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Structured expert judgement to understand the intrinsic vulnerability of traffic networks

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ABSTRACT

The concept of intrinsic vulnerability of a traffic network is defined for the first time in this paper. Intrinsic vulnerability is the susceptibility to incidents characterised by a probability of occurrence in space and time of difficult estimation, which can result in considerable reduction or loss of the system functionality. Given the nature of this type of vulnerability, its assessment might arise as a major problem. Therefore, this paper investigates the assessment of the intrinsic vulnerability of a traffic network through a set of quantifiable indicators, i.e., accessibility and reliability. Moreover, it is of interest to determine whether the selected indicators are sufficient to assess the intrinsic vulnerability or if there is any significant missing aspect to be considered. A new methodology based on structured elicitation of multivariate uncertainty from experts is presented to address these issues, allowing the estimation of the intrinsic vulnerability and its probabilistic relationship with the indicators accessibility and reliability. Although applied to the case of the metric intrinsic vulnerability, the proposed methodology emerges as an effective tool to understand other traffic descriptors of difficult evaluation such as resilience.

1. Introduction

The main function of a traffic network is to enable economic and social activity in a community. However, this functionality is threatened by hazardous events whose probability of occurrence can be estimated with some confidence, e.g. floods and earthquakes, and other type of incidents characterised by a probability of occurrence in space and time of difficult estimation, such as vehicle breakdowns and terrorist attacks, which are more challenging for the decision makers. Accordingly, the new concept *intrinsic vulnerability* of a traffic (sub-) network can be defined as the susceptibility to incidents characterised by a probability of occurrence in space and time of difficult estimation, which can result in considerable reduction or loss of its functionality. Given the nature of this type of vulnerability, its assessment is not straightforward. For that reason, the paper proposes to evaluate it through other quantifiable indicators, such as accessibility and reliability. In the case these indicators, when combined, represent a large portion of the intrinsic vulnerability, they can be used as a unified framework to assess the intrinsic vulnerability of a traffic network.

Descriptors such as intrinsic vulnerability or resilience that is used to describe the system capacity to absorb and recover from an internal or external shock, are usually adopted to encapsulate the characteristics or capabilities of a traffic network. Because of the complexity of these descriptors, they are usually estimated either through indices that rely on subjective assessments (e.g., preparedness) or by indicators (e.g., redundancy) that quantify system attributes that are assumed to be related to the descriptor (Vugrin et al., 2014; Nogal and O'Connor, 2017). Fig. 1 depicts the idea behind the assessment of a traffic network descriptor (e.g., intrinsic

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Nomenclature

b	scale factor
$dCal_e$	calibration measure for elicitation of statistical dependence associated with expert e
f_i^e	density of sample distribution provided by expert e for the quantity i
g_i	background probability density for the quantity i
n	number of nodes (random variables) of a Bayesian network
$r_{i,j}$	Spearman rank correlation coefficient for two random variables X_i and X_j
$r_{i,j\|k}$	conditional rank correlation of X_i and X_j given X_k
t_{ij}	actual travel time experienced by users travelling from origin i to the destination j
t_{ij}^0	travel time experienced by users travelling from origin i to the destination j in free-flow conditions
w_i^e	weight associated with expert e and quantity i
A_{ij}	accessibility associated with the origin–destination pair ij
A_{ij}^s	contribution of service s to accessibility associated to the origin–destination pair ij
C_ρ	bivariate Gaussian copula
CS^e	calibration score associated with expert e
D	total demand of a traffic network
D_{ij}^s	demand associated with service s, departing from node i and reaching the closest service when travelling from i to j
DM_i	Decision Maker associated with quantity i
$I(\cdot)$	relative information or entropy
IS^e	information score associated with expert e
F_X	cumulative distribution function associated with the random variable X
R_{ij}	reliability associated with the origin–destination pair ij
V_{ij}	intrinsic vulnerability associated with the origin–destination pair ij
$\mathbf{X}$	set of nodes (random variables) of a Bayesian network
$\mathcal{D}$	subset of origin–destination pairs of nodes
$\mathcal{G}$	directed acyclic graph
$\mathcal{N}$	set of nodes of a traffic network
$\mathcal{P}$	set of conditional probability densities
α	scale factor
$\rho_{i,j}$	product moment correlation for two random variables X_i and X_j
Φ^{-1}	inverse of the univariate standard normal distribution
Φ_ρ	bivariate standard normal cumulative distribution with product moment correlation ρ
Ψ_i	set of parents of node X_i in a Bayesian network
Σ_e	correlation matrix elicited from expert e
Σ_s	calibration matrix associated with the seed variables

vulnerability) through indicators.

The estimation of descriptors through indicators presents some operational issues, that is, (i) there is not a clear agreement about the most important indicators to be considered in the assessment, or even their relation with the descriptors; and (ii) some of the indicators exhibit a clear overlapping between them. In this context, a methodology is required to objectively identify the most relevant indicators related to each descriptor, and to quantify the relation descriptor-indicator and the extent of the redundant information provided by the indicators.

To address this problem, expert judgement for uncertainty quantification in proposed. One of the advantages of doing so is that it permits the assessment of uncertainty regarding variables that could be difficult to obtain otherwise, for example because experiments are too costly or data are unavailable. The type of vulnerability studied in this paper is related to incidents whose probability of occurrence in space and time are of difficult estimation. Thus, this methodology is proposed in order to determine the feasibility of assessing the intrinsic vulnerability of a traffic network. It is noted that various mathematical models for handing uncertainty and partial information exist, beyond the frequency-based approaches. With the aim of increasing their acknowledgement, Corotis (2015) proposes an interesting overview of these methods, highlighting that the integration of these approaches in the decision-making process will result in a deeper knowledge of the increasingly complex reality addressed by engineers.

Random variables are also often correlated, for this reason in addition to uncertainty quantification, expert judgement techniques

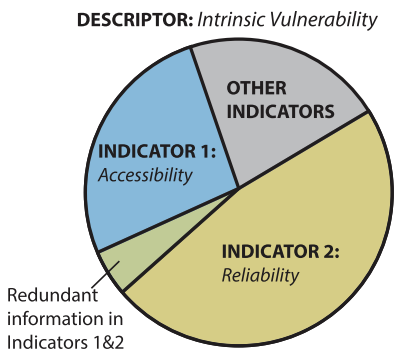

Fig. 1. Representation of the assessment of a traffic network descriptor (e.g., intrinsic vulnerability) based on a number of indicators, such as accessibility and reliability.

for dependence modelling are used to establish to what extent common indicators of the traffic network performance explain the intrinsic vulnerability. Within the list of quantifiable indicators potentially related to intrinsic vulnerability, in this paper *accessibility* and *reliability* are investigated. A justification of this selection is given in Section 2.4.

Based on the elicitation of multivariate uncertainty from experts, this paper aims to answer the following relevant questions; (a) can intrinsic vulnerability be assessed? (b) what does intrinsic vulnerability depend on? (c) can any mathematically-quantifiable indicator(s) be used as a systematic framework to evaluate it?

The study is conducted through the analysis of the National Road network of Ireland, that is, an inter-urban traffic network, without signalised intersections, under a non-congested scenario.

The paper is organized as follows; Section 2 introduces the concepts of vulnerability, accessibility and reliability, and their relationship according to the literature; a brief revision of the structured expert judgement is given in Section 3, along with the methodological framework proposed; Section 4 presents the analysis of the specific case of the Irish traffic network, whose findings are discussed in Section 5. Finally, in Section 6 some conclusions are drawn.

2. Relations between descriptors and indicators

This section reviews the definitions of vulnerability, accessibility, and reliability. The last two have been selected as potential indicators to assess intrinsic vulnerability. The operational definitions used in this paper for intrinsic vulnerability, accessibility, and reliability are also presented. Once these concepts have been stated, Section 2.4 justifies the selection of the two potential indicators to evaluate intrinsic vulnerability.

2.1. Vulnerability

According to Berdica (2002), vulnerability is the susceptibility to incidents that can result in considerable reductions in road network serviceability. Therefore, vulnerability is a measure of the ability of transportation networks to provide a service and meet its intended functions under a wide range of environmental conditions (Taylor, 2017). Existing interpretations of vulnerability include aspects such as change in welfare for passengers (Cats and Jenelius, 2012), remoteness in rural areas (Susilawati and Taylor, 2007), and loss of accessibility, serviceability and reliability (Nicholson et al., 2003; Snelder et al., 2012).

Regarding the operational approaches to assess vulnerability, some authors, such as Berdica (2002), Berche et al. (2009) and Jenelius (2010), state that traffic vulnerability depends on the scenario affecting the traffic network, and can be assessed by analysing the system response to the disruption. This implies an assumption of the location, the intensity and the duration of different hazardous events (Cho et al., 2001; Nogal et al., 2016; Nogal et al., 2017). Nevertheless, other authors (D'Este and Taylor, 2001; Taylor et al., 2006) agree that the concept of vulnerability is related to the consequences of failure, irrespective of the probability of failure. Thus, vulnerability is obtained through the system response when a partial or complete failure is given in a specific link, independent of the cause of the failure (Tampère et al., 2008; Kuang et al., 2013). In the case that intrinsic vulnerability is to be assessed, the drawback of the scenario-specific approach is that covering the full range of possible combinations location/intensity/duration becomes an impossible task. On the other hand, as noted by El-Rashidy and Grant-Muller (2014), most of the research on vulnerability measures and methodologies have focused on assessing the impact, rather than focusing on the link characteristics that lead to vulnerability. Indeed, this is the case of the second approach, which focuses on the consequences and the identification of the links in a traffic network that can potentially cause most disruption if affected (Schmöcker and Fonzone, 2015), which can be understood as a study of the criticality of the links.

Embracing the idea of a vulnerability measure independent from the concept of criticality, intrinsic vulnerability of a traffic network is formally introduced as follows; Considering a connected traffic network with set of nodes $\mathcal{N}$ and some Origin–Destination (OD) pairs of nodes, $\{i, j\} \in \mathcal{D}, i \neq j$, where $\mathcal{D}$ is a subset of $\mathcal{N} \mathrm{x} \mathcal{N}$, and an incident in a random location of the network (not necessary between the OD pair *ij*), an OD pair *ij* with a null intrinsic vulnerability, $V_{ij} = 0$, implies either that no user driving from *i* to *j* is affected by the incident, or the level of service experienced by users driving from *i* to *j* is not reduced as a consequence of the incident. An OD pair *ij* is completely vulnerable, $V_{ij} = 1$, when the OD pair *ij* loses completely its functionality as a consequence of the incident.

2.2. Accessibility

According to D'Este and Taylor (2001), accessibility is the ease for participation in activities from different specific locations using a transport system. Accessibility is a measure of the actual effort in terms of distance, time or cost, required to connect inhabitants with services and facilities. Therefore, it will depend on the existing physical connections as well as the efficiency of those connections. For instance, the congestion level of a given route will reduce its efficiency and therefore, the accessibility of the points connected by the route. In addition, the number of inhabitants affected or potentially affected by the targeted connection is often considered when evaluating accessibility (e.g., Van Wee et al. (2001, 2006)). Two type of accessibility measures exist, namely, the relative and the integral accessibility. The former focuses on a given type of service and analyse the distance, time or cost required to reach it from a given location of the transport system, whereas the latter refers to several services within a given area, and its value is assessed by integrating the relative accessibility of each individual service with respect to a given location of the transport system.

In line with the concept of integral accessibility with consideration of the portion of affected population, the following operational definition of accessibility is adopted in this paper; Accessibility of a (sub-) network is the ease for road users to reach certain services

from specific locations (origins) by using the traffic (sub-) network at a specific time. The services considered in this paper are (i) business, (ii) education, (iii) health services and (iv) interconnection with other modes. The minimum value of the accessibility is 0. A null accessibility from the origin i to the destination j, $A_{ij} = 0$, implies that users cannot reach any of the services considered when travelling from i to j. A total accessibility for a given service from the origin i to the destination j implies that the service considered is located at the origin i, resulting in a required travel time $t_i^s = 0$. Accordingly, the accessibility of the OD pair ij at the time interval studied is calculated as follows;

$$A_{ij} = \sum_s A_{ij}^s = \sum_s \left[\left(\frac{D_{ij}^s}{D} \right)^b \exp(-\alpha\, t_i^s) \right], \tag{1}$$

where D_{ij}^s is the demand associated with the service s, departing from node i and reaching the closest service when travelling from i to j. D is the total demand of the network, t_i^s is the time required to cover the distance between the origin node i and the node where the closest service s is located, and b and α are scale factors. The contribution of a specific service on the accessibility index as a function of the travel time, for different $\frac{D_{ij}^s}{D}$ ratios (contour curves) is shown in Fig. 2. It is noted that the accessibility decreases with increasing values of α, nevertheless, its influence becomes smaller with increasing values of the travel time. For example, for the represented range of D_{ij}^s/D and $\alpha = 1$ hours^{-1} (black thick line), when $t_i^s = 1$ hour A_{ij}^s is in the range of [0.09, 0.37], whereas when $t_i^s = 5$ hours A_{ij}^s is always less than 0.01. In other words, the α parameter implicitly defines a critical travel time where larger values of t_i^s cannot be considered as reasonable to reach a given service, and thus, the contribution of A_{ij}^s of the associated service to the total accessibility will be negligible. For example, in Ireland, due to the spacial arrangement of the country, a value of $\alpha = 1$ seems reasonable, however smaller values of α should be considered for countries such as Australia. Thus, its calibration should take into account the associated spatial interaction model.

The proposed formulation fulfills the collection of desirable attributes of an accessibility index discussed in Taylor (2017).

2.3. Reliability

As indicated by Mattsson and Jenelius (2015), reliability in the transportation field relates to the certainty and predictability of travel conditions. These conditions can vary due to a number of factors, for instance, normal daily demand fluctuations, man-made disasters and weather conditions. There are various valuable measures of transportation network reliability, such as capacity reliability (e.g., Chen et al. (2002)), travel time reliability (e.g., Wakabayashi (2012)) and connectivity reliability (e.g., Bell and Iida (1997)).

In this paper, the concept of travel time reliability is considered, whose operational definition is as follows; (Travel time) reliability measures the feasibility that road users reach a destination within a certain travel time under the operating conditions encountered. To measure the reliability of a given OD pair ij, the actual travel time experienced by users travelling from origin i to the destination j, t_{ij}, is compared with the associated travel time in free flow conditions, t_{ij}^0, that is,

$$R_{ij} = \frac{t_{ij}^0}{t_{ij}}. \tag{2}$$

The maximum value of the reliability associated with the OD pair ij is $R_{ij} = 1$, reached when the level of service is the optimal. Note that low values of reliability mean a lack of reliability.

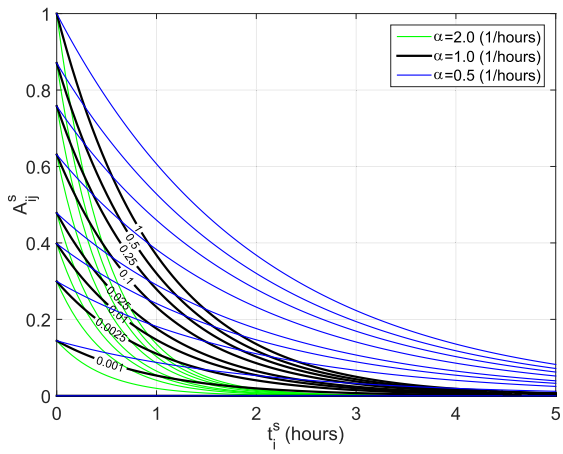

Fig. 2. Contribution of a specific service on the accessibility index as a function of the travel time, for different $\frac{D_{ij}^s}{D}$ ratios (contour curves).

2.4. Intrinsic vulnerability through accessibility and reliability

There is a clear operational overlapping between the terms vulnerability, accessibility and reliability; the reader is referred to Reggiani et al. (2015) for a comprehensive discussion on the relations between these concepts. One of the approaches commonly used to determine traffic system vulnerability is through accessibility-based methods (Taylor, 2017), which use the accessibility value as a measure of the vulnerability of the system (e.g., Taylor et al. (2006, 2013)). For instance, according to Murray and Grubesic (2007), a network node is defined as vulnerable if loss (or substantial degradation) of a small number of links significantly diminishes the accessibility of the node. The relationship between reliability and vulnerability is also acknowledged by many authors. Reliability is related to the well-functioning capacity of traffic networks, whereas vulnerability focuses on non-functioning networks' response (Husdal, 2005; Taylor, 2008), thus, there exists a certain complementarity between both terms. For instance, Watling and Balijepalli (2012) provides a methodology to determine how travel time reliability is affected by growing demand, which can be used to estimate traffic network vulnerability. Taylor et al. (2006) discuss the convenience of considering reliability or accessibility when analysing the vulnerability of traffic networks at different scale levels; in networks with a high level of connectivity, as in the case of urban networks, the level of service gains importance to evaluate the vulnerability. On the contrary, when the connectivity is lower, as in the case of inter-urban networks, the accessibility becomes a relevant indicator. It is clear that network connectivity underlies behind both accessibility and reliability.

Given the widely-accepted existing relation of accessibility and reliability with vulnerability, they will be used as potential metrics to assess the intrinsic traffic network vulnerability. Intrinsic vulnerability relates to incidents that pose a large uncertainty (in terms of space and time), regardless the intensity of the perturbation and the extent of the consequences. Because big disasters might affect the accessibility to basic services, and light perturbations result in a decrease of the reliability of the traffic systems, this initial choice allows covering both degrees of possible perturbations and consequences.

3. Structured expert judgement for characterization of multivariate uncertainty

Structured expert judgement refers to a transparent methodology for elicitation of expert opinions such that these expert judgements may be treated as (other type of) scientific data in a formal decision process. Structured expert judgment, and in particular the mathematical aggregation of expert judgments has been extensively used in different fields of application. Over thirty-three independent expert judgment studies were performed between 2006 and March 2015 and about 45 prior to 2006 (see Colson and Cooke (2017)). The great majority of these studies investigate one-dimensional (1-d) uncertainty. The investigation of intrinsic vulnerability through a set of indicators (i.e., accessibility and reliability) requires a multivariate analysis. In order to perform multivariate analysis from a probabilistic point of view, 1-d marginal distributions are required as well as a copula realizing the joint distribution. In this research, methods to elicit 1-d uncertainty regarding random variables will be used, as well as methods for elicitation of dependence. The two main advantages of the methods to be presented with respect to other traditional methods are: (i) it may be used in the absence or limited amount of field measurements, experiments or models for the particular case of interest (as is the case for concepts such as intrinsic vulnerability discussed in Section 2), ii) it allows for evidence-based scoring and combination of expert opinions. As far as the authors are concerned, this is the first time Cooke's method and its extension to the elicitation of dependence (*DCal* score in Section 3.2) are used in order to investigate a concept such as intrinsic vulnerability. Notice that despite the advantages of the methods here presented, a certain amount of data will be required as "seed variables" which can also make the general applicability of the method challenging. Moreover, structured expert judgment should not be seen as a substitute for fundamental research into driving processes. The main features of these methods are discussed next.

3.1. Expert judgement for uncertainty quantification

In this research, quantification of uncertainty through structured expert judgement is based on the classical (or Cooke's) method. This method is a performance-based linear pooling or weighted averaging model. The weights are derived from experts' calibration and information scores, as measured on seed variables. The seed variables, are variables whose realizations are known to the analysts, but not known to the experts at the moment of the elicitation. The performance-based weights use two quantitative measures of performance, namely, (a) calibration, which measures the statistical likelihood that a set of experimental results statistically correspond with the expert's assessments and (b) information, which considers how concentrated a distribution is relative to a background measure. For a complete review of the classical model, the reader is referred to Cooke (1991). For a recent Matlab implementation of the method see Leontaris and Morales-Nápoles (2018).

In this research the quantile format is used, that is, experts are asked to assess their uncertainty concerning certain continuous quantities in the form of a number of percentiles of their uncertainty distribution (5th, 50th and 95th).

In order to evaluate experts' opinions, their assessments on seed variables are analysed statistically. The assessments of each expert e are treated as a statistical hypothesis and the obtained p-value used as a calibration score (CS^e). Thus, values of calibration close to zero mean that it is unlikely that the experts' probabilities are correct.

To measure the experts' informativeness, the density of the sample distribution f_i^e provided by expert e for the quantity i is compared against a background probability density g_i (usually uniform or log uniform distribution). This comparison is carried out through the mutual entropy between densities, obtaining the information score of each expert, IS^e.

The combination of experts' assessment, or Decision Maker (*DM*), is carried out by the summation of the experts' assessments of the variables of interest, weighted according the product of scores obtained (ω_i^e), that is,

$$DM_i = \frac{\sum_e \omega_i^e f_i^e}{\sum_e \omega_i^e}. \tag{3}$$

There are different possibilities of combining the calibration and information scores to obtain the weights ω_i^e for the variable of interest i, such as, (a) considering an *optimal global* weight, $\omega_i^e = \omega^e = \delta_\alpha CS^e IS^e$ where the indicator δ_α is 1 if α is larger than a level selected to maximize the combined score of the resulting DM; or (b) considering different weights per item (*optimal item*), $\omega_i^e = \delta_\alpha CS^e I(f_i^e, g_i)$, where $I(\cdot)$ is the relative information or entropy. The results of applying any of these criteria usually provides better results than applying equal weights to all experts or even if only the best-scored expert is considered (Colson and Cooke, 2017).

A more detailed description of the process can be found in Cooke and Goossens (2008).

3.2. Expert judgement for dependence modelling

The use of expert opinions in dependence modelling is an active area still very much under development. For a recent overview on the expert judgement for dependence in probabilistic modelling, the reader is referred to Werner et al. (2016). In our research statistical dependence will be modelled through copulas, that is, positive monotonic functions in the range [0, 1], conforming a compatible multidimensional cumulative distribution function. In this case, statistical samples of joint observations or realisations are needed, however, they are usually not available, as happens in the present study. Under this perspective, the practical solution implies again the use of expert elicitation, where experts are asked about statistical dependence between variables of interest.

The dependence relations investigated in this paper through the expert judgement elicitation are based on a continuous Non-Parametric Bayesian Network (NPBN) modelling framework (for details see Hanea et al. (2015)). For a better understanding of the NPBNs, Bayesian networks, copulas and (conditional) rank correlations are previously introduced.

A Bayesian network (Pearl, 2014) is a pair $(\mathcal{G}, \mathcal{P})$, where $\mathcal{G}$ is a directed acyclic graph defined on a set of nodes $\mathbf{X}$ (the random variables), $\mathcal{P} = \{p(x_1|\pi_1), \ldots, p(x_n|\pi_n)\}$ is a set of n conditional probability functions, one for each variable, and Ψ_i is the set of parents of node X_i in $\mathcal{G}$. The set $\mathcal{P}$ defines the associated joint probability density of all nodes as

$$p(\mathbf{x}) = \prod_{i=1}^{n} p(x_i|\pi_i). \tag{4}$$

Fig. 3(a) shows a Bayesian network of three nodes, where the parents of variable ν_3 are ν_1 and ν_2, and the parent of ν_2 is ν_1.

The association between random variables can be investigated through copulas. In particular, in this research we will focus on the use of the Gaussian copula since this may be parameterised entirely with the correlation coefficient, presents no asymmetries such as tail dependence, and is perhaps the most familiar to experts. Moreover, as will be seen later, this type of copula allows measures for calibration of expert opinions (Morales-Nápoles and Worm, 2013; Morales-Nápoles et al., 2014). Let Φ_ρ be a bivariate standard normal cumulative distribution with product moment correlation ρ, and Φ^{-1} the inverse of the univariate standard normal distribution, then the bivariate Gaussian copula is defined as

$$C_\rho(u_1, u_2) = \Phi_\rho(\Phi^{-1}(u_1), \Phi^{-1}(u_2)), \tag{5}$$

where $(u_1, u_2) \in [0, 1]^2$.

On the other hand, the Spearman rank correlation coefficient for two random variables X_i, X_j with cumulative distribution functions F_{X_i} and F_{X_j} is

$$r_{i,j} = \frac{E(F_{X_i}F_{X_j}) - E(F_{X_i})E(F_{X_j})}{\sqrt{var(F_{X_i})var(F_{X_j})}}. \tag{6}$$

The conditional rank correlation of X_i and X_j given X_k is denoted by $r_{i,j|k}$.

In a NPBN, each node is associated with a continuous arbitrary invertible distribution function and each parent–child influence is represented as a (conditional) one parameter copula, parameterised in terms of the (conditional) rank correlation (see Fig. 3(b)). In Fig. 3(b), $C(\nu_3, \nu_1|\nu_2)$ represents the conditional copula of variables 3 and 1 given 2. This is parameterised by the conditional rank correlation $r_{1,3|2}$. The joint distribution of variables 1 and 3, $p(1, 3)$ will be given by the choice of the underlying copulas in the particular model. For a detailed description of the copulas/rank correlations, the reader is referred to Joe (2014) and Hanea et al. (2015).

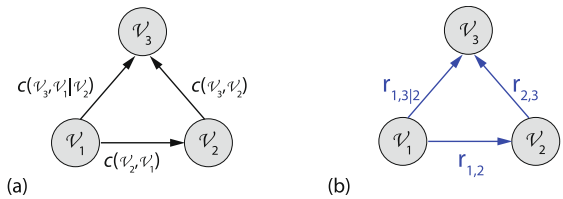

Fig. 3. Example of Bayesian network of three nodes, indicating (a) the conditional probability densities expressed in terms of copulas, and (b) the (conditional) rank correlations.

The rank correlations will be realized using bivariate (conditional) copulas. When all copulas in the assignment of a NPBN correspond to the bivariate normal copula, then a multivariate Gaussian copula with correlation matrix Σ is obtained. For instance, in the case of the NPBN of Fig. 3,

$$\Sigma = \begin{bmatrix} 1 & \rho_{1,2} & \rho_{1,3} \\ \rho_{1,2} & 1 & \rho_{2,3} \\ \rho_{1,3} & \rho_{2,3} & 1 \end{bmatrix}, \tag{7}$$

In this case the product moment correlation matrix has elements $\rho_{1,2}$, $\rho_{1,3}$ and $\rho_{2,3}$. Where $\rho_{1,3}$ is related to $\rho_{1,3|2}$ through the expression

$$\rho_{1,3|2} = \frac{\rho_{1,3} - \rho_{1,2}\,\rho_{2,3}}{\sqrt{(1-\rho_{1,2}^2)(1-\rho_{2,3}^2)}}. \tag{8}$$

and $r_{i,j} = \left(\frac{6}{\pi}\right)\arcsin\left(\frac{\rho_{i,j}}{2}\right)$.

In the case of elicitation of dependence, the calibration of the experts will be carried out by means of the D-calibration (introduced in Morales-Nápoles and Worm (2013)), i.e., a measure of the distance between a "seed" correlation matrix and the correlation matrix obtained based on experts' opinion, which is expressed as follows,

$$dCal_e = 1 - \sqrt{1 - \frac{|\Sigma_S|^{\frac{1}{4}}|\Sigma_e|^{\frac{1}{4}}}{|\frac{1}{2}\Sigma_S + \frac{1}{2}\Sigma_e|^{\frac{1}{4}}}}, \tag{9}$$

where Σ_S and Σ_e are the seed calibration matrix and the correlation matrix elicited from expert e, respectively. Properties of the *dCal* score have been investigated in Morales-Nápoles and Worm (2013), Morales-Nápoles et al. (2014); these include the fact that $dCal_e \in [0, 1]$ and will be equal to 1 iff $\Sigma_e = \Sigma_S$. Also, an expert may obtain a low calibration score if, for example, a high correlation between a pair of variables was expressed by the expert while this was not observed in the seed dependence structure Σ_S. Another property is that a necessary condition for an expert to be highly calibrated is to sufficiently approximate the dependence structure of interest element-wise.

As explained before, the expert decision can be analysed as a hypothesis testing, with null hypothesis stating that $dCal_e$ comes from the distribution of $dCal_S$. Rejecting the null hypothesis would imply that the difference between both correlation matrices might not be exclusively due to sampling fluctuation. The combination of expert opinions will be done similarly to Cooke's method but with the *dCal* score as the basis for determining weights for individual expert opinions. In this sense the procedure described here is an extension of Cooke's method in that it still uses a calibration measure for individual opinions while the calibration measure is designed specifically to score experts as assessors of dependence while Cooke's model scores experts as (one dimensional) uncertainty assessors. Next, attention will be paid to the particular case of interest.

3.3. Methodological framework

The process followed to assess the intrinsic vulnerability of a traffic network based on structured expert elicitation consists of two parts, the uncertainty quantification of the intrinsic vulnerability and the dependence modelling between accessibility, reliability and intrinsic vulnerability.

The elicitation of uncertainty quantification will provide an estimation of the intrinsic vulnerability associated with each OD pair of interest. The process is conducted as follows;

1. To determine the score of each expert to measure their capacity to express uncertainty. To do so, seed variables whose value is only known by the researcher are used. An example of the type of question related to the uncertainty distribution of the seed variables is shown in Appendix A.1;
2. To obtain from each expert their individual assessment of the intrinsic vulnerability associated with the OD pairs of interest. The type of question used to that aim is presented in Appendix A.2;
3. To determine a final estimation of the intrinsic vulnerability of the OD pairs of interest by combining the individual assessments according to Eq. (3).

The process followed to determine the contribution of the indicators accessibility and reliability into the value of the intrinsic vulnerability, the descriptor (i.e., intrinsic vulnerability) and the indicators (i.e., accessibility and reliability) are treated as random variables in a process of elicitation of statistical dependence. In that way, the statistical contribution of each indicator will be determined, including the potential redundant information between indicators. Therefore, the process will consist on the following steps;

1. To determine the score of each expert opinion according to Eq. (9), which will measure relative ability of individual opinion to express the statistical dependency between random variables. To do so, the statistical relation between seed variables, only known by the researcher, is used. Appendix A.3 provides an example of the type of question included in the questionnaire related to the dependency of the seed variables;
2. To obtain from each expert their individual assessment of dependency between the variables vulnerability and accessibility, and between vulnerability and accessibility for a given reliability associated with the OD pairs of interest. Examples of the type of questions required in

this step are also indicated in Appendix A.4. The elicited values are used to obtain the Spearman rank correlation coefficients $r_{V,R}$, $r_{A,V}$ and $r_{V,R|A}$, and the consequent product moment correlations (ρ) to build the correlation matrix in Eq. (7). Because reliability and accessibility are both quantifiable indicators, $r_{R,A}$ can be assessed directly from the analysis of the traffic network;

3. Once the correlation matrix associated with each OD pair is determined, the corresponding multivariate Gaussian copula is defined, meaning that the multivariate probability distribution function of intrinsic vulnerability, accessibility and reliability is known for each OD pair of interest.

It is noted that the median value of the intrinsic vulnerability is used to determine the relation between variables during the elicitation of dependence (see Appendix A.4), therefore, the elicitation of uncertainty quantification should be conducted before the elicitation of statistical modelling.

The analysed indicators might account for an important portion of the intrinsic vulnerability of the traffic network, therefore the intrinsic vulnerability can be systematically evaluated by introducing the value of the two quantifiable indicators into the obtained multivariate probability distribution function. In the case that accessibility and reliability are not representative enough of the total intrinsic vulnerability, other indicators should be included following the same process. In order to determine to which extent the selected indicators account for a representative portion of the total intrinsic vulnerability, the value of the intrinsic vulnerability obtained with the multivariate probability distribution function should be compared against the value previously obtained through the elicitation of uncertainty quantification.

4. Assessment of vulnerability by means of accessibility and reliability

4.1. Case study and elicitation process

The Irish traffic network presented in Fig. 4 is under study during the interval of time 8.00 a.m. to 9.00 am. The length of the links are

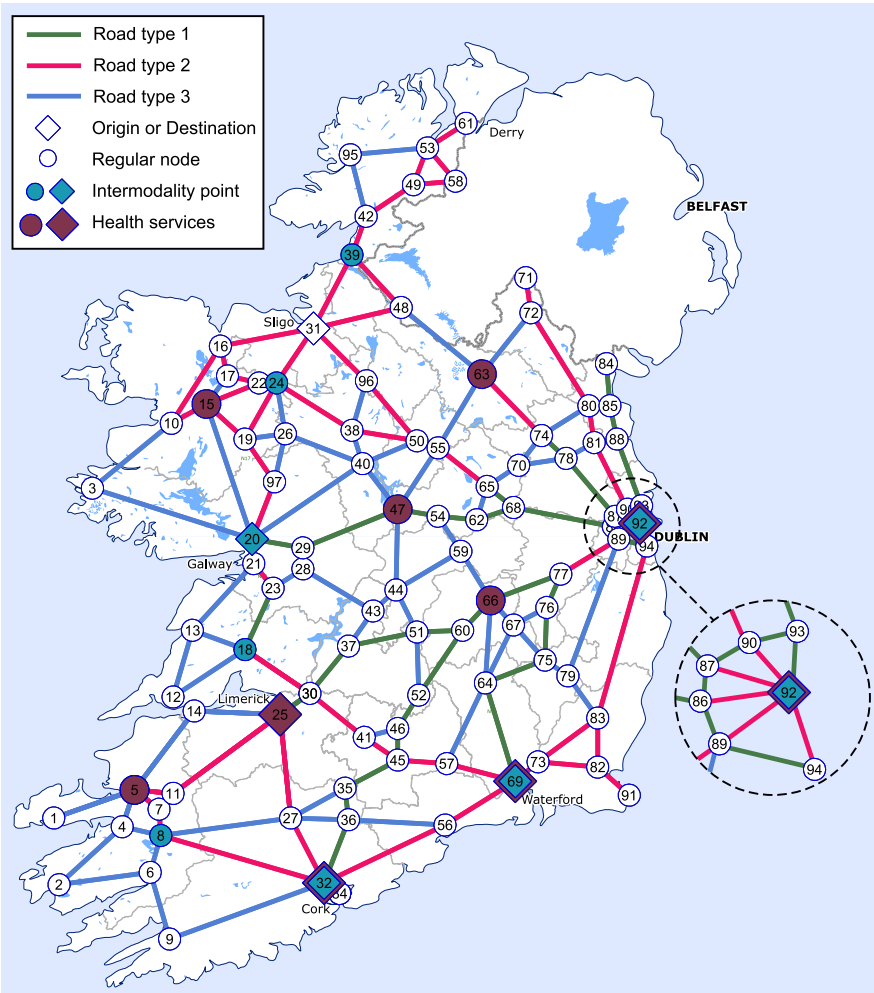

Fig. 4. Traffic network under study; Irish case study.

proportional to the real length of the roads connecting the corresponding nodes, and all links represent bidirectional roads. For the sake of simplicity, the typology of the roads has been reduced to three generic types, i.e., highways, primary and secondary roads. Their characteristics for good ambient conditions are given in AECOM and ESRI (2014) and shown in Fig. 5. The OD pairs and the probabilistic distribution of demands associated with each OD pair in the interval of time studied are given in Fig. 6. These values have been obtained considering the NRA traffic data corresponding to the working days of January 2016.

A C-logit stochastic user equilibrium model is used to reproduce the traffic behaviour during the period of interest, based on the formulation proposed by Zhou et al. (2012). It is assumed that, during the time interval analysed, the 60% of the demand is travelling because of business reasons, 12% because of educational purposes, 3% are the potential users of the health services, and 8% of the demand are the potential users of the intermodality facilities. Using the Monte Carlo method, 10, 000 simulations were carried out. The combination of different traffic demands were introduced to obtain the travel time and the link flow associated with the links and routes of the traffic network. For each simulation, the indices accessibility and reliability associated with the set of OD pairs were computed, using Eq. (1) and (2), respectively, with $b = 0.2$ and $\alpha = 1$ hours^{-1}. The obtained data were used for the preparation of the elicitation process.

For the sake of clarification, it is noted that the minimum value of the mean of accessibility corresponds to the OD $31 - 92$ (Sligo-Dublin), whose value is 0.16. This is due to the combination of two facts, namely, a low OD demand and the complete absence of intermodality and health services in node $i = 31$ (Sligo). On the contrary, the largest value of the mean of accessibility corresponds to the OD $92 - 69$ (Dublin-Waterford), whose value is 0.88. In this case, the demand is very large and the intermodality and the health services are both in the origin node ($t_i^s = 0$). The effect of the other two services, business and education, is shown when analysing the OD pairs $92 - 31$ (Dublin-Sligo) and $92 - 63$ (Dublin-Cavan). In both cases the demand and the contribution of services health and intermodality are similar, nevertheless, the distance between nodes $i - j$ makes the mean of accessibility different, that is, $A_{92-31} = 0.61$ and $A_{92-63} = 0.76$. The large difference in the accessibility of the users travelling to Sligo from Dublin and vice versa makes clear the influence of both, the demand and the proximity to services on the accessibility value.

As sketched in Section 3.3, the elicitation consists of two parts, (a) elicitation of uncertainty, where Cooke's method is applied to determine the intrinsic vulnerability associated with different OD pairs, and (b) elicitation of probabilistic dependence, where the statistical relations between intrinsic vulnerability, accessibility and reliability are studied. The dependence relations between variables are modelled directly by Gaussian copulas, and the score of the experts will be computed considering the D-calibration equation.

Therefore, the questionnaire consisted of 10 questions for calibration of uncertainty, 6 questions for calibration of dependence, 5 questions related to the variables of interest and finally, 10 questions on dependence between variables. Table 1 summarizes the structure of the questionnaire. The questions related to dependence elicitation are asked following Morales et al. (2008).

The calibration variables for dependence modelling shown in Table 1 indicate that the accessibility indices for the analysed ODs are not highly correlated, whereas a higher correlation exists between the reliability indices.

A number of 5 experts on Transportation with deep knowledge of the terms discussed, participated in the elicitation process. The country where they are based on and their background are indicated in Table 2. It is noted that they were not aware of the real location of the traffic network to avoid that previous experience on aspects such as the geographic features or the existing black spots, could influence their answers.

4.2. Results

The results obtained from the elicitation process are presented and discussed in this section. First, the capacity to estimate the intrinsic vulnerability is analysed in the uncertainty-elicitation phase. In the dependence-elicitation phase, the dependence structure between intrinsic vulnerability, accessibility and reliability is addressed.

The calibration scores, CS^e obtained by the five experts during the uncertainty elicitation are given in the 2nd column of Table 3. The information scores, IS^e, in all variables, and in seed variables only, are given in columns 3 and 4 of the same table respectively. The combination of the expert opinions according to the "equal weights" decision maker is used as a baseline to compare the

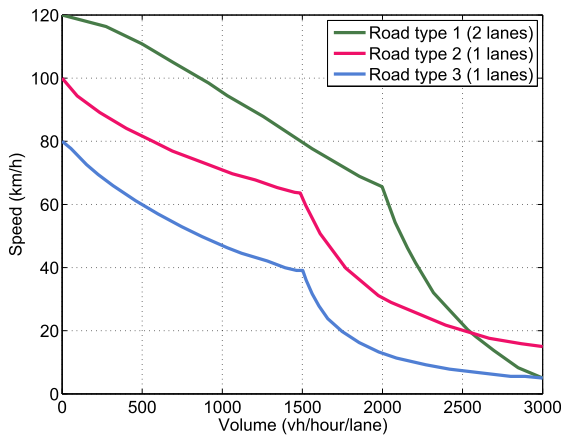

Fig. 5. Characteristics of the road types.

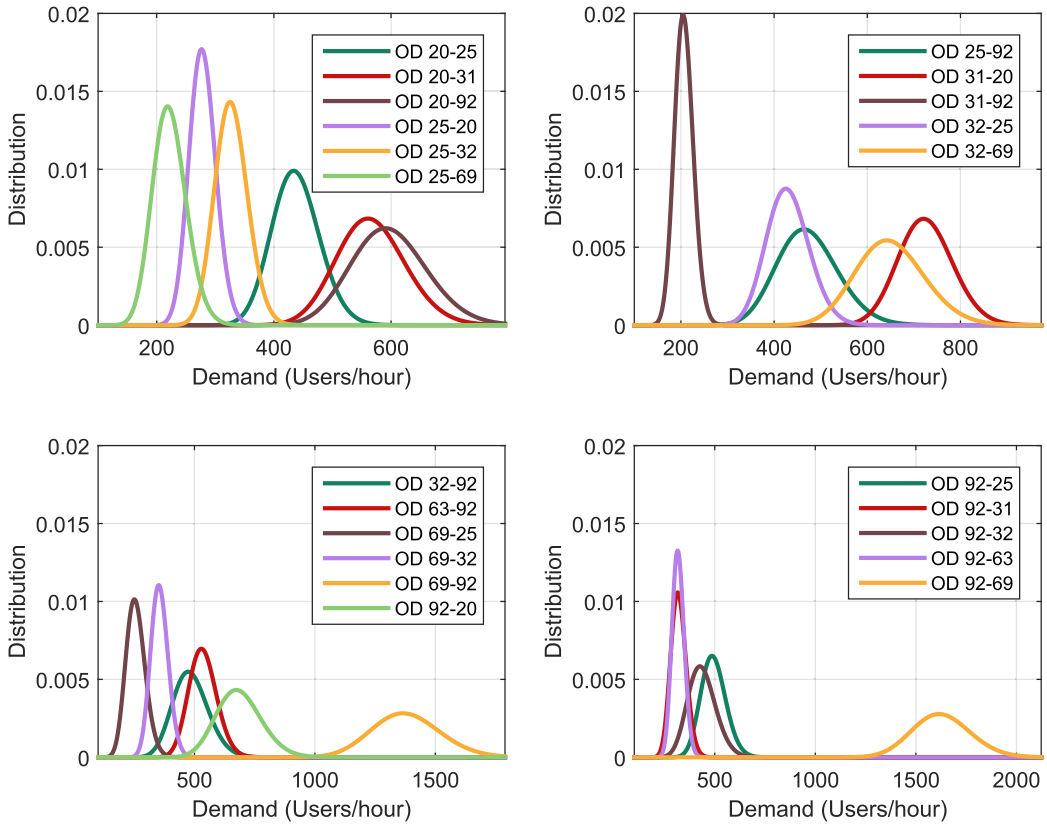

Fig. 6. Probabilistic distribution of the traffic demands.

Table 1
Structure of the questionnaire. A_{ij}, R_{ij} and V_{ij} denote accessibility, reliability and intrinsic vulnerability associated with the OD pair *ij*.

ELICITATION OF UNCERTAINTY			
	Calibration Variables		Variables of Interest
ODs	$Max[A_{ij}]$	$Min[R_{ij}]$	V_{ij} (percentiles 5, 50 and 95)
20–25 (Galway-Limerick)	0.590	0.889	*Unknown values*
25–69 (Limerick-Waterford)	0.457	0.845	
32–69 (Cork-Waterford)	0.816	–	
32–92 (Cork-Dublin)	0.636	0.826	
69–92 (Waterford-Dublin)	0.851	0.777	
All ODs	–	0.755*	–

ELICITATION OF DEPENDENCE MODELLING				
Calibration Variables		Variables of Interest (percentile 50)		
		ODs	$Prob(V_{i,j}\|A_{i,j})$	$Prob(V_{i,j}\|A_{i,j}, R_{i,j})$
$Prob(A_{25,69}\|A_{32,92})$	0.499	20–25		
$Prob(A_{32,92}\|A_{69,92})$	0.455	25–69		
$Prob(A_{25,69}\|A_{32,92}, A_{69,92})$	0.500	32–69	*Unknown values*	
$Prob(R_{25,69}\|R_{32,92})$	0.575	32–92		
$Prob(R_{32,92}\|R_{69,92})$	0.871	69–92		
$Prob(R_{25,69}\|R_{32,92}, R_{69,92})$	0.563			

* The minimum reliability of the network is included to increase the variability of the elicited values.

performance of the other decision makers. Column 5 shows the product of columns 2 and 4 for experts whose weight is different than zero (see Section 3.1). Column 6 shows the ratio of the information score for calibration variables to the information score for calibration variables of the equal weight decision maker. Notice that one individual expert (the one with the best calibration score)

Table 2
Country and background of the experts involved in the elicitation process (sorted by alphabetical order).

Country	Background
Japan	Reliability of transportation networks, transit assignment models, passenger behaviour.
Malaysia	Transport modelling, reliability, vulnerability modelling, GIS, SCATS.
Sweden	Traffic safety, vulnerability in transportation systems, traffic and transport planning.
Sweden	Transportation, networks, resilience, data analysis, public transport.
The Netherlands	Transport systems, transport modelling, traffic flows, choice behaviour, network resilience.

has an information score lower than the equal weight combination.

Column 7 shows the given weights (equal weights), and the 6th row the reference values in regards to the calibration and information for the equal-weights DM. When selecting the optimised global weights decision maker, the obtained calibration score (0.6827) is 6 times better than the reference case (0.1135). Moreover, this combination improves any of the individual scores. In the case of the optimised item weights decision maker, its calibration score is 2.55 times better than the reference case and still larger than the calibration scores of individual experts.

The performance based combinations all have a larger information score than the equal weight combination. The best performance DM in terms of information as explained earlier is given by the optimal-item combination, with a 78% of improvement, meanwhile the optimal global reaches 46% higher information score than the equal-weights combination. Taking into account all information available, the optimised global weights decision maker is recommended as the criterion to combine the experts' opinions, being the combination of experts C and E identified as the optimum. Their corresponding weights, w_i^e are given in the last column of Table 3.

Fig. 7 depicts the 5th, 50th and 95th percentiles given by the experts when quantifying the uncertainty of intrinsic vulnerability. The same percentiles are also shown for the three DMs analysed, namely, equal, optimal global and optimal item. Both, the values given by the group of experts identified as the best set for the uncertainty quantification at the calibration stage, and those obtained when applied the selected DM (optimal global) are represented with a thicker line. Note that the selection of the optimal-item DM would provide similar assessment of the vulnerability (see Fig. 7). In Appendix B, Figs. 11 and 12 show the quantification of uncertainty of accessibility and reliability given by the experts. These were used to evaluate experts' performance and uncertainty assessors (see Section 3.1).

From the results depicted in Fig. 7, two important conclusions can be drawn in this phase; (a) given the range of definition of the intrinsic vulnerability, the level of uncertainty expressed by the experts is very high, and (b) despite the uncertainty exhibited, OD $32-92$ (Cork-Dublin) and OD $69-92$ (Waterford-Dublin) seem to be the most vulnerable ODs in the Irish road network according to the value of the medians.

Regarding the second phase, i.e., the dependence modelling, Table 4 shows the dependence calibration score of the experts when assessing dependence according to the questions in Table 1. In the 2nd column a *dCal* score (Eq. 9) for each expert is obtained from the correlation matrix corresponding to the accessibility variables. Column 3 gives a score for the reliability variables and column 4 for a correlation matrix containing both. Columns 5 to 7 provide the normalized weights when applying the criterion of *optimal global*. Notice that the optimal global in this case is constructed with the *dCal* score and not with the calibration score in Cooke's sense.

On expressing the dependence relation of the variables studied, experts A and B exhibit good performance in relation to the accessibility, and expert D in reference to the reliability. Nevertheless, expert D is selected as the best combination of experts to assess the dependence for the variables of interest vulnerability, given the score obtained when considering the combination of both variables.

Expert C provided values inconsistent with the underlying assumptions of the model proposed (that is the assessment based on a multivariate Gaussian copula, see Morales et al. (2008)) and therefore no score has been assigned to this expert. Finally, as explained before, the criterion of optimal global is shown to provide better (or equal) dependence calibration than other combination criteria such as the equal weighted (6th row) or any individual score. Fig. 8 depicts similar conclusions, through the representation of the conditional probability distribution of the reliability for the OD $25-69$ (Limerick-Waterford) when considering the equal-weights combination, the optimal global combination, and independently experts' opinion, in comparison with the true distribution. The DM optimal global (or expert D) provides better assessment of the dependence that the other individual opinions and slightly better than the DM equal weights.

Therefore, taking into account the conditional probabilities given by expert D, $Prob(V_{i,j}|A_{i,j})$ and $Prob(V_{i,j}|A_{i,j}, R_{i,j})$, the correlation

Table 3
Scores obtained by experts and DM analysis.

Expert ID	Calibration	Mean relative information		Weight	Information relative to Equal	Normalized weight	
		All variables	Calibration variables			Equal	Optim. Global
A	1.58E-5	2.010	1.911	0	6.87	0.20	0
B	9.86E-7	1.570	1.482	0	5.33	0.20	0
C	0.0275	1.917	1.650	0.0454	5.94	0.20	0.4861
D	0.0082	1.272	1.408	0.0116	5.06	0.20	0
E	0.2441	0.394	0.197	0.0480	0.71	0.20	0.5139
Equal	**0.1135**	**0.347**	**0.278**	**0.0316**	**1.00**		
Opt. Global	**0.6827**	**0.292**	**0.406**	**0.2770**	**1.46**		
Opt. Item	**0.2895**	**0.371**	**0.496**	**0.1436**	**1.78**		

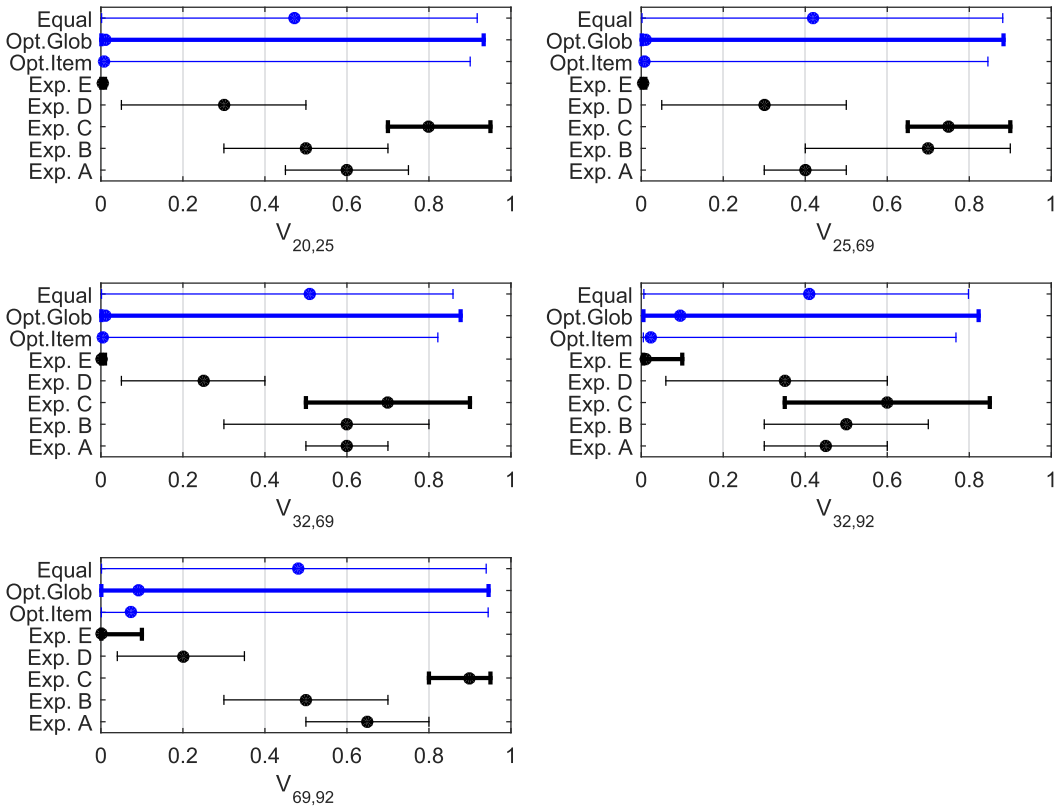

Fig. 7. Quantification of uncertainty (5th, 50th and 95th percentiles) of the intrinsic vulnerability by the experts (assessment stage).

Table 4
D-Calibration of the experts.

	D-Calibration			Weight (Opt. Global)		
	Acces.	Reli.	Both	Acces.	Reli.	Both
Exp. A	0.63	0.38	0.31	0.50	0	0
Exp. B	0.64	0.49	0.41	0.50	0	0
Exp. C	-	-	-	0	0	0
Exp. D	0.57	0.75	0.51	0	1.00	1.00
Exp. E	0.54	0.59	0.41	0	0	0
Equal	**0.64**	**0.62**	**0.49**			
Opt.Gl	**0.69**	**0.75**	**0.51**			

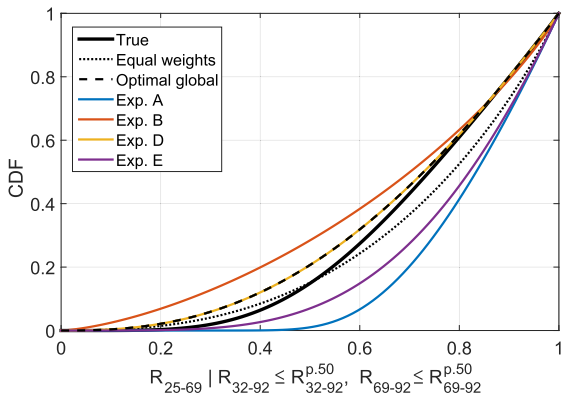

Fig. 8. Conditional probability distribution of reliability of OD 25 – 69 when considering the equal-weights combination, the optimal global combination, and the experts' opinion independently, in comparison with the true distribution.

matrix (Eq. 7) can be obtained for each OD. Table 5 shows the elements of the correlation matrix for the ODs studied and the required conditional correlation for the BN of interest. It is noted that in the case of the OD $20 - 25$ (Galway-Limerick), it was not possible to evaluate the dependence relation given that the provided conditional probabilities were also inconsistent with the underlying assumptions, which is likely due to the high absolute value of the correlation between accessibility and reliability in this case. According to the results, accessibility and intrinsic vulnerability are negatively correlated, and the reliability index is also negatively correlated to intrinsic vulnerability. Moreover, accessibility and reliability explain a low percentage of the vulnerability of the network; nevertheless, in the case of the OD $32 - 92$ (Cork-Dublin), reliability presents a higher correlation with vulnerability according to the combined expert opinion.

Based on the data shown in Table 5 and the marginal distribution of the intrinsic vulnerability elicited through Cooke's method, the joint probability distribution function of intrinsic vulnerability, accessibility and reliability is defined through a Gaussian copula. In that way, the conditional probability of the intrinsic vulnerability, V, can be determined for different values of A and R. For instance, Fig. 9 shows the conditional probability distribution of the intrinsic vulnerability for low values of accessibility and reliability (lower than their 25th percentiles).

5. Discussion

The opinions of experts have exhibited high uncertainty when assessing the intrinsic vulnerability. This may be because this type of vulnerability aims at considering all type of incidents characterised by an occurrence probability in terms of time and space of difficult estimation. Indeed, it resulted to be very challenging for the experts to consider an undefined incident, or maybe, all of them.

The present study is the first step towards a more resilience-based analysis of traffic networks. A resilient view implies the consideration of not only typically measured hazardous events, e.g. extreme weather events, allowing the assessment of the associated probability and the consequent risk analysis; but also the emerging threats, such as terrorist attacks, whose uncertainty in terms of location and intensity challenges the risk-based approach. Therefore, the identification of the most vulnerable ODs when there is not a clear identification of the potential hazard provides very relevant information (see Caschili et al. (2015), Modica and Reggiani (2015) for a more detailed discussion on resilience and vulnerability). In the case of the Irish traffic network, two ODs has been clearly identified as more *intrinsically* vulnerable than the other ODs studied.

It is also interesting to know the factors considered by the experts when assessing the intrinsic vulnerability, which are sorted in Fig. 10 according to their relative importance. Aspects such as the redundancy and the type of roads lead the listing, whereas none of them considered the type of potential incident.

In relation to the dependence modelling, the correlation values obtained allow the following conclusions; (a) reliability and accessibility are both valid indicators to assess the intrinsic vulnerability of the network, and (b) given the low correlation obtained, other indicators are required to explain a larger portion of the intrinsic vulnerability. The last conclusion can be understood when analysing Fig. 9, which shows that the OD pair $32 - 92$ is the most vulnerable, followed by $32 - 69$. However, the analysis of uncertainty showed that the most vulnerable OD pairs were $32 - 92$ and $69 - 92$. This discrepancy highlights the need of adding other indicators to estimate the intrinsic vulnerability.

The experts usually expressed themselves more confident in assessing the relation between vulnerability and the other indicators, rather than estimating the uncertainty distribution of the vulnerability. This fact points out that the elicitation of dependence is an effective tool to identify the set of indicators that better explain a large portion of some system descriptors, such as intrinsic vulnerability or resilience.

6. Conclusions

In this paper, a new methodological approach has been presented to gain some understanding of intrinsic vulnerability of traffic networks. The approach is based on the quantification of multivariate uncertainty. The 1-d uncertainty distributions are assessed through Cooke's method while statistical dependence is investigated through the extension of Cooke's method by the *DCal* score. The advantage of this approach, as shown along the paper, is that it allows the operational definition of a concept such as intrinsic vulnerability. It also allows for the quantification of its uncertainty and the dependence of this concept on other variables that partially explain this uncertainty such as accessibility and reliability as defined in Sections 2.2 and 2.3. Hence this approach is especially valuable in contexts where the characterization of the state of knowledge regarding a particular subject (intrinsic vulnerability in this case) needs to be characterized. It should be noticed that this approach is not meant as a replacement for fundamental research on the topic but rather as complementary. The uncertainty regarding the location and duration of the incidents generating such a vulnerability makes other existing approaches for vulnerability assessment very challenging. As a result of this approach, the identification of the most intrinsically vulnerable OD pairs of a traffic network has been realised. For the case discussed in this paper, reliability and accessibility are proposed as explanatory variables for intrinsic

Table 5
Dependence modelling of vulnerability given by the elements of the correlation matrix (Eq. 7).

OD	$r(R, A)$	$r(A, V)$	$r(V, R)$	$r(R, V\|A)$
20–25 (Galway-Limerick)	−0.97	−0.15	–	–
25–69 (Limerick-Waterford)	−0.64	−0.12	−0.17	−0.32
32–69 (Cork-Waterford)	−0.68	−0.15	−0.20	−0.42
32–92 (Cork-Dublin)	−0.71	−0.21	−0.35	−0.74
69–92 (Waterford-Dublin)	−0.72	−0.12	−0.17	−0.38

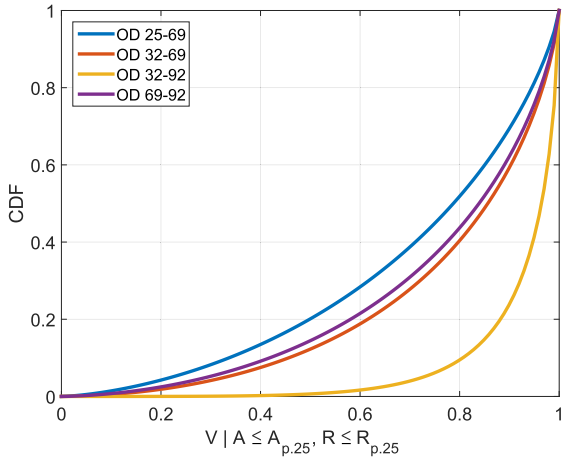

Fig. 9. Conditional probability distribution of the intrinsic vulnerability given an accessibility and a reliability ⩽ their 25th quantiles.

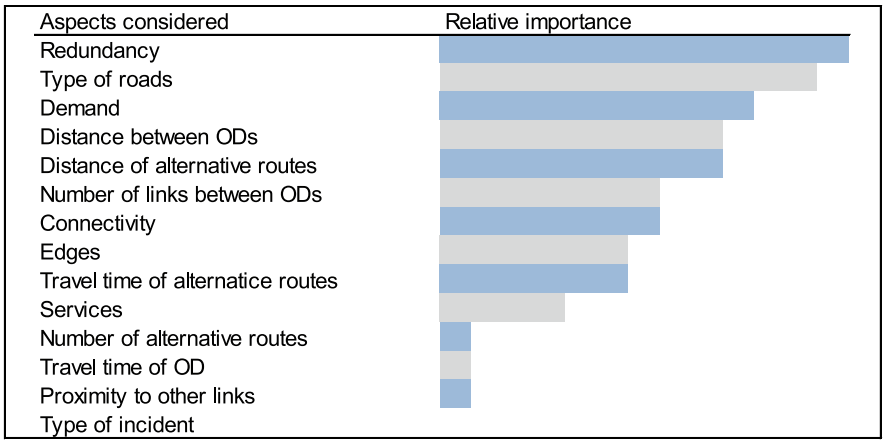

Fig. 10. Aspects considered by the experts when assessing the intrinsic vulnerability.

vulnerability and their individual contributions to the intrinsic vulnerability have been established. Nevertheless, other quantifiable indicators that when combined explain a large portion of the intrinsic vulnerability should be investigated. Given that complete scrutiny of all potentially related indicators is unapproachable, the identification of the most relevant indicators describing intrinsic vulnerability is important. Therefore future research will be oriented to the identification of other characteristics (measured by quantifiable indicators) that lead to such a vulnerability.

Once the probabilistic relationship between the descriptor and the indicators is established, the set of indicators can be used as a systematic framework to evaluate the descriptor. The framework is expressed in terms of a multivariate probability distribution function. The multivariate probability distribution function relating intrinsic vulnerability with reliability and accessibility is a straightforward tool to assess the impact of different strategies aiming to improve the intrinsic vulnerability. In that case, the values of accessibility and reliability associated with each strategy would be computed and introduced in the multivariate probability distribution function to directly determine the intrinsic vulnerability. In addition, an analysis of the multivariate probability distribution function will provide interesting insights regarding the sensibility of the intrinsic vulnerability to specific changes in the reliability and the accessibility indexes.

The method presented in this research can be applied to avoid the commonly-used approach of evaluating a descriptor, for instance the resilience of a given system, through a weighted summation of performance indicators. Without entering into discussion on how performance indicators and weights are selected, it is clear that the weighted summation does not allow the identification and removal of redundant information. For example, the connectivity degree is a factor common to both accessibility and reliability, thus a weighted summation would overweight the contribution of the connectivity.

It is noted that the proposed approach can be also applied to determine the relationship between a descriptor whose value is known and a list of indicators. In such a case, the elicitation of uncertainty may not be required.

Acknowledgment

The research has been partially founded by the COST Action IS1304. The authors are especially thankful to all experts that participated in this research for their cooperation, and to Professor Ross Corotis for his valuable inputs.

Appendix A. Questionnaire

Examples of the questions used to conduct the structured elicitation process are shown. To allow experts to provide their best estimates, definitions of intrinsic vulnerability, accessibility and reliability are given along with a detailed description of the case study. Note that the numerical values used in the elicitation of dependence refer to those presented in Table 1.

1. Elicitation of Uncertainty. Variables for calibration

Uncertainty distribution of $\mathbf{max}[\mathbf{A}_{20,25}]$: What is the maximum value of the accessibility associated with the OD 20–25, $max\,[A_{20,25}]$? Provide the 5th, 50th and 95th percentiles of the uncertainty distribution.

2. Elicitation of Uncertainty. Variables of interest

Uncertainty distribution of $\mathbf{V}_{20,25}$: What is the intrinsic vulnerability associated with the OD 20–25, $V_{20,25}$? Provide the 5th, 50th and 95th percentiles of the uncertainty distribution.

3. Elicitation of dependence. Dependencies between variables for calibration

$\mathbf{Pr}(\mathbf{A}_{25,69} > \mathbf{0.46}|\mathbf{A}_{32,92} > \mathbf{0.64})$: What is your estimate that the accessibility associated with OD 25–69 is larger than 0.46 given that the accessibility associated with OD 32–92 is larger than 0.64?

$\mathbf{Pr}(\mathbf{A}_{25,69} > \mathbf{0.46}|\mathbf{A}_{69,92} > \mathbf{0.85}, \mathbf{A}_{32,92} > \mathbf{0.64})$: What is your estimate that the accessibility associated with OD 25–69 is larger than 0.46 given that (a) the accessibility associated with OD 32–92 is larger than 0.64, and (b) the expected value of the accessibility associated with OD 69–92 is larger than 0.85?

4. Elicitation of dependence. Dependencies between variables of interest

$\mathbf{Pr}(\mathbf{V}_{20,25} > \mathbf{med}|\mathbf{A}_{20,25} > \mathbf{0.60})$: What is your estimate that the intrinsic vulnerability associated with OD 20–25 is larger than your estimation of the median of $V_{20,25}$ given that the accessibility associated with OD 20–25 is larger than 0.60?

$\mathbf{Pr}(\mathbf{V}_{20,25} > \mathbf{med}|\mathbf{A}_{20,25} > \mathbf{0.60}, \mathbf{R}_{20,25} \leqslant \mathbf{0.90})$: What is your estimate that the intrinsic vulnerability associated with OD 20–25 is larger than your estimation of the median of $V_{20,25}$ given that, for OD 20–25 the accessibility is larger than 0.60 and the reliability is smaller than or equal to 0.90?

Appendix B. Graphical results of the elicitation of uncertainty

Figs. 11 and 12 depict the 5th, 50th and 95th percentiles given by the experts when quantifying the uncertainty of accessibility and reliability, respectively. The values given by the group of experts identified as the best set for the uncertainty quantification at the calibration stage, and those obtained when applied the selected DM (optimal global) are represented with a thicker line.

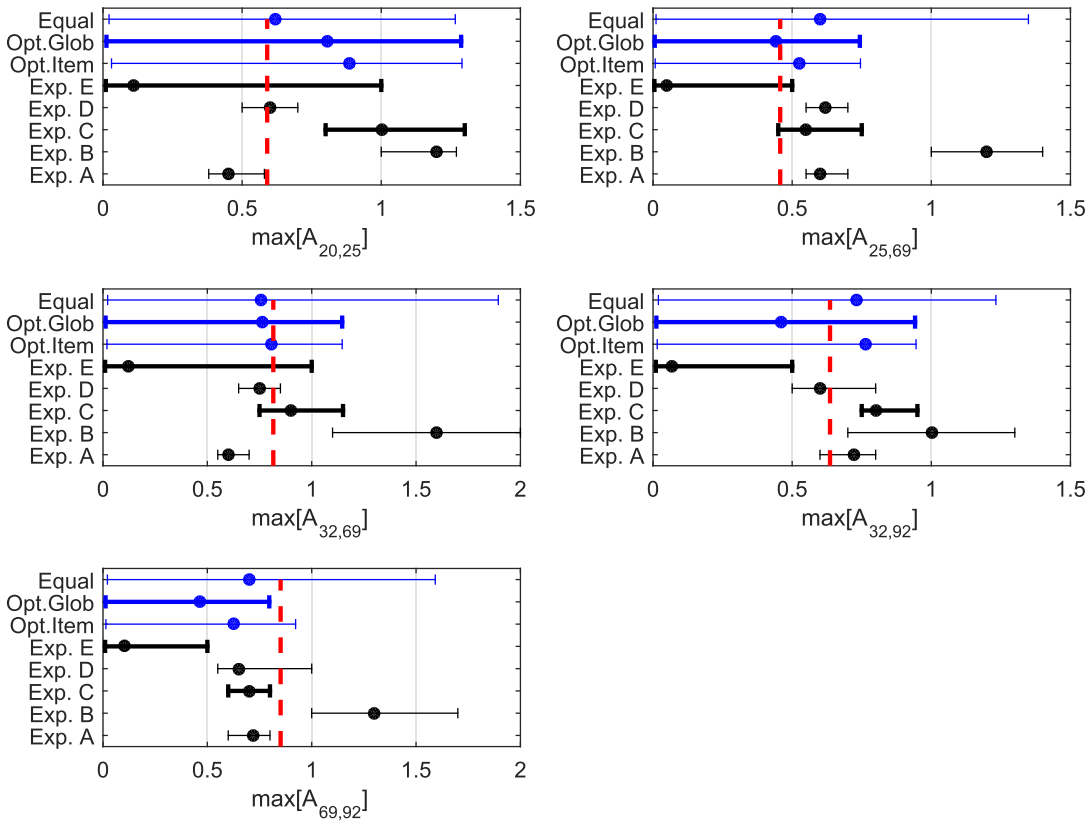

Fig. 11. Quantification of uncertainty (5th, 50th and 95th percentiles) of the accessibility by the experts (calibration stage).

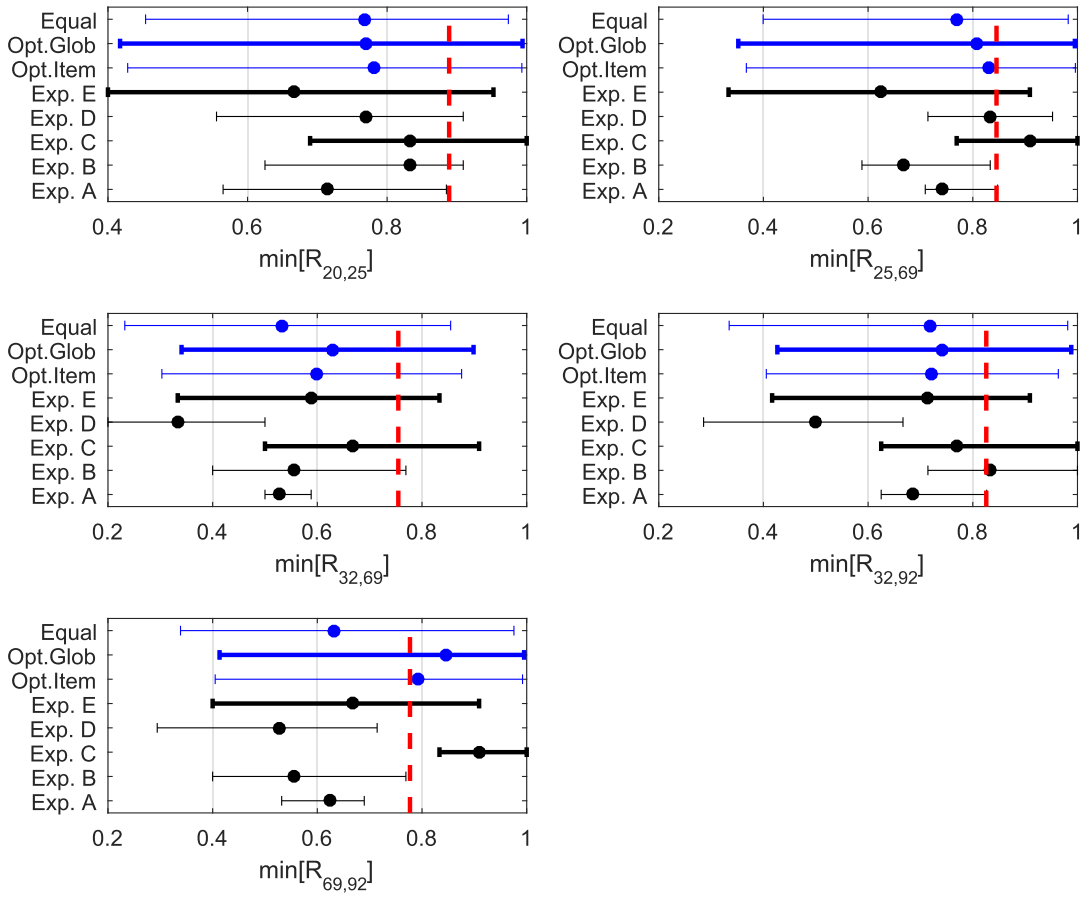

Fig. 12. Quantification of uncertainty (5th, 50th and 95th percentiles) of the reliability by the experts (calibration stage).

Appendix C. Supplementary material

Supplementary data associated with this article can be found, in the online version, at https://doi.org/10.1016/j.tra.2019.07.006.

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