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# INTEGRATED METHOD FOR VIRTUAL STRENGTH PREDICTION OF TIMBER

**Ani Khaloian[1], Jan-Willem van de Kuilen [2]**

**ABSTRACT:** Before timber boards can be used for engineering applications, they need to be strength graded. This step is currently done based on visual or machine grading methods. Each approach may face problems such as frequent measurement (human) errors or problems in dynamic measurements due to missing density values, respectively. To increase accuracy of the predictions, an advanced numerical method has been developed based on FE-analysis to predict tensile strength of the boards. By simulating the tensile test procedure virtually, stress developments around wood heterogeneities have been analysed, and identifying parameters (IPs) have been provided, representing the stress concentrations in 3D anisotropic space. Virtual dynamic-MoE has been derived after performing the stress-wave analysis, to be used as another IP for strength predictions. These parameters have been used in a non-linear multiple regression analysis with the tensile strength for the predictions. Similar approach has been performed, using the parameters of the visual and machine grading methods. The quality of strength prediction based on virtual method was in the same level/slightly higher than recently available methods, depending on the wood species. The model has been developed by considering a scatter for the quality range of 450 spruce, Douglas fir and beech boards. For the model verification, the approach has been used for strength prediction of a group of ash and maple boards, which provided satisfactory results.

**KEYWORDS:** FEM, Strength, Stress concentration factor, Dynamic MoE, Prediction

## 1 INTRODUCTION

Safety of timber structures depends on many factors from which one of the initial and most important steps is the assignment of correct strength grades. Strength grading of timber is generally performed based on visual or machine grading methods [1-7], based on which timber is allocated to different strength classes to which the mean and characteristic values of the mechanical and material properties are assigned [8]. In contrast to the visual grading method, which is based on the visual inspections, machine strength grading is based on using large variation of non-destructive technologies, allowing the measurement of knottiness parameters, density and eigenfrequency. For this kind of grading, statistical relationships between the parameters of non-destructive measurements and the mechanical properties are used. Currently, different machines are used in timber industry to grade and classify timber [2, 9-11].

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Knots are the critical points in especially softwood tensile boards, causing strength reduction and failure of these samples. These points are influencing the mechanical behaviour of the boards by causing localized fiber deviations. The features of these local points have been mathematically studied during the last decades [9, 12-17], based on which diverse models have been provided for predicting the mechanical response of wood under different loading conditions.

By using the finite element approach in this study, an advanced integrated numerical method is provided for tensile strength prediction of timber. The 3D geometrical reconstruction of boards solely based on the surface information of knots facilitates more accurate prediction of the fiber pattern. Developed model covers a complete quality range of wood including softwoods and hardwoods. Despite the biological variety within and in between different wood species, the generalization of the model to predict tensile strength without knowing the wood species is promising for grading of timber.

## 2 NUMERICAL PROCESS

ABAQUS and python are tools, used for simulations and for the numerical analysis in the current study. In total 450 samples were analysed in this study, and the simulations were performed for each single board. Boards were selected in a way to cover the natural scatter of wood. Spruce, Douglas fir, beech, ash and maple boards are the species that were investigated in this study. The boards were geometrically reconstructed solely based on the surface information of knots. The local fiber pattern was predicted and its deviation around these heterogeneities was estimated.

Average properties of the main samples of this study are presented in Table 1. It is shown in this table that a relatively strong variation exists in between the quality of different wood species. Variation of the material and mechanical properties is not only in between different wood species, but also among different samples of one species. It is shown in Table 1 that CoV (coefficient of variation) values higher that 0.4 are not unexpectable for the strength values. Therefore, development of a numerical approach to cover this scatter, automation of the prediction process and generalization of the approach to a species-independent model is not an easy step.

***Table 1:*** *Comparison of the mechanical properties of different wood species*

| **Species** | | **Spruce** | **Douglas fir** | **Beech** | |
|---|---|---|---|---|---|
| | | | | $Beech_1$ | $Beech_2$ |
| **Number** | | 103 | 151 | 100 | 100 |
| **Length** (mm) | avg. | 4101 | 4467 | 3102 | 3102 |
| | CoV | 0.008 | 0.02 | 0.003 | 0.002 |
| **Thickness** (mm) | avg. | 40 | 46 | 24 | 24 |
| | CoV | 0.014 | 0.01 | 0.01 | 0.01 |
| **Width** (mm) | avg. | 150 | 146 | 151 | 100 |
| | CoV | 0.002 | 0.002 | 0.003 | 0.002 |
| **Strength** (MPa) | avg. | 29.19 | 19.31 | 31.11 | 34.56 |
| | CoV | 0.34 | 0.47 | 0.43 | 0.44 |
| **Density** ($Kg/m^3$) | avg. | 462 | 490 | 758 | 773 |
| | CoV | 0.13 | 0.09 | 0.05 | 0.05 |
| $\mathbf{MoE_{static}}$ (MPa) | avg. | 12000 | 10200 | 11100 | 11300 |
| | CoV | 0.20 | 0.25 | 0.18 | 0.24 |
| **TKAR** (-) | avg. | 0.3 | 0.35 | - | - |
| | CoV | 0.32 | 0.31 | - | - |
| **DEB** (-) | avg. | 0.22 | 0.23 | 0.18 | 0.21 |
| | CoV | 0.36 | 0.26 | 0.55 | 0.55 |
| **DAB** (-) | avg. | 0.40 | 0.44 | 0.20 | 0.24 |
| | CoV | 0.38 | 0.29 | 0.61 | 0.59 |

Beside the material properties, the level of heterogeneity of different species of this study was very different. Some samples/boards (for example the Douglas fir boards analyzed in this study) had relatively complicated geometrical configurations due to many natural material imperfections, whereas some others had few. Figure 1 shows examples of softwood and hardwood boards with different levels of heterogeneities.


***Figure 1:*** *Comparison of the geometrical features of softwoods and hardwoods*

The developed numerical approach involves the following individual steps:

1. The registration of board data on the basis of visual surface information and the transmission of the data to a central computer system
2. Computational construction of a three-dimensional mechanical-geometrical model of each single board
3. The performance of virtual tensile tests on each individual board and the registration of prime strength and stiffness determining parameters
4. Application of experimentally verified mathematical equations for board classification
5. Data output for further processing

The mathematical model is based on the above-mentioned approach and includes aspects such as: uncertainty modelling in geometrical reconstruction, which was done by adding a random error in the model and influencing the coordinates of the weak spots in boards [13].

The 3D geometrical reconstruction was performed by incorporation of the above-mentioned steps: one and two. After reading the registered data of the knots, including their surface information and their geometrical configuration, the 3D geometrical configuration of each board was estimated. TKAR, DEB and DAB [1] were the identifying parameters (IPs) from the visual board assessment method that were used in the first part of this study for validation of the results of the tensile strength predictions.

In above-mentioned step three of the analysis two steps were performed numerically on the geometrically reconstructed boards to extract the strength and stiffness properties:

- Quasi-static virtual tensile tests [13]
- Explicit dynamic stress wave analysis [14]

The complex geometrical configuration of natural imperfections in wood as well as the strong fiber deviations in the vicinity of these features strongly influenced the uniformity of stress distribution in boards. By analyzing the maximum stresses around each features in a virtual tensile analysis as well as the average stresses in the bulk material, numerical methods were developed for calculation of the Stress Concentration Factors (SCFs) in a 3D anisotropic and heterogeneous space. These equations are provided in Equation 1 [13]. Figure 2 represents the influence of the nonlinear function of the interacting stresses, simplified in a 2D space for wood.

$$
\begin{aligned}
SCF_1 &= \max(\sigma_{sim} \cdot \frac{A_{knot}}{A_{total}}) \\
SCF_2 &= \sigma_{avg} .(\frac{A_{total}}{A_{total} - A_{knot}}) \\
SCF_3 &= \sigma_{avg} .(\frac{A_{total}}{\left| A_{total} - A_{projected} \right|})
\end{aligned}
\tag{1}
$$

In Equation 1, $\sigma_{sim}$ is the maximum-$\sigma_{11}$ stress around each knot. Parameter $\sigma_{avg}$ is the average stresses that are developing in the bulk material/clear wood. $A_{knot}$ is the biggest total cross sectional area of each knot on its central axis. $A_{total}$ is the cross section of the board.

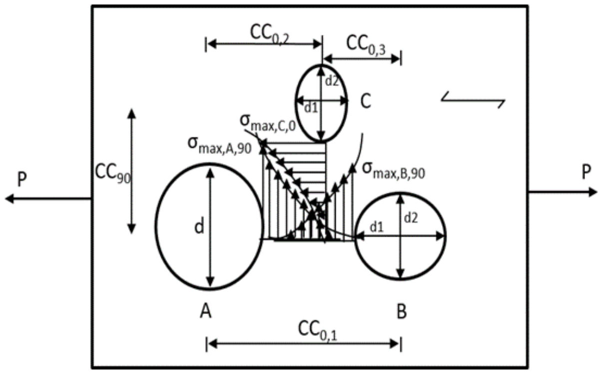


***Figure 2:*** *Non-linear function of 3D interacting stresses around knots with different geometries under tension parallel to the fiber direction*

Stress wave analysis has been performed for calculation of the virtual dynamic modulus of elasticity (MoE). For this reason, velocity of the stress wave has been calculated after one complete round of wave forth and back in the board. Therefore, by knowing the density ($\rho$) and the eigenfrequency ($f$) of each board, the dynamic MoE of each board has been derived (Equation 2) [14]. To reduce dependency of the simulations to the input parameters and to the density of single boards, average density of each set of species has been used for the calculation of MoE.

However, this parameter can be updated any time with the actual density for the calculation of dynamic MoE, if the actual density of each board is known.

$$E_{dyn} = 4.l^2.f^2.\rho \tag{2}$$

Therefore, by considering the interest of timber industry to take in to account the density of timber as a parameter showing the natural scatter of this material, the developed parameter was adapted to this requirement. In this case, the local influence of the developed SCFs were globalized for timber boards by taking the density of each single board into account. Reformulation of the previously defined $SCF_1$ parameter is presented in Equation 3. Similar approach is also valid for $SCF_2$ and $SCF_3$.

$$SCF_1 = \max(\sigma_{sim}.\frac{A_{knot}}{A_{total}})/\rho \tag{3}$$

After outputting the IPs of the numerical process, these parameters were used in linear and non-linear multiple regression analyses, in the fourth step of the above mentioned numerical process for tensile strength predictions. Based on this approach, a mathematical equation has been provided for tensile strength prediction of wood. The model has been validated with the experimental results and measurements.

In contrast to the homogeneous and isotropic materials, point-wise density and stiffness map of wood is strongly varying from one point of the material to another. Much stronger variation can be observed, comparing between

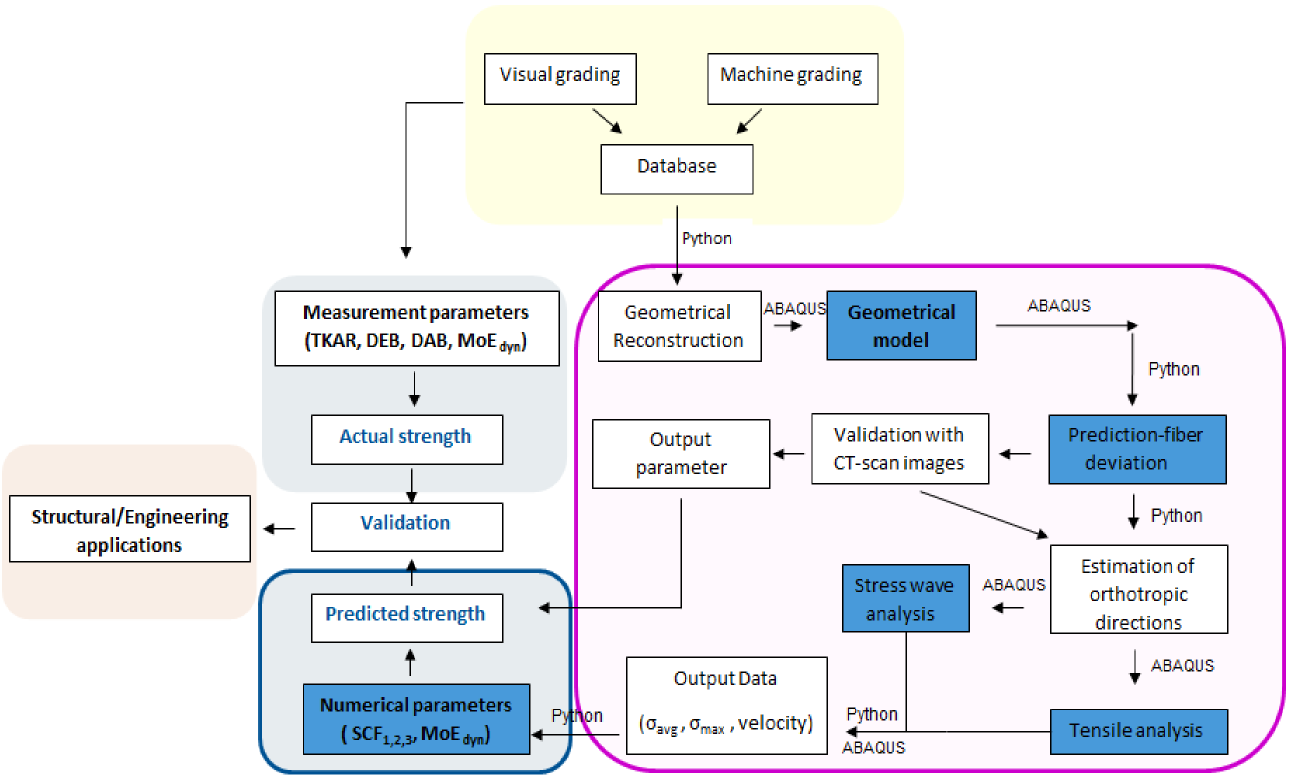


***Figure 3:*** *Flow of the developed method and different steps*

different samples and species. Adaption of the model to cover this natural scatter may require high scanning technologies and costs. Therefore, a species independent model was developed in this study, where different species were categorized in a general ‘wood’ group. In this case, the application of average properties may speed up the prediction process and globalize the dependency to species dependent parameters.

The general structure of this work is presented in Figure 3. The purple box represents the complete numerical process, based on which the mathematical model is developed. This process contains the reconstruction of boards, estimation of the material’s orthotropic directions, virtual tensile and stress wave analysis and extraction of the IPs for strength predictions. The yellow box on top shows the currently available grading methods and possible measurement parameters that are stored in a central computer system for grading and for tensile strength predictions. The blue boxes represent the IPs of both methods (currently available ones based on measurements: visual and machine grading methods, and the ones that have been extracted/calculated from the developed numerical approach) to be used in statistical regression analysis for prediction of the tensile strength. The orange box on the left side of this flowchart shows the applicability of the approach for the engineered wood products and structural applications after validation of the developed approach.

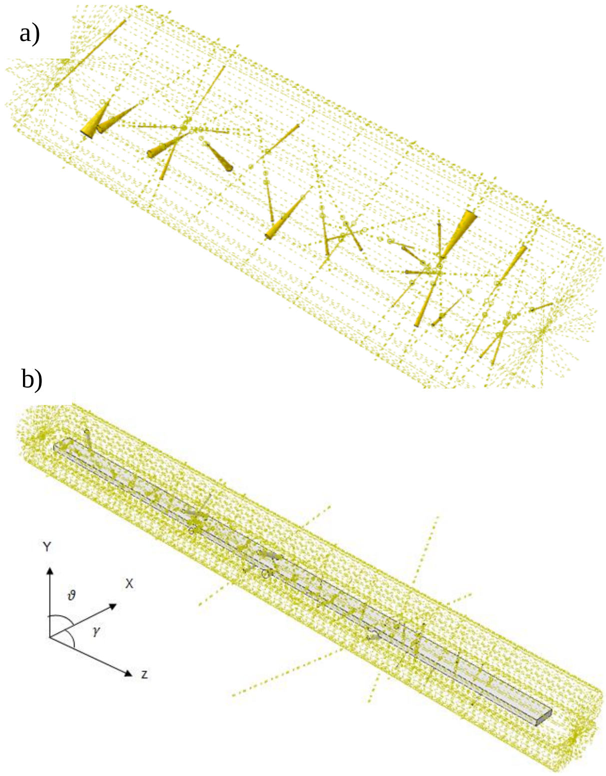

***Figure 4:*** *Reconstructed 3D geometrical model of example boards with complicated geometrical configuration. a) Knots in a spruce board, b) an example Douglas fir board*

## 3 RESULTS

### 3.1 GEOMETRICAL RECONSTRUCTION

By back-engineering of the registered surface information of the boards in a central computer system, a full 3D geometrical model of the boards have been reconstructed. The model contains the central axis of rotation of each knot that simplifies the recognition of these spots. Therefore, different material properties can be assigned to these geometrical features. Reconstructed geometrical model of two example boards are presented in Figure 4.

Since these features are the weak points, where failure mainly occurs especially under tension, in wood consideration of the localization of the stresses caused by the strong local fiber deviation and the geometrical configuration of these natural features is an important step for wood. Based on the estimation of fiber field and the rotation of fibers around these natural features using the concepts of computational fluid analysis [14, 18], point-wise orthotropic coordinate rotations have been implemented for wood. Results of the structural model have been validated with the CT-scan images.

Figure 5 represents a stepwise numerical procedure for an example spruce board from: board reconstruction, prediction of the fiber pattern, virtual tensile test to the development of stresses for prediction of the SCFs.

It is shown in Figure 5 that the location with higher stresses around natural geometrical features corresponds well with the actual location of the failure in board.

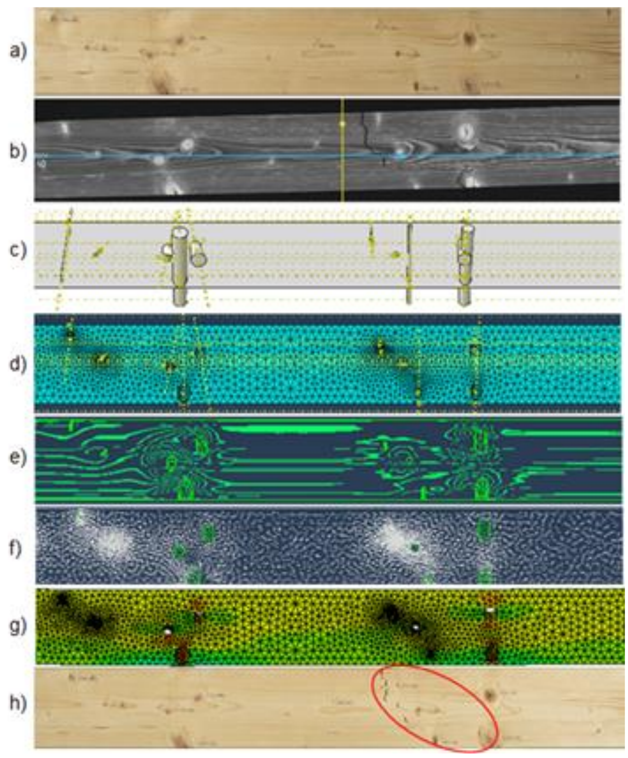

***Figure 5:*** *Modelling procedure. a) actual board with knots, b) CT-scan and fiber pattern, c) 3D geometrical reconstruction, d) discretized heterogeneous FE-model, e) numerical prediction of fiber pattern, f) local coordinates and definition of the orthotropic stiffness, g) virtual tensile test and stresses parallel to the fibers, h) actual failure location corresponding to the locations with higher stresses*

### 3.2 MULTIPLE REGRESSION ANALYSIS FOR TENSILE STRENGTH PREDICTION OF LAMELLAS

Parameters of the visual, machine and virtual grading methods were used in a multiple regression analysis to find the relation of each parameter with the tensile strength. SCF parameters together in one set of analysis provide much higher accuracy for strength predictions than the parameters of visual grading method (TKAR, DEB, DAB) ($R^2$>0.5 and $R^2$>0.2 respectively). Comparisons of the relationships between the numerical and recently available grading IPs to the tensile strength are presented in Table 2. Simulation parameters in this table include the SCF parameters and the simulated dynamic MoE, without taking of the actual density of boards in to account.

***Table 2:*** *$R^2$ values of the non-linear multiple regression analysis for three sets of parameters*

| | Simulation Parameters | Knot Parameters | Tested Parameters |
|---|---|---|---|
| **Spruce** | 0.71 | 0.35 | 0.60 |
| **Douglas fir** | 0.66 | 0.27 | 0.67 |
| **Beech** | 0.59 | 0.18 | 0.51 |

Knot parameters in Table 2 are the parameters of visual grading (TKAR, DEB and DAB). Tested parameters include visual knot parameters, density and the measured dynamic MoE.

It was found that each single SCF parameter has a power function with tensile strength. Similar relation/function was identified when comparing the multiple relation of virtual knot parameters (SCF parameters) without consideration of the dynamic MoE with the tensile strength. Such behaviour was previously presented in the Figure 2 of the current paper (and in Figure 8 for a general "wood" group).

By using the numerical parameters (SCFs, together with the virtual dynamic MoE) in a non-linear multiple regression analysis with tensile strength, an optimum combination of the parameters was found for strength prediction of each single species. Equation 4 shows the nonlinear equation with an exponential function of the SCFs for tensile strength prediction of wood. These relations are graphically shown in Figure 6.

$$f_1 = \sum_{i=1}^{n} a_i . e^{b_i . SCF_i} + c . MoE_{dyn} + d \qquad (4)$$

The coefficients of Equation 4 for each of these samples are provided in the studies of Khaloian and Van de Kuilen 2019c [18].

Predictions based on measured (experimental) parameters (including dynamic MoE, density and knots), provided coefficients of determination of $R^2$=0.60, $R^2$=0.67, $R^2$=0.51 for spruce, Douglas fir and beech respectively. Therefore, it is shown that considerable improvements are obtained in the quality of the strength predictions with the developed numerical method, compared to the current visual and machine grading methods.

Although modification of the SCFs with density improves quality of the strength predictions for single species (to be seen in Table 3), their influence in a nonlinear multiple regression analysis (Equation 4) is negligible for each species. However, it is shown in Figure 7 that usage of these parameters as extra single parameters in a multiple regression analysis together with non-modified SCFs and dynamic MoE slightly improves the quality of the tensile strength predictions.

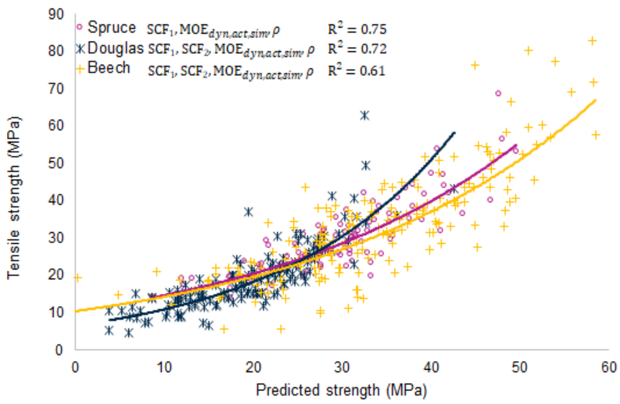


***Figure 6:*** *Strength predictions based on developed virtual method with the optimized number of numerical parameters*

The developed method is verified for small groups of ash and maple boards. $R^2$ value of 0.66 between the predicted and actual tensile strengths represents the quality of the predictions.

***Table 3:*** *Comparison of coefficient of determination when using SCFs or the modified SCFs*

| $R^2$ | $SCF_1$ | Modified $SCF_1$ with density |
|---|---|---|
| **Spruce** | 0.51 | 0.55 |
| **Douglas fir** | 0.47 | 0.50 |
| **Beech** | 0.42 | 0.43 |

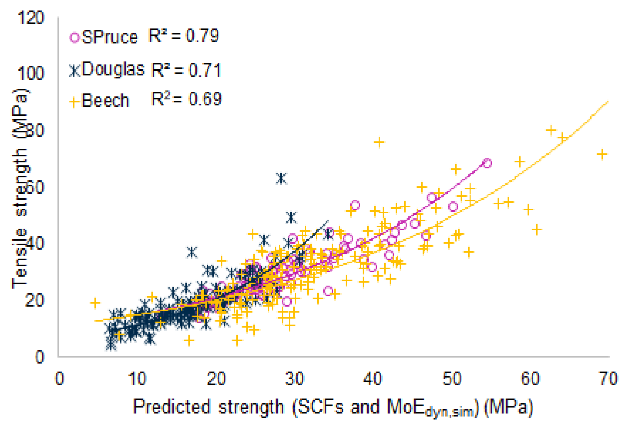


***Figure 7:*** *Tensile strength predictions when using SCFs and modified SCFs with density together with simulated dynamic MoE*

After developing the model for single species, all species were analyzed as a group of 'wood' to further develop the model for species-independent case. For this reason, the relation of the tensile strength with the initially calculated SCFs, as well as the SCFs that were modified with the density parameter were compared for the general wood group.

By comparing the SCFs and the modified SCFs for the general model it is shown that the relation to the tensile strength is lower in the case of $SCF_1$ compared to the modified $SCF_1$. This is due to the local (but not global) influence of the $SCF_1$ parameter. In the case of $SCF_1$, in contrast to the modified $SCF_1$, stresses are considered separately around each single knot and the maximum value is then selected.

Additionally, this parameter considers the biggest knot surface in a cross-cut instead of the ratio of the bulk material around the heterogeneities. Therefore, this parameter has a relatively high relation with tensile strength for each of the single species. Yet, higher scatter of this parameter in the generalized model results in a lower relation to the tensile strength. Modification of this parameter by generalizing the stress-influence based on the density of each board results in a higher coefficient of determination in the generalized model. These comparisons are shown in Figure 8.

The coefficient of determination, while using the virtual parameters including the SCFs and the dynamic MoE in a non-linear multiple regression analysis with tensile strength is $R^2$=0.60 for the generalized model. Addition of the modified SCF parameters to the same analysis results in a $R^2$=0.63 for the general group (Figure 9).

In the case of using the parameters of currently available visual and machine grading methods for strength predictions, coefficient of determination of $R^2$=0.59 was obtained. The $R^2$ value from numerical parameters shows that the quality of tensile strength predictions based on the virtual parameters for wood is approximately 10% better than the quality of predictions

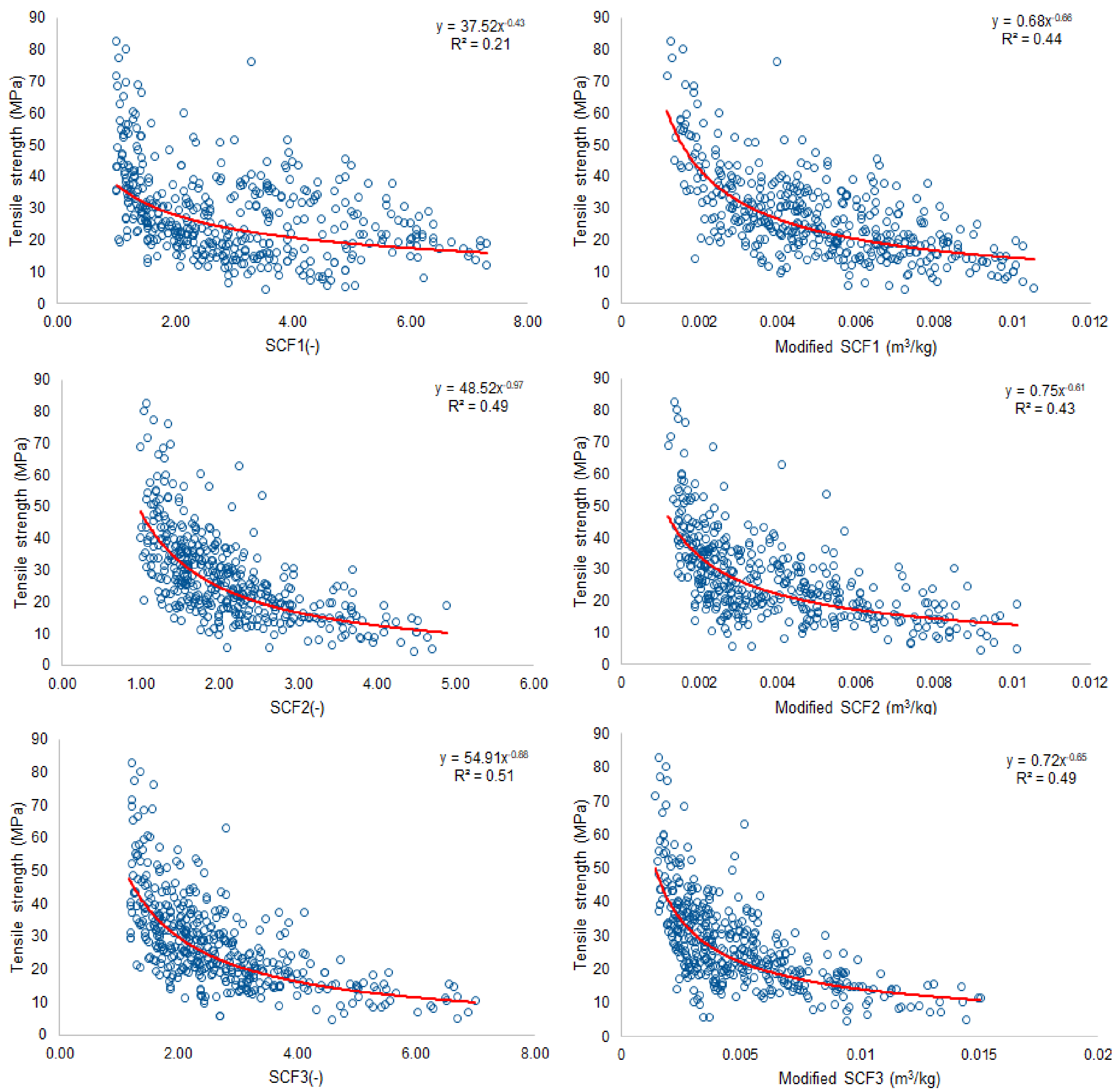


***Figure 8:*** *SCFs and modified SCFs in the generalized model*

based on the parameters of currently available grading methods.

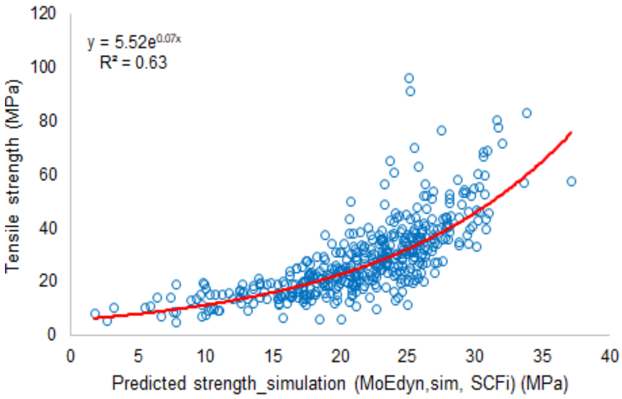


***Figure 9:*** *Tensile strength predictions based on numerical parameters*

## 4 DISCUSSION

In this study a method has been suggested that generalizes the strength predictions for the natural scatter of wood. Previously, studies of Olsson et al. [11] provided detailed information about the local fiber orientation and its distribution over the board surfaces based on laser scanning of surfaces of the boards. Combining this parameter with the axial dynamic MoE, an IP has been provided in their study, which gave a high coefficient of determination with the bending strength ($R^2$=0.72, $R^2$=0.62, $R^2$=0.59) for spruce, Douglas fir and oak samples with different dimensions, respectively.

A regression analysis using the static MoE with the bending strength provided a $R^2$ value of 0.71 for spruce boards [19]. By performing the dynamic measurements and consideration of the heterogeneities, Olsson et al. [2] came up with a coefficient of determination of 0.75 for bending strength predictions.

Lukacevic et al. [10] provided a method by considering the quadratically weighted knot information on boards surfaces which gave a high coefficient of determination to bending strength ($R^2$=0.76). Studies on tensile strength are relatively limited. In the same study [10] a $R^2$ of 0.92 was provided for spruce boards, considering the knot area ratio and fiber deviation area ratio for 29 spruce samples. However, it is expected that the correlation is reduced by increasing the number of samples, or by considering a bigger scatter of heterogeneities for wood. Khaloian and Van de Kuilen [14, 19] provided a virtual method, considering the numerical stress concentration factors ($SCF_{1,2,3}$) and the virtual dynamic MoE in a non-linear multiple regression analysis with tensile strength, which resulted in a $R^2$ value of 0.71, 0.66 and 0.59 for spruce, Douglas fir and beech respectively. It is shown in the current study that addition of the modified SCF parameters with density to the non-linear multiple regression analysis improves the quality of the predictions to $R^2$=0.79, 0.71, 0.69 for spruce, Douglas fir and beech respectively. This approach improves the quality of strength predictions by an average of 12%.

As already shown, the previous studies concentrated on strength predictions based on specific wooden species. Here, the method has been generalized for tensile strength prediction of total scatter of wood. Considering only the relation of numerical/virtual parameters with the tensile strength for the scatter of wood, a $R^2$ value of 0.63 has been obtained. The prediction quality is in the same level/slightly higher than the prediction quality based on the measured parameters ($R^2$=0.58). This value shows the high capability of the developed model for tensile strength prediction of wood.

## 5 CONCLUSION

Despite new machinery development in the timber industry during last decades, it is still difficult to assign a clear method for classification of timber by covering the natural scatter of this material. Due to strong anisotropy and heterogeneity of wood, some IPs may be strong strength predictors for many species, leading to failure in estimation for the others. By focusing on the previously developed virtual methods, and further developing the numerical method to generalize the IPs to cover the natural scatter of the material, a species-independent method has been provided for tensile strength prediction and classification of timber. It is shown that the quality of predictions with a $R^2$=0.63 based on the virtual method, is slightly improved compared to the predictions based on the currently available visual plus machine grading methods ($R^2$=0.58) in the generalized method that neglects the categorization of the wood species. Therefore, based on the capability level of the industry for machinery (for visual and machine grading methods) or computer power (for virtual/numerical method), different parameters can be extracted and be used for tensile strength predictions. In this context, a mathematical equation is provided for strength prediction of wood, independent of wood species and their heterogeneity levels. By far, the developed method is shown to be the strongest generalized method that is provided for strength prediction of wood.

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