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Stochastic Model Predictive Control: uncertainty impact on wind farm power tracking

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Abstract— **Active power control for wind farms is needed to provide ancillary services. One of these services is to track a power reference signal with a wind farm by dynamically de- and uprating the turbines. Due to the stochastic nature of the wind, it is necessary to take this stochastic behavior into account when evaluating control signals. In this paper we present a closed-loop stochastic wind farm controller that evaluates thrust coefficients providing power tracking under uncertain wind speed measurements. The controller is evaluated in a high-fidelity wind farm model simulating a 9-turbine wind farm to demonstrate the stochastic controller under different uncertainty levels on the wind speed measurement and different controller settings. Results illustrate that a stochastic controller provides better tracking performance with respect to its deterministic variant.**

I. INTRODUCTION

A large part of the clean energy currently generated is harvested by wind farms that extract energy from the wind [1]. A wind farm is a collection of wind turbines placed in each other's proximity to, among others, reduce maintenance and cabling costs. However, a wake develops downstream of each turbine, which is a region that is characterized by a flow velocity deficit and an increased turbulence intensity [2]. Since wind turbines are placed together in a farm, the wakes of upstream turbines influence the performance of downstream turbines. For example, the flow velocity deficit influences the power production of downstream turbines [3] while an increased turbulence intensity increments the turbine's fatigue loads as suggested in [4], [5], which possibly reduces the turbine's lifetime.

The objective of wind farm control is to reduce the levelized cost of wind energy by intelligently operating the turbines inside the farm. Sub-goals may include the increase of the farm-wide power generation, the reduction of turbine fatigue, and the integration of energy from wind farms with the electricity grid. This integration is related to the provision of ancillary services. One example is secondary frequency regulation (a subclass of active power control) in which the objective is to have the wind farm's power generation track a power reference signal generated by transmission system operators, during a time span of several minutes [6]. We call this power tracking and turbines need to dynamically de- and uprate their power output during this time span such that tracking at a farm level is ensured. Since the power reference signal is below the maximum possible power that can be harvested, the tracking problem has a set of solutions. For example, for generating an equal amount of power with the farm, one could uprate the downstream turbines while derating the upstream turbines or the other way around. It is therefore possible and necessary to add, besides tracking, another performance measure, such as the decrease of control signal variations over time and/or the increase of available power in the farm. Two actuation methods to ensure these objectives are axial induction and wake redirection control. In the former, generator torques and pitch angles or thrust coefficients are utilized as control variables while in the latter, the yaw angles are utilized as control variables [7].

Results that provide power tracking using axial induction actuation can be found in [8], [9], [10]. More precisely, in [8] and [9], the authors each propose a different wind farm power tracking solution while minimizing the axial force exerted by the flow on the turbines. However, as stated in [5], the dynamical turbine loading is a better measure of fatigue than static turbine loading. In [10], the authors propose a distributed controller providing tracking while minimizing variation in the axial force (dynamical loading) that is exerted by the turbine on the flow. The work presented in [11], [12] demonstrates an optimization algorithm that provides power tracking while minimizing the added turbulence intensity or maximizing the available power, respectively. However, all the above proposed controllers are evaluated in a simplified wind farm model [13]. The questions remains if similar results can be obtained when a more realistic dynamical wind farm model, such as a Large-Eddy Simulation (LES) based wind farm model, is utilized for the evaluation of the controllers. The authors in [14] propose a distributed tracking controller that contains a simplified wind farm model to evaluate control signals, whereas the controller is also evaluated in a relatively simple farm model.

A controller that is tested in an LES based wind farm model and employs axial induction actuation providing power tracking can be found in [15]. The optimization problem solved in [15] contains dynamical wake and turbine models, but the only objective is tracking and no constraint regarding, *e.g.*, dynamical loading is included. The controllers presented in [16], [17] are also tested in an LES, but neither a wake model nor constraints were taken into account. The controller provides tracking and the wind farm power reference signal is distributed heuristically among the turbines without taking a measure of dynamical loading into account. This controller has also been evaluated in a wind tunnel [18].

After this literature survey we conclude that results, which are obtained with a closed-loop controller providing power tracking while taking uncertain measurements into account,

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are not yet available in current literature. Since wind farm measurements such as wind speed are in general uncertain quantities, it is important to take this into account in the controller while providing power tracking.

In this paper we propose a wind farm power tracking stochastic model predictive controller (SMPC) [19]. The main objective of the controller is to provide power tracking while taking uncertain wind speed measurements into account. While [20] presents a deterministic MPC, this work is considered as a stochastic extension and we will study the impact of uncertain wind measurements on the tracking performance. This work illustrates that measurements that deviate from the "true" value will deteriorate the controller's performance. However, when considering the measurement as a stochastic variable in the controller, its performance can be increased significantly. The controller is evaluated in a high-fidelity wind farm model simulating a 9-turbine wind farm. For this case, the effect of the wind speed measurement's uncertainty level and settings of the stochastic controller are investigated. This work illustrates that a stochastic controller can provide power tracking in a high-fidelity wind farm model under uncertain wind speed measurements.

This paper is organized as follows. In Section II, the high-fidelity simulation environment is briefly introduced. In Section III, the stochastic controller is introduced. More precisely, Section III-A describes the parameter-varying controller model and Section III-B the control formulation. This is followed by the presentation and discussion of simulation results in Section IV and conclusions in Section V.

II. SIMULATION MODEL

The high-fidelity "PArallelized Large-eddy simulation Model (PALM)" [21] is used to evaluate the proposed controller. In this work, PALM includes the actuator disk model (ADM) [22] to determine the turbine's forcing terms acting on the flow and power generation. A consequence of choosing the ADM is that the control signals for turbine i are the disk-based thrust coefficient $C'_{T_i}(t)$ following [23], [24] and yaw angle $\gamma_i(t)$. Both of these signals can be used to manipulate the turbine thrust force and power generation (see (1)). In this work, the measurements at time t are 1) the power generated by a turbine $P_i(t)$ and 2) a mean rotor-averaged wind velocity $\bar{v}_i(t)$ for $i = 1, 2, \ldots, \aleph$ with $\aleph$ the number of turbines. In this work we consider the rotor-averaged wind velocity as an uncertain parameter and it is therefore defined as the random variable $v_i(t) \sim \mathcal{N}(\bar{v}_i(t), \sigma_v)$ with σ_v the standard deviation of the (assumed to be) Gaussian randomly distributed signal $v_i(t)$. Note that the standard deviation can be seen as a measure of turbulence intensity, *i.e.*, a higher σ_v models a higher turbulence intensity.

III. STOCHASTIC MODEL PREDICTIVE CONTROLLER

In this section, a Stochastic Model Predictive Controller (SMPC) is formulated. Prior to that, the controller model employed in the SMPC is defined.

A. Controller model

Axial induction based wind farm power tracking results that are presented in, *i.e.*, [20] indicate that flow dynamics could be neglected and a wind farm can be modeled as $\aleph$ uncoupled subsystems when power tracking is the objective. Each subsystem consists of a dynamical turbine model based on the actuator disk theory. While wake effects are neglected in the surrogate dynamical model, the turbine dynamics are still affected by the local flow conditions. Hence, the turbine models are updated according to the local rotor-averaged wind velocity, which in reality may be affected by upstream turbines inside the farm. In this work, the following model for turbine i is employed

$$\begin{aligned} P_i(t) &= \frac{\pi D^2}{8}\Big(v_i(t)\cos[\gamma_i(t)]\Big)^3 \hat{C}'_{T_i}(t), \\ C'_{T_i}(t) &= \tau \frac{\mathrm{d}\hat{C}'_{T_i}(t)}{\mathrm{d}t} + \hat{C}'_{T_i}(t), \end{aligned} \tag{1}$$

for $i = 1, 2, \ldots, \aleph$, with D the rotor diameter, $P_i(t)$ the power generated by turbine i, $C'_{T_i}(t)$ its control signal, $\hat{C}'_{T_i}(t)$ the first-order filtered control signal that is applied in the high-fidelity wind farm simulator, $\gamma_i(t)$ the yaw angle and $v_i(t)$ the Gaussian distributed rotor-averaged wind speed perpendicular to the rotor. We furthermore have $\tau \in \mathbb{R}^+$, the time constant of the filter that acts on the control signal. In previous work [20], the yaw angle was considered not to be equal to zero. In this work however, we assume $\gamma_i(t) = 0$ and we therefore neglect its dependency in the sequel of this paper.

Temporally discretizing (1) at sample period Δt using the zero-order hold method yields the following state-space representation of turbine i

$$\boldsymbol{x}_{i,k+1} = A_i \boldsymbol{x}_{i,k} + B_i(v_{i,k}) C'_{T_i,k}, \quad \boldsymbol{y}_{i,k} = \boldsymbol{x}_{i,k}, \tag{2}$$

with

$$\begin{aligned} A_i &= e^{-\Delta t/\tau} I_2 \in \mathbb{R}^{2\times 2}, \\ B_i(v_{i,k}) &= \int_0^{\Delta t} \frac{1}{\tau} e^{-s/\tau} \mathrm{d}s \begin{pmatrix} \frac{\pi D^2}{8} v_{i,k}^3 \\ 1 \end{pmatrix} \in \mathbb{R}^2, \\ \boldsymbol{x}_{i,k}^T &= \begin{pmatrix} P_{i,k} & \hat{C}'_{T_i,k} \end{pmatrix} \in \mathbb{R}^2, \qquad C'_{T_i,k} \in \mathbb{R}. \end{aligned} \tag{3}$$

Lifting the state variables of the turbines and adding the wind farm power error signal to the state variable results in the following wind farm state-space model:

$$\begin{aligned} \boldsymbol{x}_{k+1} &= A\boldsymbol{x}_k + B(\boldsymbol{v}_k)\boldsymbol{C'_{T,k}}, \\ e_k &= P_k^{\text{ref}} - \sum_{i=1}^{\aleph} P_{i,k}, \end{aligned} \tag{4}$$

with wind farm power reference signal $P_k^{\text{ref}} \in \mathbb{R}$ and tracking error signal $e_k \in \mathbb{R}$. Note that the latter signals is stochastic due to the occurrence of $\boldsymbol{v}_k$ in the system matrix $B(\boldsymbol{v}_k)$ as defined in (4). Furthermore we have:

$$
\begin{aligned}
\boldsymbol{v}_k^T &= \begin{pmatrix} v_{1,k} & v_{2,k} & \dots & v_{\aleph,k} \end{pmatrix} \in \mathbb{R}^{\aleph}, \\
\boldsymbol{x}_k^T &= \begin{pmatrix} \boldsymbol{x}_{1,k} & \boldsymbol{x}_{2,k} & \dots & \boldsymbol{x}_{\aleph,k} \end{pmatrix} \in \mathbb{R}^{2\aleph}, \\
\boldsymbol{C'_{T,k}} &= \begin{pmatrix} C'_{T_1,k} & C'_{T_2,k} & \dots & C'_{T_\aleph,k} \end{pmatrix}^T \in \mathbb{R}^{\aleph}, \\
\boldsymbol{\hat{C}'_{T,k}} &= \begin{pmatrix} \hat{C}'_{T_1,k} & \hat{C}'_{T_2,k} & \dots & \hat{C}'_{T_\aleph,k} \end{pmatrix}^T \in \mathbb{R}^{\aleph}, \\
A &= \texttt{blkdiag}\Big(A_1, A_2, \dots, A_\aleph\Big) \in \mathbb{R}^{2\aleph \times 2\aleph}, \\
B(\boldsymbol{v}_k) &= \dots \\
&\quad \texttt{blkdiag}\Big(B_1(v_{1,k}), \dots, B_\aleph(v_{\aleph,k})\Big) \in \mathbb{R}^{2\aleph \times \aleph},
\end{aligned}
$$

where $\texttt{blkdiag}(\cdot)$ denotes block diagonal concatenation of matrices or vectors. The model described above will be employed in the controller that is presented in the following subsection.

B. Stochastic control formulation

Stochastic Model Predictive Control (SMPC) [19] is an advanced control technique that can determine control inputs by solving a finite-horizon stochastic constrained optimization problem at each sampling time. Then, following the receding horizon concept, the first step of the optimal control sequence is applied after which new measurements are taken and the optimization procedure repeats itself. We employ SMPC using the presented dynamical model in (4) for a specified finite-horizon (from k_0 to $k_0 + N_h$) such that the following cost function is minimized:

$$
V(e_k, \boldsymbol{C'_{T,k}}) = \sum_{k=k_0}^{k_0+N_h} e_k^T Q e_k + \boldsymbol{C'_{T,k}}^T R \, \boldsymbol{C'_{T,k}}, \qquad (5)
$$

with N_h the prediction horizon and can be regarded as, together with Q, R, controller tuning variables. Increasing Q relative to R results in better tracking, but decreases the importance of finding low energy control signals, and visa versa. Note that the proposed cost function is uncertain as it is a function of the wind farm power tracking error signal e_k. The stochastic constrained optimization problem for wind farm power tracking is now defined as follows:

$$
\begin{aligned}
\min_{\boldsymbol{C'_{T,k}}} \quad & \mathbb{E}\left[V(e_k, \boldsymbol{C'_{T,k}})\right] \\
\text{s.t.} \quad & \boldsymbol{x}_{k+1} = A\boldsymbol{x}_k + B(\boldsymbol{v}_{k_0},)\boldsymbol{C'_{T,k}}, \\
& e_k = P_k^{\text{ref}} - \sum_{i=1}^{\aleph} P_{i,k}, \;\; \boldsymbol{v}_{k_0} \sim \mathcal{N}(\bar{\boldsymbol{v}}_{k_0}, \sigma_v) \qquad (6) \\
& C'_{T,\text{min}} \leq C'_{T_i,k} \leq C'_{T,\text{max}}, \\
& |C'_{T_i,k} - C'_{T_i,k-1}| < \mathrm{d}C'_T, \\
& k = k_0, k_0+1, \cdots, k_0+N_h \;,
\end{aligned}
$$

where $\mathrm{d}C'_T$ is an upper-bound on the maximum variation of the control actions between two time sequences. In (6), the thrust coefficient is considered as decision variable while the yaw angles take on constant values. Moreover, the mean value $\bar{\boldsymbol{v}}_{k_0}$ of the uncertain variable is considered as the "true" wind speed.

To solve the problem formulation given in (6), one should calculate a multidimensional integral to obtain the value of the cost function (5), which is in general computationally expensive. In the interest of having a computational efficient controller, we follow a sample-based approach, where N_s samples (realizations) of the random variable $\boldsymbol{v}_{k_0}$ are generated from the assumed distribution to approximate the value of the cost function (5). The number of samples N_s should be considered as a tuning variable. More precisely, if N_s increases, then the gap between the true cost function and the approximated version decreases.

An approximated counterpart of the optimization problem in (6) is defined as follows:

$$
\begin{aligned}
\min_{\boldsymbol{C'_{T,k}}} \quad & \frac{1}{N_s} \sum_{j=1}^{N_s} V(e_k^j, \boldsymbol{C'_{T,k}}) \\
\text{s.t.} \quad & \boldsymbol{x}_{k+1}^j = A\boldsymbol{x}_k^j + B(\boldsymbol{v}_{k_0}^j)\boldsymbol{C'_{T,k}}, \\
& e_k^j = P_k^{\text{ref}} - \sum_{i=1}^{\aleph} P_{i,k}^j, \;\; \boldsymbol{v}_{k_0}^j \sim \mathcal{N}(\bar{\boldsymbol{v}}_{k_0}, \sigma_v) \qquad (7) \\
& C'_{T,\text{min}} \leq C'_{T_i,k} \leq C'_{T,\text{max}}, \\
& |C'_{T_i,k} - C'_{T_i,k-1}| < \mathrm{d}C'_T, \\
& k = k_0, k_0+1, \cdots, k_0+N_h \\
& j = 1, 2, \cdots, N_s \;,
\end{aligned}
$$

where $\boldsymbol{v}_{k_0}^j$ is one realization of the random variable $\boldsymbol{v}_{k_0}$ and $V(e_k^j, \boldsymbol{C'_{T,k}})$ the cost evaluated for the realization $\boldsymbol{v}_{k_0}^j$ for $j = 1, \dots, N_s$. The proposed problem in (7) is then solved at each sampling time and performance of the SMPC is addressed in the following section for different values of N_s and different values of σ_v in one case study.

IV. SIMULATION RESULTS

A. Simulation initialization

Simulations are initialized as follows: a fully developed flow field is generated in the precursor such that the free-stream wind speeds are U_∞=8 [m/s] and V_∞=W_∞=0 [m/s] in the longitudinal, lateral and vertical direction, respectively, and a turbulence intensity of approximately 6% at hub-height in front of the farm. The flow is then propagated N seconds in advance with 9 turbines in the flow field, and with $C'_{T_i,k} = 2$ (corresponding to the Betz-optimal value) and $\gamma_{i,k} = 0$ for $i = 1, \dots, \aleph$. Hence for each case study, the wakes are fully developed at the start of each simulation.

The greedy power (P^{greedy}) is defined as the time-averaged wind farm power harvested with $C'_{T_i,k} = 2$ and $\gamma_{i,k} = 0$ for $i = 1, \dots, \aleph$ and N seconds of simulation starting with the previous described initial flow field. With unyawed turbines, a wind farm can potentially harvest above the P^{greedy} threshold for only a relatively short period of time. This period is defined by the wake propagation time. Table I presents a summary of both case studies.

The sample period Δt is chosen such that in both case studies, the Courant condition [25] holds. The simulation domain is indicated by the parameters L_x, L_y, L_z and represent the length in the x- y- and z-direction, respectively. The

TABLE I

SUMMARY OF THE 9-TURBINE SIMULATION CASE STUDY.

$L_x \times L_y \times L_z$	$15.3 \times 3.8 \times 1.3$ [km^3]
D, z_h	120, 90 [m]
$\Delta x \times \Delta y \times \Delta z$	$15 \times 15 \times 10$ [m^3]
$U_\infty, V_\infty, W_\infty$	9, 0, 0 [m/s]
Δt	1 [s]
TI_∞,	6%
N, τ, N_h	850, 5, 10 [s]
$C'_{T,\max}, C'_{T,\min}, \mathrm{d}C'_T$,	2, 0.1, 0.2

grid dimensions in the x- y- and z-direction are represented by $\Delta x, \Delta y, \Delta z$, respectively and the turbulence intensity is denoted as TI_∞. The controller parameters τ, N_h are chosen following [20] and provide power tracking, and $\mathrm{d}C'_T$ prevents the actuator from changing too fast. In this work, no sensitivity analysis is done on the controller parameters. Instead, sensitivity analysis on the SMPC parameter N_s and the level of uncertainty in the wind σ_v are presented for two case studies.

B. Performance measure

To assess the controller's tracking performance, the following indicator is introduced

$$e_{\mathrm{rms}} = \sqrt{\frac{1}{N}\sum_{k=1}^{N} |e_k|^2}, \tag{8}$$

with tracking error signal as defined before, *i.e.*, $e_k = P_k^{\mathrm{ref}} - \sum_{i=1}^{\aleph} P_{i,k}$. A smaller e_{rms} indicates better tracking performance.

C. Nine-turbine case study

The turbine spacing is $5D \times 3D$ [m], *i.e.*, the turbines are separated $5D$ in the longitudinal direction from each other and $3D$ in the lateral direction (see Fig. 1 for the initial flow field). The controller parameters are $Q = 1$ and $R = I_\aleph$ (with $I_\aleph$ the identity matrix with dimension $\aleph$) and chosen such that tracking performance is maximized and the control signal amplitudes are minimized.

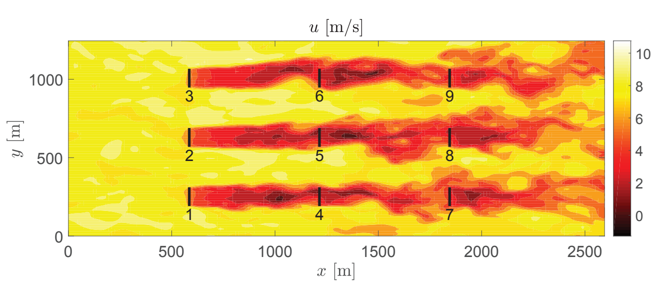

Fig. 1. Initial longitudinal flow velocity component at hub-height. The flow is going from west to east and the black vertical lines represent the wind turbines.

The wind farm power reference signal is defined as:

$$P_k^{\mathrm{ref}} = 0.7P^{\mathrm{greedy}} + 0.3P^{\mathrm{greedy}}\delta P_k, \tag{9}$$

with δP_k a normalized "RegD" type AGC signal [26] coming from an operator and $P^{\mathrm{greedy}} \approx 11.3$ [MW]. Tracking results for different values of N_s and $\sigma_v = 0.3$ are depicted in Fig. 2.

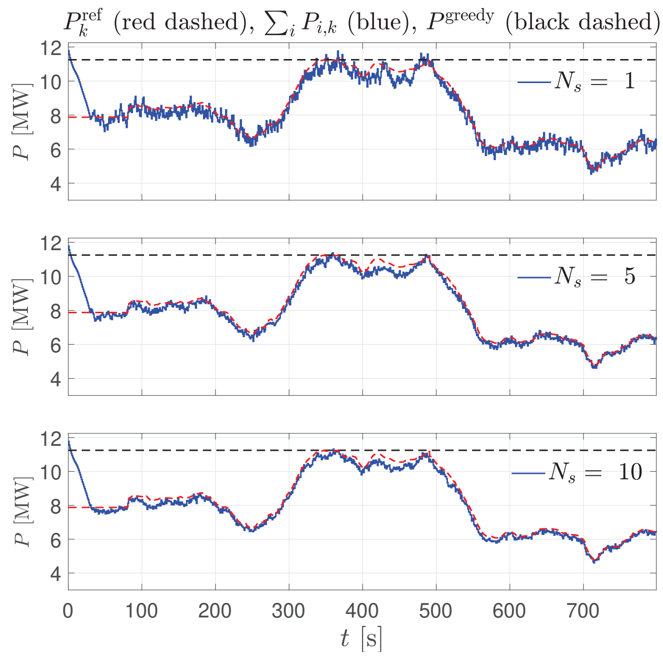

Fig. 2. Tracking results for different N_s (number of samples) values and $\sigma_v = 0.3$.

It can be observed that tracking is ensured. However, the wind farm power signal in the $N_s = 1$ case exhibits severe oscillations. In fact, this case can be considered as the deterministic case since one wind speed measurement $v_{k_0}^1 \sim \mathcal{N}(\bar{v}(k_0), \sigma_v)$ is used to construct the controller model (see (4)). Therefore, in each time-step, the controlled wind farm power is in general not following the reference, but is fluctuating around it due to the noise definition (its mean value is $\bar{\boldsymbol{v}}_{k_0}$). This fluctuating behavior is undesired due to the consequent fluctuations in the control signals. This case illustrates the negative effect of using uncertain measurements for the evaluation of control signals. The cases $N_s > 1$ are SMPC cases and an increase in N_s results in a decrease of these oscillations, and also better tracking performance. This can be observed in Table II where e_{rms} values for different N_s are given.

TABLE II

THE e_{RMS} VALUES FOR DIFFERENT N_s AND $\sigma_v = 0.3$.

N_s	e_{rms} [MW]	N_s	e_{rms} [MW]
1	0.5656	15	0.5087
5	0.5181	20	0.5082
10	0.5103		

When the "true" wind speed $\bar{\boldsymbol{v}}_{k_0}$ is used in the controller model, $e_{\mathrm{rms}} = 0.4463$ [MW]. However, as indicated before, assuming perfect wind speed knowledge is from a practical point of view not realistic hence the introduction of a stochastic controller.

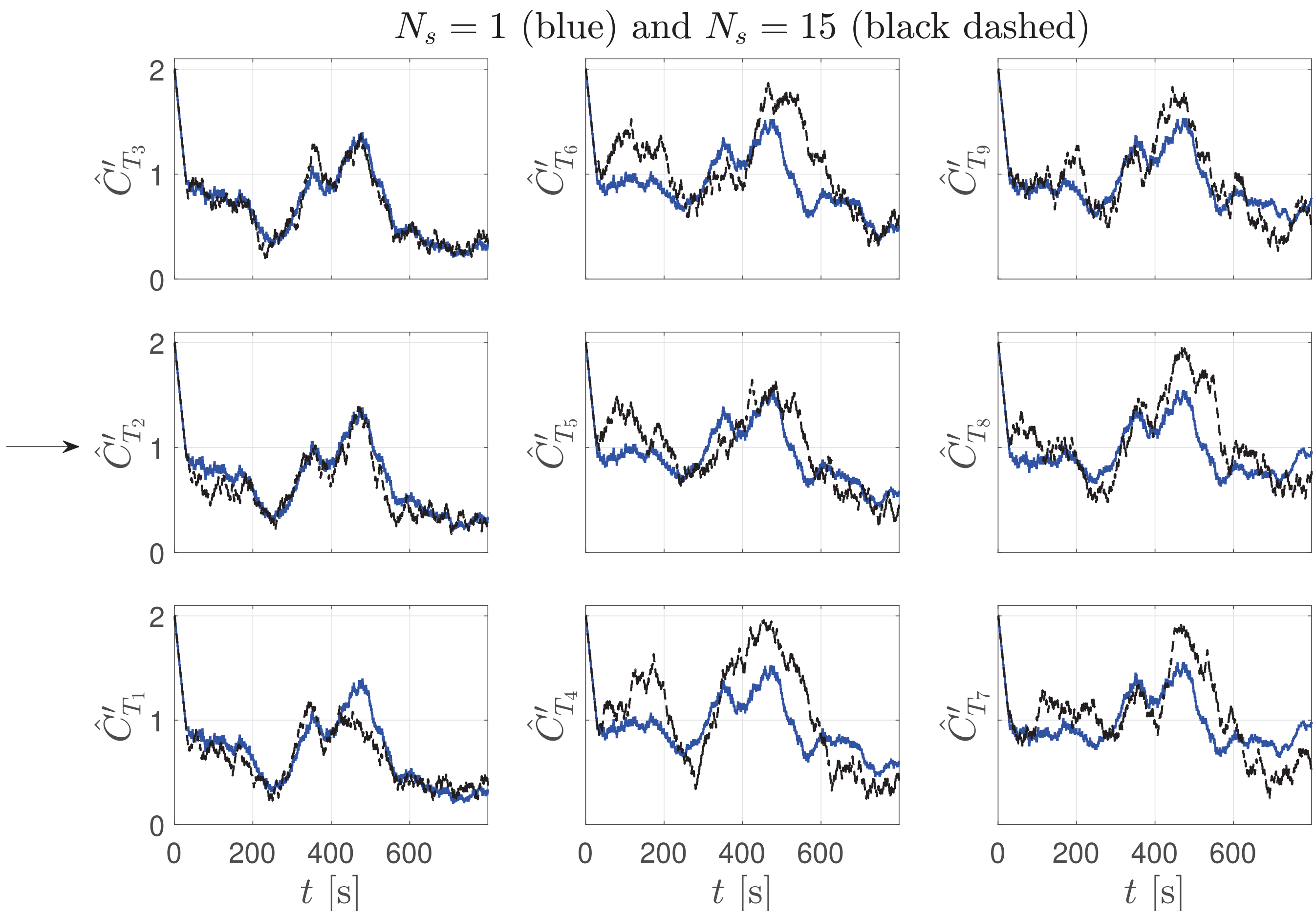

Fig. 3. Control signals for two different N_s values. The wind is coming from the west as indicated by the black arrow.

Figure 3 depicts the control signals for two N_s values. It can be observed that different control signal distributions are found that provide power tracking. Additional simulations are performed for different σ_v values. Table III presents e_{rms} for these different cases with $N_s = 10$. Table III presents expected results since e_{rms} increases as the uncertainty level of the wind speed increases. Remaining open questions are what the exact value of σ_v and the minimal required tracking performance are. These questions are not touched upon in this work. Nevertheless, a stochastic controller is presented that can provide power tracking under different levels of uncertainty in the wind speed measurement, and it can improve tracking performance by tuning the controller settings.

TABLE III
THE e_{RMS} VALUE FOR DIFFERENT σ_v AND $N_s = 10$.

σ_v	e_{rms} [MW]	σ_v	e_{rms} [MW]
0.1	0.4736	0.4	0.5434
0.2	0.4872	0.5	0.5908
0.3	0.5103	0.6	1.2517

V. CONCLUSIONS

Power tracking is an important ancillary service that needs to be delivered by a wind farm. This control task is not trivial due to, among others, the stochastic behavior of the wind in a farm. In order to ensure power tracking in such a stochastic environment, this paper proposed a sample-based stochastic model predictive controller. The effectiveness of the controller has been demonstrated in one case study, a 9-turbine wind farm, and the controller has been evaluated in a high-fidelity simulation environment. This paper presents a sensitivity analysis on the number of samples that are taken into account in the controller, and the level of uncertainty in the wind speed measurement. It has been shown that an increase in the number of samples results in an improvement of the tracking performance. It has furthermore been shown that an increase of the uncertainty level results in a reduced tracking performance. Hence, it can be concluded that an increase of the uncertainty level demands for an increase of the number of samples in the controller to ensure a non-decreasing tracking performance.

Future work can entail the development of a wind speed estimator that can predict the wind speed N_h steps ahead. This prediction can consequently be used in the controller's prediction horizon, and should improve tracking performance. This prediction can come from, among others, a full wake model that can be included in the controller (rather then the simplified controller model proposed in this work). Although tracking is ensured, including a full wake model in the developed stochastic controller is still interesting to research the potential improvements that then can be achieved. Furthermore, the inclusion of a controller model that takes on pitch and generator torque will bring the control strategy closer to practical implementation. Such a model can easily be implemented in the proposed control strategy. Nevertheless, the purpose of this paper is to propose for the first time a stochastic wind farm power tracking controller.

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