Delta G_\mathrm{P,L}^q, \quad G_\mathrm{P,R}^q = -G_\mathrm{P,t0}^q + \Delta G_\mathrm{P,R}^q. \tag{28}$$

The force applied by the fluid in the left and right chambers on the system is

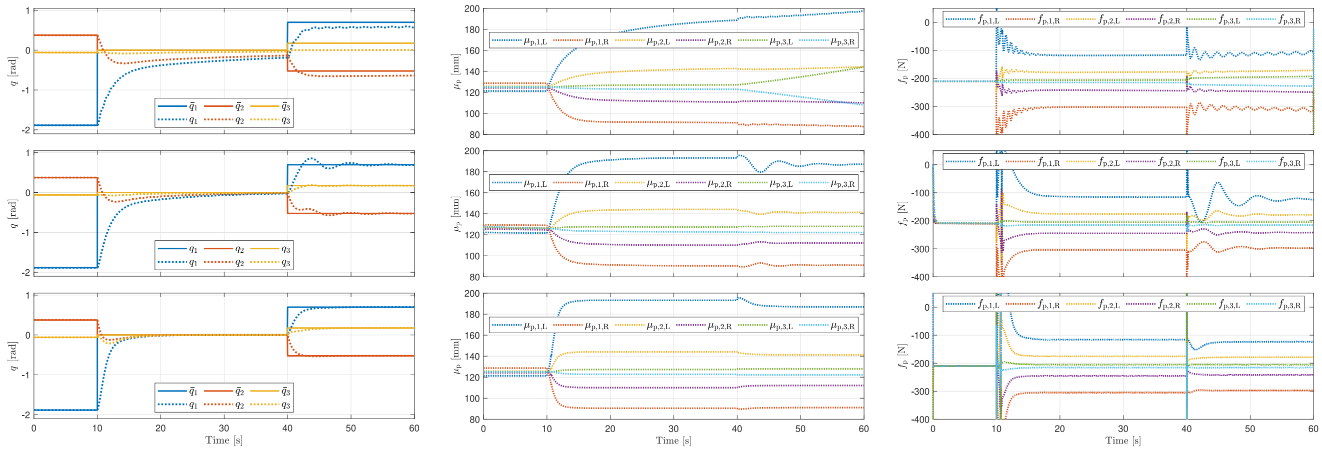
$$G_\mathrm{P}^q(q, \mu_p) = G_\mathrm{P,L}^q + G_\mathrm{P,R}^q = \Delta G_\mathrm{P,L}^q + \Delta G_\mathrm{P,R}^q. \tag{29}$$

We re-arrange to find an expression for the desired conservative force offsets which equally distribute a commanded torque $\bar{\tau}$ to the fluid in both chambers. Setting $\Delta\bar{G}_\mathrm{P,L}^{\bar{q}} = \Delta\bar{G}_\mathrm{P,R}^{\bar{q}} = 0.5\,\bar{\tau}(q, \bar{q})$ results for the chosen set point controller $\bar{\tau}$ and a diagonal elastic matrix K with elements k_i in:

$$\bar{G}_{\mathrm{P},j}^{\bar{q}}(q, \bar{q}_i) = \pm G_{\mathrm{P,t0},i}^q - 0.5(k_i\bar{q}_i + G_i(q)). \tag{30}$$

Equation (6) is inversed to compute the desired piston position $\bar{\mu}_\mathrm{p} = \Gamma(q, \bar{q})$:

$$\Gamma_j(q, \bar{q}) = \frac{1}{A_{\mathrm{p},j}}\left(\frac{\alpha_{\mathrm{air},j}\, \partial_{q_i} V_{\mathrm{C},j}}{p_\mathrm{atm}\, \partial_{q_i} V_{\mathrm{C},j} - \bar{G}_{\mathrm{P},j}^{\bar{q}}(q, \bar{q})} - V_{\mathrm{C},j}(q)\right). \tag{31}$$



Configuration q Piston position μ_p Actuation force f_p

Fig. 4. Simulation of posture regulation under PCC approximation comparing the performance of an end-to-end PID baseline controller (1st row), with a coupling-aware PID controller (2nd row) and the nonlinear backstepping controller (3rd row). The set-point reference configuration is shown with solid lines.

Backstepping: The system is now in the form of (11), so Theorem 1 can be invoked and a specialized version of (12) can be derived for the PCC-case and our chosen set-point controller of (26). The partial derivative of the Lyapunov function of the soft system controller evaluates to $\partial_{\dot{q}} H(q, \dot{q}) = \dot{q}^{\mathrm{T}} B(q)$, which allows to re-formulate (12) into:

$$\Pi = \dot{\Gamma} - K_1(\mu_p - \Gamma) + S^{\mathrm{T}}\dot{q},$$
$$\Psi = G_{\mathrm{P}}^{\mu_{\mathrm{p}}} + D_{\mathrm{p}}\Pi + M_{\mathrm{p}}\dot{\Pi} - K_2(\dot{\mu}_{\mathrm{p}} - \Pi) - (\mu_{\mathrm{p}} - \Gamma). \quad (32)$$

V. SIMULATIONS

A. System

We consider a planar soft robot arm consisting of three independently actuated CC segments, modeled upon the second half of the robot in [4]. Segments have equal length $l_0 = 11\,\mathrm{cm}$, uniform mass density $\rho = 0.99\,\mathrm{kg/m}$ concentrated on the central axis. The stiffness K and damping D matrices are diagonal with constants 0.01 N /rad and 0.01 Ns /rad. The segment has a diameter of 44.5 mm. Based on CAD analyses of a real system, we take $d_{\mathrm{C,a}} = 7.14\,\mathrm{mm}$, $d_{\mathrm{C,b}} = 20.19\,\mathrm{mm}$, and $b_{\mathrm{C}} = 8.07\,\mathrm{mm}$. A positive curvature and positive configuration q_i correspond to bending counter-clockwise. The straight configuration of the robot along the x-axis is perpendicular to gravity acting in negative y-direction as shown in Figure 3, so that gravity tends to induce clock-wise bending. Moving to the pistons, $A_{\mathrm{p}} = 7.9\,\mathrm{cm}^2$, $m_{\mathrm{p}} = 0.19\,\mathrm{kg}$, $l_{\mathrm{p}} = 0.5\,\mathrm{m}$ are chosen. We consider a damping matrix D_{p} with damping constants $d_{\mathrm{p}} = 10\,\mathrm{kN\,s\,m^{-1}}$ along the diagonal. The pistons are filled with air at $\mu_{\mathrm{p},0} = l_{\mathrm{p}}$ and $p_{\mathrm{atm}} = 1\,\mathrm{bar}$ and subsequently pre-loaded to $\mu_{\mathrm{p,t0}} = 0.25\,l_{\mathrm{p}}$. We set the backstepping gains to $K_1 = 6000\,\mathrm{s}^{-1}$ and $K_2 = 4.5\,\mathrm{kN\,m^{-1}}$.

B. End-to-End PID

We first introduce an end-to-end PID controller, which will serve as a baseline

$$\Delta f_{\mathrm{p}} = K_{\mathrm{p}}(\bar{q} - q) + K_{\mathrm{i}} \int_0^t (\bar{q} - q)\,\mathrm{d}t' - K_{\mathrm{d}}\,\dot{q}, \quad (33)$$

where $K_{\mathrm{p}}, K_{\mathrm{i}}, K_{\mathrm{d}} \geq 0$ are scalar gains. $\Delta f_{\mathrm{p}} \in \mathbb{R}^{n_q}$ is the scalar offset from the actuation force $f_{\mathrm{p,t0}}$ corresponding to the pre-loaded pressure p_{t0}. Analogue to (30), Δf_{p} can be equally distributed on both chambers within a segment. The PID gains have been selected so to achieve a similar transient behaviour as for the backstepping controller and are equal to $K_{\mathrm{p}} = 200\,\mathrm{N\,rad^{-1}}$, $K_{\mathrm{i}} = 7\,\mathrm{N\,rad^{-1}\,s^{-1}}$, and $K_{\mathrm{d}} = 200\,\mathrm{N\,s\,rad^{-1}}$.

C. Coupling-Aware PID

Next, we implement a control strategy that takes advantage of the understanding of the potential coupling and uses a PID for low-level control of the pistons

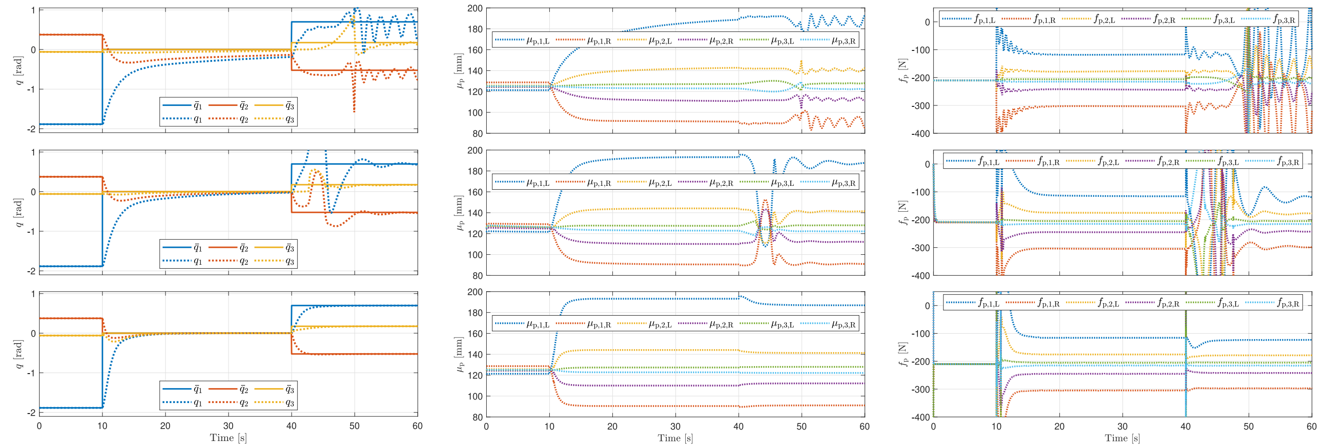
$$f_{\mathrm{p}} = K_{\mathrm{p}}\big(\Gamma(q, \bar{q}) - \mu_{\mathrm{p}}\big) + K_{\mathrm{i}} \int_0^t \big(\Gamma(q, \bar{q}) - \mu_{\mathrm{p}}\big)\mathrm{d}t' - K_{\mathrm{d}}\,\dot{\mu}_{\mathrm{p}}. \quad (34)$$

Here, $K_{\mathrm{p}}, K_{\mathrm{i}}, K_{\mathrm{d}} \geq 0$ are scalar gains, and $\Gamma(q, \bar{q})$ is the correction on (26) which takes the coupling defined in (31) in account. The PID gains are tuned similarly to the coupling-aware PID and are equal to $K_{\mathrm{p}} = 150\,000\,\mathrm{N\,m^{-1}}$, $K_{\mathrm{i}} = 15\,000\,\mathrm{N\,m^{-1}\,s^{-1}}$, and $K_{\mathrm{d}} = 100\,\mathrm{N\,s\,m^{-1}}$.

D. Results

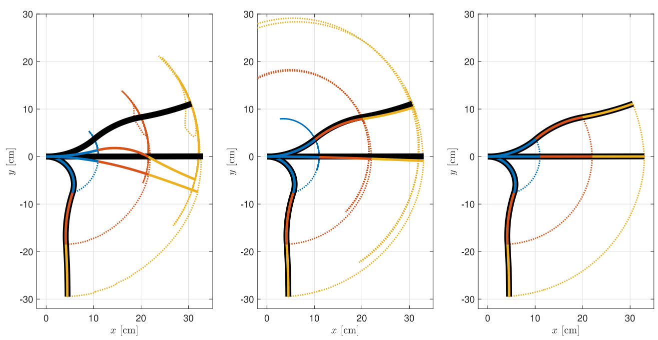
We simulate the response of the closed loop generated by all three controllers to a sequence of step references. The segments are initialised at the equilibrium configuration. At 10 s, the reference is moved to the straight configuration $\bar{q} = 0$. After another 30 s, we change it again to $\bar{q} = \begin{bmatrix} 0.6981\,\mathrm{rad} & -0.5236\,\mathrm{rad} & 0.1745\,\mathrm{rad} \end{bmatrix}^{\mathrm{T}}$. Figure 4 shows that the backstepping controller is approaching the set-point reference with no oscillations nor overshooting. These are instead visible for coupling-are PID controller after the second change in reference configuration. The end-to-end PID controller does not converge to the desired configuration within 60 s as it does not take into account gravity.

Next, we increase the inertia of the actuation system by setting the piston mass m_{p} to 0.5 kg. We leave both the backstepping and the PID gains unchanged. Figures 5-6(a) demonstrate that the backstepping-based approach is able to adapt to the new system, while the end-to-end PID shows large



Configuration q Piston position μ_{p} Actuation force f_{p}

Fig. 5. Simulation of posture regulation under PCC approximation for an actuation system with increased inertia ($m_{\mathrm{p}} = 0.5$ kg) comparing the performance of an end-to-end PID baseline controller (1st row), with a coupling-aware PID controller (2nd row) and the nonlinear backstepping controller (3rd row). The set-point reference configuration is shown with solid lines.



(a) End-to-end PID (b) Coupling-aware PID (c) Backstepping

Fig. 6. Cartesian evolution of the soft robot for an actuation system with increased inertia ($m_{\mathrm{p}} = 0.5$ kg). All gains remain unchanged and are tuned for the original system with $m_{\mathrm{p}} = 0.19$ kg. The dotted lines mark the evolution of the tip of the segments. The soft robot consists of three segment (blue, orange and yellow). The reference configuration at the three set-points is marked with a thick black line.

oscillations at 50 s and the coupling-aware PID displays significantly overshoot in curvatures and piston positions. Note that the latter are especially dangerous in real experiments, since they may signify that pistons reaches their limits.

VI. Conclusion

This letter proposed a model for soft robots actuated using pneumatic fluidic drive cylinders, and introduced a model-based controller to take actuators' dynamics into account. The stability of this backstepping-based control strategy was proven using a Lyapunov argument. As an example of application, model and control strategy have been specialized for the planar PCC-case. We also proposed a coupling-aware extension of the standard hierarchical PID strategy as a middle-ground solution. Future work will focus on applying this strategy to more sophisticated models and controllers, and on experimental validation in a lab environment.

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