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Topology Optimization considering design-dependent Stokes flow loads

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1. Abstract
This article presents an evolutionary topology optimization method for mean compliance minimization of structures under design-dependent viscous fluid flow loads. The structural domain is governed by the elasticity equation and the fluid by the incompressible Stokes flow equations. When the modelling of a system consists in the interaction of multiple domains, the classic density-based topology optimization methods become arduous within the framework of dealing with the moving multi-physics loads and interfaces, due to the considerable volume of intermediate density elements. Herein it is suggested an alternative methodology to handle this type of loading problems. With an extended Bi-directional Evolutionary Structural Optimization (BESO) method, design-dependent Stokes flow loads are modelled straightforward during the optimization procedure. The discrete nature of the method allows both fluid and structural domains to be modelled separately in each step of the optimization. In order to validate the methodology, only small structural displacements and a simple staggered fluid-structure interaction algorithm are considered in this paper. Primary results are shown for a 2D flexible structure immersed in an incompressible viscous flow channel.
2. Keywords: Topology Optimization; BESO Method; Design-dependent loads; Fluid-structure interaction; Stokes flow.

3. Introduction
In order to improve the structural design in the field of engineering, Structural Topology Optimization [1,2] has been developed. The idea is to find optimal structural topologies inside predefined design domains concerning objective functions and constraints.

Through the last years, topology optimization has been under some strong scientific effort to be extended for different classes of engineering systems [3-5]. Some of them include fluid flow or even multiphysical effects, such as fluid-structure interaction problems [6,7,8].

Only a few authors have studied the topology optimization of FSI coupled systems. The classic element density-based topological optimization methods become arduous when dealing with FSI problems within the framework of separated domains with explicit boundaries. That is because this kind of analysis methods requires predefined explicit interfacing boundary descriptions for the coupling boundary conditions. Thus, it is necessary to devise new computational techniques to overcome this limitation [8].

In this context, the presented work proposes the extension of the Bi-directional Evolutionary Structural Optimization (BESO) [9] method for FSI systems design. The discrete nature of the evolutionary methods imply that no intermediate density elements are allowed during the optimization procedures. Thus, fluid-structural boundaries are always explicit and the coupling boundary conditions evaluation is straightforward. To the best of the authors' knowledge, fluid-structure interaction problems still have not been treated with the evolutionary topology optimization methods..

4. Governing equations and finite element model

4.1. Fluid domain
In this work we shall consider fluids with the following properties:

- The medium is incompressible.
- The medium has a Newtonian character.
- The medium properties are temperature independent and uniform.
- The flow is laminar and at steady-state.
- Inertia forces are not considered.

- There are no body forces.

For the above restrictions, the governing partial differential equations for the motion of the fluid can be expressed as the incompressible Stokes flow equations

$$\begin{cases} -P_{,i}^{f} + \mu v_{i,jj} = 0 \\ v_{j,j} = 0 \end{cases} \tag{1}$$

where μ is the dynamic viscosity and P and v are the pressure and velocities on the fluid domain, respectively. The boundary conditions applied in this work are

- No-slip condition: $v_j = 0$ at fluid flow walls.
- Velocity profile given at inflow: $v_j = v_j^0$
- Pressure value given: $P^f = P_0$

4.2. Solid domain

Herein we shall consider linear elasticity for the solid domain under fluid flow loads. Neglecting body forces and any acceleration, the linear structural analysis is governed by

$$\sigma_{ij,j}^{s}(u) = -\left(-P_{,i}^{f} + \mu v_{i,jj}\right)^{fsi} \tag{2}$$

where σ_s is the Cauchy stress tensor, u is the displacement field and the superscript fsi denotes the vector with the loads from the fluid flow. Equation (2) is also given as the fluid-structure interface boundary condition. For the solid domain, the following Dirichlet boundary condition is applied:

$$u_i = 0 \tag{3}$$

4.3. Finite element model

A mixed finite element is chosen to model Stokes flow equations, in which velocities and pressures from the fluid domain are interpolated in the same finite element. With the correct shape functions these elements are stable and satisfy compatibility conditions [10]. Although they are too costly for large-scale problems, they showed to be effective for the cases explored in this work. The finite element used herein is known as Q2P1, in which velocities are interpolated with quadratic shape functions and pressures with bilinear shape functions in isoparametric axes.

The finite element matrices for solving (1) are

$$\begin{bmatrix} \mathbf{K}_f & -\mathbf{Q} \\ -\mathbf{Q} & \mathbf{0} \end{bmatrix} \begin{Bmatrix} \mathbf{v}_f \\ \mathbf{P}_f \end{Bmatrix} = \begin{Bmatrix} \mathbf{0} \\ \mathbf{0} \end{Bmatrix} \tag{4}$$

where $\mathbf{P}_f$ and $\mathbf{v}_f$ are the pressure and velocities vector, respectively. The fluid stiffness matrix $\mathbf{K}_f$ and the incompressibility matrix $\mathbf{Q}$ are evaluated as follows:

$$\mathbf{K}_f = \mu \int_{\Omega_e} \mathbf{B}^T \mathbf{I}_0 \mathbf{B} d\Omega_e \tag{5}$$

$$\mathbf{Q} = \int_{\Omega_e} \nabla \mathbf{N}_v^T \mathbf{N}_P d\Omega_e \tag{6}$$

where the matrices $\mathbf{N}$ contain the shape functions for velocities and pressures with the correspondent v and P subscripts, respectively. The matrix $\mathbf{B}$ contains the partial derivatives of the shape functions and, for 2D cases,

$$\mathbf{I}_0 = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{7}$$

Considering only Stokes flow loads, the finite element model for the structure is expressed as

$$\mathbf{K}_s \mathbf{u}_s = -\left(\mathbf{K}_f \mathbf{v}_f - \mathbf{Q}\mathbf{P}_f\right)^{fsi} \tag{8}$$

where $\mathbf{K}_s$ is the finite element matrix for the structure and $\mathbf{u}_s$ is the displacements vector. The fluid loads $\left(\mathbf{K}_f \mathbf{v}_f - \mathbf{Q}\mathbf{P}_f\right)^{fsi}$ are evaluated at the fluid-structure interfaces.

In order to model the fluid-structure interaction, some assumptions were made:

- Incompressible fluid and structure are at steady state.
- The control volumes of a fluid domain before and after structural deformation shall be distinguishable.
- Small structural displacements are considered. However, fluid-induced forces in the linear elasticity equation are dependent on the structural displacements.

5. Optimization problem and sensitivity analysis

The examples considered in this work concern compliance minimization with volume constraints of structures under fluid flow loading. The fluid model considered is the Stokes flow. The objective is to find the distribution of a given amount of solid material to obtain a structure with maximum stiffness (or minimum compliance C). The evolutionary topology optimization problem for this case can be stated as:

$$\begin{aligned}
\min_{x_i}: \quad & C(x_i) = \tfrac{1}{2}\mathbf{u}_s^T \mathbf{K}_s \mathbf{u}_s \\
\text{subject to:} \quad & h = V(x_i)/V_0 = V_s \\
& \mathbf{K}_s \mathbf{u}_s = -\left(\mathbf{K}_f \mathbf{v}_f - \mathbf{Q}\mathbf{P}_f\right)^{fsi} \\
& x_i = [0,1]
\end{aligned} \tag{9}$$

where C is the structural compliance, V_0 is the full design domain volume, V_s is the prescribed final solid volume, nel is the number of elements inside the design domain and x_i represents the discrete design variables, in which 1 is a solid element and 0 is void or fluid.

The sensitivity of the structural compliance due to an element removal can be obtained by its derivative:

$$\frac{\partial C}{\partial x_i} = -\mathbf{u}_s^T \frac{\partial \mathbf{K}_f}{\partial x_i}\mathbf{v}_\mathbf{f} + \mathbf{u}_s^T \frac{\partial \mathbf{Q}}{\partial x_i}\mathbf{P}_\mathbf{f} - \frac{1}{2}\mathbf{u}_s^T \frac{\partial \mathbf{K}_s}{\partial x_i}\mathbf{u}_s \tag{10}$$

We assume that the first term from the sensitivity expressed in (10) is zero at the element level, since there are no-slip boundary conditions at the walls ($\mathbf{v}_\mathbf{f} = \mathbf{0}$). For the second term, the derivatives of the incompressibility matrix is expressed as

$$\frac{\partial \mathbf{Q}}{\partial x_i} = \mathbf{Q}^i \tag{11}$$

where $\mathbf{Q}^i$ is the fluid elemental incompressibility matrix. It represents an addition of a fluid matrix in the problem, once the solid element is removed. The variation of the stiffness is defined by the derivatives of the material as

$$\frac{\partial \mathbf{K}_s}{\partial x_i} = \mathbf{K}_s^i \tag{12}$$

when $x_i = 1$ and null when $x_i = 0$ (fluid or void elements). $\mathbf{K}_s^i$ is the ith element stiffness matrix.

Thus, the sensitivity numbers for stiffness maximization of structures under Stokes fluid flow loads are

$$\alpha_i = -\frac{\partial C}{\partial x_i} = \begin{cases} \frac{1}{2}\mathbf{u}_i^T \mathbf{K}_s^i \mathbf{u}_i - \mathbf{u}_i^T \mathbf{Q}^i \mathbf{P}_i & \text{if} \quad x_i = 1 \\ 0 & \text{if} \quad x_i = 0 \end{cases} \tag{13}$$

where the subscript i indicates the values of $\mathbf{u}_s$ and $\mathbf{P}_f$ at the element level.

6. The extended BESO method for fluid-structure interaction problems

The following algorithm lists the steps of extended the BESO method for steady state and small displacements fluid-structure interaction problems.

1. Define design domain, loads and boundary conditions.

2. Define BESO parameters.
3. Discretize the design domain using a FE mesh for the given fluid and structure domains.
4. Apply the fluid boundary conditions and solve fluid flow FE Equations (4).
5. Identify the fluid flow loads considering the fluid-structure boundary conditions, apply the solid boundary condition and solve structural FE Equation (8).
6. Calculate the sensitivity numbers according to Equation (13).
7. Apply a filter scheme. Project the nodal sensitivity numbers on the finite element mesh and smooth the sensitivity numbers for all (fluid, void and solid) elements in the design domain.
8. Average the sensitivity numbers with their previous iteration ($n-1$) numbers and save the resulting sensitivity numbers for the next iteration.
9. Determine the target structural volume V_{n+1} for the next iteration.
10. Construct a new fluid-structure design by switching design variables x_i from 1 to 0 and from 0 to 1, tracking the advance of the fluid-void regions. Details of the material update scheme can be found in [7,9].
11. Assemble the global matrices according to the change of the current design.
12. Repeat steps 2-12 until the following stop criterion is satisfied:

$$error = \frac{|\sum_{k=1}^{5} C_{n-k+1} - \sum_{k=1}^{5} C_{n-5-k+1}|}{\sum_{i=1}^{5} C_{n-k+1}} \leq \tau \tag{14}$$

7. Numerical results

The studied example considers a fluid channel with a flexible structure obstructing the flow. The physical model is shown in Figure 1. The fluid flows through the channel with an inlet velocity v_j = 0.0001 m/s. The pressure boundary condition is imposed at the outlet as P^f = 0 and no-slip conditions are imposed at the fluid flow walls. The fluid density is chosen to be ρ_f = 1000 kg/m^3 and its viscosity as μ = 0.001 kg m^{-1} s^{-1}. The structural design domain is represented by a flexible structure of 60×60 μm, including a rectangular area of 50×10 μm considered as a solid box in order to avoid trivial solutions or a void structure. The elasticity modulus of the structure is chosen to be as $E = 3 \times 10^9$ N/m^2 and the Poisson's ratio $\nu = 0.3$. This example is similar to the one presented by [8] for compliance minimization considering design-dependent fluid flow loads.

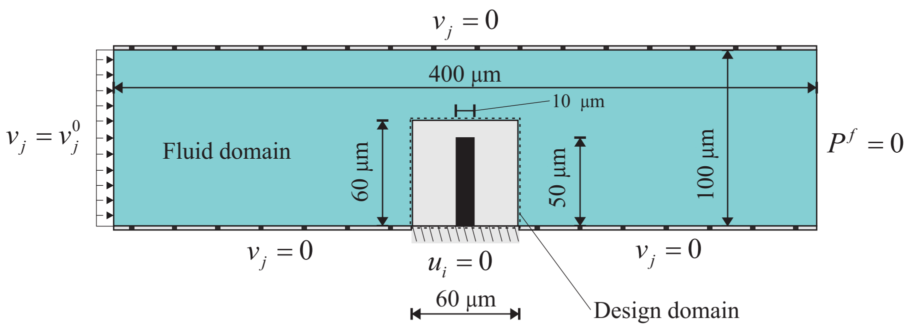

Figure 1: Structural design problem for a fluid flow channel.

The model was discretized with 25600 finite elements in total, being 2304 solid elements and the other 23296 ones modeling the fluid flow. The BESO method started from the initial full design domain with an evolutionary ratio $ER = 2\%$, i.e., removing 2% of the initial structural volume each iteration until the prescribed volume fraction, taken as $V_s = 30\%$ from the design domain. The other BESO parameters are chosen to be the maximum admission ratio $AR_{max} = 1\%$, filter radius $r_{min} = 7.5 \times 10^{-6}$ m and the convergence error tolerance $\tau = 0.001$. Figure 2 shows the structural topology solution, as well as streamlines, velocity and pressure fields of the fluid domain after the structural optimization process. Figure 3 presents the evolutionary history of the structural mean compliance (objective function) along the optimization. The final solution presents a structural topology with mean compliance $C = 2.0800 \times 10^{-20}$.

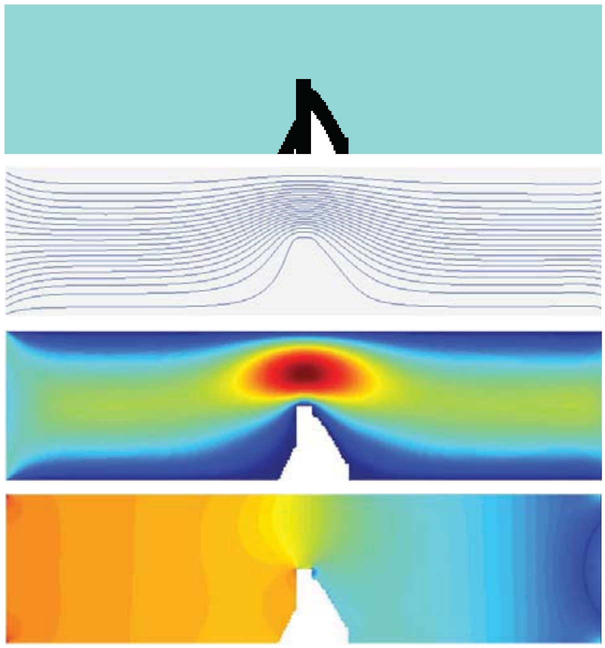

Figure 2: Final topology solution for the structure after the optimization and fluid streamlines, velocity and pressure fields.

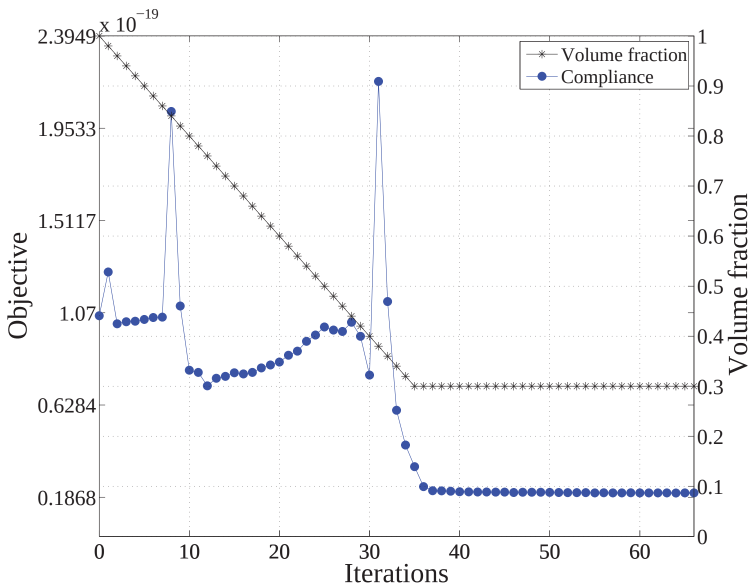

Figure 3: Evolutionary history of the structural mean compliance during the optimization.

The evolutionary procedures showed to be effective in dealing with the moving fluid-structure interfaces. Solid elements were replaced by fluid ones and the fluid flow advanced into the structural design domain. This represents a great potential for design problems considering design-dependent FSI loads and topology optimization. The new term $\mathbf{u}_i^T \mathbf{Q}^i \mathbf{P}_i$ on the sensitivity showed similar behavior as the pressure loading sensitivities from design-dependent pressure loading problems presented in [7]. A greater portion from the regions with low strain energy were first removed due to the subtraction of this new term on the sensitivity.

10. Conclusions

This paper described a topology optimization problem of structures under viscous fluid flow loading. A new sensitivity analysis is presented. The features of the evolutionary methods allow the switch between fluid and solid elements, which address the main challenge of dealing with moving fluid-structural boundaries during the optimization procedures. This presents some potential use for the area of fluid-structure interaction systems design and it might be of some impact in the research of structural topology optimization. The results considered a flexible structure in contact with stokes fluid flow. The structural topology was designed considering compliance minimization and design-dependent FSI loads. The ongoing research expects to explore new results and bring further discussions on fluid-structure interaction problems considering topology optimization.

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