# Traffic Control and Route Choice: Occurrence of Instabilities

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# Traffic Control and Route Choice:
# Occurrence of Instabilities

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## Abstract

Traffic control and traveller's behaviour are two processes which influence each other. The two processes have different objectives that the actors in these processes try to achieve: the road manager will try to achieve a system optimum and the road users will search for their own optimum. Decisions taken by the road manager in traffic control have an influence on the possibilities for travellers to choose their preferred mode, route and time of departure, and vice versa. Road managers and road user have different objectives. The control problem is to optimise traffic control in such a way that the system is at an optimum, taking into account the reaction of travellers.

Some simple examples were taken to study the result of the optimisation of the different actors on the dynamics of the system and the existence of stable situations where each actor has no possibility to optimise his decisions anymore. This has been done with analysis and microscopic simulation.

Under certain conditions multiple stable situations are possible, but some of these situations are sensitive to small disturbances, by which the system moves away from the original situation. There appears a non-linear relationship between system parameters and the character and location of the equilibrium situations.

# 1 Introduction

There is a tradition in traffic control to adapt control structure and parameters to the actual traffic flows and conditions, such that delays, queues or some other collective objective function is optimised. For fixed-time and traffic actuated controllers, methods and guidelines have been developed. For example: Webster's formula for cycle time and green splits, the rule to minimise delays by choosing green splits which minimise the maximum degree of saturation, the rules for maximum green times and maximum gap times for vehicle actuated controllers, etc.

Apart from the optimisation of the total network, often a special treatment is given to certain groups of road users. In urban areas pedestrians may get a preferred treatment, at bus routes the bus may get a priority treatment and on crossings of cycle tracks cyclist may get more frequently a green phase. Such priority control is the consequence of a policy to assign the use of traffic space especially to certain preferred groups of road users because they play an important role in the local situation. In the case of priority for public transport, reason might be that this minimises the total waiting time of all road users and improves the operating speed for busses and trams. Another reason is to reduce the operating costs of a transport mode, which is heavily subsidised by public funding.

There is also a certain expectation from the authorities that a preferential treatment of certain traffic classes - and in most cases one has public transport and pedestrians in mind - will reduce the growth of car traffic and will influence the modal choice in favour of collective public transport, cycling or walking. In each case it is possible to influence route choice and time of departure by traffic control.

Because traffic control has an influence on travel behaviour, a change in traffic control may have the impact that traffic volumes change. If traffic control is modified such that congestion on a certain route disappears and delays on intersections decreases, traffic might be attracted from other links where congestion still exists or which are part of a longer route. This might have the consequence that queues, which originally disappeared, return. Delays may come back on the original levels. The question is whether there still is a net profit for the traffic system as a whole.

If we assume that a modification in traffic control gives a change in travel behaviour, it is necessary to anticipate this change. If we want to optimise delays, it should be done for the traffic volumes that will be present *after* the introduction of the optimised traffic control and not for the traffic volumes which existed *before* the implementation. Of course, it is possible to follow an interactive approach, where after each shift in traffic volumes the control scheme is adjusted until equilibrium has been reached, or one may use self-adjusting traffic control. However, it can be shown, for certain examples, that the process of the adjustment of traffic control, followed by a shift in traffic volumes, does not necessarily lead to a system optimum. It is even possible that the system oscillates between two or more states.

The objective of the paper is to show that even small problems with simple assumptions about day-to-day route choice can have multiple solutions. This can even lead to oscillations or even chaotic behaviour of the system. The complicated problem of day-to-day route choice in combination with optimisation of control is not discussed here, but is a topic for further research.

In this paper we start with the description of a simple route choice model which displays the occurrence of oscillations. In section 3 we will study a simple road network with traffic control on one of the links and we investigate the occurrence of stable, consistent equilibrium situations. First we give the results of a simulation study. This shows the more or less conventional pattern that one stable condition exists where flows are consistent with the travel times and the traffic control is optimised with respect to total delay. A further analysis of a slightly simplified situation shows that there is a possibility of the occurrence of oscillation in route choice for a simple road network. In section 4 we study the combined route choice and traffic control optimisation problem in some more detail. It is shown that in the two-level optimisation problem optimum solutions exist which can be interpreted as meta-stable saddle-points. This means that there is equilibrium between traffic control and route choice, but small disturbances in the conditions will cause the system to slide to another state. Finally, we draw some conclusions and we make some remarks on the existing problems and the directions for further research.

# 2 Route choice based on day-to-day experience

It is assumed that the route chosen by travellers depends on their perception of travel times on different alternative routes. Normally, travellers have imperfect knowledge about the actual travel times and have to rely on experiences in the past. It has been shown that route choice based on historical knowledge can lead to oscillating behaviour, where on one day one route is preferred and the next day most of the traffic follows another route (Nakayama et al. 1999). It is also known (Horowitz, 1984 and Watling, 1999) that a traffic assignment based on stochastic route choice does not necessarily lead to equilibrium.
A simplified model, which describes the dynamics of route choice with two alternatives parallel routes, is the following. We assume that on day *n* the volume on route 1 is proportional to the volume on the previous day on the same route (conservative force) and also proportional to the volume on the other route (the more volume on the other route, the higher the travel times and the more travellers will switch). This model is

$$V_{n,1} = \alpha V_{n-1,1}\ V_{n-1,2} = \alpha V_{n-1,1} (V - V_{n-1,1}) \qquad (1)$$

where $V = V_1 + V_2$ and $\alpha$ is a constant which can be determined from observations, e.g. from the equilibrium state when $V_n = V_{n-1}$. Equation (1) is an example of the logistic equation, which is known to give equilibrium states for certain values of $\alpha$, oscillating behaviour for other (higher) values of $\alpha$ and chaotic behaviour if $\alpha$ comes above a certain value.
If in equation (1) the transition is made from volumes to fractions of traffic choosing a route, *x*, the equation becomes

$$x_n = \alpha' x_{n-1} (1 - x_{n-1}) \qquad (2)$$

where $\alpha' = \alpha V$.

In Figures 1 and 2 the dynamics of route choice is illustrated for different values of $\alpha'$. Transition between different dynamic patterns can be described as splitting of the number of states between which the route choice oscillates. The transition to chaotic behaviour is characterised by the fact that there is an unlimited number of states between which the route choice is moving.
The difference between random and chaotic behaviour is, that chaotic behaviour has certain regularities, such as the quasi-periodicity and (strange) attractors. Furthermore, chaos behaviour often has the property that small changes in behaviour result in large changes in the future state. The similarity is that both random and chaotic behaviour cannot be predicted. Important properties of systems with chaotic properties are:

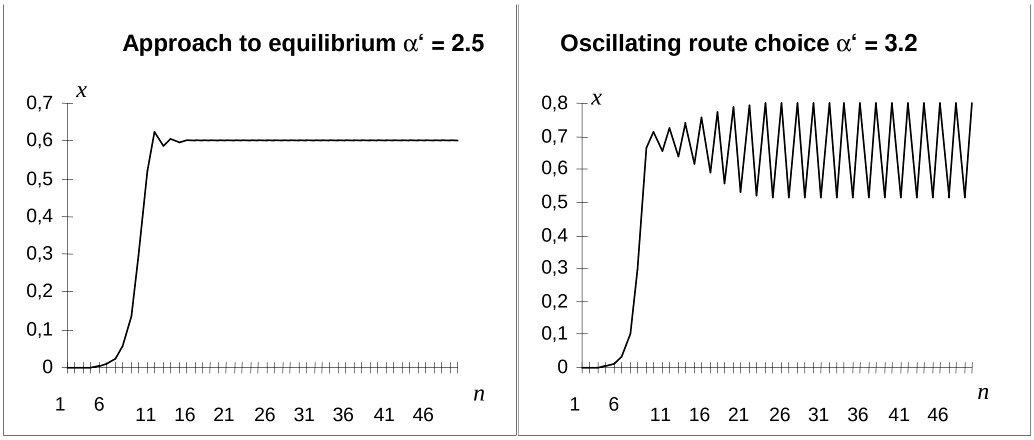


**Figure 1:** Route choice between two alternative routes, equilibrium and alternating behaviour

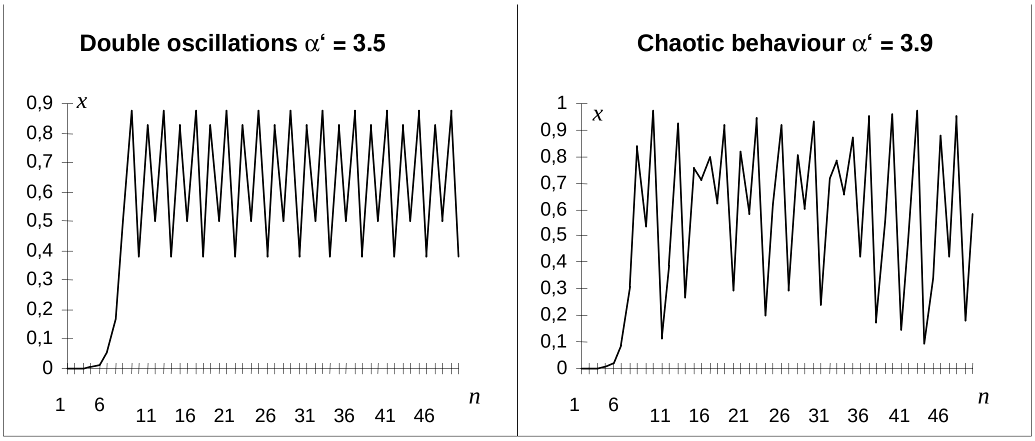


**Figure 2:** Route choice: oscillations with four states and the transition to chaotic behaviour

- they have non-linear dynamics
- positive feedback exists, by which certain changes are enhanced
- negative feedback exists, which drives the condition of the system from a dynamics of unlimited growth in one direction.

These three properties can clearly be seen in the example in this section. Equation 2 is non-linear, for small values of $x$ there is positive feedback proportional to $x$ and for values of $x \approx 1$, the negative feedback pushes $x$ back in the direction of smaller values.

Of course this is a simple and maybe even an unrealistic model of route choice behaviour and much depends on the parameters $\alpha'$, but it shows that even simple examples of non-linear systems with positive feedback can behave in a very complex way.

# 3 The combined traffic assignment and control optimisation problem

## 3.1 Introduction

Already in the seventies Allsop and Charlesworth (Allsop 1974, Charlesworth, 1977) showed the relevance of the interdependence of traffic control and route choice. The problem to be solved was initially to search for a traffic control scheme that optimises total delay for traffic volumes which are consistent with the travel times influenced by the control scheme, i.e. a traffic condition where no traveller can improve his travel time by choosing another route (Wardrop's first principle). Two parties try to achieve their own goals, each with its own objective function and space of choices. The infrastructure manager tries to optimise the road system, for example by maximising the utilisation and minimising total delays and stops. A part of the available instruments is the setting of traffic signals, but also other measures are possible. The second group of actors are the drivers who choose their routes such that they minimise their travel time. The travel times are partly determined by the traffic signal settings; while the traffic signal settings are optimised for certain traffic flows that are the consequence of the behaviour of the drivers.
Fisk (1984) showed that this situation could be seen as an example of a non-co-operative game, in which two players have their own objectives and their own strategies. The strategy is known and the choices are predictable, such that it is possible to choose an optimal strategy, taking into account the predictable reaction of the other party. Road users can choose their route under the assumption that traffic control will be optimised for the total delays. The infrastructure manager can optimise the traffic control knowing that the road users will shift their roads after the modification of the control scheme.

## 3.2 An example

A first attempt to analyse this problem was by simulation. The objective was to analyse the distribution of flows between origin A and destination B on two alternative routes. One route has a controlled intersection; the other one is a bypass (see figure 3)

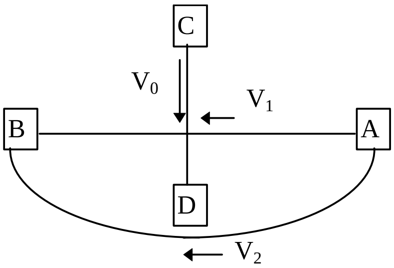


**Figure 3**:  Simple road network

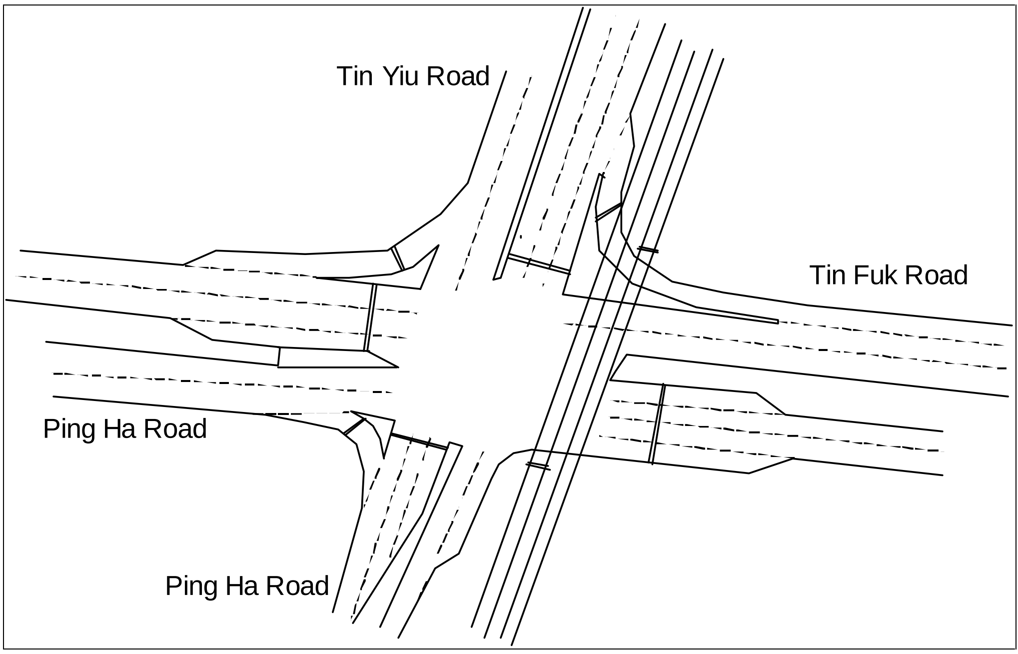


**Figure 4:** The controlled junction on the route between A and B.

A junction as sketched in figure 4 was chosen for this study (from a traffic network study in Hong Kong). For this junction a two-lane bypass for the north-south movement was created. This bypass is situated on the western side of the junction and is 2.5 kilometres longer than the route across the junction The free speed for the bypass is 100 km/hr and for the route with the junction 50 km/hr. The flows for the AM-peak are as given in figure 5.

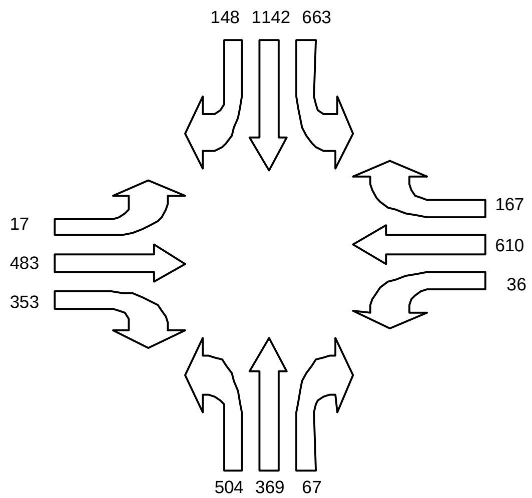


**Figure 5:** Traffic flows, AM-peak

For this situation a number of control strategies were simulated to see how they affected route choice. For the calculation of travel times, the microscopic simulation model FLEXSYT-II- was used. The Transport Research Centre (AVV) of Rijkswaterstaat developed FLEXSYT-II-. A more detailed description of the model and its validity can be found in (Taale and Middelham 1995 and 1997) and (Taale and Scheerder 1998). The travel times have been calculated as the travel time at cruising speed plus the average delay at the intersection.
Because FLEXSYT-II- has no assignment, other than specified by the user, route choice in this study was investigated by changing the splitting rate. First, it was assumed that only 1% of the traffic took the bypass (to measure travel time), then the splitting rate was increased to 10%, 20%, etc. In this way it was possible to find the equilibrium. This was done for four control types. First, the existing fixed-time control plan was simulated for all splitting rates. Then the same was done for optimised fixed-time control. Using Webster's formula for cycletime and greentimes, the optimised control plan was derived for the original flows. Then, the existing vehicle actuated control plan was used and finally an optimised vehicle actuated control. This was done also by using Webster's formula and putting the greentimes as maximum greentimes in the control plan.
In figure 6 the travel times on both routes are shown for all control types. The travel time for the bypass is the same for all types.

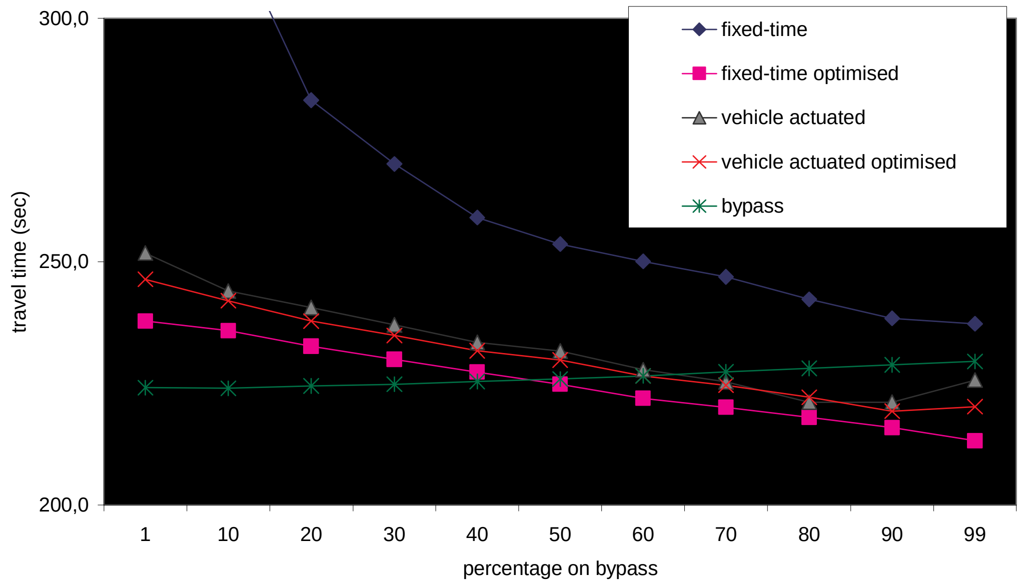


**Figure 6:** Travel times for different distributions of the flow on the controlled route and the bypass

For normal fixed-time control equilibrium is never reached. For optimised fixed-time control the equilibrium is around 50%. For both vehicle actuated control types equilibrium is reached around 60%. The total delay is shown in figure 7.

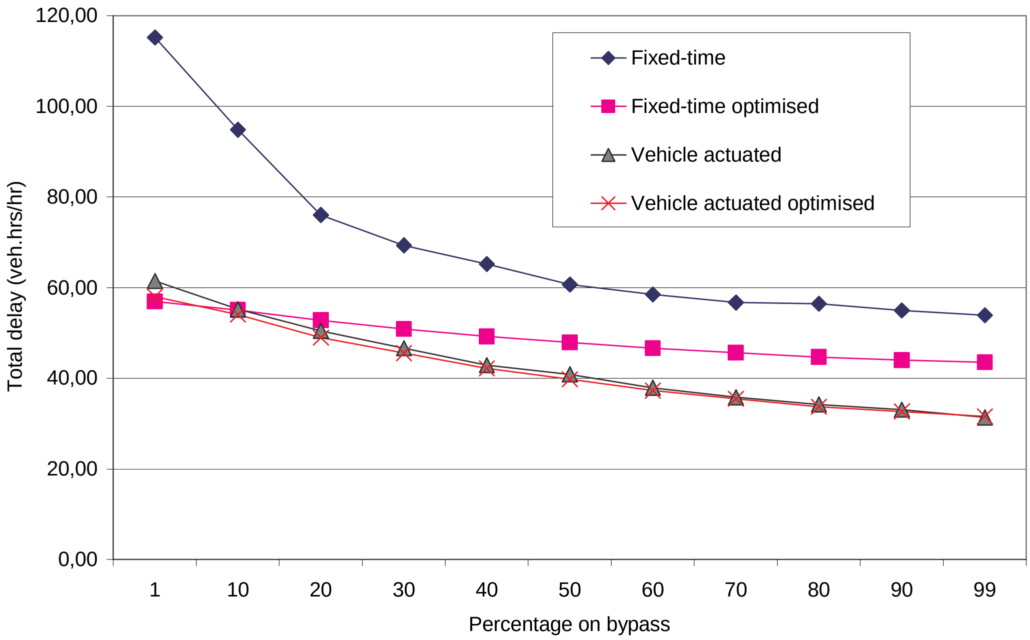


**Figure 7**: Total delay in the network

The figure shows that a user optimum does not necessarily means a system optimum. From the previous figure it was clear that a user optimum was located around 50% or 60%, but from figure 7 it can be seen that a system optimum is reached when as much traffic as possible uses the bypass.
So far, the green times were optimised only for the situation in which only 1% of the traffic uses the bypass. When the optimisation is done for both equilibrium situations, the following table is the result.

**Table 1:** Results before and after optimisation at equilibrium

| | Fixed-time (splitting rate 50%) | | Vehicle actuated splitting rate 60%) | |
|---|---|---|---|---|
| | *before* | *after* | *before* | *after* |
| total distance travelled (veh.km/hr) | 10888.97 | 10823.72 | 11080.76 | 11080.47 |
| travel time bypass (sec.) | 225.9 | 225.8 | 226.5 | 226.5 |
| travel time junction (sec.) | 224.8 | 224.9 | 226.5 | 226.3 |
| total delay (veh.hrs/hr) | 47.91 | 39.14 | 37.25 | 37.26 |

Table 1 shows that for fixed-time control the travel time for the two separate routes does not change and that the total delay decreases with 18%. For vehicle actuated control the situation does not change at all: the travel times on the two routes and the total delay stays the same.

## 3.3 Multiple solutions

The result of the simulation study showed one single solution for the combined assignment and traffic control problem. Multiple solutions were obtained for a similar problem as figure 3:

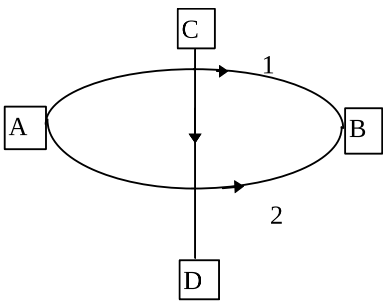


**Figure 8:** Example of a network with symmetric choice possibilities

The example of figure 8 is symmetric. Both routes between A and B cross the route between C and D at a controlled intersection. The traffic control at these intersections is fixed time optimised with Webster's method. It appears that, depending on the magnitude of the flows and the internal lost time, several different solutions exist:

- the symmetrical solution with a 50 - 50% distribution between route 1 and 2
- an asymmetrical solution where more drivers choose for one of the two routes.

Of course, for every asymmetrical solution a 'mirror' solution exists: if a stable solution is obtained with x% on route 1 and 100 - x% on route 2, another solution is the distribution 100 - x% on route 1 and x% on route 2.

In figure 9 the difference in travel time between route 1 and 2 is given for different values of the percentage of the traffic, which is using route 1. The travel time difference is the same as the difference in delay and that was calculated using the two-term delay formula derived by Webster (Webster, 1958, see also chapter 4).
The stable situations occur if the difference in travel time is zero. Figure 9 gives the symmetrical situation where the equilibrium and optimum is obtained for 50% distribution:

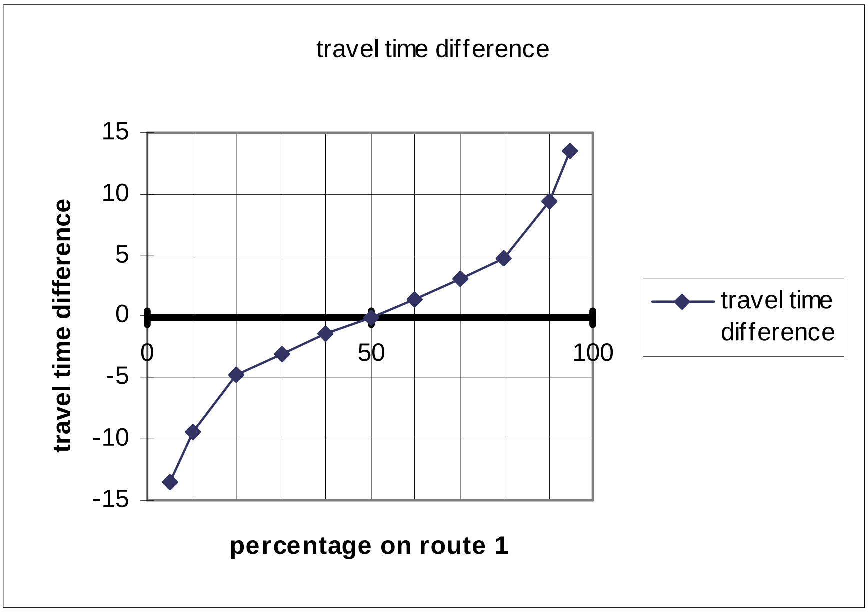


**Figure 9:** Travel time difference between route 1 and 2, giving one single symmetric equilibrium

The situation was calculated for 1000 veh/h flow from A to B, 500 veh/h from C to D, saturation flows of 1800 veh/h and internal lost times of 9 seconds, with a minimum green time of 6 seconds. If we change the flow from A to B to 600 veh/h, the picture changes a lot:

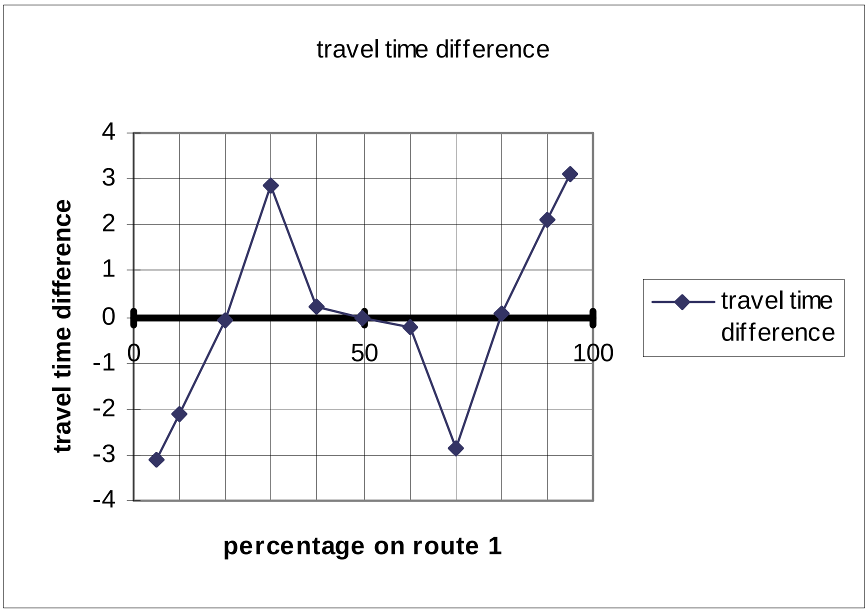


**Figure 10**: Travel times differences for route 1 and 2 with asymmetric equilibrium

Two asymmetric stable equilibrium states exist: the 20 - 80% and the 50 - 50% distribution. If we have the system in the 50-50% state and small changes occur in this distribution, the change is enhanced by the subsequent adaptation of the traffic control scheme: the control scheme for the route with the largest flow gives shorter average delays. The total travel time is in both cases at a minimum for the 50 - 50% distribution.
Changing the parameters of the control scheme (lost time or minimum times) or changing the flow or saturation flows, changes the appearance of the time-difference curves significantly, so that a small change of the parameters can have the result that the equilibrium states move over large distances and the asymmetric solution disappears suddenly.
Apparently the system of route choice and traffic control is under certain circumstances critically dependent on system parameters. In the following chapter we shall investigate this behaviour in some more detail for the original network of figure 3.

# 4 Further analysis

The assignment problem in the case of deterministic route choice based on individual shortest routes can be formulated mathematically as

$$\min_{(V)} \; Z(V_i) \tag{3}$$

where $V_i$ is the volume on link $i$. The function $Z$ is defined as

$$Z = \Sigma_i \int T_i \,(z,C,t_g\,)\;\; \mathrm{d}z \tag{4}$$

and $T_i$ is the traveltime on link $i$ including delays for volume $z$, cycle time $C$ and greentime $t_g$. The minimisation of the delays on controlled intersections can be represented by the following formal expression

$$\min_{\{t\}} \Sigma_i \, D_i \,(V_i,\, t_j) \tag{5}$$

where $t_j$ are the time parameters of the traffic control, $D_i$ is the delay for link i and $V_i$ represents the volumes to be calculated from the solution of equation (3).
In order to get some more insight in the characteristics of the problem, we shall reduce the traffic control problem to one single dimension. The combined assignment and optimisation problem can be visualised in a two dimensional space which makes further analysis easier. We assume that the cycle time remains fixed, the only parameter left is the green split. The delay $d$ for a single controlled flow is given by (Webster, 1958):

$$d(V,\, C,t_g) \approx 0.9\; [½\; (C - t_g)^2\; (1 - V\,/\,s)^{-1}\; C^{-1} + ½\, x^2\; /V\; (1 - x\;)\;] \tag{6}$$

with

$V$ = volume
$x = (V\,/\,s)\;(C\,/\,t_g)$
$s$ = saturation flow
$C$ = cycle time
$t_g$ = green time.

For both approaches of the intersection together the delay is given by

$$\begin{aligned} D \approx 0.9[&V_1\,\{½(C\text{-}t_{g1})^2(1\text{-}V_1\,/s_1)^{-1}C^{-1}+½\;(V_1C/t_{g1}s_1)\;^2/V_1(1\text{-}V_1C/t_{g1}s_1)\} + \\ &V_0\{½(C\text{-}t_{g0})^2(1\text{-}V_0\,/s_0)^{-1}C^{-1}+½(V_0C/t_{g0}s_0)^2/V_0(1\text{-}V_0C/t_{g0}s_0\;)\}] \end{aligned} \tag{7}$$

with:

$0 \le V_1 \le V$
$t_{g1} + t_{g0} = C - t_l$ ($t_l$ is the internal lost time of the control scheme)

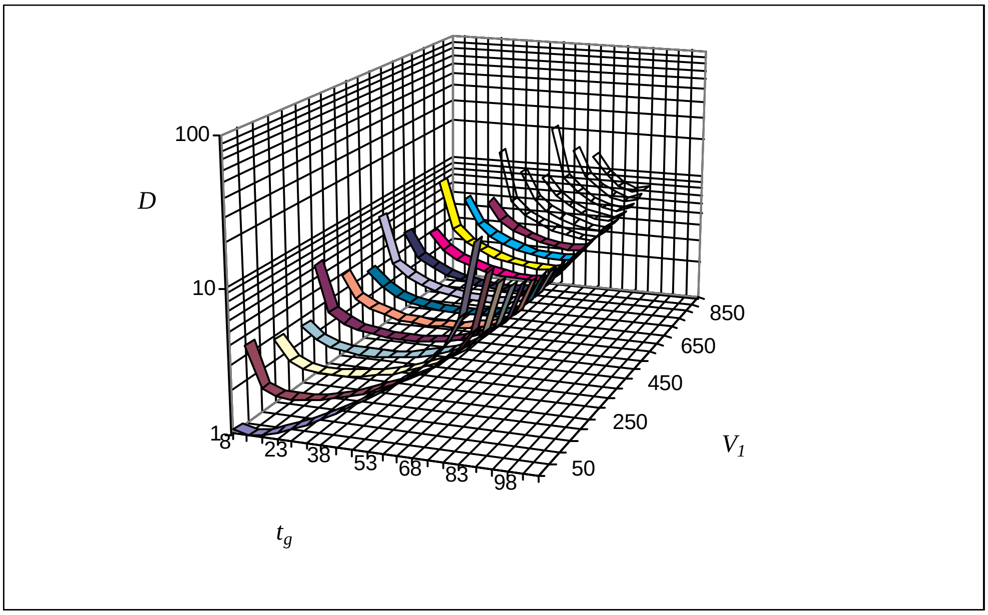


**Figure 11:** Total delay as a function of the green time $t_g$ and volume $V_1$.

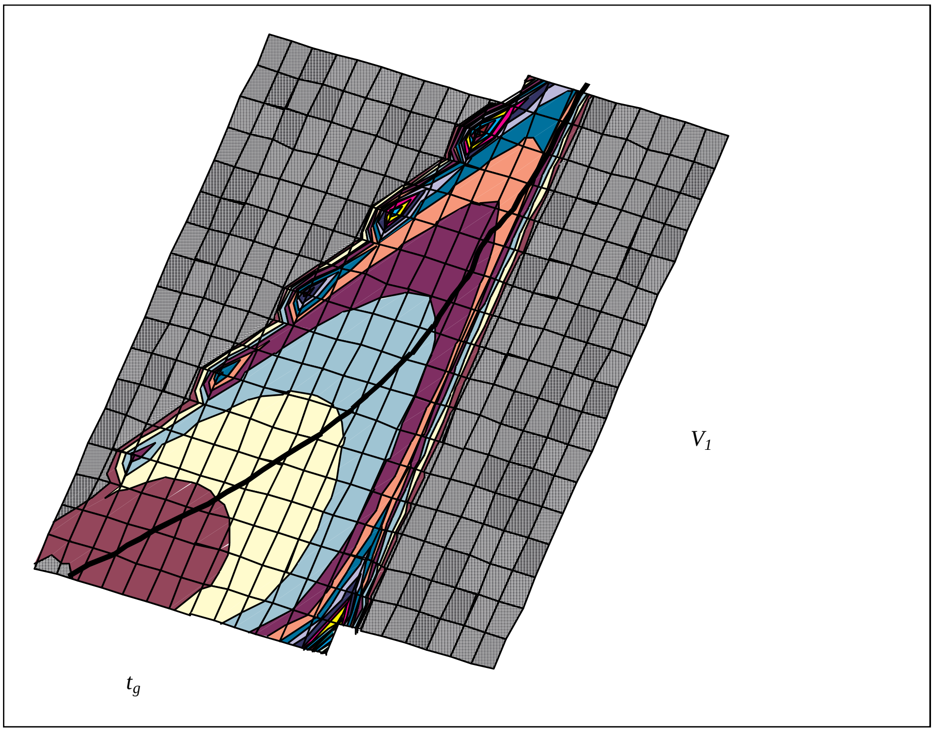


**Figure 12**: Iso-curves with equal total delay in the $t_g$-$V_1$ plane. The thick line gives the green time that minimises the total delay for a given $V_1$

The route choice problem can be formulated in the following way

$$\min_{\{V\}} Z\{V_1, V_2\} \text{ with } V_1 + V_2 = V \tag{8}$$

where $Z$ can be elaborated into

$$\begin{aligned} Z &= V_1 L_1/v + V_2\, L_2/v + 0.45[(C-t_{g1})^2/C \int(1-z/s)\mathrm{d}z + (C/s.t_{g1})^2 \int z\,(1-z\,C\,/s.t_{g1})\mathrm{d}z] \\ &= V_2 L_2/v + V_1\, L_1/v + 0.45[s(C-t_{g1})^2/C\, \ln(1-V_1\,/s)^{-1} - V_1\, C/s.t_{g1} - \ln(1-V_1\, C/s.t_{g1})] \end{aligned} \tag{9}$$

With the boundary condition $V_1 + V_2 = V$, the function $Z$ becomes also a function of two variables, $V_1$ and $t_g$.

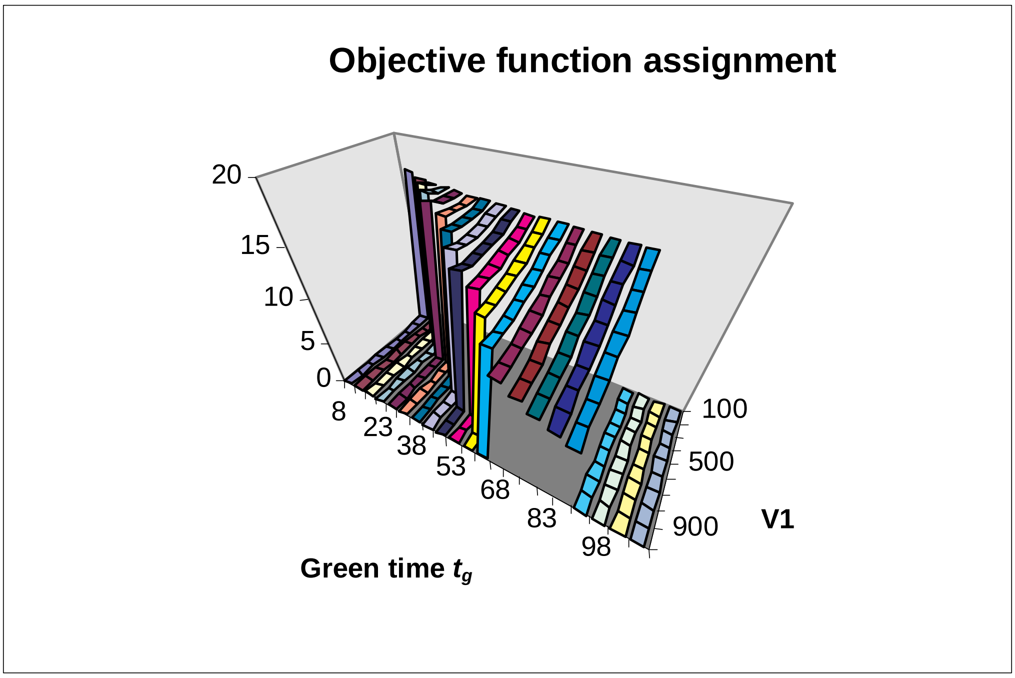


**Figure 13**: Objective function $Z$ as a function of $t_g$ and $V_1$

Figure 13 gives a graphical representation of $Z(V_1\,, t_g)$ The equilibrium solutions are on the line drawn in figure 14 where $\delta Z / \delta V_1 = 0$.

If we combine the lines, which give the optimum green split (from figure 12) and the equilibrium assignment (figure 14), we obtain figure 15:

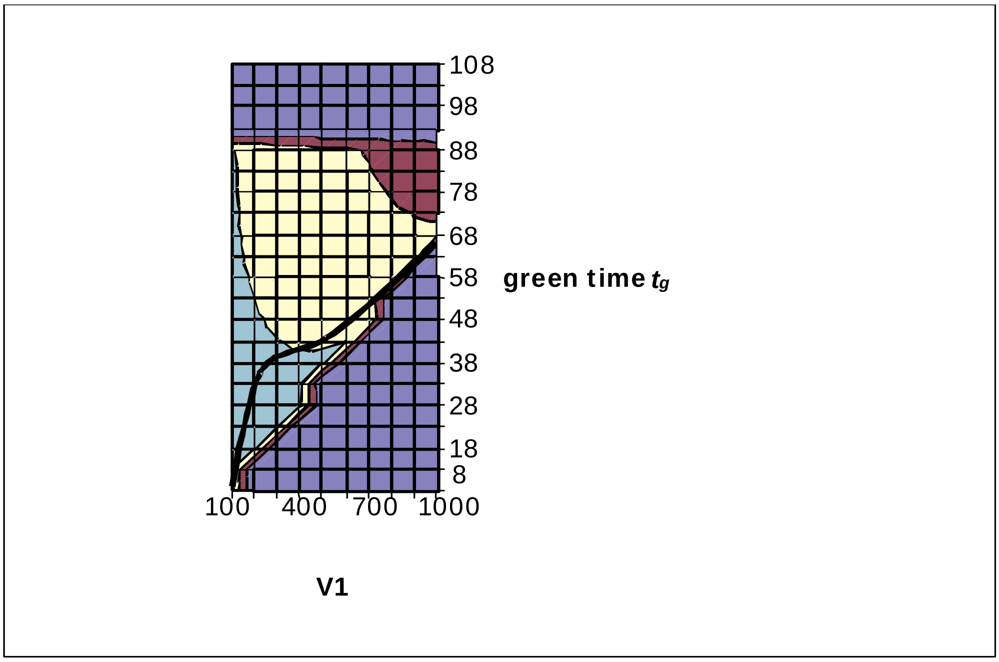


**Figure 14**: Iso-lines with equal values of the objective function Z and the (thick) line giving the equilibrium assignment for a given green time.

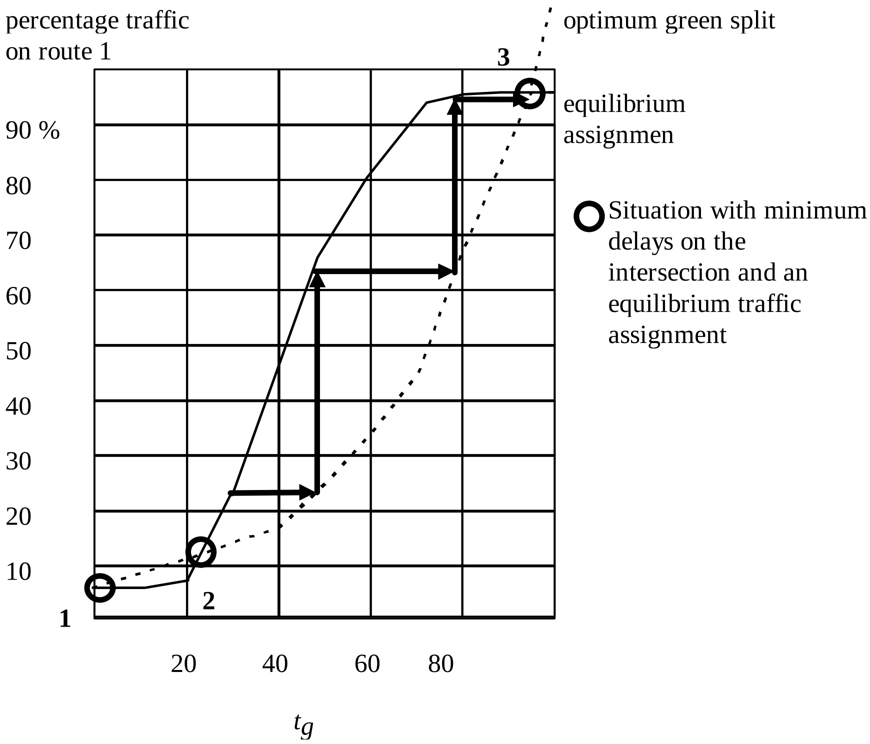


**Figure 15:** Optimum green time (dotted line) and equilibrium assignment (drawn line) with three equilibrium situations

We see that in this example three situations exist were the traffic control is optimised with respect to the traffic volumes and the traffic volumes are consistent with the travel times. If we assume that the process of adjustment of traffic control and route choice are iterative, we find patterns given by the arrows in figure 15: an adjustment in traffic control will give a change in travel time, with the consequence that some drivers choose another route. The changed traffic volumes make it necessary to adjust the control scheme etc. The process stops if a situation has been reached where the drawn and dotted curves intersect (i.e. points 1, 2 and 3).
If in situation 2 the route choice would slightly change and the traffic control is adapted to the changed flows, we see that a positive feedback mechanism exists: a small variation in route choice is reinforced by the mechanism in which more traffic leads to more green time which reduces delay and attracts more traffic, etc. Only at the extremes, where all traffic chooses the same routes or where congestion prevents further growth, the positive feed will disappear. So in this example two stable situations exist: 1 and 3. The total travel time is minimal (for this example) in situation 3.
Also in this case the form of the two curves of figure 15 depends critically on control parameters and (saturation) flows. The optimal green split depend on minimum green time and the internal lost time, which makes that the shape of the curves in figure 15 is determined for a great deal by the boundaries of the space of feasible solutions. Changes in the boundaries will change the shape of the curves, which can have the consequence that the curves intersect on one, two or more points and that the intersection point can move irregularly after small changes in the system parameters or boundary conditions.
The increasing en decreasing difference between the curves of the optimal green split and the equilibrium assignment is due to the non-linear behaviour of the delay function. Small increases in green time lead to large decreases in delay and thus a large switch in traffic from route 2 to route 1. This effect is larger than the increase in delay due to the increase in traffic flow. But at a certain level the increase in traffic will compensate the decrease in delay caused by the larger green times and the difference between the curves will be smaller.

# 5 Final remarks

## 5.1 Conclusions

In a few simulation studies and a further analysis of the problem it is shown that in rather simple traffic situations very complex processes can arise, if we let the system move to equilibrium. The equilibrium situation is not always uniquely determined and it is even possible that oscillations occur (Chen and Wang 1999). The equilibrium situation that is achieved after an iterative adjustment of traffic control to changing route choice is not always a system optimum. This leads to the conclusion that the traffic dependent optimisation of traffic control may result in a sub-optimal situation and it might be better to use traffic control as a management tool to steer the traffic flows, more than as a mean to accommodate traffic volumes.

## 5.2 Further research

The necessary analytical tools for such a strategic approach are still limited: the combination of dynamic traffic control and dynamic traffic assignment that supports the search for a system optimum is still to be developed and is an important topic for further research.
Furthermore, the knowledge about the occurrence of the instabilities is very limited yet. Empirical data on this subject may exist; there are even real time systems that estimate travel behaviour from real time traffic data (e.g. Bell and Grosso 1998). However, as far as the authors know, no analysis of the existing traffic data has been reported which looks for the existence of multiple stable equilibria or to the possibilities to increase system performance by changing traffic control and route choice simultaneously.
The problem becomes more complex even if we realise that in the real world the degree of freedom for travellers is much larger than just route choice. The influence of traffic control on time of departure, modal choice, frequency of travelling, choice of destination etc. has been quantified by a few researchers (Mokhtarian and Raney 1997, Mogridge 1997), but more should be done to apply these results to a method which optimises traffic control taking into account the expected behavioural response, including day-to-day dynamics. Apart from practical tools, which make it possible to optimise traffic control, predict the impact on travel behaviour and to anticipate on future change in behaviour, there is also a need for an analytical framework to study the existence of equilibrium conditions in a system of traffic control and individual travellers.

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