Sub-scale flight test model design
Developments, challenges and opportunities

Raju Kulkarni, A.; La Rocca, G.; Veldhuis, L. L.M.; Eitelberg, G.

DOI
10.1016/j.paerosci.2021.100798

Publication date
2022

Document Version
Final published version

Published in
Progress in Aerospace Sciences

Progress in Aerospace Sciences 130 (2022) 100798

Contents lists available at ScienceDirect

Progress in Aerospace Sciences

journal homepage: www.elsevier.com/locate/paerosci

Sub-scale flight test model design: Developments, challenges and opportunities☆

A. Raju Kulkarni *,1, G. La Rocca, L.L.M. Veldhuis, G. Eitelberg

Delft University of Technology, Kluyverweg 1, 2629HS Delft, The Netherlands

ARTICLE INFO

Keywords:
Sub-scale Flight Testing (SFT)
Classical similitude theory
Governing equations based similitude
Computational scaling
Sub-scale model design methods
Flight dynamics assessment

ABSTRACT

Growing interest in unconventional aircraft designs coupled with miniaturization of electronics and advancements in manufacturing techniques have revived the interest in the use of Sub-scale Flight Testing (SFT) to study the flight behaviour of full-scale aircraft in the early stages of design process by means of free-flying sub-scale models. SFT is particularly useful in the study of unconventional aircraft configurations as their behaviour cannot be reliably predicted based on legacy aircraft designs. In this paper, we survey the evolution of various design approaches (from 1848 to 2021) used to ensure similitude between a sub-scale model and its full-scale counterpart, which is an essential requirement to effectively perform SFT. Next, we present an exhaustive list of existing sub-scale models used in SFT and analyse the key trends in their design approaches, test-objectives, and applications. From this review, we conclude that the state-of-the-art sub-scale model design methods available in literature have not been used extensively in practice. Furthermore, we argue that one sub-scale model is not sufficient to predict the complete flight behaviour of a full-scale aircraft, but a catalog of tailored sub-scale models is needed to predict full-scale behaviour. An introduction to the development of such a catalog is presented in this paper, but the development of a formal methodology remains an open challenge. Establishing an approach to develop and use a SFT catalog of models to predict full-scale aircraft behaviour will help engineers enhance confidence on their designs and make SFT a viable and attractive testing method in the early stages of design.

1. Introduction

The scarcity of fossil fuels and the tremendous growth in air-traffic is a major cause of concern for the aviation industry [1–4]. These concerns must be addressed swiftly and effectively to ensure sustainable air-travel. Many studies claim that unconventional aircraft designs, incorporating novel technologies, can offer a solution towards sustainable air-traffic growth [5–11]. Most of these claims are based on "paper" designs. Moreover, past experience shows that many designs, once manufactured at full-scale and flight-tested, demonstrated deficiencies in their flight behaviour, such as stability and control (S&C) characteristics and handling qualities, which lead to costly rework and the eventual deterioration of the overall aircraft performance [12,13]. Worryingly, most of these configurations that needed re-design were not even unconventional designs but derivatives of existing aircraft.

If the aerospace industry needs to shift towards unconventional configurations, the ability to accurately evaluate the aircraft flight-behaviour in the early stages of design cycle is essential not only to prevent costly last-minute rework but to assess their viability too. Flight testing of a prototype (*the physical device or system for which the predictions are to be made* i.e., a full-scale aircraft in aerospace applications) is the best way to ascertain the flight behaviour of a radical aircraft design [14]. However, the cost of manufacturing a prototype is estimated to be in the order of 300%–800% of the market price of aircraft currently in operation [15,16], which would make full-scale flight testing economically unviable.

In the past, as an alternative to unaffordable full-scale aircraft, sub-scale models have been used for flight testing (refer to Section 3 for historical review). These *"models" are physical devices or systems that feature similarities to the full-scale prototype such that observations on their behaviour may be used to predict the performance of (some aspects of) the prototype* [14].

Scaled models are commonly used by companies and research groups to understand the behaviour of a prototype. In aerospace industry, scaled models are used in different phases of the aircraft design

☆ This document is the result of the research project funded by the framework of the Clean Sky 2 Large Passenger Aircraft (CS2-LPA-GAM-2019) project.
* Corresponding author.
E-mail address: A.Rajukulkarni@tudelft.nl (A. Raju Kulkarni).
1 Researcher.
2 The model has its own power-plant and is not supported by stings, tethers or ground vehicles such as cars and trucks.

https://doi.org/10.1016/j.paerosci.2021.100798
Received 26 November 2021; Received in revised form 17 December 2021; Accepted 26 December 2021
Available online 31 January 2022

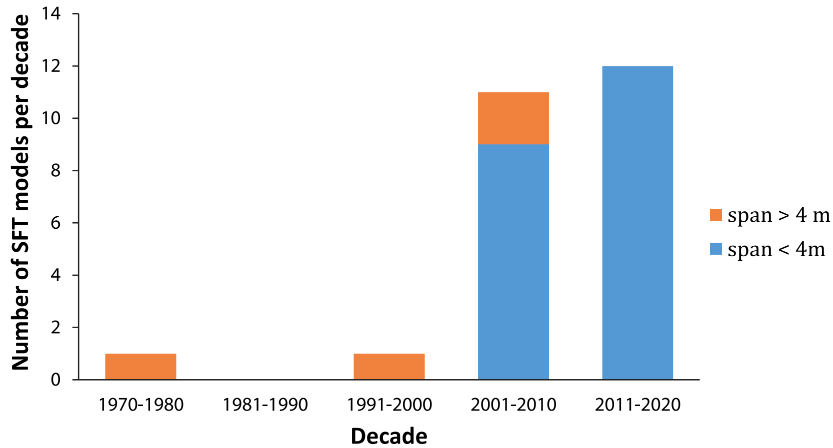


Fig. 1. Growth in the number of SFT models reported in the literature since 1970 and the reduction in the size of the SFT model after 2005.

process, such as wind-tunnel testing, drop-testing and flight testing with the intent to predict and assess different aspects of the full scale aircraft behaviour (details in Section 2). In most literature and this paper, flight testing performed in open atmosphere with powered[2] sub-scale aircraft models is referred to as Sub-scale flight testing (SFT) [17–19]. These smaller models can be manufactured quickly and at a fraction of the cost of the prototype. Besides, the risk of performing experiments with such models is much lower than the full-scale aircraft.

The literature reviewed for this paper indicates that SFT has been used in a wide-range of flight tests to study dynamic stability, control characteristics, effect of novel technologies on flight behaviour, effect of power-plant on landing and take-off distance, systems integration feasibility and as a proof-of concept for unconventional designs.

The first reported SFT dates back to 1970 but more than 90% of the reviewed SFTs have been performed after 2005 (Fig. 1). The tests performed before 2005 involved the construction of large models whose size ranged between 30% and 50% of the full-scale aircraft. Such large models (typically span size larger than 4 m) were needed to accommodate the large components and measuring devices necessary for testing. Early tests were expensive and therefore limited to well funded research organizations and large commercial entities such as National Aeronautics and Space Administration (NASA), Airbus, Boeing, Lockheed Martin etc.

The sharp increase in SFT after 2001 can be attributed to four main factors, namely, (i) miniaturization and improved performance of electronics, (ii) increased affordability of electronics, (iii) availability and affordability of Commercial Off The Shelf components (COTS) such as landing gear, ducted fans, small jet engines, pressure probes, (iv) surge in Advanced Air Mobility (AAM) and (v) increased use of rapid prototyping techniques. The impact of these developments on SFT will be discussed in detail in Section 3.

The miniaturization of electronics and COTS components in the recent years has opened up avenues for sub-scale model designs whose size can now vary over a range from 50 cm span (with micro-measurement devices) to over 4 m (with powerful yet small jet or electric engines) [20–23]. This opens new regions of sub-scale model design space which can be exploited to diversify SFT applications. Moreover, the reduction in the cost of equipment used in SFT has allowed the larger scientific and engineering community to perform SFT. As a consequence of these developments interest in SFT has grown.

Yet, the role and the actual value of SFT in the overall aircraft design process remains unclear. This is primarily due to the limited availability of dedicated literature. Chambers [24] provides a broad and non-technical monograph on tests conducted using sub-scale models at NASA until 2013. Wolowicz et al. [25] provide a generalized treatment of sub-scale model design methods using governing equations without addressing the challenges of SFT model design. Bushnell [26] discusses scaling of wind-tunnel models to primarily support aerodynamic testing. Several other review papers provide sub-scale model design approach as a side-note to another primary topic [27,28]. For example, Coutinho et al. [27] focused their review on scaling models for structural testing. Casaburo et al. [28] extend the review of Coutinho et al. with additional articles on structural sub-scale model testing in the intervening period (2016–2019). Recently, Sobron et al. [29] have published a review paper that mainly focuses on the practical challenges (such as planning and execution of flight test, data analysis, operational constraints) in performing SFT.

Apart from these publications, a comprehensive review of the role of SFT in the overall aircraft design cycle, developments and challenges in SFT model design and a well-formalized methodology to support the design of future SFT models is missing. The objective of this paper is to fill this gap by providing an up-to-date review of current trends and emerging methods in SFT model design and to support aircraft design teams.

To this purpose, we address five main aspects of SFT and its model design:

1. The comparison of SFT with other computational and experimental testing methods to identify unique features of SFT, which can be exploited to improve the aircraft design process (Section 2)
2. All the sub-scale aircraft models that have been realized till date, their key design features, and interesting trends (Section 3)
3. The evolution of various aircraft sub-scale model design approaches and their relative merits and demerits (Section 4)
4. The challenges posed in the integration of sub-scale model design, manufacturing and flight testing and the potential solutions based on literature survey (Section 5)
5. The non-technical barriers impeding the widespread use of SFT (Section 6)

Finally, we summarize our findings and offer concluding remarks on the review of sub-scale model designs and their design approaches (Section 7).

2. Positioning SFT in the aircraft design process

In this section, we discuss the capabilities and application of different testing methods employed in the design of an aircraft. Many of these testing methods (such as wind-tunnel testing, material testing) have been used for a long time to evaluate the in-flight behaviour and performance of a design. Discussing the relative merits and limitations of various testing methods helps in understanding the potential of SFT and to pin-point the key stages in the aircraft design process where

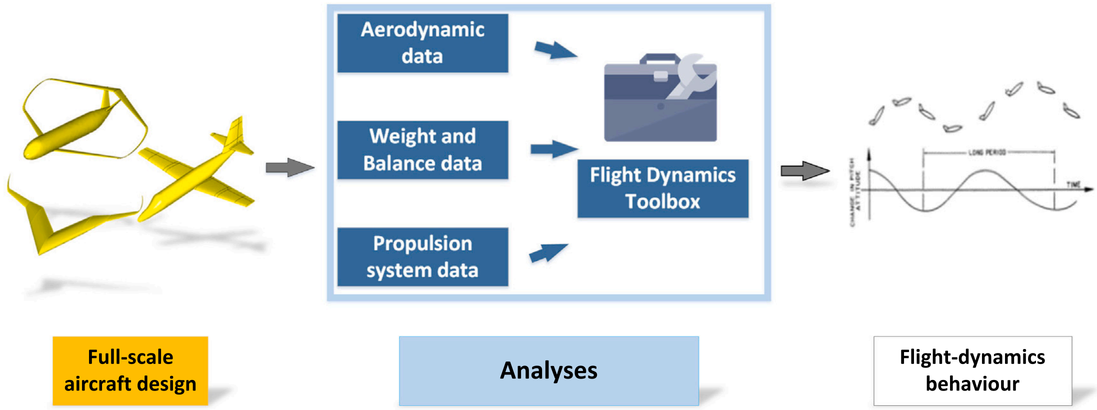


Fig. 2. Methodology to estimate flight dynamics behaviour using computational simulation or ground based testing (discussed in Section 2.1.2.1) by studying the aircraft behaviour per discipline and then combining the disciplinary analysis using a flight-dynamics tool box.

SFT can offer most benefits. In addition, we classify different testing methods based on the test objective and position them within a typical design life-cycle of an aircraft.

2.1. Testing techniques in aircraft design process

In most literature, the value and benefits of performing SFT vis-à-vis other testing methods have not been discussed. Computational simulation, experimental simulations using sub-scale models (including ground-based testing methods such as wind-tunnel testing, impact testing, etc.) and full-scale flight testing are the three main testing methods. However, full-scale flight testing is impractical in the conceptual and preliminary design stages of an aircraft. Therefore, we limit the discussion to computational and experimental simulations in this section.

2.1.1. Computational simulation

Computational simulation uses software to analyse the behaviour of a prototype. Software such as Computational Fluid Dynamics (CFD), Finite Element Methods (FEM), Multi-body Dynamics (MBD) etc. help predict the flight behaviour of an aircraft [30–32]. Such methods typically discretize a complex geometry into a sub-set of simpler geometrical entities (such as quadrilateral and triangular faces) and apply governing equations to each discrete entity to predict the aircraft behaviour. The discrepancies introduced by discretization, the assumptions and approximation in the governing equations used in the simulation and the numerical noise, all together, lead to errors in the predicted results [33,34].

Very fine discretization with higher order governing equations can alleviate these inaccuracies, but significantly increases the computational cost, rendering the use of such simulations untenable in the conceptual and preliminary design stages, when many different aircraft configurations and variants need to be investigated. For example, in Direct Numerical Simulation (DNS), one of the most accurate CFD simulation technique, the computational effort scales with the third power of Reynolds number, which typically ranges from 5 to 30 million depending on aircraft size and operating conditions. On the other hand, most numerical methods based on lower order and semi-empirical equations are predominantly developed and validated for conventional designs. Thus, their applicability to unconventional aircraft designs is unclear.

Aircraft design is a typical multidisciplinary problem. However, many computational methods are generally used for mono-disciplinary analysis. For example, CFD is used to study aerodynamics and FEM is used for structural analysis. Thus, designers are faced with the task of combining multiple disciplinary results to estimate the design behaviour such as the study of flight dynamics or aeroelasticity. In the process, the errors, assumptions and uncertainties in each of these disciplinary analyses may propagate downstream in design process and lead to erroneous conclusions about a prototype's behaviour. For example, in the estimation of aircraft flight dynamic behaviour, different types of disciplinary data (aerodynamic, weight and balance, propulsion etc.) must be provided to a flight-dynamics toolbox (Fig. 2). However, the inaccuracies in these disciplinary analyses can result in cases where stable designs are deemed unstable and vice-versa. To prevent this, engineers need to acquire clear understanding of the capabilities and limitations of their computational simulations by validating computational results with experimental simulations.

2.1.2. Experimental simulation

As discussed in Section 1, sub-scale aircraft models can be employed to study the behaviour of the full-scale aircraft [18,24,25,35,36]. These smaller models can be manufactured quickly and at a fraction of the cost of the prototype (typically less than 0.01% of the market price of an aircraft depending on the test [16,29,37–40]). Moreover, the risk of performing experiments with such models is much lower than the full-scale aircraft. Sub-scale models can either be used in ground-based facilities or for free-flight tests (see Fig. 3). Irrespective of the type of tests, the results of experimental simulations performed using sub-scale models must be ***scaled-up***. Scaling-up the model test results involves correcting all the discrepancies that occur in the simulation of prototype behaviour due to the differences in the size, shape and mass of prototype and model. These discrepancies are known as ***scale-effects***. A detailed treatment of scale effects and different methods to address them is provided Section 4.

2.1.2.1. Ground based testing. Ground-based tests are performed in large facilities (such as wind-tunnels, material testing laboratories and (aero-) engine test rigs [41–44]) that simulate the prototype operating environment artificially to extract high-quality data to predict prototype behaviour at specified flight conditions. These facilities enable better control on the test conditions than free-flight tests, thereby reducing the uncertainties in the measurements. These tests are mainly used to obtain a general understanding of the implications of new-technologies and innovations in design, where, numerical or analytical methods are unreliable due to the limited knowledge of the underlying phenomenon (see examples in Table 1) [45,46]. The models used in these tests are generally intended as proof of concept and do not mimic a specific full-scale design or vehicle (neither geometry nor in-flight behaviour). In certain cases, ground based testing is used to simulate the behaviour of specific vehicle/design [39,47,48]. However, these testing methods have limited applicability as explained in the following sections.

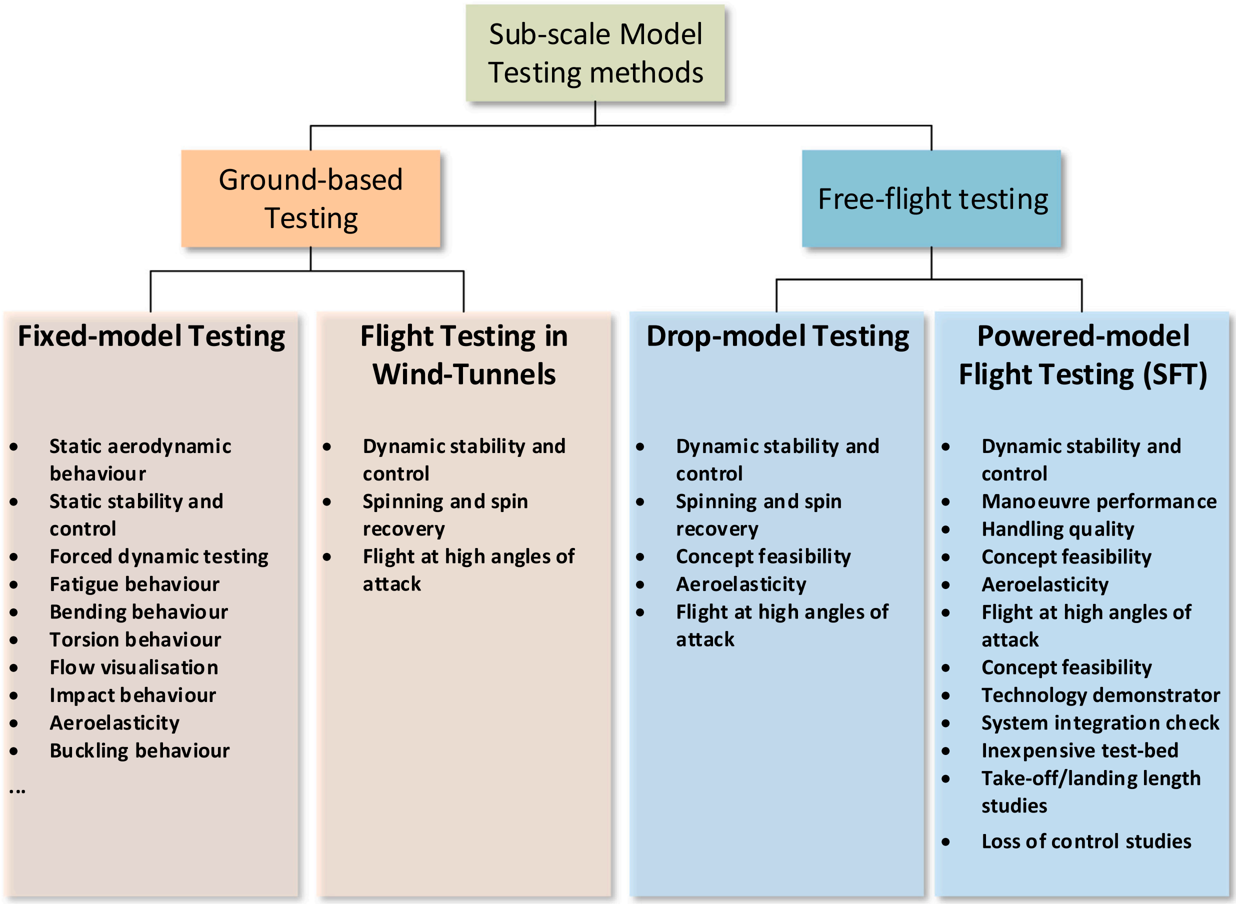


Fig. 3. Classification of testing methods that are based on sub-scale models and their common applications found in the literature [24,28,29].

2.1.2.2. Fixed-model testing. Ground-based Testing is classified into two sub-groups, namely, fixed-model testing and flight testing inside wind-tunnels (Section 2.1.2.3). In fixed-model testing, sub-scale models are attached to the test equipment with rigid supports that limit the motion of the model. Typical facilities allow one or two degrees of freedom but the sophisticated testing facilities such as those operated by NASA and DNW (German Dutch Wind Tunnels) allow up to six degrees of freedom. Some applications of fixed-model tests are listed in Fig. 3. Fixed model tests are used to evaluate both static and dynamic behaviour of a model. Examples of fixed model static tests used in aircraft design cycle are tabulated in Table 1.

The fixed model testing methods have evolved so much in the last century that they are able to generate high-quality data [24,25,39,62] that can be directly used to evaluate the prototype static behaviour. In addition to these static tests, fixed model tests are also used to perform dynamic tests. These tests are known as forced dynamic tests as the models are forced to perform a dynamic manoeuvre using an actuator [39,48,62]. Typically, all dynamic tests that require aerodynamic forces are performed in the wind-tunnels. These forced dynamic tests can be divided into two sub-groups as follows:

1. **Flight dynamics** : involves the study of combined effect of aerodynamic forces and inertia forces acting on the model (see Fig. 4). Forced-dynamic tests are used to estimate aerodynamic derivatives such as the variation of force and moments due to pitch, roll and yaw rates of the aircraft. The aerodynamic derivatives are then used with propulsion and weight & balance database of the aircraft in a flight dynamics model to determine prototype flight dynamics behaviour as shown in Fig. 2.
This method would work well if the aerodynamic derivatives predicted using the wind-tunnel testing could be directly used in flight dynamics toolbox. In practice, this is not possible due to two reasons. First, the motion of the models is forced (in one or more degrees of freedom as allowed by the testing facility). As a result, the natural dynamics response of the model is not studied (see example in Fig. 5). Second, the models are attached to the wind-tunnel using stings that affect the flow around the model and thus the aerodynamic forces and moments (see Fig. 6) [39,64]. If connected at the centre of gravity (CG) they affect the aircraft aerodynamics and thus the flight dynamics.
In a study performed by NASA with the X-48B model, Vicroy [63] found that the effect of the shape of the attachment sting on the pitching moment of the aircraft is significant. Three different types of stings were used in the wind-tunnel to determine the pitching moment coefficient as shown in Fig. 7 and compared with the (averaged) data obtained from 50 different flight tests at similar conditions. The y-axis of the figure is redacted for confidentiality reasons. However, the scale of the graph shows that support stings affect both the magnitude and the trend of pitching moment.
Therefore, the stings are often connected behind the CG to ensure that the perturbance of the stings on the flow dynamics is limited. However, this comes with negative consequences. For example, when pure pitching moment is desired, connecting the sting behind the CG will also induce plunging motion as a result, such tests cannot be directly used to predict full-scale aircraft behaviour [39].
The effect of attachment location on the aerodynamic derivatives is best illustrated with an in-house study where a RANS simulation of the pitching moment of a swept wing was performed. The pitching moment was studied about five different attachment

Table 1
Overview of static testing methods used in ground-based testing.

Test type	Description	References
Aerodynamic	Typically performed using a wind-tunnel. The goal of such tests is to acquire high-quality data that provides information on the force and moments acting on the model in different flight conditions (angles of attack, side-slip angles, flight mach number, movable deflections). In addition, such tests are also used to study the flow around the model.	[24,40,47–49]
Structural	Provide insights into the structural (bending, torsion, fatigue and buckling) strengths of different components of a model for the first few modes as the scaled model is stiffer than the full-scale aircraft. Structural engineers gain insights from these tests to predict full-scale structural behaviour.	[42,50–52]
Propulsion	The propulsion tests are classified into two categories: • Isolated tests: to study the performance of engines at different flight conditions which is usually performed in specific engine test chambers. • Engine integration tests: inclusion of a propulsion unit often modifies the airflow around the airframe and therefore the associated forces and moments acting on the model. This is generally studied in wind-tunnel.	[41,46,53–55]
Aeroelastic	Study the coupled effect of aerodynamic forces and elastic forces. These aeroelastic phenomena are shown using Collar's triangle of forces in Fig. 4.	[56–58]
Aeroacoustics	To identify noise sources in an aircraft and quantify their magnitude. Such tests are typically performed in wind-tunnels using a microphone array. For nuisance effects, both in-flow microphones and far-field scans are evaluated.	[59–61]

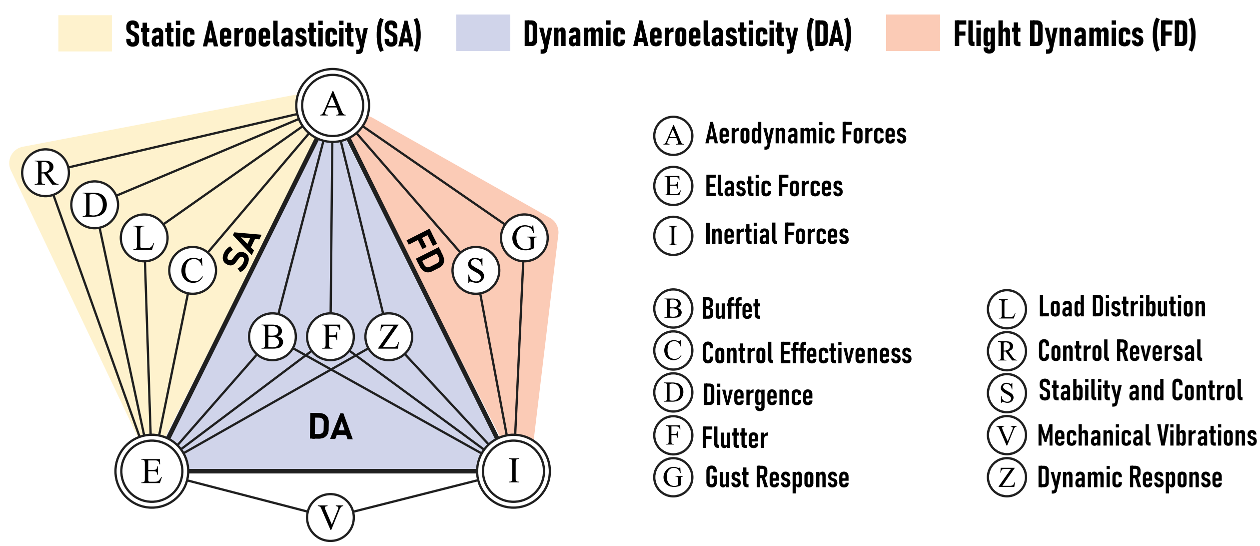


Fig. 4. Collar's Triangle of Forces showing aeroelastic and flight dynamics phenomena occurring as a consequence of interaction between aerodynamic, elastic and inertial forces [57]. Tests are performed to study each of these phenomena to ascertain the in-flight behaviour of a design..

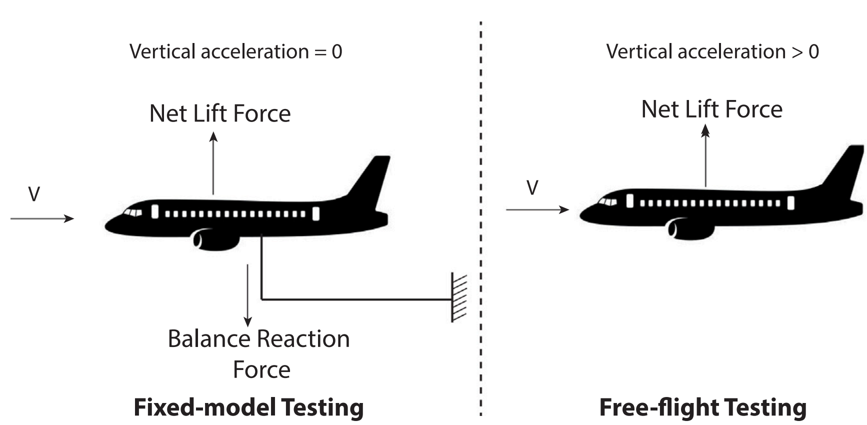


Fig. 5. Example comparing a fixed-model testing (degrees of freedom < 5) and free-flight testing. Here, the balance reaction force counteracts the net lift force of the model which results in zero vertical acceleration, which is not included in the dynamic response. Whereas, in free-flight testing (discussed in Section 2.1.2.4), the effect of non-zero vertical acceleration is accounted.

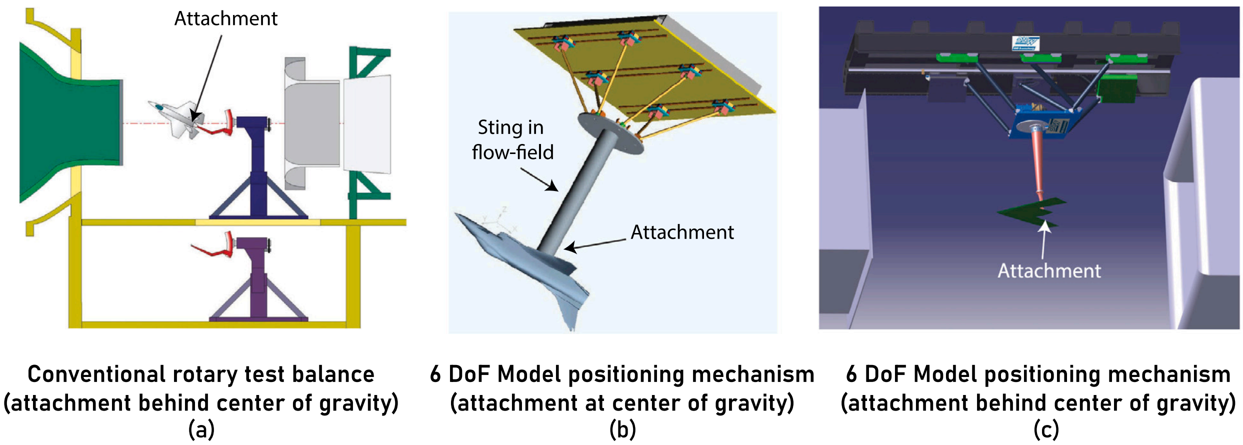


Fig. 6. Different test rigs available at DNW's state of the art wind-tunnels to perform forced dynamic motions, where, the stings are either far behind the CG (Figure (a) and (c)) or the sting perturbs the flow-field (Figure (b)) affecting the prototype behaviour estimation [39,48].

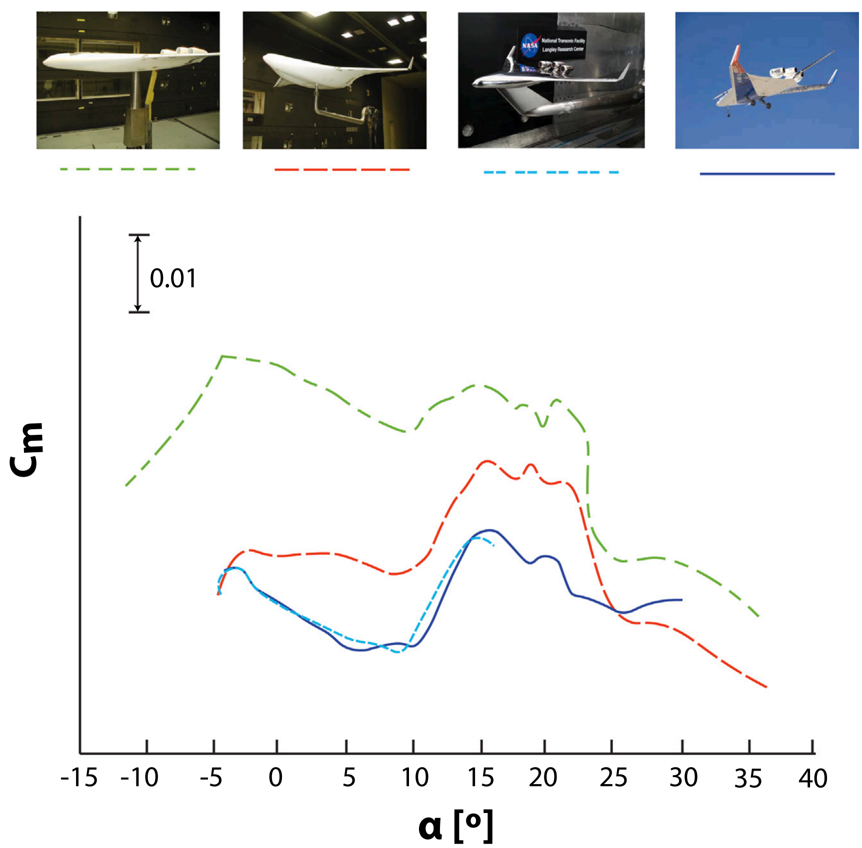


Fig. 7. The variation of moment coefficient with respect to angle of attack tested at Reynolds number of 6 million when three differently shaped stings are used and compared with a free-flight test model [63].

points as shown in Fig. 8. The motion of the root section of the wing at different time instances for these attachment points is depicted in Fig. 9.

Consequently, the movement of the attachment point changes the static and dynamic derivatives used in estimating the flight-dynamics behaviour. The longitudinal derivatives for different locations of attachment point are as shown in Fig. 10. The graph shows that the static derivatives C_{M_α} (moment derivative with respect to angle of attack) changes by 10% when the attachment point is moved from leading-edge to the trailing edge of the wing and C_{Z_α} (z-force derivative with respect to angle of attack) by 0.5%. However, the dynamic derivatives $C_{M_q} + C_{M_{\dot\alpha}}$ (moment derivative with respect to rotation rate and rate of change of angle of attack) change by nearly 35% and $C_{Z_q} + C_{Z_{\dot\alpha}}$ (z-force derivative with respect to rotation rate and rate of change of angle of attack) change by 110%. Thus, the impact of attachment location on dynamic derivatives is much more significant than the static derivatives. It is important to note that these results only depict the situation of a single-wing simulation. The changes in the moment derivatives due to shift in attachment point of a full aircraft is expected to be higher due to larger moment arm.

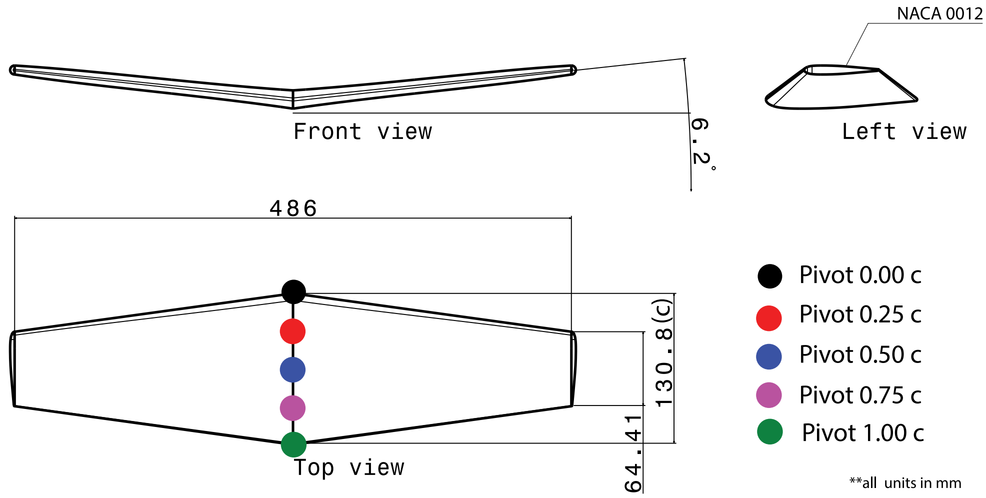


Fig. 8. Geometry of the model used in RANS simulation to evaluate the effect of five different attachment location on aerodynamic derivatives.

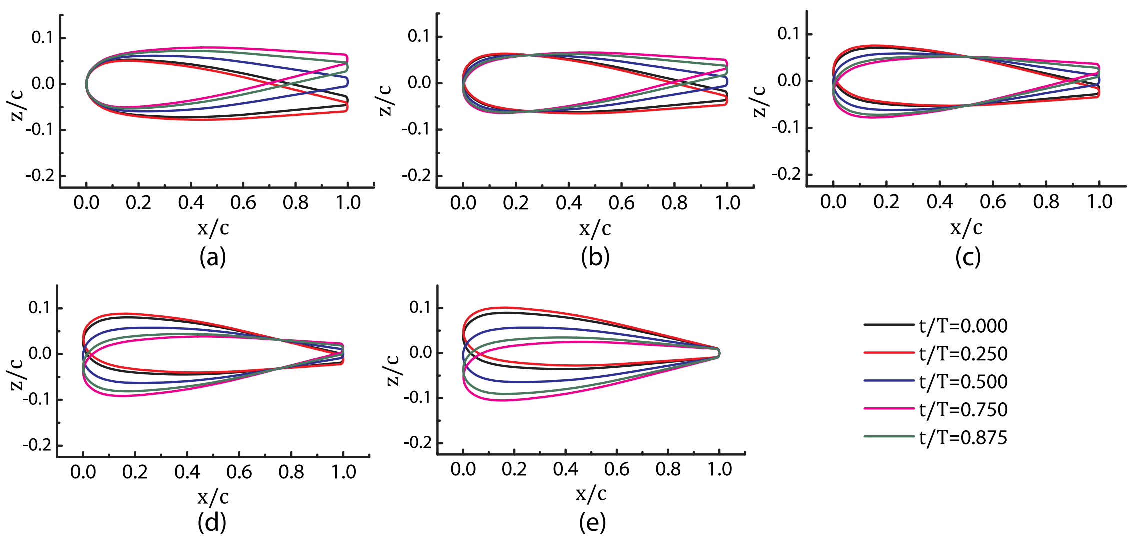


Fig. 9. Motion of root section of the wing at different time instances where the attachment point is at (a) leading-edge of the section, (b) quarter-chord of the section, (c) half-chord of the section, (d) three-quarter chord of the root section and (e) trailing-edge of the section.

Although the 6 degrees of freedom wind tunnels [39,48] are capable of complex simultaneous multi-axes motions and single axis constant amplitude and frequency sinusoidal motions, they can only solve the first challenge of degrees of freedom. The challenge associated with sting attachment induced effect persists.

2. **Aeroelasticity** : involves the study of combined effect of aerodynamic, inertial and elastic forces on the model as shown in Fig. 4 (e.g., buffet, flutter and dynamic response). These critical aeroelastic conditions can be found by observing either the free oscillation of the structure following an initial disturbance or by the response of the structure to an external periodic excitation. In the former method, the airspeed is increased until a maintained oscillation of a specific amplitude occurs. In the latter method, one or several exciters (eccentric rotating masses, air-pulse exciter, etc.) are used to excite the oscillation. At each airspeed, the amplitude response is recorded for varying exciter frequencies. The critical condition is found when the amplification becomes very large [56]. Forced dynamic testing is often used to determine the optimum location of engines or external fuel tanks. It has also become clear [56] that rigid body degrees of freedom (i.e., translation and rotation of the aeroplane as a whole) have an influence on the flutter of swept wings and tails. However, it is impractical to allow all the degrees of freedom corresponding to free flight conditions in a wind-tunnel. Thus, engineers simplify the problem by separating the constituent motion of the aeroplane in the symmetric and asymmetric types and examining them separately.

2.1.2.3. Flight testing in wind-tunnels. In order to overcome the challenges in forced dynamic testing, engineers have moved towards indoor (wind-tunnel) flying sub-scale models. Tests that allow free-flight of sub-scale models inside the wind-tunnels are known as indoor-model flight tests. Here, the models perform free motions in the available space (i.e., the models have all six degrees of freedom). Spin tests, vertical drop tests, and hover tests performed in wind-tunnel are examples of indoor-model flight testing [65–67]. Most of these tests are not truly free-flight tests because they use strings to prevent damage to the model and the facility. However, it has been found that such string supports tend to alter the dynamic behaviour of the model too [24,64].

The specifications and the capabilities of the test-facility determine the size of the model, the test-conditions, and the manoeuvres that can be performed. As per estimates by Owens et al. [62], for free-flight tests in wind-tunnel, the largest model dimension should be 1/5th of the wind-tunnel length to ensure sufficient manoeuvring space.

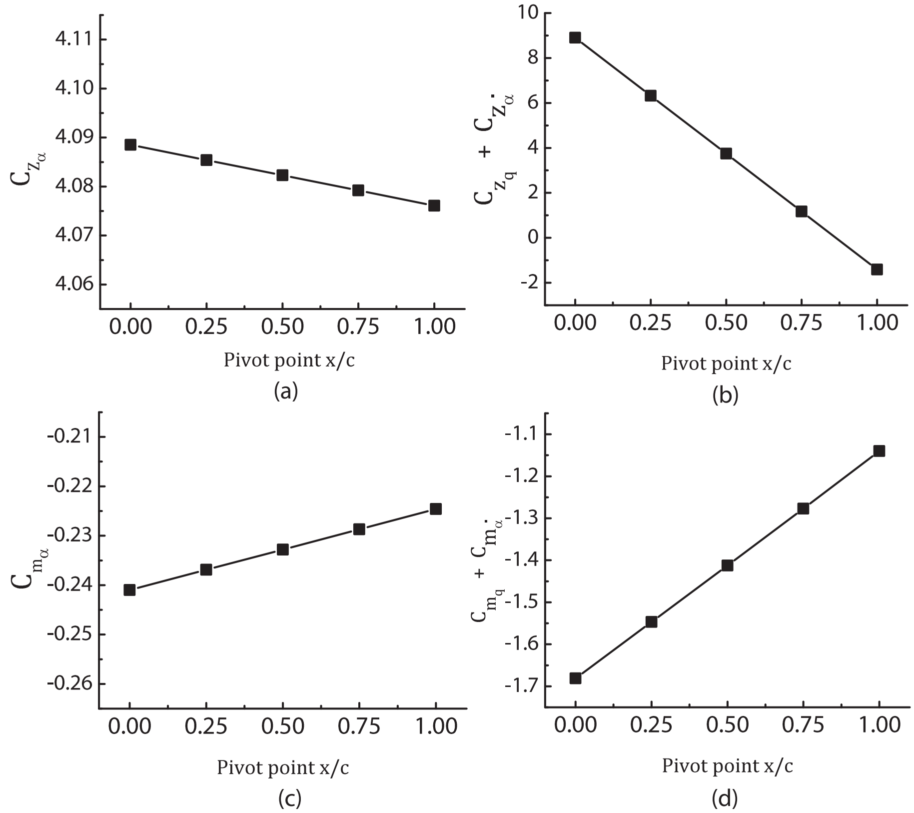


Fig. 10. Variation of longitudinal aerodynamic derivatives at different attachment locations shown in Fig. 8.

Consequently, the models end up being so small (typically less than 1 m) that they are prone to significant *scale-effects* (details in Section 3).

Thus, the constraints imposed by the test-facility either prevent large models from performing the full-range of motions necessary to study the prototype behaviour or demand the use of smaller models, thereby leading to scale effects. Notable exceptions are the very large wind-tunnels operated by NASA and DNW, which can house sufficiently large models to prevent scale effects [39,48,62].

These state of the art ground-based testing facilities offer better control over the test conditions than smaller wind-tunnels and free-flight testing methods, which reduces uncertainties in the measurements. Additionally, ground-based measurement equipment potentially have higher sensitivity, which ensures better resolution of model output (i.e., they are more accurate). However, even these state of the art methods have technical limitations as described in the preceding paragraphs. Besides, the increased control on the test conditions and improved accuracy of measurement comes at a cost. In addition to the cost of the model, per-day cost of these facilities can run into thousands of euros, where, a test-campaign generally ranges from 2–3 weeks. Moreover, the waiting time to access such scarce test facilities can stretch to months. To overcome these limitations, free-flight testing has been used an alternative.

2.1.2.4. Free-flight testing. In free-flight testing, experiments are carried out in open atmosphere with all the measurement apparatus inside the model. Free flight testing always provides 6 degrees of freedom, which allows the study of the coupled effect of all forces acting on the model. This feature is the reason why free-flight testing is especially used to study the *dynamics* of aircraft model. Furthermore, unlike ground based testing or computational simulation, there is no need to combine the results of individual disciplinary analyses, as the relationship between the tightly coupled disciplines is directly manifested in its flight behaviour. For example, unlike the process shown in Fig. 2, the flight dynamics behaviour of the sub-scale model can be directly evaluated during a free-flight test, as shown in Fig. 11. The flight dynamics response is a coupled reaction to aerodynamic, inertia and elastic forces acting on the model. In addition, these results can be further analysed using an appropriate system identification process to arrive at disciplinary data that can be used to validate other experimental or computational methods [68,69].

2.1.2.5. Drop-model testing. Free-flight tests are classified into drop-model tests and powered-model flight tests as shown in Fig. 3. In drop tests, the model is launched into the atmosphere with external aids such as launchers (ground-based or rocket) or dropped from another aircraft or helicopter [70–72].

Until 2000, drop-model testing was the preferred free-flight testing method because miniature equipment (such as flight control system, compact propulsion system, radio-controlled actuators, etc. suitable for sub-scale model) was not available in the market and most well-funded research organizations like NASA (which were predominantly involved in past free-flight tests) had easy access to drop/launch vehicle. At the same time, the need of expensive drop/launch equipment is the main reason this testing approach is scarcely employed. Moreover, this testing method ignores the effect of airframe/propulsion integration which can be a relevant contributor to the model behaviour. To overcome these challenges, powered models are used, as discussed in the following section.

2.1.2.6. Sub-scale Flight Testing (SFT). Tests with models that perform the mission with an on-board power-plant are called SFT [17–19]. Since SFT does not require a launch equipment, it can be a cost effective testing approach to study aircraft dynamics. However, fitting all the measurement equipment, energy source for the power-plant such as fuel or batteries and the flight control equipment in the limited space of the model is a challenge in the execution of SFT. As discussed in Section 1, in the last decade, the improvements and miniaturization of electronics and COTS components, complemented by advancements in rapid prototyping techniques, have opened up avenues to exploit

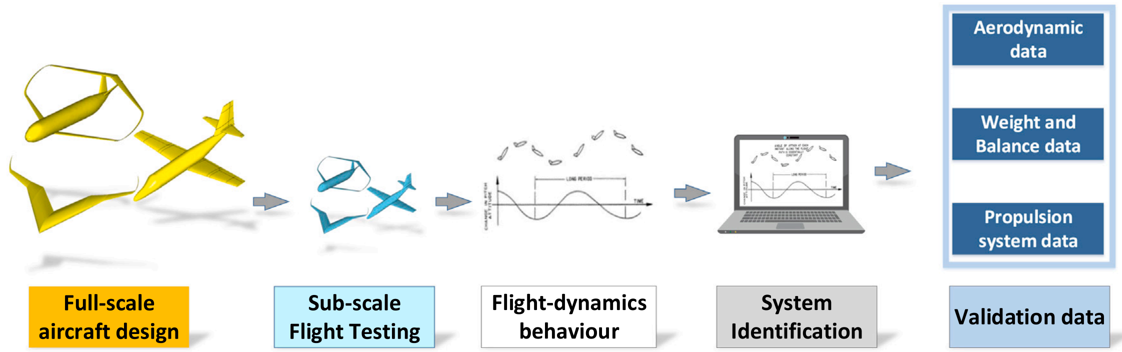


Fig. 11. Methodology to estimate flight dynamics behaviour using sub-scale flight testing, where, the coupled effects of aerodynamic and inertial forces are directly manifested in the model flight dynamics behaviour. Further insights on model's performance per discipline can be obtained by performing system identification studies.

SFT. For example, with the availability of miniature battery powered engines, it is possible to have model installations with a wide-range of thrust over weight ratio, as the weight of the scaled model is much lower than the full-scale aircraft. Thus, these latest developments offer engineers the flexibility to perform varied SFTs while still maintaining the necessary similarity requirements (discussed in Section 4).

Nevertheless, SFT is afflicted by the problem of scale effects like all sub-scale model experimental testing methods. In addition, engineers wishing to perform SFT must consider three other factors:

1. The model flies in open atmosphere, which introduces uneven turbulence, gusts, etc. that affects the model behaviour.
2. The model needs to be certified by competent authorities before it can be flown for testing. Certification of models for SFT has not reached the universality that has been achieved for full-scale aircraft. Every country has its own set of rules and local certification authorities to oversee SFT activities. In general, these authorities assess the potential damage in case of a crash and check whether the model can safely complete the required mission. Thus, based on their risk perception, local authorities can impose restrictions on the model size and test conditions. For example, in the Netherlands, where the rules are derived from European regulations for drones [73], SFTs require a certificate before flight, which are categorized into *open* (mass $< 25\,\mathrm{kg}$), *specified* (mass $> 25\,\mathrm{kg}$ and span $< 3\,\mathrm{m}$) or *certified* category ($25\,\mathrm{kg} <$ mass $< 150\,\mathrm{kg}$ and span $> 3\,\mathrm{m}$). More details can be found in the Dutch Government website [74].
3. SFT models are completely unsupported. Therefore they must be able to take off and land (with appropriate consideration for the required landing gear) and enable the test pilot to fly the test mission in an accurate and repeatable manner (by accounting the required model flying qualities) .

These critical considerations determine the very feasibility of SFT and eventually the accuracy with which full-scale aircraft behaviour and performance can be predicted. Thus, these factors must be taken in careful consideration while designing the sub-scale model to ensure that the actual value of SFT is harnessed.

These issues combined with the limitations imposed by COTS components make SFT challenging. For example, the restrictions imposed by the authorities, combined with the (limited) range of operations of radio-controlled devices that are typically used in SFT, make the simulation of the complete mission including transonic cruise impossible. Nevertheless, SFT can be used to study certain parts of the mission, where all the disciplines are coupled and the results of multiple SFTs can be used in conjunction with other testing methods to predict full-scale flight behaviour (Section 5).

2.2. Testing methods and the aircraft design process

In the early stages of aircraft life-cycle, several tests must be performed from the conception till first flight to analyse its performance and in-flight behaviour. These tests consists of physical experiments or computer-based simulation, as described in the preceding sections. Each of these tests can be broadly grouped into one of three categories based on the test objective [24,75,76], namely:

1. **Phenomenological tests:** these preliminary tests intended to improve fundamental understanding of the underlying phenomena or evaluate the impact of new technology and innovations on the prototype behaviour. Such tests are a part of fundamental research and generally not intended for the evaluation of a specific prototype design.
2. **Demonstrator tests:** to provide a proof-of-concept of new designs and novel technologies and to show that different aspects of the model can be integrated together in flight. These tests are intended to enhance the confidence of various stakeholders such as investors, airlines and the general public.
3. **Simulation tests:** to simulate the full-scale flight behaviour and draw relevant correlation to prototype flight behaviour. These tests are intended to evaluate the performance of specific vehicle design.

This classification of tests is mapped over the key steps in the development of an aircraft as shown in Fig. 12. Fundamental research, innovative ideas and advancements in technologies (both in aerospace industry and the allied fields) trigger the development of a new aircraft that incorporate these progresses. Typically, such breakthroughs are an outcome of phenomenological tests. These tests are performed to get a preliminary understanding of the physics and to estimate the potential gains from incorporating such developments.

For example, distributed electric propulsion shows promise in improving aircraft performance [11]. However, the aerodynamic interaction between the wing and the propellers is not well understood [46,54,55]. To improve their understanding, engineers perform a preliminary test with a (simple) wing and a propeller. The models used for such tests are not designed based on a specific vehicle but are minimum viable products to study a specific behaviour.

Ground-based tests are the most used methods for phenomenological tests [46,54–56,58]. Although computational simulation is used for phenomenological testing, they are generally accompanied by ground-based validation tests to quantify the impact of the underlying assumptions and approximations in the computational simulation [54,77]. Until the last decade, free-flight testing was not used for phenomenological tests, as the method was considered too expensive for preliminary studies and the accuracy of measurements from such tests was insufficient to make pertinent conclusions [24]. This has changed in the recent years and engineers are employing this method much

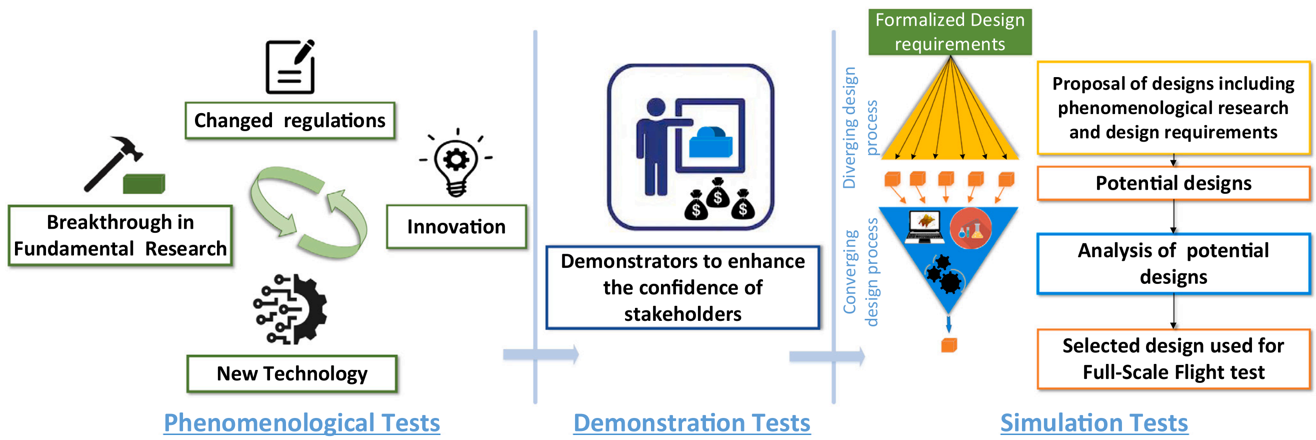


Fig. 12. An overview of different types of test methods and their role in different phases of development of an aircraft from conception till the first full-scale flight.

Table 2
Comparison of different testing methods described in Section 2.1 based on their applicability to different test objective.

Test-type	Ground-based Testing	Computer Simulation	Sub-scale Flight testing
Phenomenological Tests	✓	?	✓
Demonstrator Tests	✗	✓	✓
Simulation Tests	✓	?	✓

✓ is Applicable, ✗ is Not applicable and ? is Applicable after validation with other tests

more frequently in phenomenological tests [17,78–80]. Examples are shown in Section 3.

Since phenomenological tests are based on mono-disciplinary analysis and the models used in such tests are minimum viable products, engineers are confronted with two main questions:

1. Are the results of such tests sufficient to evoke the confidence of stakeholders such as airlines, investors and the general public?
2. Do these (mono-disciplinary) benefits actually translate into meaningful gains when all the sub-systems are integrated in an aircraft where multi-disciplinary effects are in action?

In the design of conventional aircraft, these question can be answered by experienced aircraft manufacturers as they are able to estimate the impact of phenomenological benefits (determined using ground-based testing and computational simulation) on the overall aircraft performance and behaviour based on legacy information. Furthermore, the stakeholders are not sceptical about such improvements as the topology of the aircraft does not change significantly.

However, in the case of unconventional designs, where legacy information is missing, demonstrator tests are used by engineers (see Fig. 12) to enhance the confidence of the stakeholders by demonstrating the effect of the proposed improvements on the relevant disciplines. This type of test can be performed using a sub-scale model in free-flight as they offer a natural environment to perform multi-disciplinary analysis as explained in Section 2.1. In the last decade, computational methods have been used to demonstrate multi-disciplinary aircraft system integration and are generally called Digital System Models or Digital Twins [81]. And, to the best of author's knowledge, ground-based testing methods have not been used for demonstrator testing.

Once engineers are satisfied with the results of phenomenological and demonstrator tests, they formalize the set of requirements based on which the aircraft is designed. Numerous designs, that incorporate the design requirements and the novel technology that has been studied using phenomenological tests (see Fig. 12), are proposed at the start of the design. However, not all designs can be brought into production. Thus, multiple rounds of trade-off studies are performed, where, every round reduces the contending designs until a handful of designs remain, which are evolved, matured and optimized until one design comes out.

These trade-off studies are based on simulation tests. In the past, ground-based tests, computer simulations and sub-scale flight tests have been used for simulation tests and their specific applications have been listed in Fig. 3. The initial trade-off rounds are generally based on lower order computer simulations [82,83]. However, as the design pool becomes smaller, higher order computational methods, ground-based tests and SFTs are employed [17,24,39,48,64,79,80,84,85]. Here, the ground-based tests are generally used to perform mono-disciplinary analysis and the computer simulations and SFT are used for multi-disciplinary analysis. These higher order analysis can reveal shortcomings of designs unseen in the early phases of design and prevent expensive last-minute rework.

Based on the discussion provided in this section, the applicability of different testing methods per test objective is summarized in Table 2. Notably, sub-scale flight testing is the only testing method employed for all three types of tests. The ground-based tests are not used for demonstrator tests and the computer simulations are generally not trusted without a validation using sub-scale model test. The possibility of using SFT in different phases of aircraft development makes it a useful tool in the aircraft design process. However, owing to the reasons described in Section 2.1.2.6, the widespread use of SFT has not been possible. In the following section we summarize the pros and cons of SFT, based on which, a detailed review of developments in SFT will be performed and a proposal for future progress in SFT will be made.

2.3. Summary: SFT in aircraft design process

On the basis of what has been discussed so far, the strong points of SFT, in contrast to other testing methods can be summarized as follows:

1. SFT enhances the accessibility of dynamic testing environment (for aeroelasticity and flight dynamics) to the larger engineering community, as it does not require investment is expensive infrastructure, such as wind tunnels.
2. SFT demonstrates the potential to improve the quality of dynamic testing, as no supporting (and perturbing) devices such as stings and strings are necessary to constrain the model in SFT.

3. SFT provides a natural setup for simulating aircraft behaviour that is influenced by multiple disciplines.
4. SFT is a method that can be used for phenomenological tests, demonstrator tests and simulation tests. It is useful in all phases of aircraft development, which is not always possible with other testing methods.

In order to harness these benefits, ***challenges*** posed by SFT must be overcome. The key challenges of SFT can be summarized as follows:

1. SFT is prone to scale-effects, which must be accounted for in the design of the sub-scale models.
2. compared to other testing methods, SFT poses additional constraints in the design phase as engineers must ensure the model can safely complete the required mission, as the model remains unsupported for the entire duration of the test.
3. if local authorities impose any constraints on the size and test conditions of SFT, they must be accounted in the design phase.
4. despite miniaturization of COTS components and electronics, fitting them within the limited space of a SFT model while accounting for mass and inertia (discussed further in Section 4) is a formidable task for the designers.
5. SFT model's range of operations is largely limited to sub-sonic conditions.[3]
6. SFT is performed in open atmosphere which introduces errors and uncertainties in measurements due to gusts, uneven turbulence, etc.

These challenges can be broadly classified into three categories, namely, the challenges that affect the multi-disciplinary design of the sub-scale model (1–5); the challenges posed by the limitations in the available technologies or equipment(5); and those that are inherent to the testing method (6).

Whilst the practical challenges in performing SFT and the limitations imposed by the equipment and electronics used in SFT are largely discussed in literature, the developments and challenges associated with the design of sub-scale model are not [29,37,86]. Thus, for the remainder of this paper, we mainly discuss the challenges associated with the multi-disciplinary design of the sub-scale model with relevant references to other practical challenges where necessary. This original contribution is one of the main objectives of this paper. In the following sections, we discuss the past applications of SFT, the design strategies used in those tests, the state of the art design methods that are available today and how they can be exploited to improve the applicability and value of SFT.

3. Historical perspective on sub-scale flight testing

In the preceding sections, an overview of the conceptual design cycle and the role of SFT was provided. In this section we look at key milestones in the field of SFT (see Table 3) and review the existing literature (in English Language) on SFT models that were manufactured and tested. In the process, we also look at developments in allied fields that have helped the growth of SFT.

The key milestones in the history of SFT are shown in Table 3. Although numerous models were built and flown by hobbyists, which added to the aircraft design knowledge base and furthered the understanding of aircraft behaviour, most of the efforts were not recorded in the literature. The first reference in the literature of the use of SFT in aircraft design process to simulate full-scale behaviour was done in 1979 as part of the HiMAT (Highly Manoeuvrable Aircraft Technology) program by NASA [87–90].

This aircraft was a departure from small balsa wood models, where 44% scaled composite model of a fighter aircraft weighing 450 kg was used to test aircraft behaviour. The HiMAT model was designed to reproduce the manoeuvre and pull-up behaviour at high loading conditions (12 g at sub-sonic and 10 g at supersonic speeds).

Such (large) SFT models, which were built before 2005, were affected by long manufacturing times and high costs. In 1990s, industrial techniques known as rapid prototyping were developed to manufacture models quickly at low cost. The materials used in rapid-prototyping in the 1990s were typically plastics. As a result, the properties and structural behaviour of the scaled model were markedly different from that of the prototype. Cho et al. [91] were among the first to develop similarity rules (discussed in Section 4) in manufacturing. These techniques were quickly adopted in the construction of wind-tunnel models. Chuk and Thomson [92] compared different manufacturing techniques used in wind-tunnel model design. However, the models developed using these methods were limited to testing under low loads as the materials used in rapid-prototyping were low strength plastics. Casaburo et al. provides an insight into rapid prototyping techniques used for construction of models. In the last decade, composite layup and 3D printing of metals [94] have become popular in model construction, which allows high loads on model during tests. In the case of SFT, composite layups and metal 3D printing have been predominantly used since 2005.

The next breakthrough came in the form of miniature models, i.e., models less than 30% size of the prototype, that were made possible by the miniaturization of on-board equipment [19,62,95]. These include miniature turbo-jet engines, landing gear, inertial measurement units, pneumatic systems, servomotors, etc. Before 2000, these components had to be manufactured specially for SFT, which increased the cost and waiting times for tests.

Today, a large variety of such equipment is available commercially off-the-shelf [20–23]. Since these components are manufactured in bulk, their costs have come down significantly. Furthermore, designers have a large catalogue at their disposal to choose the right equipment for the test at hand. Finally, a lot of effort also goes into optimizing the shape, size and weight of such equipment. This improves the performance of the equipment and consequently enables tests in wider range of flight conditions such as speed, altitude, angles of attack etc.

At the core of these components lie a set of integrated circuits (ICs) that are essential to enable communication between the pilot and the model in flight, power the sensors needed to measure various flight parameters and store the measured data for further analysis. The size of these ICs and their mass greatly affect the design of SFT model. Broadly, the size of an IC is determined by two factors, namely, the maximum number of transistors that can be fit inside an IC and the size of each transistor (known as minimum feature size). The review by Kurzweil [96] shows that there is an exponential drop in the size and cost of ICs and the storage capabilities of ICs have grown exponentially (Figs. 13 and 14).

All these developments translated into improvements in the electronic products used in SFT. For example, micro Secure Digital memory cards (SD cards), when introduced in 2005, had memory of up to 125 megabytes(mb) [97,98]. This low storage capacity was one of the bottlenecks in the use of SFT. For example, a typical SFT of 30 min measuring at 100 Hz with aeroprobe (approximately 20 mb), inertial measurement unit (IMU) (approximately 30 mb) and 10 unsteady pressure sensors measuring at 50 kHz (approximately 300-500 mb) surpasses the capabilities of SD card from 2005. Thus, all the measurements cannot be made simultaneously in one SFT. Today, SD cards have storage capacity of 1 terabyte, while their size has almost remained the same. Thus, many more parameters can be measured and stored in a single flight without storage capacity limitations.

Not only has the miniaturization improved the capabilities of different devices but it has also enabled tests which were not possible using sub-scale models. For example, Bunge et al. [99] installed 16 differential pressure sensors on the wings of 1/4 scale PA-18 Super Cub,

[3] This is based on the current limitations imposed by COTS equipment and certification authorities. In the future, with sufficient improvements in technology and relaxation in certification requirements, miniature models might be able fly long distances at transonic and possibly supersonic speeds.

Table 3
Timeline of key milestones in the realization of SFT based on the information available in the literature.

Milestone	Year/Inception Date	References
Tests with balsa models that have geometrical shape similar to the prototype (results not recorded in the literature)	**pre-1979**	–
First SFT model reported in the literature (model having 10 m span and 450 kg mass)	**1979**	[87–90]
Introduction of rapid prototyping and composite materials in the manufacture of sub-scale models	**1998**	[28,91,92]
First miniature SFT model built and tested (span smaller than 4 m)	**2006**	[62]
First computationally scaled model designed and tested	**2010**	[93]

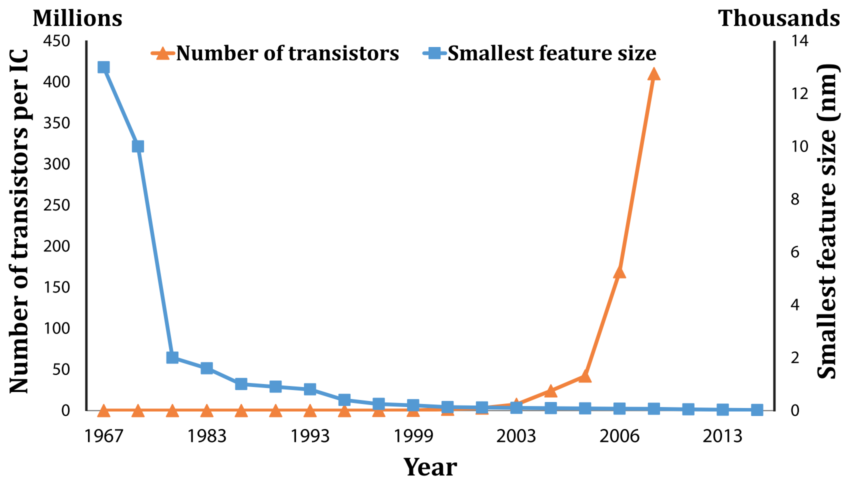


Fig. 13. Miniaturization of electronics due to exponential increase of number of transistors per integrated circuit (IC) and the reduction in the size of transistors.

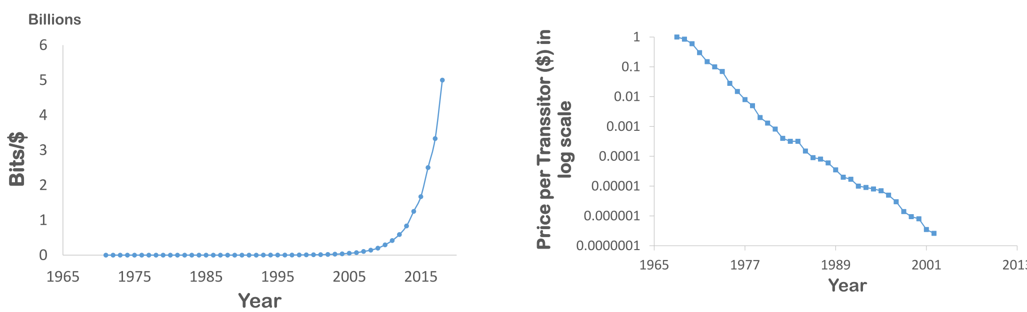


Fig. 14. Improvements in cost-effectiveness of electronics.

which proved to be sufficiently accurate. Such in-flight surface pressure measurement capabilities on SFT models were non-existent before 2005. Furthermore, miniature autopilot systems like Pixhawk, which have come into existence after 2008, have opened up the possibility of autonomous flight and on-board computer-vision [100]. Such systems are enabling widespread use of SFT by reducing the pilot effort and improving the safety of the SFT models.

Another major reason for the surge in SFT models after 2005 is the increasing prevalence of Unmanned Aerial Systems (now called Advanced Air Mobility (AAM)), which are smaller (compared to general aviation aircraft) and cheaper than conventional aircraft [101,102]. These are intended to improve the urban mobility of the future. The AAM aircraft design process generally includes the flight test of both sub-scale models and full-scale aircraft, whose size is often comparable to SFT models of conventional passenger/military aircraft. Thus, the growing interest in AAM aircraft has also added to the increase in number of SFTs and resulted in the improvement of SFT components and design practices.

In the last decade (2010–2020), utilizing these advancements, many tests have been conducted using varied design approaches, manufacturing methods, and test objectives. Tables 4–6 list different sub-scale aircraft models that have been recorded in the literature and specifically used for SFT. While these tables present a brief description of individual tests, the detailed treatment of manufacturing methods, control-laws, and specifications of on-board equipment are not included. In the remainder of this section, we analyse the overall trends in SFT concerning the objective of the test and the applications of SFT. Based on this analysis, we formulate the key tasks that must be performed to successfully complete SFT.

3.1. Test-objective based classification of SFT

As discussed in Section 2, SFT can be classified into three categories, namely, demonstrator, phenomenological and simulation tests. Of all the SFT reviewed in the literature addressed here, 52% of the models were used for demonstration, 20% to study specific phenomena, and 28% to simulate the full-scale flight behaviour. This is shown in Fig. 15. In the following paragraphs, we categorize different classes of SFT applications based on the type of test they require.

3.1.1. Applications requiring demonstration tests

Applications that use demonstration SFT models are categorized into the following classes:

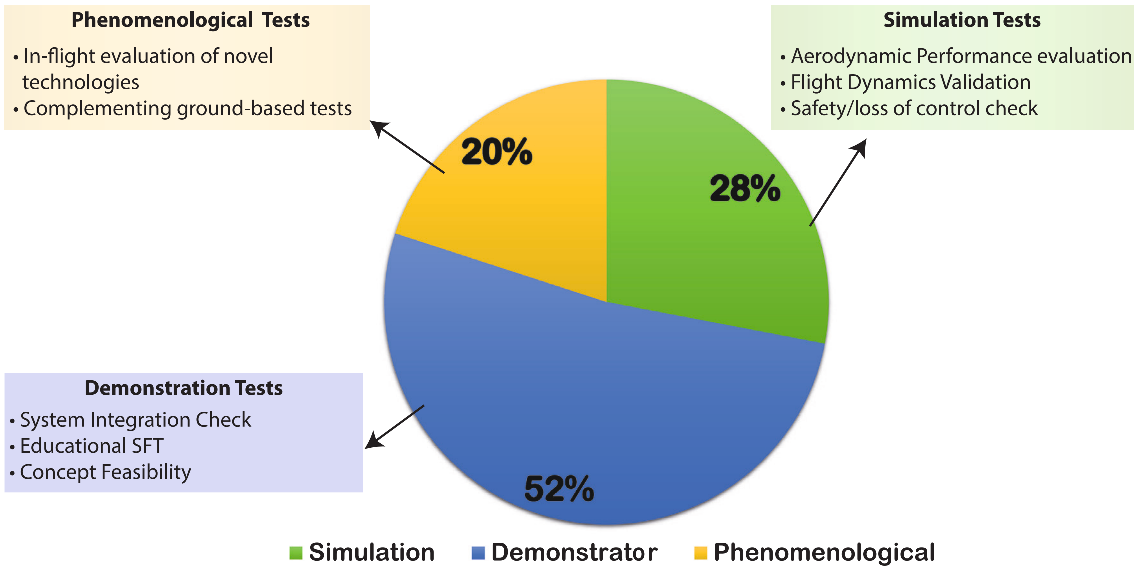


Fig. 15. Distribution of SFT based on test objective and classification of applications per test objective.

1. **System integration**: to study whether different sub-systems can work together while ensuring a safe and efficient flight for a given design. Examples of such system integration tests are the Faser project [62], DUUC project [103,104] and the Eclipse Project [36,95]. Such tests are also used to improve SFT techniques by assessing the performance of different COTS components in flight. For example, SFT to develop data acquisition, transmission and storing techniques also belong to this category.
2. **Concept feasibility**: models that are not designed with any particular prototype design as a reference. They merely act as an engineer's impression of prototype design.
3. **Educational SFT**: employed to teach students the principles of flight, aircraft design, and manufacturing techniques.

Examples of SFT intended for demonstration are shown in Table 4. The design effort in these tests is not as high as the simulation tests and the phenomenological tests because the model is not required to behave exactly as the prototype (even if its design exists at the time of demonstration tests). Accommodating all the on-board equipment within the model and ensuring that the model can complete the mission safely are the only requirements. Therefore, in most cases, engineers do not use a specific design approach when designing demonstrator SFT model (as shown with NA in Table 4) but only strive to design a model which can complete the mission safely.

3.1.2. Applications using phenomenological tests

Phenomenological SFT models are employed in two main types of applications:

1. **In-flight evaluation of novel designs and technologies**: preliminary test to understand the potential impact of unconventional design, novel control-system or technology on the overall flight performance and behaviour. For example, to validate flight control laws or to study the noise of emission of an unconventional aircraft during approach.
2. **Complementing ground testing**: tests that cannot be performed using ground based tests such as the evaluation of the effect of a propulsion unit design on take-off length. [18,119]

Here, design is more involved than demonstration tests, as the specific phenomenon being tested must be replicated in addition to satisfying the requirements of a demonstrator test. There are a number of design methods that are typically used to design sub-scale models (discussed further in Section 4). For phenomenological tests, engineers often apply the simplest scaling method, known as geometric scaling. The results of such tests are only applicable to the prototype if there are no scale effects, which is difficult to avoid in practice. Nevertheless, geometrically scaled models are considered to be sufficient to get a preliminary qualitative understanding of the prototype behaviour [24,25]. Table 5 lists examples of SFT used in phenomenological testing.

3.1.3. Applications employing simulation tests

Simulation tests are used to study the performance or in-flight behaviour of specific full-scale aircraft design after sufficient design maturity has been attained (Fig. 12). Although the applications of simulation tests appear similar to that of phenomenological tests, the difference is that the results of the former are only applicable to a specific vehicle configuration and the latter is intended for generalized understanding of the underlying phenomenon. Simulation SFT have been used in the following applications:

1. **Aerodynamic performance evaluation**: drag estimation, high angle of attack behaviour, and high side-slip angle characteristics
2. **Flight dynamics evaluation**: estimation of stability and control derivatives, manoeuvre performance, and handling qualities
3. **Safety and loss of control situation**: simulate extreme flight envelope scenario to determine the safety of a design

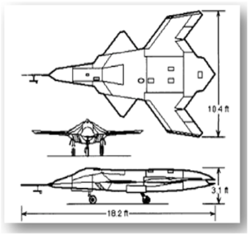
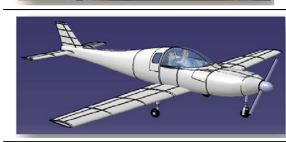
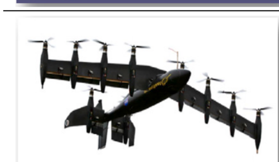
It is interesting that computationally scaled models (discussed further in Section 4) have been mainly used in simulation SFT cases, which makes scaling for simulation very challenging. Despite numerous simulation SFT tests, only a handful of simulation SFT results have been validated. In some cases such as X-48B, e-Genius Mod and NEXST [17,80,124], the authors claim similitude between the model and the prototype but very little quantitative information is provided. Examples of past simulation SFTs are shown in Table 6.

3.2. Key tasks in SFT

Based on the review of the literature, we have identified and classified the key tasks that must be performed to accomplish SFT. These tasks can be grouped into four main sub-categories (Fig. 16), as follows:

1. The design of the SFT model
2. Manufacture of the model and installation of COTS components

Table 4
Past sub-scale flight tests used for demonstration testing.

Model Image/Drawing	Model name and goal	Organizations	Design approach	References
	**X-36b** (1997) Demonstrate tail-less stealth design using a model which was approximately 565 kg and a span of 5.3 m	NASA	Geometric Scaling (28%)	[105,106]
	**FASER** (2006) System integration test to study flight data acquisition system and techniques on a conventional aircraft	NASA	NA	[62]
	**RAVEN** (2008) Teaching aid to educate students on the principles of SFT design, manufacture and testing	Linköping University	Geometric scaling (13.8%) & Froude number scaling	[19]
	**ECLIPSE** (2009) Flying demonstrator for circulation control devices and fluidic thrust vectoring to replace the conventional ailerons	BAE Systems, Imperial College London, Universities of Cranfield, Leicester, Liverpool, Manchester, Nottingham, Southampton, Warwick, Wales and York	NA	[35,36,95,107,108]
	**ECO-Sport** (2010) Teaching aid to educate students on the principles of SFT design, manufacture and testing	Linköping University	NA	[109]
	**GL-10** (2010) Demonstrate the transitions from hover to wing borne flight and from wing borne flight back to hover in a reliable and repeatable way	NASA	NA	[110]

(*continued on next page*)

3. Planning and execution of flight test
4. Interpretation and scale-up of SFT results to predict the prototype behaviour.

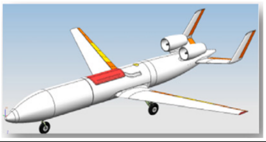
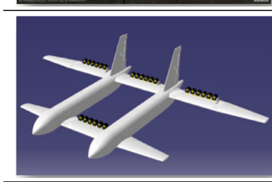
While many works in the literature deal with one or two of the these categories, all of them are rarely treated together. Nevertheless, it is important to note that these tasks are inter-linked. A bad design makes the model unsuitable for flight or poorly similar to the prototype whereas bad realization (i.e., manufacturing or flight test) of SFT will adversely affect the quality of the results, thus, rendering the test useless. Thus, every SFT design must holistically deal with these aspects to improve the applicability and use of SFT. This is discussed further in Section 5. Each sub-category of tasks is briefly described in the following sections.

3.2.1. SFT model design

Certification Compliance: Unlike other testing methods, no direct constraints are imposed on the size of the model in SFT as long as the model can take-off, perform the required mission and land safely. To alleviate the risk of damage in the event of crash, most governmental authorities impose certification requirements, which typically include the maximum weight of the model and the dimensions of the flight box of the model (i.e., the farthest distance in the airspace to which the model can be flown from the point of lift-off). These requirements must be taken into consideration during the design of the model. For example, if the flight-box is small, the model needs to make sharp turns, which can lead to high forces on the model and must be accounted in the structural design of the model.

Furthermore, the mission profile of SFT (i.e., a detailed description of an aircraft's flight path and its in-flight activities) needs to be such that the model can perform the manoeuvres of interest within the available airspace. For example, in the study of flight dynamics behaviour, the SFT model must be able to return back to its equilibrium after being perturbed from steady level flight within the flight-box to successfully complete the test. Inability to finish the required motion or manoeuvre within the flight box, renders the SFT unusable to for that study.

Table 4 (*continued*).

Model Image/Drawing	Model name and goal	Organizations	Design approach	References
	**Flexi-Bird** (2010) Sub-scale model to study environmental and safety issues	Warsaw University of Technology, University of Stuttgart, ONERA, Airbus, NLR, FOI Stockholm	NA	[111,112]
	**DUUC** (2016) Concept feasibility study demonstrating propulsive empennage	TU Delft	Geometric Scaling (5.5%)	[9,103,104]
	**Avistar Elite** (2019) System integration test to validate flight data acquisition system in flight	Technical University of Munich	NA	[113]
	**ALBATROSS** (2019) Concept feasibility study of semi aero-elastic wing-tips for improved efficiency	Airbus	NA	[114]
	**MAVERICK** (2020) Concept feasibility study of blended wing body that promises environmental performance benefits	Airbus	NA	[115]
	**DEP STOL** (2020) Concept feasibility study to explore aero-propulsive coupling effect in distributed electric propulsion aircraft	Northwestern Polytechnical University	NA	[116]
	**Flying-V** (2020) Concept feasibility study of a Flying-V aircraft	TU Delft	Geometric Scaling (4.65%)	[117,118]

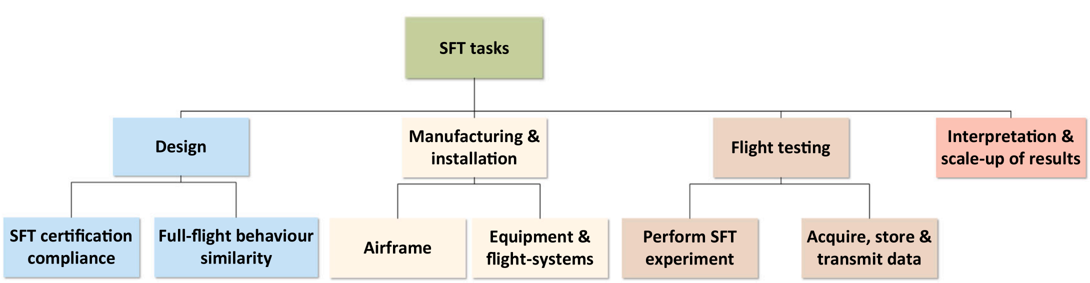
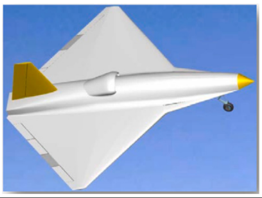
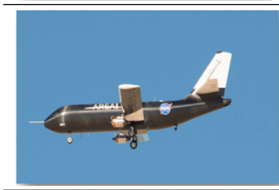


Fig. 16. Overview of key tasks in SFT.

Similarity to the prototype: The key aim of a sub-scale model simulation test is to predict prototype behaviour, which is possible only if the behaviour of the sub-scale model is similar to prototype. In the context of similarity, numerous questions must be answered. For example, when is a model said to be similar to a prototype? What methods can be used to make sure that a model behaviour is similar to the prototype? Furthermore, even in case of similarity between a model and the prototype, how can the results of SFT be used to predict full-scale behaviour.

These questions have been asked for over a century in numerous fields. Section 4 provides a detailed overview of the evolution of

Table 5
Past sub-scale flight tests for phenomenological testing.

Model Image/Drawing	Model name and goal	Organizations	Design approach	References
	**Super-Ximango** (2008) Test to characterize the aerodynamic performance and stability	University of Arizona, Advanced Ceramics Research	Geometric Scaling (20%)	[120]
	**DEMON** (2009) Extension of ECLIPSE model to understand aerodynamic phenomena	BAE Systems, Imperial College London, Universities of Cranfield, Leicester, Liverpool, Manchester, Nottingham, Southampton, Warwick, Wales and York	Computationally scaled model of ECLIPSE model	[35,36,95,107,108]
	**PTERA** (2014) Test to characterize aerodynamic performance and stability	NASA	Geometric Scaling (11%)	[18]
	**GA-USTAR** (2017) Test to understand stall/upset aerodynamic behaviour of a Cessna 182 model	University of Illinois at Urbana–Champaign	Geometric Scaling (20%)	[121]
	**Super-STOL** (2019) Test-bed to determine the effect of propeller, wing, and flap design on maximum achievable lift coefficient	Massachusetts Institute of Technology	Geometric Scaling (30%)	[119]

similitude criteria and how they can be used in the design of sub-scale model to ensure with the prototype. Despite numerous similitude criteria, the applicability of the these criteria is often limited owing to vast differences in the test conditions of the model and the prototype.

3.2.2. Manufacture of airframe and installation of COTS components

Once designed, the SFT must be manufactured, which involves two main tasks, namely, manufacture of the airframe and installation of equipment necessary for SFT (Fig. 16). The airframe typically includes the skins, ribs, spars, frames, bulkhead etc. Often, for sub-scale models under 0.5 m span, the monocoque structure is sufficient (i.e., skins carry all the structural loads) [128]. However, for larger models, other structural components might be necessary depending on the mission. The equipment includes the engines, landing gear, batteries, actuators, flight controller etc. and the components needed to measure the flight behaviour such as pitot tubes, inertial measurement units, accelerometers etc. Depending on the requirements and scale of the model, these components are either purchased commercially off the shelf or manufactured in-house to meet specific requirements.

3.2.3. Flight test and data acquisition

At the end of manufacturing phase, the SFT must be performed which includes training the pilot, multiple ground tests and performing the required mission. Furthermore, appropriate mechanism should be set in place to capture, store and transmit data collected by various equipment in flight. Reader is referred to the work of Sobron et al. [29], Jordan et al. Jordan et al., Kuehme et al., Kuehme et al. and Hueschen [129] for more details on the selection and integration of equipment and its use for testing.

3.2.4. Interpretation of the SFT results

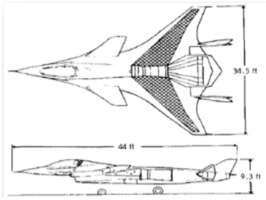
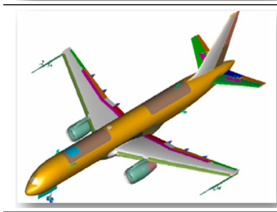
After the required measurements are recorded, they must be used to predict the prototype behaviour (in simulation and phenomenological tests). This process is known as ***scale-up*** of results. As a general rule, scale-up is only possible if the model experiences streamlines, forces and moments that are similar to the prototype (i.e., their ratios are equal) as discussed in Section 4.

Often, complete similarity of streamlines, forces and moments is not possible due to differences in shape, size and test-conditions of the model. In such cases, engineers utilize partially similar models (i.e., only certain forces, moments and streamlines are similar) to study a specific aspect of full-scale aircraft behaviour. Furthermore, the methodology of scaling up the SFT results in case of partially similar models also depends on the design approach. A detailed treatment of different SFT model design strategies, different types of partial similarity (also known as scaling laws), the conditions for similarity per SFT model design approach and the corresponding methodology for the scale-up of results is discussed in Section 4.

4. Sub-scale model design approaches

A sub-scale model should be designed such that its in-flight behaviour is similar to the prototype, at least for those features that must be studied. In order to determine similarity (or the lack thereof), numerous theories, also known as similitude theories, are available. Attempts to develop such theories started well before the inception of flight, when engineers looked at approaches to avoid *"costly mistakes"* in the design of hydraulic structures, channels and harbours, hydraulic

Table 6
Past sub-scale flight tests used for simulation testing.

Model Image/Drawing	Model name and goal	Organizations	Design approach	References
	**HiMAT** (1979) Simulate the manoeuvre performance of the prototype with a model that handles 12 g at sub-sonic and 10 g at supersonic speed	NASA	Computationally Scaled (44%)	[87–90]
	**NEXST-1** (2005) Simulate the supersonic aircraft drag prediction.	Japan Aerospace Exploration Agency, Sankoh Software DEPT. Co. LTD	Geometric Scaling	[80]
	**AirSTAR** (2006) Simulate prototype flight dynamics and loss-of control situations	NASA	Geometric Scaling(5.5%) & Froude number scaling	[79,122]
	**X-48B** (2007) Simulate flight performance, stability characteristics, high angle of attack behaviour, and high side-slip angle behaviour of blended-wing body	NASA, Boeing	Geometric scaling (8.5%)	[78,123,124]
	**Generic Future Fighter** (2010) Test to study vortex induced at the canard of a fighter-aircraft and its effects on the aircraft	Linköping University	Computationally Scaled (13%)	[125,126]
	**Cirrus SR22T** (2018) Test for dynamics model validation by matching inertia	University of Illinois at Urbana-Champaign	Geometric Scaling (21%) & Mass Scaled	[69,127]
	**e-Genius Mod** (2019) Test with modified airfoils to match full-scale glider behaviour	University of Stuttgart	Computationally Scaled (33.3%)	[17]

machines and ships [76]. The design approaches for similitude are not limited to SFT. Indeed, most of the concepts discussed in this section originated from the design of sub-scale models other than SFT, although they are applicable to SFT too.

Some of the pioneers who developed theories for model testing were famous scientists such as Froude, Stokes and Reynolds (Section 4.1) [130–132]. While their scaling laws are used till date, they were not an integral part of the model design process until the Buckingham π-theorem [133] and fractional analysis [134] were conceived to formalize the idea of similitude (Section 4.2). These two developments were expanded to formulate the classical similitude theory (Section 4.4). Kline [135] proposed the use of governing equations and approximation theory to establish similitude (Section 4.5). However, this similitude theory had its challenges (Section 4.5.3), which were later overcome by the computational similitude theory developed in the 1990s (Section 4.6). A timeline of evolution of these theories is shown in Table 7.

4.1. Dimensional analysis

Dimensional Analysis is a general method by which we deduce information about a phenomenon based on the premise that a phenomenon can be described by a dimensionally correct equation constructed using physical parameters that influence the phenomenon [75, 76,135]. This method can be used to simplify the high-dimensional

Table 7
Evolution of sub-scale model design approaches.

Design approach	Year of inception	References
Dimensional Analysis	1761	[130–132,136]
Model Laws	1915[a]	[75,75,76,76,133–135,135,137,137–139]
Scaling Laws	1951[b]	[14,24,25,75,75,76,135,140,141]
Classical Similitude Theory	1950	[14,24,25,75,76,140,142]
Similitude using governing equations and approximation theory	1965	[75,135,137]
Computational Similitude Theory	1990	[93,137,143–145]

[a]Numerous model laws were formulated much before 1915. However, their utility in establishing similitude and generalization only happened after the introduction of Buckingham's π-Theorem.

[b]The idea of scaling laws seemed to be present earlier than this. However, the first formal application and articulation of this idea appears in the work of Langhaar [76].

problems by reducing the number of system parameters, thereby reducing the number of variables to be considered in a test. In addition, dimensional analysis is useful in establishing dimensionless numbers [75,76,135] that are convenient figures of merit used to compare the characteristics of a prototype and its model, irrespective of their size. In aerospace applications, dimensional analysis has been used to establish various dimensionless numbers to compare aircraft of varying sizes. Coefficient of lift (C_L), coefficient of drag (C_D), moment coefficient (C_M), coefficient of pressure (C_P), coefficient of thrust (C_T), Reynolds number (Re), Froude number (Fr), Strouhal number (Str) are some commonly used dimensionless numbers.

4.2. Model laws

Dimensional homogeneity is a sufficient condition to establish a dimensionless number. However, not all dimensionless numbers are meaningful. A dimensionless number can only be used if it influences the phenomenon being tested. For example, non-dimensional boundary layer thickness expressed as a ratio of boundary layer thickness to length of a runway is not useful, as the size of the model has no impact on this dimensionless number. However, when the dimensionless number is expressed as the ratio of boundary layer thickness to the mean aerodynamic chord of the model, the dimensionless number can be effectively used to compare model and prototype behaviour.

Mathematically, model laws can be expressed using the following relationship:

$$N_{model_i} = N_{prototype_i} \tag{1}$$

where, $i = 1, \ldots, m$, N is a relevant dimensionless number and m is the number of relevant dimensionless terms that are necessary to evaluate the similarity between the model and the prototype [75,76,133–135,137]. Examples of dimensionless numbers used in model laws are Reynolds number, Euler number, Mach number, Froude number, Strouhal number etc. Model laws have been well documented for numerous engineering problems. Eventually, these laws can be easily used without performing laborious dimensional analysis. Some examples of dimensionless numbers commonly used to establish model laws are shown in Table 8. There are two methods to determine the dimensionless numbers that must be used in model laws, namely:

1. Fractional Analysis: Rayleigh [134] proposed that the key forces (occasionally energy terms) that affect the phenomenon must be selected by *intuitive* reasoning. The ratios of these forces are then used to predict the model laws using dimensional analysis
2. π-theorem [133,135]: If m different parameters affect a phenomenon being studied, where the parameters are defined as $q_1, q_2, q_3, \ldots, q_m$ and can be represented as follows:

$$f(q_1, q_2, q_3, \ldots, q_m) = 0 \tag{2}$$

then, Eq. (2) can be re-written as:

$$F(\pi_1, \pi_2, \pi_3, \ldots, \pi_n) = 0 \tag{3}$$

where, $n = m - k$, k is equal to the number of parameters in Eq. (2) that do not combine into non-dimensional form and $\pi_1, \pi_2, \pi_3, \ldots, \pi_n$ are non-dimensional parameters. Depending on the phenomenon being studied, the transformation from Eq. (2) to Eq. (3) can be mathematically complex. For more details on this transformation in π-theorem, the reader is referred to the work of Kline [135], Langhaar [76], and Buckingham [133]. These π-terms, when resolved properly, can be used as the dimensionless numbers in the model laws. Thus, the term N in Eq. (1) can be substituted by these π-terms.

4.2.1. Challenges in establishing model laws

Experimenters need to know *"a priori"* all the physical variables ($q_1, \ldots, q_m$) that influence the test in order to establish the model laws that must be used. In case the model laws are obtained using π-theorem one could end up with π-terms that are completely meaningless if one or more physical variables are not considered. Often, these π-terms are useful when they are defined in hindsight, after understanding the underlying phenomena. For example, if a model is used to predict the drag of a prototype, it is important to include the boundary layer thickness as a physical variable in the application of π-theorem. Without boundary layer thickness, Reynolds number will not be a π-term, which results in a flow dissimilarity between the model and the prototype. In the case of fractional analysis, model-laws are established purely based on intuition of experimenters, which could lead to erroneous conclusions as demonstrated by Kline [135].

Furthermore, once these model laws are defined, engineers aim to make sure that all the model laws are satisfied [75,76,135,137–139]. However, matching all model laws may not be possible owing to differences in the model, shape, size and test conditions. For example, a football and a section of a wing can have the same Reynolds number. However, the development of boundary layer is not same for the two because of the dissimilarity in their geometrical shapes which results in differences in the airflow around them and the forces acting on them. Thus, the differences in geometrical shapes and the consequent affect of the flow around the body and forces must be accounted for, which is done using scaling laws.

4.3. Scaling laws

Scaling laws are necessary to define the relationship between a prototype and its model. Such laws are useful in describing their relative geometrical shapes, the flow around their bodies and ratios of forces acting on the model and prototype, such that they are not vastly different in their behaviour as explained in the preceding section. Langhaar [76] mathematically described the scaling laws between a prototype and a model using the following relations:

$$x' = K_x x, \quad y' = K_y y, \quad z' = K_z z, \quad t' = K_t t, \quad m' = K_m m \tag{4}$$

where (x, y, z) and (x', y', z') are the Cartesian reference frames of the prototype and the model respectively in which each point on the prototype and the model adhere to the relationship in Eq. (4). t and t' are the

Table 8
Dimensionless numbers commonly used to establish model laws in different simulation problems.

Problem being studied	Dimensionless numbers
Incompressible flow	Reynolds number, pressure coefficient, Froude number, Weber number
Compressible flows	Reynolds number, Mach number, Prandtl number, specific heat ratio
Flow-excited vibration	Strouhal number
Internal compressible flows	Reynolds number, Mach number, pressure coefficient
Boundary layer thickness	Reynolds number, Womersley number

time-periods of the motion of prototype and the model. m and m' are the masses of prototype and model. K_x, K_y, K_z, K_t and K_m are constants and known as *scale factors*. The exact value of K_x, K_y, K_z, K_t and K_m are chosen based on the objective of SFT and the test conditions.

These scaling laws are general conditions that must be satisfied when the values of model laws of the prototype and the model are the same. It is not necessary to use all scaling laws simultaneously. Only those scaling laws pertaining to the phenomenon being studied must be used. Six main groups of scaling laws can be identified as follows [14,75,76,135,137–140]:

1. **Geometric scaling law:** If the prototype and the model have the same shapes, the model is said to be geometrically scaled. Mathematically, $K_x = K_y = K_z$ condition must be satisfied to obtain a geometrically scaled model.
2. **Mass and Inertia scaling law:** When the ratio of masses of all homologous parts of the prototype and the model are kept equal, the two systems are mass scaled. Achieving this type of scaling is challenging due to two reasons. First, mass distribution directly affects inertia which must be accounted for when rotational motion is involved. Second, mass is a function of material density and volume, which in-turn is a function of geometry. Thus, the masses of prototype and model are directly affected by their geometry and the choice of material.
3. **Time scaling law:** For cyclic phenomenon, time scaling is equal to the ratio of time-period of motion of the model and the prototype. For non-cyclic processes, the model is time scaled if both the prototype and the model move such that the ratio of the time needed to complete any given fraction of the total path to the total time of the motion are equal for the two systems. In other words, K_t is constant throughout the experiment.
4. **Kinematic scaling law:** If the prototype and the model have the same shape of streamlines, they are said to be kinematically similar. Mathematically, kinematic similarity is achieved when every fluid particle around the prototype and the model satisfies the following equations:

$$u' = \frac{K_x}{K_t}u, \qquad v' = \frac{K_y}{K_t}v, \qquad w' = \frac{K_z}{K_t}w \tag{5}$$

$$a'_x = \frac{K_x}{K_t^2}a_x, \qquad a'_y = \frac{K_y}{K_t^2}a_y, \qquad a'_z = \frac{K_z}{K_t^2}a_z \tag{6}$$

where, u', v', w' and u, v, w are the velocities and a'_x, a'_y, a'_z and a_x, a_y, a_z are the accelerations of the fluid particles around the model and the prototype in x, y, z directions respectively.
5. **Dynamic scaling law:** If the homologous parts of a prototype and its model experience forces whose ratio is constant, the two systems are dynamically scaled. Mathematically, based on Eq. (4), dynamic similarity can be expressed as follows:

$$F'_x = \frac{K_m K_x}{K_t^2}F_x, \qquad F'_y = \frac{K_m K_y}{K_t^2}F_y, \qquad F'_z = \frac{K_m K_z}{K_t^2}F_z \tag{7}$$

where, F'_x, F'_y, F'_z and F_x, F_y, F_z are the net forces experienced by the fluid particles moving around the model and the prototype in x, y, z directions respectively. By combining Eqs. (6) and (7), we can conclude that dynamic similarity exists, if the systems are kinematically similar and the mass distributions are similar (i.e., mass scaled).
6. **Structural scaling law:** The scaled model closely reproduces the structural response of the full-scale vehicle [146]. Here, the structural deformation of the model must be similar to the prototype, which is only possible if the stiffness of the model (ratio of the force applied and model deflection) is the same as the prototype at all locations in the model. [141,143,146–149] The original structural scaling laws proposed by Goodier and Thomson [141] did not include aeroelastic effects, which are critical in aircraft problems. Wissmann [146] proposed scaling laws for aeroelastic problems in the 1960s. Although structural scaling has been used in ground testing of aircraft, it has not yet been used for aeroelastic testing in flight.

4.3.1. Scaling laws implementation challenge

The implementation of different scaling laws is difficult, especially kinematic and dynamic scaling, owing to challenges in estimating the parameters in Eqs. (5) - (7) . This has led scientists to excessively rely on geometric scaling, which allows them to easily fix the scaling factors before the start of the experiment. Before 1960, most authors recommended the use of geometric scaling, claiming that similar shapes implied similar flow properties. In fact, this is not true because the stream lines around the model do not scale geometrically with the size of the model. This is because the flow field does not scale as per euclidean geometry but a type of non-euclidean geometry known as differential geometry [137]. Consequently, Eqs. (5)–(7) cannot be satisfied.

For example, for a model with lower Reynolds number (i.e., smaller model with lower testing velocity) as compared to the prototype, the boundary layer changes the flow field so much that it is not representative of the flow field around the prototype. For models with lower Reynolds number, the flow has lower momentum and separates when the flow slows down and pressure increases. This is shown in Fig. 17, where, two models are scaled geometrically (30% & 60%). The results of SFT using a 30% scaled model in this case will not be representative of the prototype behaviour due to separation. The 60% scaled model has similar shape of streamlines as the prototype but the transition location is different, which much must be accounted and corrected using appropriate numerical or analytical methods to predict prototype behaviour. Thus, (complete) geometric scaling is usually neither necessary nor sufficient condition to ensure similitude [137].

Furthermore, surface finish (i.e., surface roughness, debris, insect remains, etc.) also affects the transition location [150–152]. Typically, the effect of surface roughness, which is an artefact of manufacturing and maintenance, is not included in geometric scaling. In fact, ensuring the geometric scaling of the roughness can be challenging and would significantly escalate the manufacturing cost of the model.

Additionally, the flow fields (artefact of geometrical shape) also have a significant impact on static and dynamic stability, required control power, propulsion, etc. These secondary effects must be carefully assessed during SFT model design. For example, flow separation might make the control surfaces ineffective leading to the complete loss of model.

4.4. Classical similitude theory

The classical similitude theory is one of the most widely used method in sub-scale modelling problems. [14,24,25,75,76,138–140,

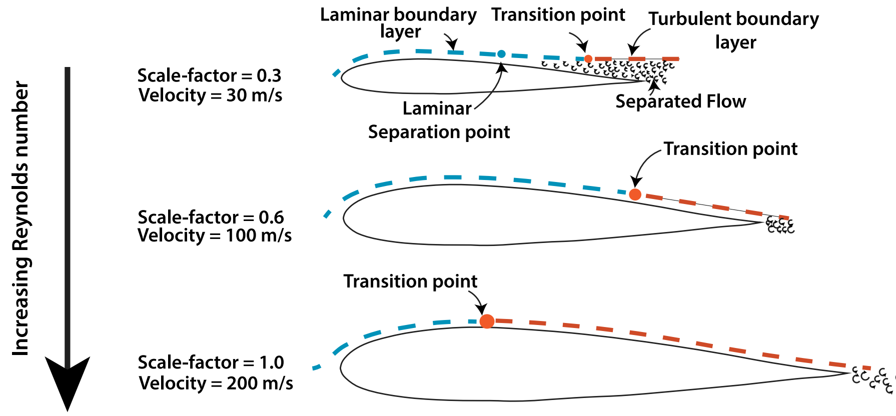


Fig. 17. Geometrically scaled models do not guarantee geometrically scaled shapes of the boundary layer. This is because the model scales as per euclidean geometry whereas the flow-field does not [137]. Here, the 30% scaled model has completely different flow characteristics as compared to the prototype due to separation. In case of 60% scaled model, the streamlines have similar shapes but different transition point, which must be corrected to predict prototype behaviour.

142] The concept of classical similitude theory is captured best by the definition provided by Langhaar [76], which is as follows: ***A function f' is similar to function f, provided the ratio for f'/f is a constant, when the functions are evaluated for homologous points and homologous times. The constant ratio, $f'/f = K_f$, is called the scale factor.***

f and f' are abstract scalar function defining a state of the prototype and the model. In this definition, the homologous point refers to the same relative position on the reference frames in which the two systems are described and the homologous time refers to the same fraction of time period in which the two systems describe the paths of their trajectories. Notably, homologous times are only used to study time variable states.

This definition of Langhaar [76], though accurate, is rather abstract. The challenge of reducing the complex design aspects of models and prototypes into an *"abstract scalar function"* makes the use of this definition difficult. Years of research in the field of sub-scale flight testing have been invested in finding the right scalar function (i.e., similitude criteria). [14,75,76,138–140,142]

In essence, classical similitude theory is the process of reducing complex design parameters associated with prototypes and their models into tangible scalar functions, such that they can be measured and effectively used to compare the behaviour of model and prototype. In this paper, we attempt to summarize and unify the different versions of classical similitude theory using one set of nomenclature and definitions and then maintain it consistently to allow readers to easily understand the state of the art.

The model laws and scaling laws are primitive attempts at arriving at the scalar function described by Langhaar [76]. However, individually these laws cannot be used effectively to design a sub-scale model owing to the limitations listed in the preceding sections. Classical similitude theory combines model laws and scaling laws to establish similitude. The model laws and scaling laws that must be satisfied for a phenomenon being studied are determined by studying a large number of past experiment and are known as ***similitude criteria***. The similitude criteria needed for common applications are well documented by Wolowicz et al. [25] and forms the basis for a majority of sub-scale model tests.

Once the similitude criteria that affect the phenomenon being tested are selected, they lead to a set of equations whose solution determines the size, shape, mass and inertia of the model and the test conditions. The general methodology of implementing classical similitude theory is shown with the Unified Modelling Language (UML) activity diagram in Fig. 18. The classical similitude theory can be used for a myriad of problems, one such problem is shown with Example 1 in Appendix A to help the reader get an understanding of classical similitude theory.

4.4.1. *Limitations of classical similitude theory*

While the key benefit of using classical similitude theory is that the labour involved in determining the similitude criteria is very small and the associated mathematics required to solve them is not rigorous [135], it has a practical limitation that constantly challenges the scientists known as the *scale effects*. All relevant model and scaling laws (called similitude criteria) must be solved together to arrive at a sub-scale model whose size, shape, and the test conditions are such that its behaviour is similar to the prototype. However, solving such similitude criteria leads to an over constrained problems. In particular, model laws cannot be satisfied simultaneously due to certification requirements, limitations of the testing equipment, cost limitation, etc. as shown in Appendix A.

These over-constrained problems are often solved by selecting some of the model law(s) based on experience and ignoring others (see Appendix A). The discrepancy in the results owing to the ignored model laws are termed as scale effects. These scale effects are corrected either using legacy information from previous tests (if available) or by resorting to the experience of the engineer. This might be possible for conventional designs where flight data of similar full-scale aircraft are available. Whereas, for unconventional designs, due to lack of data, results prone to scale effects cannot be scaled up.

4.5. *Similitude theory using governing equations and approximation theory*

4.5.1. *Similitude theory based on governing equations*

Kline [135] proposed the use of analytical methods (governing equations) to overcome the limitations of classical similitude theory. He argued that a similitude criterion that is applied to every infinitesimal element of a model will apply to the whole body, as long as both the model and the prototype belong to the same class of problems. He defined *a class of problems* as a group of problems that obey the same governing equations and boundary conditions. Kline defined similitude between any two systems as follows [135]:

"If two systems obey same governing equations and boundary conditions and if values of all coefficients in these equations and boundary conditions are made the same, then the two systems must exhibit similar behaviour provided a unique solution to this set of equations and boundary conditions exist".

As a consequence, even without solving the governing equations, sufficient information to establish a similitude between the model and the prototype can be obtained by comparing the coefficients of the normalized governing equations. An obvious problem in comparing the coefficients in the governing equations is the variation of values due to the differences in the size of the model and the prototype. Kline further proposed a two-step approach to normalize governing equations and the associated boundary conditions to allow the comparison of coefficients as follows [135]:

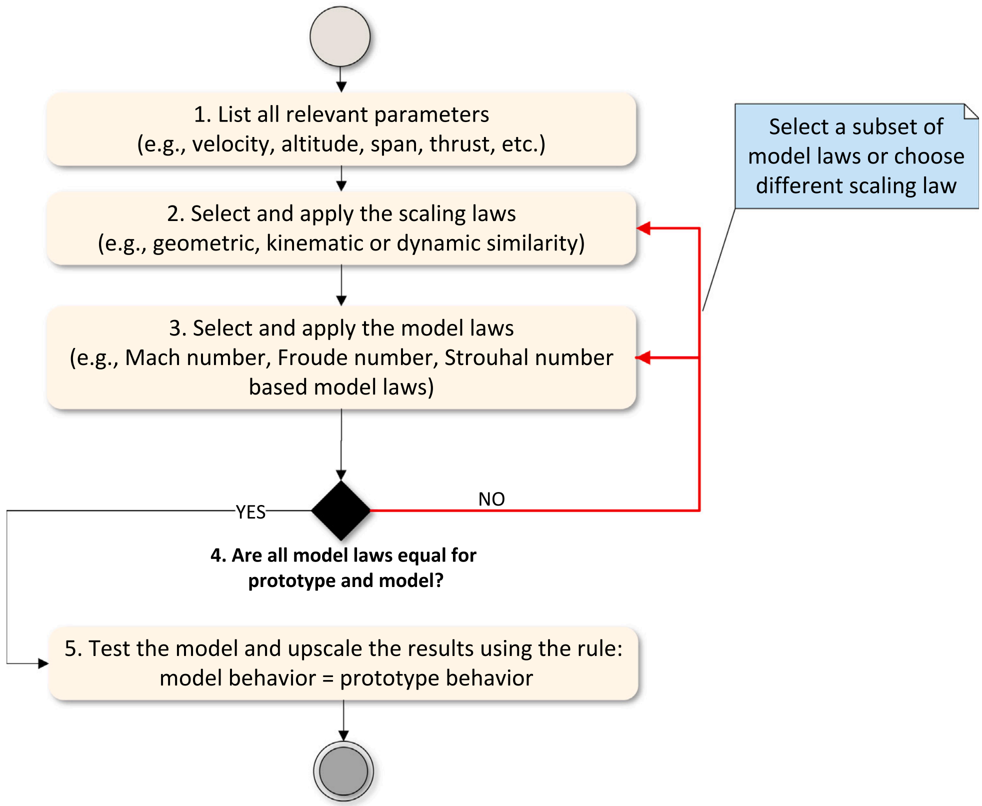


Fig. 18. UML activity diagram capturing the tasks in classical similitude theory.

1. make all the variables in the governing equations dimensionless
2. make all the equations dimensionless

He called this approach normalization of governing equations. A normalized equation contains two sets of terms. The first set is composed of dimensionless independent variables that affect the phenomenon under study (i.e., variables in the original dimensional governing equations). The second set is made up of dimensionless physical parameters which are system properties or physical constants. For example, the Navier–Stokes equation, ignoring the time dependent terms and the z-components terms, is reduced to dimensionless form as follows: x-momentum equation:

$$\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{u}}{\partial \bar{y}} = -\frac{\Delta p_L}{\rho U^2}\left(\frac{L}{\delta}\right)^2 \frac{\partial \bar{p}}{\partial \bar{x}} + \frac{\nu}{UL}\left(\frac{\partial^2 \bar{u}}{\partial \bar{x}^2} + \left(\frac{L}{\delta}\right)^2 \frac{\partial^2 \bar{u}}{\partial \bar{y}^2}\right) \tag{8}$$

y-momentum equation:

$$\bar{u}\frac{\partial \bar{v}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{v}}{\partial \bar{y}} = -\frac{\Delta p_L}{\rho U^2}\left(\frac{L}{\delta}\right)^2 \frac{\partial \bar{p}}{\partial \bar{y}} + \frac{\nu}{UL}\left(\frac{\partial^2 \bar{v}}{\partial \bar{x}^2} + \left(\frac{L}{\delta}\right)^2 \frac{\partial^2 \bar{v}}{\partial \bar{y}^2}\right) \tag{9}$$

here, the dimensionless independent variables are

$$\bar{x} = \frac{x}{L}, \qquad \bar{y} = \frac{y}{\delta}, \qquad \bar{u} = \frac{u}{U}, \qquad \bar{v} = \frac{v}{V}, \qquad \bar{p} = \frac{p}{\Delta p_L} \tag{10}$$

and the dimensionless physical parameters are

$$\pi_1 = \frac{UL}{\nu}, \qquad \pi_2 = \left(\frac{L}{\delta}\right)^2, \qquad \pi_3 = \frac{\Delta p_L}{\rho U^2} \tag{11}$$

where, L is the length of the object, δ is the boundary layer thickness, U and V are the x- and y-component of the velocity far upstream, Δp_L is the largest pressure difference between two points on the body, x is the x-coordinate, y is the y-coordinate, u is the x-component of the velocity, v is the y-component of the velocity, ν is the kinematic viscosity, ρ is the density, and p is the pressure. It can be noted that π_1 is the Reynolds number and π_3 is the coefficient of pressure.

Thus, if the model and the prototype belong to the same class of problems and have a unique solution in the domain of the tests, the similitude criteria are the coefficients of the normalized governing equations and the associated boundary conditions. These similitude criteria must then be satisfied by performing appropriate *transformations*, thus altering the shape and size of the model and varying the test conditions suitably so that model and prototype have the same coefficients. This is shown using the activity diagram in Fig. 19. In addition, the application of this iterative process is shown using Example 2 in Appendix B.

A key consequence of the application of this theory is that the model and prototype, in general, do not retain the same shape i.e., the model is not geometrically scaled. Consequently, these changes may result in model designs that do not belong to the same class of problems as the prototype. In other words, the model and the prototype may not follow the same governing equations. For example, if inviscid theory governing equations are used to model the flow properties, it might be applicable to the full-scale aircraft which has a thin boundary layer. However, for a distorted sub-scale model, the same governing equations might not be applicable anymore. Thus, similitude cannot always be established. In some cases, the reverse is also true, i.e., the model and the prototype belong to the same class of problems only after transformation. Kline [135] demonstrated specific cases where the model and the prototype belong to the same class of problems after the transformation. For example, he showed how distorting the models helps in capturing compressibility effects (see Appendix B) [135].

However, despite the best efforts in transforming the model, similitude criteria cannot be satisfied in many cases [153]. Consequently, like the classical similitude theory, the issue of scale effects persists as all coefficients cannot be matched simultaneously. Nevertheless, when sub-scale models are designed using similitude criteria based on governing equations, better results than classical similitude theory can be obtained, as the former takes the entire flow field into account in the design, unlike the latter, which uses generalized laws. Furthermore, the

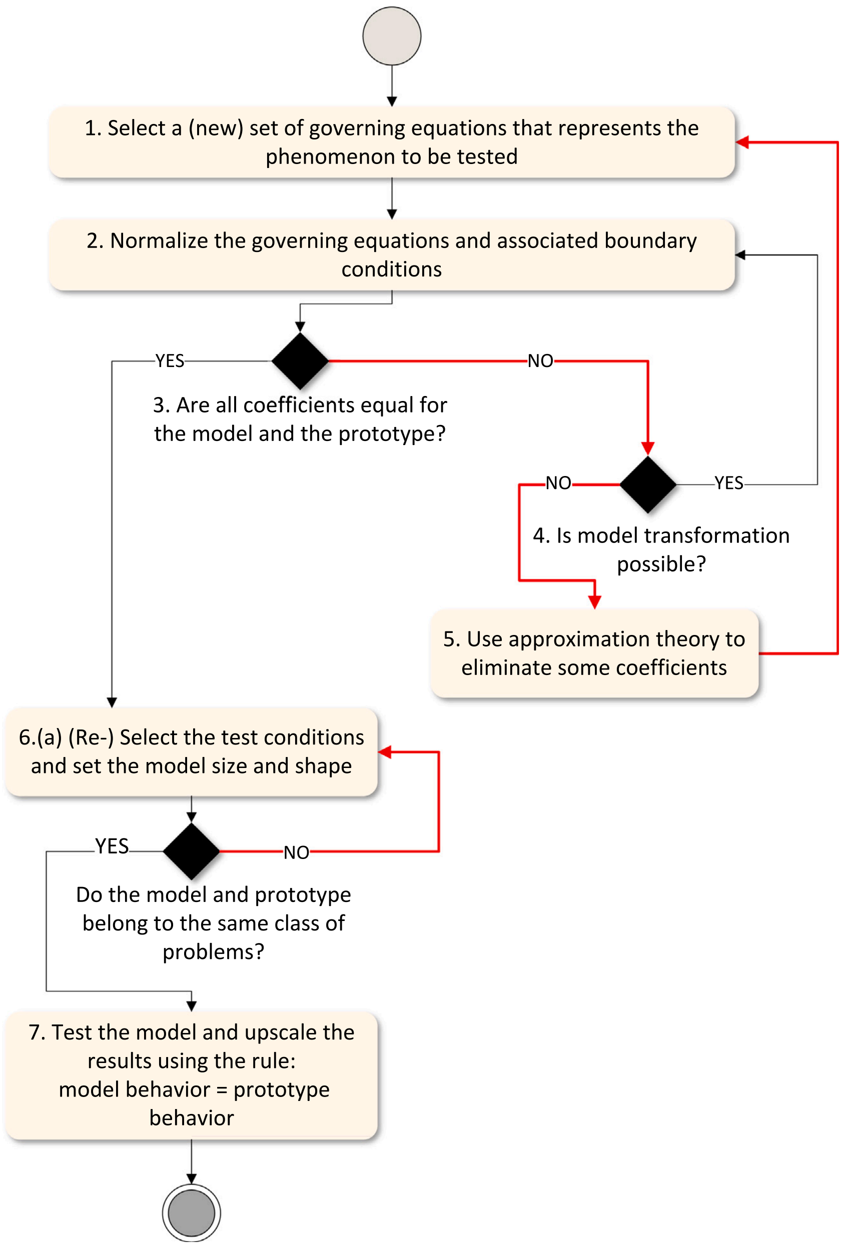


Fig. 19. UML activity diagram of similitude theory using governing equations and approximation theory.

method employing governing equations provides insights into which similitude criterion might be ignored as explained in the following section.

4.5.2. Governing equations and approximation theory to establish similitude

The coefficients of governing equations in Kline's [135] method are analogous to model laws and scaling laws in classical similitude theory [75]. Just as all model laws and scaling laws cannot be matched, all the coefficients in the governing equations cannot be matched. To overcome this problem, Kline introduced approximation theory [135] that allows the experimenters to ignore those model laws that are not completely necessary for the simulation of the phenomenon. Application of approximation theory is shown as part of Example 2 in Appendix B.

Kline [135] proposed a careful consideration of the normalized governing equations. If any coefficient in the normalized governing equation is insignificant as compared to the scale of the equations (i.e., $\ll 1$), the coefficient is ignored as it has little or no effect on the similitude. This method of simplifying the similitude criteria is known as approximation theory [135]. Thus, after setting appropriate test conditions and selecting the model shape and size, if all the coefficients cannot be matched, certain terms that do not contribute to the phenomenon being tested are neglected after careful mathematical analysis. If no coefficient can be eliminated by applying the approximation theory, test conditions and/or model shape and size must be altered (i.e., transformed, see activity diagram in Fig. 19).

One could argue that neglecting terms in governing equations is comparable to ignoring model laws in classical similitude theory. Nonetheless, there is a significant difference. In classical similitude theory, we ignore model laws by intuitive feeling. However, Kline's approximation theory neglects coefficients only after analysing its significance in the governing equations. For example, in the normalized Navier–Stokes equation (see Eq. (8) and (9)), for very high Reynolds number problem, one might be tempted to ignore the terms multiplied by the inverse of Reynolds number in the absence of governing equations. This approximation is not correct because L/δ term may have a high numerical value. Consequently, the product of L/δ and $1/Re$ shown in Eq. (8) and (9) may not be insignificant. Since insights about the combined influence of various π terms derived using the π-theorem is not available in classical similitude theory, ignoring π terms becomes challenging.

4.5.3. Limitations of similitude theory with governing equations and approximation theory

Including governing equations and approximation theory in establishing similitude criteria offers several improvements over classical similitude theory such as improving the rationale in selecting dimensionless numbers, reducing dependence on legacy information and experience of the experimenters, and developing appropriate justification for discarding dimensionless numbers from similitude criteria. Nevertheless, establishing similitude criteria using governing equations and approximation theory poses other challenges, namely:

1. The governing equations are not always available. Even if the governing equations are obtained, ensuring that model and prototype belong to the same class of problems is difficult.
2. The governing equations combined with boundary conditions often do not have a unique solution (indicated using red arrows in Fig. 19). Thus, similarity criteria cannot be established with the selected governing equations and a different set of governing equations must be selected or developed.
3. It is often impossible to match all similitude criteria by transforming the coefficients of governing equations (by altering the model geometry and test conditions). Even in cases where it is possible, it is a laborious and time-consuming task.
4. Normalizing governing equations is a rigorous mathematical effort. Furthermore, establishing normalized governing equations and the associated approximation theory for multiple disciplines is often impossible.
5. Combined use of governing equations and approximation theory cannot guarantee similitude as all the coefficients for both model and prototype must be equal. Only in some specific cases, similitude is possible. In other words, following all the steps shown in Fig. 19 does not guarantee a similar sub-scale model.
6. This method provides a Boolean output, i.e., whether similitude is achieved or not. It does not express the extent of similitude when all the similitude criteria are not satisfied. For example, it would be useful for experimenters to know that their model has attained 80% of similarity for the phenomenon being tested, which can be used in the scale-up process by assigning uncertainty values to scaled-up values of performance parameters. Unfortunately, this figure of merit cannot be extracted from any of the methods describe thus far.

4.6. Computational similitude theory

The method proposed by Kline [135] focused on establishing normalized governing equations and then utilizing them without actually solving these equations. An alternative approach is to solve these governing equations to determine the similitude criteria. Baker et al. [75] were the first to demonstrate the use of the solution of governing equations to establish a similitude relationship. While the examples shown by Baker et al. [75] could be solved analytically, unfortunately, most similitude problems are complicated and not easily solvable.

With improvements in computing power and the development of powerful solvers, many governing equations have become numerically solvable, albeit approximately through discretization such as Computational Fluid Dynamics (CFD) and Finite Element Methods. This opens up the possibility of comparing the behaviour of the model and the prototype to ensure they match before starting with complex and expensive manufacturing and testing activities. Although these computational methods have their limitations as detailed in Section 2.1, they can be used for preliminary design of sub-scale models. After execution of SFT, the results can be used to study the uncertainty of the methods and validity of underlying assumptions in the computational simulations. Bushnell [26], in his review, opines that "*numerical simulations can increasingly include the influences of the various scaling issues. Computational methodologies are becoming the approach of choice for (flight behaviour) prediction, with the wind tunnel increasingly relegated to a supporting computational tool validation role*". Thus, these computational methods can be used as a bridge to link the in-flight behaviour of the sub-scale model and the full-scale aircraft as explained in the following paragraphs.

The approach of solving governing equations to arrive at an approximate solution using computational analysis is a significant departure from the method proposed by Kline, where, similitude is established without actually solving the governing equations. In this section, we group all the sub-scale model tests that use computational analysis to establish scaling laws into a broad category known as *computational scaling laws*. The models developed using computational scaling laws are called computationally scaled models. For problems with single disciplines, many authors refer to computational scaling with the name of the discipline. For example, aerodynamic scaling, structural scaling, thermal scaling, etc. [28].

Despite the capabilities of computational tools, comparing the behaviour of the prototype and its model is challenging. Similar to Kline's approach, results of computational analysis of the prototype and its model cannot be directly compared owing to differences in scale. Thus, the results of the computational analysis must be non-dimensionalized to enable the comparison. Furthermore, depending on the problem, the result of the computational analysis might be a very large data-set. For example, in the case of aerodynamic analysis, the result includes forces and moments on the body at different location, velocity scans around the model, the pressure distribution, boundary layer information etc. Comparing such large data-sets is challenging in itself, let alone arriving at a figure of merit that establishes an extent of similitude between the model and the prototype that can be used by the designer to alter model shape and size to enhance the similitude.

Thus, establishing a function, which is composed of non-dimensional parameters affecting the phenomenon being tested, to quantify the *extent of similitude* between prototype and scaled model is a key step in computational scaling approach. This aspect of computational scaling was already recognized by engineers at NASA in the 1970s when they tried to design a 44% sub-scale fighter aircraft which mimicked the aerodynamic behaviour of its full-scale counterpart [87–89]. However, the methods developed were only suitable for the specific models developed in HiMAT. After this, most of the SFTs were largely designed using classical similitude theory as shown in Section 3 and no developments have been seen with respect to computational scaling until 2005.

In allied sub-scale model testing fields such as wind-tunnel testing, new methods to estimate the extent of similitude have been developed in the last two decades. Pettersson and Rizzi developed functions based on coefficient of lift, drag, and moments to design wind-tunnel test models whose behaviour was similar to the prototype [154]. Similar functions have been used to design models for sub-scale flight testing by Bergmann et al. in glider design [17]. Functions to study aeroelastic similitude have been formulated by multiple authors such as French, Mas Colomer et al.,Pereira et al. and Ricciardi et al. [93,143,145,148]. Ricciardi et al. established a fundamental criterion to compare the extent of similitude, called Model Assurance Criterion (MAC), between the structural mode shapes of model and prototype.

Most of these functions are aimed at specific application of aerodynamics or aero-structural analysis using a specific testing methods such as wind-tunnel testing. However, such functions can be formulated for SFT too. As formulating a function of extent of similitude for different applications (e.g., flight dynamics simulation, wind-tunnel testing, etc.) is a time-consuming effort, designers would benefit from a generalized function such as the **Degree of Similitude (DoS)** which we formulated in a previous work [155]. DoS is a figure of merit to establish the extent of similarity between a prototype and its model and defined as ***the weighted sum of normalized virtual scaling errors.*** The virtual scaling error is the difference between the dimensionless coefficients of the prototype and the model, as estimated using multi-disciplinary computational analysis. Mathematically, DoS is given as follows:

$$DoS_{test} = 1 - \frac{1}{n}\sum_{i=1}^{n} w_i * \frac{|C_{i_p} - C_{i_m}|}{|C_{i_p}|} \tag{12}$$

where, n is the number of dimensionless coefficients, C_{i_m} is the ith relevant dimensionless coefficient of the model obtained using a computational analysis, C_{i_p} is the ith relevant dimensionless coefficient of the prototype obtained using computational analysis and w_i is the Degree of Influence (DoI) of the ith coefficient on the phenomenon being tested.

DoI is a weighting factor used to quantify the influence of each of the dimensionless coefficient on the phenomenon being tested. For example, among $C_{m_{\dot{\alpha}}}$ and C_{m_q}, latter has greater influence on the damping and time period of the short period motion than the former. If both $C_{m_{\dot{\alpha}}}$ and C_{m_q} of the model and the prototype cannot be matched simultaneously, matching the values of C_{m_q} is much more important in increasing the extent of similitude for damping and time period of the short period motion. Thus, the relative importance of different coefficients used in DoS is quantified using DoI. The degree of influence of different aerodynamic coefficients can either be determined quantitatively (governing equations) or qualitatively (expert opinion).

Depending on the phenomena being tested and the flight behaviour to be studied, different formulations of DoS (and DoI) can be used to design numerous models, where, each model predicts specific aircraft characteristics. In case DoS = 1 is achieved, perfect similarity exists between model and prototype for the aircraft characteristics being studied. Once a figure of merit such as DoS is formulated to estimate the extent of similitude, designers can iterate over the design of their models by varying test conditions, size and shape of the model till they arrive at a design which is similar to the prototype for the phenomenon being tested (i.e., DoS =1). This is illustrated using the activity diagram in Fig. 20. Different techniques have been used for design iterations of sub-scale models, as discussed in Section 4.6.1.

Even after using the state of the art iterative methods, DOS of 1 may not be achieved. Here, engineers can use the flexibility of DoS to their advantage by reducing the scope of the test (by reducing the number of coefficients in DoS) to build different models, each having DOS which is almost equal to 1. The results of the individual tests can then be combined to determine the overall aircraft behaviour. To date, the exact methodology for the composition of results from multiple tests to completely predict prototype behaviour remains an open challenge as discussed in Section 5.3.

The key difference between the preceding similitude theories and the computational similitude theory is that, in the latter case, the iterative design cycle does not result in a generally applicable scaling. For example, the model laws in classical similitude theory shown in Table 8 is applied to any model for a given type of problem. In computational scaling, for a specific combination of prototype design and the test objective, a unique scaling law is established to arrive at a model whose response can be scaled up to predict a specific feature (or set thereof) of the prototype behaviour, i.e. the feature defining the test objective. Some examples of test objectives include, simulation of short-period motion, simulation of dutch-roll, study of spin characteristics, study of flutter behaviour or combinations thereof. In general, these computational scaling laws are based on iterative procedures to transform model design and/or test conditions to ensure similitude, as explained in the following section (see red line in Fig. 20).

4.6.1. Iterative methods for computational scaling

Numerical optimization is an obvious choice to enable efficient modification of the sub-scale model design and test conditions to achieve the highest similarity with the prototype. French [93] was one of the first to use optimization in the application of structural similitude. He demonstrated the use of optimization to match the stiffness distribution over the wing. Here, the figure of merit for the extent of similitude was the difference between the normalized deflection along the span of the wing of the model and the prototype. French [93] showed with physical testing that such an optimization-based scaling technique was indeed effective in achieving similarity. While French [93] specifically used this method for the design of wind-tunnel models, similar techniques can be used to design models for SFT.

Most engineering problems are multi-disciplinary. As a consequence, the use of the methods described in Sections 4.4 and 4.5 can be challenging, as multiple governing equations and a broader set of similitude criteria must be satisfied to design a model similar to prototype. Optimization based scaling laws are much more versatile as they rely on Multi Disciplinary Analysis and Optimization (MDAO) strategies to account for the coupled effects of the various disciplines. In such MDAO problems, the objective function is the figure of merit quantifying the extent of similitude (such as DoS), the design variables are the parameters defining the geometry of the model and the test conditions, and the constraints are a combination of manufacturing and mission requirements, including those set by certification authorities.

For example, Pereira et al. [145] used MDO in aeroelasticity problems by ensuring homologous pressure distribution over the model and the prototype while matching the reduced natural frequencies. The design variables were the rib thicknesses under manufacturing constraints. Many other complicated similitude problems in the field of aeroelasticity are solved using optimization [143,148,156] as described in the review by Mas Colomer et al. [143].

The applicability of computational scaling is not limited to aeroelasticity. For example, this method shows great potential in the study of aircraft flight dynamics behaviour, where the model must demonstrate multiple-disciplinary similarity with the prototype (i.e., similar aerodynamic behaviour, structural behaviour and mass distribution). Other studies which include aeroacoustics, aero-propulsive interactions, aero-thermal design, unmanned aerial vehicle design, etc. can also benefit from this method. The computational scaling method is largely unexplored, but demonstrates very high potential. A full exploitation will depend on the ability to address the challenges discussed in Section 4.6.2.

4.6.2. Challenges in computational scaling

Computational scaling laws are the state of the art in sub-scale model design method for all physical testing methods including SFT. Nevertheless, the state of practice lags behind. There are numerous reasons for this which are detailed as follows:

1. The formulation of the objective function to establish scaling laws is challenging because the design space may be large when all the relevant parameters that affect the phenomenon are selected. Furthermore, quantifying the degree of influence per test adds to the complexity of objective function formulation (Eq. (12)).
2. Developing (accurate) computational disciplinary analysis tools, which can be used in an optimization, is a knowledge and labour intensive task.
3. Most computational analyses require repetitive pre/post-processing activities (e.g., generation of computational grids for CFD and FE analysis, post-processing of flow analysis, etc.), which are laborious, time-consuming and error-prone. Because, an iterator is used to modify the design, these pre/post-processing activities must be automated which requires non-trivial investment of time and resources.

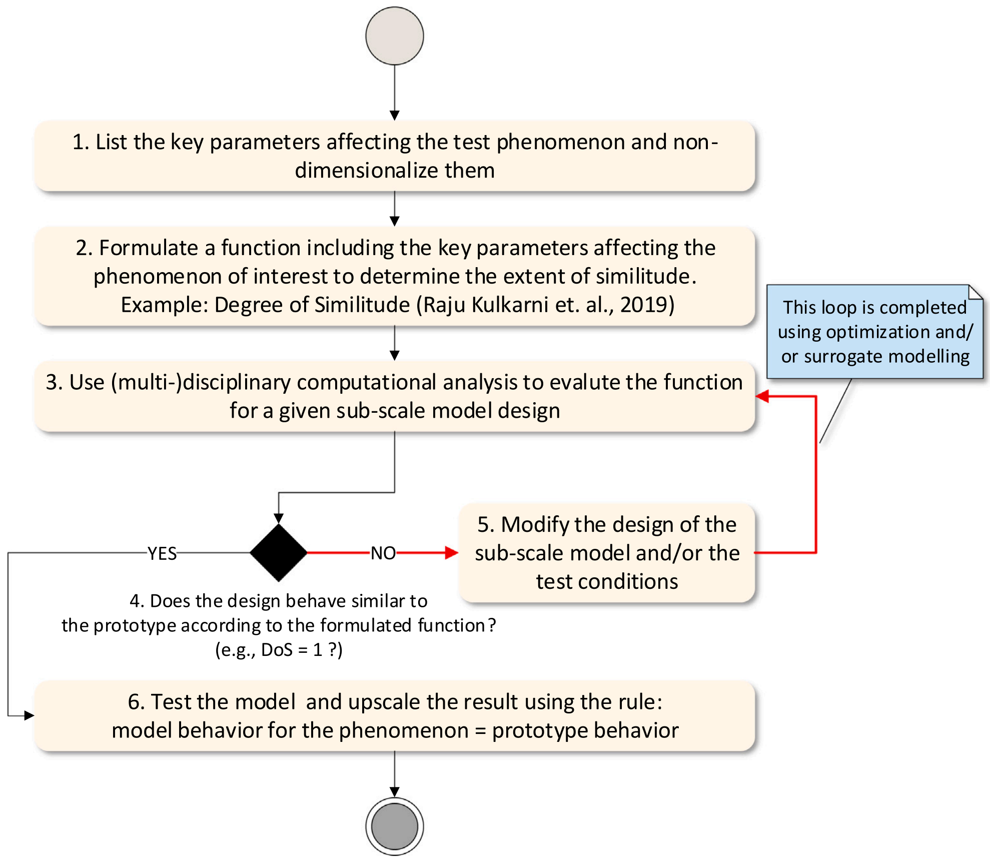


Fig. 20. UML activity diagram of methodology for computational scaling.

4. Even if the disciplinary analysis tools are available and automated, combining them together in a multi-disciplinary analysis framework, selecting the right MDO architecture and optimization algorithm can be challenging tasks, which requires specific knowledge of MDO and numerical optimization techniques. In other words, barriers associated with MDO must be lowered for experts in the field of SFT to make full use of its potential.
5. The benefits of optimization based scaling laws can be negatively affected by the use of high-order, high-fidelity and time-consuming analyses in the optimization process. To keep the computational time compatible with the usability of SFT in the design process, surrogate-model techniques can be very effective. These surrogate models are analytical approximations of the actual high-fidelity analysis and are orders of magnitude faster to evaluate, thus making the optimization effort time manageable.

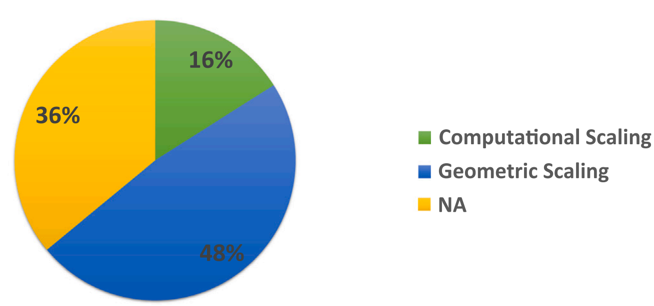


Fig. 21. Distribution of 25 SFTs (reviewed in this paper) based on design approach. Here, 'NA' implies scaling criteria are not used at all as there is no full-scale aircraft design and the SFT is solely intended for demonstration tests.

4.7. Design methods employed in SFT

Despite the increased use of SFT to study unconventional aircraft designs, it is interesting to note that many SFTs have no equivalent full-scale counterpart, i.e., there is no full-scale aircraft design (about 36% of the 25 SFTs reviewed). As a result, they do not use any of the similitude methods discussed in Section 4. About 48% of the 25 SFT models reviewed in this paper were geometrically scaled (shown in Fig. 21). The primary reason for this is the ease of applying geometrical scaling as compared to other scaling laws. Nonetheless, the response of geometrically scaled models is often prone to scale effects, which leads to significant uncertainty in results. Thus, the results of tests are mostly relevant to demonstration tests. Finally, 16% of the models that were studied were computationally scaled. This type of scaling ensures that the specific disciplinary behaviour of the model is similar to that of the prototype. It is interesting to note that engineers working on HiMAT [88,89] utilized principles of computational scaling as early as 1976. However, the complexity of performing computational analysis in those days prevented the widespread use of this method. With tremendous improvements in computational power, researchers have been using computational scaling more frequently in the recent years.

Review of design approaches shows that engineers performing SFT recognize the importance of sub-scale model design approach for a given test objective. The distribution of SFT based on design approach per test objective is shown in Fig. 22. It is observed that engineers avoid complicated and resource-intensive computational scaling when designing models for demonstration tests. In most cases (69%), no scaling criteria are used at all. Conversely, in simulation and phenomenological tests, either geometrical scaling or computational scaling is used to design the sub-scale model, which is essential in scaling up the results of SFT. Nevertheless, in both simulation and phenomenological tests,

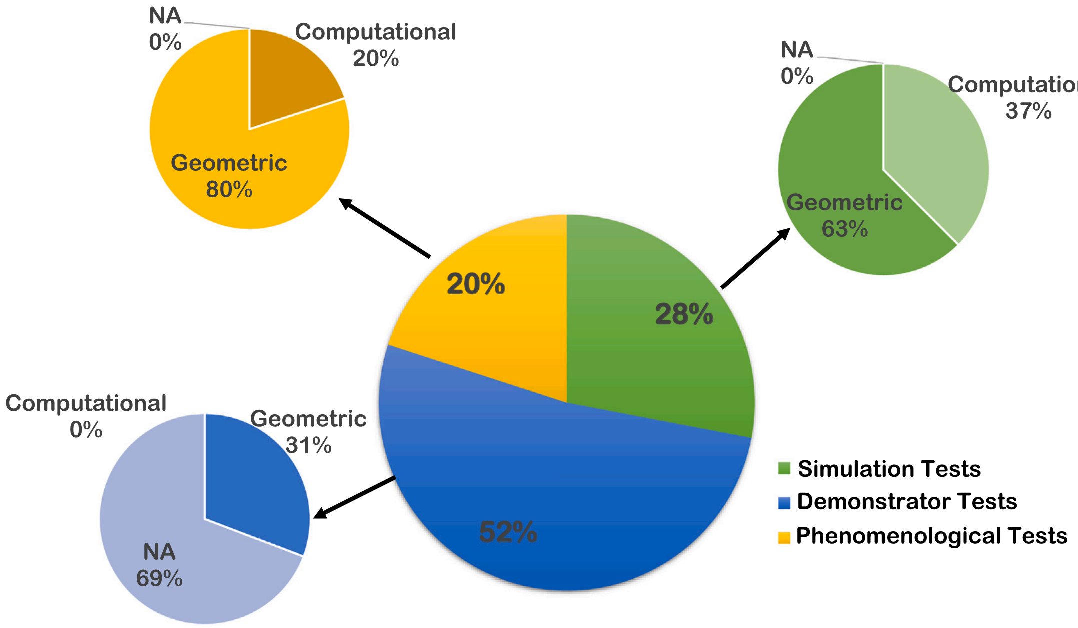


Fig. 22. Distribution of SFT based on design approach per test objective.

the majority of the models are designed using geometric scaling despite their susceptibility to scale effects.

For aeroelastic tests performed in wind-tunnels, the benefits of computationally scaled models over geometrically scaled models have been demonstrated [93]. Similar studies have been performed for SFT models by Bergmann et al. [17], where they demonstrated the improved similarity in lift, drag and moment with the prototype of the computationally scaled model as compared to geometrically scaled model.

However, research work demonstrating improvements in similarity with computational scaling in SFT models are limited to static characteristics and have not yet demonstrated improvements in dynamic behaviour (e.g., flight dynamics). This is primarily because SFT was considered too complicated in the past (before 2005) for most research entities as discussed in Section 1 and has not received the same attention and funding as other testing methods to perform comparative studies of different scaling approaches. Furthermore, the impact of the inaccuracies in computational methods and the longer time needed to perform the optimization on the overall SFT process has not been studied. Nevertheless, based on the evidence provided by ground based tests, computational simulations and static SFTs [17,27,28,93,149,157], computational scaling shows promise in improving SFT, which must be investigated further by scientific community.

5. Discussion: Moving beyond the state of the art in SFT

Despite the development of computational scaling, only a minority of the designs use this state of the art methodology. Besides, even where computational scaling is employed, the sub-scale model designs are based on mono-disciplinary computational analysis. In this section, we discuss the different disciplines that must be considered in the SFT model design cycle, the associated technological barriers in integrating these disciplines in an MDO framework, and how these barriers can be lowered. These ideas will support the widespread use of computational scaling that will in-turn improve the quality of SFT simulation necessary to predict prototype behaviour.

5.1. Improving SFT model design cycle considering multi-disciplinary requirements

Despite developments in design approaches, manufacturing techniques, COTS equipment and the flight-test data acquisition individually, the interaction between these lines of research is not very significant. This is because SFT is generally performed by a small team of engineers, who can either focus on designing similar models or deal with other practical considerations (manufacturing, flight testing, etc.) that arise during SFT in the limited design lead-time. Thus, last-minute design modifications and re-work becomes necessary to account for factors that were not addressed during the design phase, which might nullify all the efforts of designing similar models.

Although, the multi-disciplinary analysis based computational scaling approach offers the possibility to include all the requirements of SFT during the design phase, the SFT models designed so far do not include them owing to the lack of clarity on which requirements must be used. Based on the review of past tests, a brief description of key requirements in SFT model design cycle is provided. These typically include requirements on testing, equipment constraints, manufacturing constraints, and pilot effort that are not included in the design of sub-scale models. Following sections describe the specific aspects of realizing SFT that are incorrectly neglected in SFT design cycle.

5.1.1. Inclusion of mission safety in SFT model design process

As discussed in Section 3.2.1, there are two critical tasks in the design of SFT models. First, ensuring similarity of behaviour between model and prototype. Second, ascertaining that the model remains controllable throughout the flight and stays within the designated flight box (i.e., model can safely perform the required mission). The former requirement can be satisfied with one of the approaches discussed in Section 3. However, for the latter, one must assess static and dynamic stability, controllability, flying and handling qualities of the model at different phases of flight, accurately and quickly, to make SFT feasible. This requires the inclusion of flight dynamics analysis in the design of the scaled model. Such an analysis requires a detailed information on aerodynamic performance, structural design, selection and placement of on-board equipment, and the power output of the propulsive unit, which, in-turn requires the development and integration of dedicated disciplinary tools. Little work has been done so far that shows a detailed implementation and integration of all these aspects. However, the use of a flight-dynamics toolbox in the design cycle to assess the model flight behaviour using simplified analysis tools has been shown by Raju Kulkarni et al. [158] and Dantsker et al. [159].

5.1.2. Equipment constraints

A lot of on-board equipment is used in SFT models. Besides performing their required functionality, these components affect the overall model design in two ways. First, one or more of the selected components may not fit inside the model whose accommodation might result in the modification of the external aerodynamic shape. Second, the mass of the components and their relative position in the model affects the weight, balance and inertia of the model (in-turn affecting the flight dynamics). These issues must be carefully assessed during the design phase to prevent costly last-minute rework. These effects of component selection, which are rarely evaluated in current design cycle, should be incorporated to enhance the effectiveness of SFT in the future.

5.1.3. Manufacturing constraints

With the advancements in manufacturing and material technology, one can manufacture sub-scale models in various ways using different materials. The selection of the manufacturing technique depends on the time available for manufacturing, the structural requirements of the mission, the budget, required precision and the materials that can be used in manufacturing.

The manufacturing technique and the material used in the process directly affect the SFT model design features such as surface finish, trailing edge radius, gap between movable and the fixed part of the wing, etc. These features affect the aerodynamics and the weight and balance of the model, thereby, affecting the flight behaviour. Nevertheless, the implication of manufacturing technique and material are generally not included during the design phase for two reasons. First, rules of thumb to predict the implication of manufacturing techniques on model design are not available. Second, such rules of thumb cannot be generalized due to substantial variation in requirements from one design to another (especially with novel aircraft configurations). This can only be overcome by using computational analyses to evaluate the impact of designers decision on the overall model design, which is enabled by computational scaling approach.

5.1.4. Pilot effort

SFT requires a pilot to fly the sub-scale model. The pilot can either be a human or a trained computer that performs the test. However, in most SFTs, there is little or no time available to build an auto-pilot system, especially if the goal is to simulate prototype behaviour in the conceptual design phase. In such cases, it is essential to design sub-scale models whose handling and flying qualities are of a certain level such that the pilot effort remains acceptable during the entire SFT mission. However, the handling qualities and the pilot effort are quite different for remote-controlled models as compared to a prototype owing to the differences in their size, mass and inertia, in addition to the fact that pilot is not on-board the aircraft.

Williams [160] proposed standards for specifying Unmanned Aerial Vehicles (UAV) handling qualities. Unlike SFTs, UAVs in general do not have similarity requirements that alter the handling-qualities, which allows engineers to improve handling qualities without being constrained by similarity requirements. Thus, improving SFT model handling qualities is much more challenging than for a normal UAV. Nevertheless, the proposed requirements can be formalized and included in the optimization loop for computational scaling to ensure that pilot does not get overwhelmed. Dantsker et al. provide an example for such flight testing automation [159].

5.1.5. Overcoming uncertainties in model response

Use of SFT is often discouraged because of uncertainties in the response of the model. Undoubtedly, ground-based testing methods can use much more sophisticated and precise measurement instruments as compared to SFT because there are no limitations on the weight, size, and shape of these equipment in ground-based testing. Moreover, the uncertainties posed by atmospheric turbulence in SFT can be lowered in controlled environment of ground-based testing. Nevertheless, ground-based testing methods are ill-equipped to study several aspects of flight-behaviour as discussed in Section 2. For example, the tests conducted in AirSTAR [79] program to reduce fatal accidents requires the simulation of different flight phases serially with no time lag, which is challenging and often impossible in ground-based testing. Engineers are working towards reducing the uncertainties by improving the precision of the on-board measurement equipment [37,113], developing mathematical models to correct the effects of atmospheric turbulence [161] and repeating the tests multiple times [63]. Therefore, instead of rejecting SFT for its uncertainty, these developments should be included in the SFT life-cycle to improve its quality.

5.2. Overcoming technological barriers in computational scaling

Of the challenges mentioned in Section 4.6.2, two challenges can be classified as technological challenges, namely, automating disciplinary analysis and managing and executing MDO frameworks effectively. Researchers are making advances in allied engineering fields which can be effectively used in SFT to lower the technological barriers. Following paragraphs briefly discuss these developments.

5.2.1. Automating disciplinary analysis

A typical sub-scale model design cycle which utilizes computational scaling with medium or high fidelity analysis involves numerous tasks such as geometry manipulation, discretization (meshing), generation of specialized input data for discipline-specific analysis, and post-processing the results. There are very few commercial software packages where all these capabilities are bundled into one. Thus, designers need to use multiple software while performing all the intermediate data manipulations to adhere to their input requirements. This makes computational scaling, laborious, time-consuming and error-prone. The overarching solution to this problem is to automate the entire design workflow. However, this automation process in itself is non-trivial.

Whenever a design case is highly rule-driven, multidisciplinary, repetitive and demands geometry manipulation and product (re)configuration as in the case of SFT model design, Knowledge based engineering (KBE) is likely to be the best possible technology at hand [162]. KBE applications an be developed, which, based on user input, automatically generates models of a specific family of products [163,164], such as a family of sub-scale model designs. For each model, the application automatically creates the abstractions required by the various analysis tools in the design cycle. An example of such a KBE tool has been demonstrated by Raju Kulkarni et al. [155,158].

5.2.2. Managing and executing a complex MDO workflow

Availability of automated disciplinary analysis tools alone is not sufficient. The disciplinary analysis tools must be combined to perform MDO. The integration tasks, especially when large number of disciplines are involved, can scale-up quickly making the entire process unmanageable. Estimates indicate that setting up first executable MDO problem takes 60%–80% of the project time. [165] Furthermore, it cannot be reconfigured easily based on the insights obtained from initial MDO execution owing to the massive effort needed for rework and limited time available for the project. These problems are also typical of the SFT model design.

Several Process Integration and Design Optimization (PIDO) systems are available on the market, that offer support in setting up optimization workflows.[166–173] However, setting up a workflow always demands a large amount of manual, tedious and error prone operations through the PIDO tool's user interface. Furthermore, most of them do not provide any support in terms of the formulation of MDO problems, such as the selection (and automatic integration) of the most convenient MDO architecture and/or optimization algorithm.

To support the existing PIDO tools, recent developments such as InFoRMA (Integration, Formalization and Recommendation of MDO Architectures) [174] and KADMOS (Knowledge- and graph-based Agile Design for Multidisciplinary Optimization System) [175] drastically improve the MDO accessibility by supporting users with formalization and execution of MDO problems. These emerging methods coupled with PIDO tools have demonstrated their value in several aerospace use-cases for prototype design [174,175]. Computationally scaled SFT model design can definitely benefit from such methods that lower the barriers of implementing MDO for both novice and experienced designers.

5.3. Catalog of sub-scale models to mitigate scale-effects

In spite of using the appropriate sub-scale design approach and incorporating all the requirements, a critical problem in sub-scale flight testing lies in overcoming scale effects. While scale-effects cannot be eliminated in all the problems, they can be mitigated by not *overloading* the similitude problem. With overloading, we intend the situation where one sub-scale model is expected to replicate many more non-dimensional parameters than physically possible for the combination of model size and test conditions. For the example shown in Appendix A, no suitable model can be found that simultaneously replicates the Reynolds number and Froude number of the prototype; such a similitude problem is said to be overloaded. Consequently, one sub-scale model cannot completely simulate the prototype behaviour for the phenomenon being tested.

Szücs proposed the theory of partial modelling to overcome the overloading problem in sub-scale model testing [137]. Partial modelling involves the sub-division of a complex system into sub-systems called partial models and studying each of the partial models separately to understand a specific aspect of prototype behaviour. Szücs then proposed the combination of results of partial model tests to predict the behaviour of the complex system. When this concept was first proposed in 1980, the implementation was rather abstract without a concrete methodology.

With the introduction of computational scaling, we can use Szücs' postulate to simulate prototype behaviour by designing, manufacturing, and testing a ***catalog of sub-scale models*** i.e., multiple sub-scale models, each one designed to offer the best similarity as required to a specific test condition or phenomenon. The results of these tests are then integrated to determine the overall prototype behaviour. However, integrating the results of sub-system can be quite challenging to implement. Thus, for the progress of SFT research, an appropriate methodology to create the catalog of sub-scale models must be identified and formalized.

One approach is to use equations of motions to list all the parameters relevant to characterize prototype behaviour and then classify them into sub-groups. Thereafter, per sub-group, a scalar function like the DoS (Eq. (12)) must be formulated. For each DoS, an optimal model (i.e., DoS = 1) must be designed using computational scaling and tested. The results of these tests can be combined together to predict prototype behaviour.

For example, the aerodynamic derivatives ($C_{m_q}, C_{z_q}, C_{z_\alpha}$ and C_{m_α}), non-dimensionalized mass and inertia can be used in the formulation of DoS for one model to study short-period motion and aerodynamic derivatives (C_z, C_{x_u} and C_{z_u}) can be used in the formulation of DoS for another model to study phugoid motion. Where, C_{m_α} and C_{m_q} are the derivatives of moment with respect to angle of attack and rotation rate respectively, C_{z_α}, C_{z_u} and C_{z_q} are the derivatives of force in z-direction with respect to angle of attack, velocity and rotation rate respectively and C_{x_u} is the derivatives of force in x-direction with respect to velocity. The behaviour of the two models can be studied together to predict the longitudinal behaviour of the prototype. For the detailed description of this approach, the reader is referred to previous work by the authors [155].

The number of designs in a catalog directly impacts the overall cost, effort and time needed to simulate prototype behaviour. Hence, a cost–benefit analysis of utilizing a catalog of sub-scale models must be performed before embarking on the process. If the size of this catalog is too large, SFT is not viable as its unique selling proposition of being an affordable simulation method is lost. The catalog size can be decreased by reducing the number of governing parameter sub-groups and thereby the number of designs. Besides, each sub-scale model should be manufactured modularly. As a result, if two or more models have similar components, they can be reused. For example, if the tail design changes, while the rest of the components are unaltered, modular design can be used to just replace the tail. Thus, a catalog of modularly designed sub-scale models using computationally scaling has the potential to mitigate scale-effects.

6. Non-technical barriers in simulation SFT

In the preceding section, different technical challenges in the current state of the art in SFT are discussed, in this section we discuss some non-technical hurdles that are commonly observed in SFT:

1. **Insufficient time for the design of SFT model:** The preceding discussion reveals that SFT model design is an involved and meticulous process which requires careful considerations. In many aspects, SFT model design cycle can be compared to the prototype design cycle with the major difference being their design objectives. While the prototype designs are aimed at demonstrating an improved performance, sub-scale models are aimed at simulating prototype behaviour before actually building the prototype. Despite the importance of designing similar model, only a small portion of the available time is allocated to design because engineers have to dedicate larger share of the lead time to overcome practical challenges such as manufacturing, pilot training and development of auto-pilot. For example, in many cases, preparation for manufacturing and acquisition of on-board equipment is started simultaneously with the design of the SFT model. As a result, the sub-scale model designs might be over-constrained by the pre-conceived decisions and unable to replicate prototype behaviour.
2. **Small SFT teams:** Often, the design teams working on SFT are limited to a handful of members without much help from disciplinary experts. Insufficient resources coupled with limited time for SFT creates tremendous pressure on small teams to "fly" the model without systematically exploring the design space. Consequently, the model cannot adequately simulate the prototype behaviour.
3. **Psychological reliance on geometric scaling:** Many experimenters who have expertise in phenomenological testing often have a psychological reliance on geometric scaling. They often raise the question, *"How can two models be similar if they are not geometrically similar?"* In fact, this question is not just encountered in SFT but is often seen in other sub-scale model testing methods where scale effects are prominent. As a result of this bias, difficulties arise in scaling up results. As discussed in Section 4 and demonstrated with an example in Appendix B, geometric similarity is neither necessary nor sufficient condition to prove similitude [137].

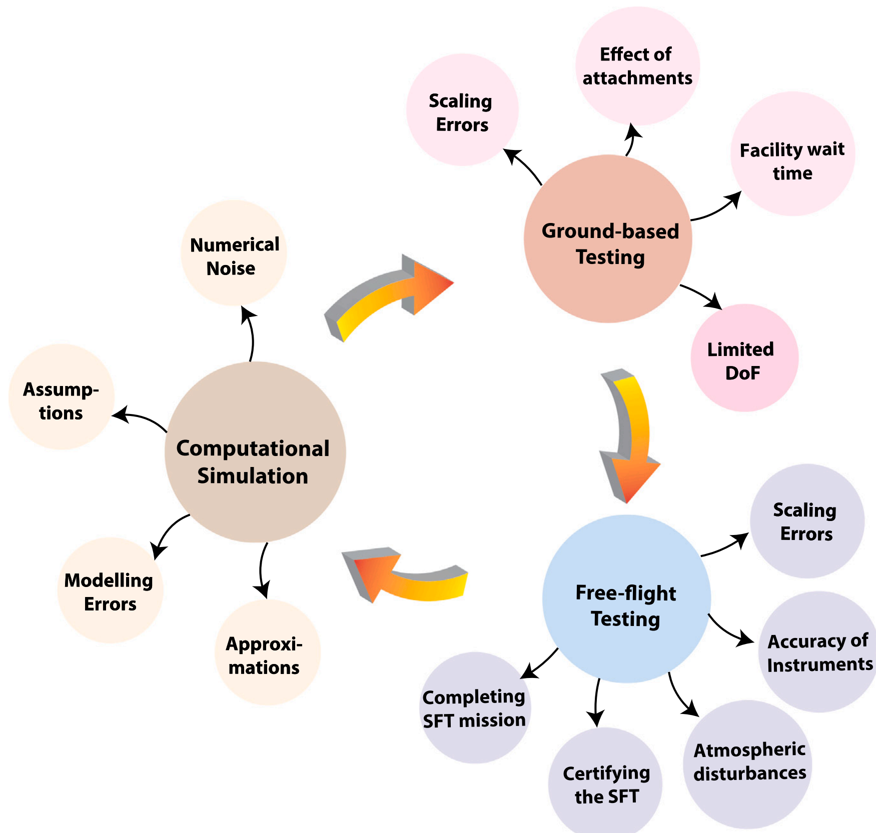


Fig. 23. Challenges in computational simulation, ground-based testing and free-flight testing.

4. **Perceived threat to ground-based testing methods:** SFT is often viewed as an attack on the conventional ground-based testing methods such as wind-tunnel testing. However, this apprehension is not well-founded. From the discussion in Section 2, we establish that SFT is used in specific dynamic tests to complement the static tests from ground based testing methods. SFT is only utilized in those cases where the ground-based testing infrastructure cannot adequately recreate the flight-conditions necessary to replicate prototype behaviour.
Not only are the objectives, applications, and implementation of past SFTs (see Sections 2 and 3) complementary to those of ground based testing, but also the associated limitations of different testing methods are complementary. Fig. 23 summarizes the inadequacies of different testing methods. With the exception of scaling-errors, computational simulation, SFT and ground-based testing are faced with different challenges. Moreover, these limitations can be overcome be systematically using a combination of testing methods. For example, the assumptions and modelling errors in computational methods can be reduced by validating the results using experimental methods. Similarly, the uncertainties induced by of atmospheric disturbances in SFT can be quantified using computational simulation and wind-tunnel testing. Finally, the problem of scaling errors can be mitigated by computational scaling. Thus, it is important to dispel this misconception and emphasize that the combination of SFT and ground-based testing is symbiotic in aircraft design cycle and not parasitic.

7. Conclusions

The main purpose of this paper is to provide an entry point into the field of SFT to engineers who are well versed in the fundamentals of aerospace engineering but not yet with the intricacies of this testing method. To this end, we reviewed publicly available literature on SFT (98 papers and books) and developments in allied fields from 1848–2021. In this study we assess four main aspects of SFT, namely, the role of SFT in aircraft design cycle, the value of using a systematic approach to design similar SFT models, developments in design approaches for SFT model and the (potential) impact of including other considerations such as manufacturing, equipment selection, etc. SFT model design process.

In this paper, SFT has been discussed in parallel to other main techniques to predict prototype behaviour, such as computational simulation and ground based testing methods such as wind tunnel testing. Based on this review, we established that SFT is best suited for the analysis of dynamic behaviour of aircraft (for both flight mechanics and aeroelasticity), as this testing method uniquely allows (unconstrained) large range of motion. Furthermore, due to the improvements and availability of miniature electronics and components, SFT can be used in all types of tests (demonstrator, phenomenological and simulation tests), which is not always possible with other ground-based testing methods or computer simulation. Majority of SFTs are used as concept demonstrators (52%) as they are adequate in arousing the interest of the scientific community while not requiring a cumbersome design approach to establish similitude with the prototype. So far, less than 30% of the total tests were used to actually simulate prototype behaviour.

The authors believe that using SFT models for the sole purpose of demonstrators, as mostly found in literature, is a major limitation of SFT used till date. More emphasis on simulation and phenomenological tests will not only enhance confidence on unconventional configurations but also reduce development cost and lead times, thereby making SFT a viable and attractive testing method in early stages of design. Thus, development of a methodology to design similar sub-scale model is the most urgent challenge to overcome to make SFT a powerful assessment method for aircraft designs.

The study of the evolution and developments in SFT model design approaches brings to light three main methods, namely, classical similitude theory, similitude theory with governing equations and approximation theory, and computational similitude theory. The salient features of each of the methods with their relative merits and challenges were discussed. The survey of past SFTs reveals that in cases where a formal design approach was considered, 75% of sub-scale models were geometrically scaled because its application is simple and time-efficient. The state-of-the-art computational scaling approach, which shows most promise in accurately scaling up SFT results, has only been used in 16% of all SFT models.

Although computational similitude theory shows significant promise in the design of similar sub-scale models, its widespread use is limited by the development complexity of the required computational systems and their computational efficiency. These barriers can be lowered by strengthening the interaction between the design, manufacturing, instrumentation and flight-testing activities by using design automation technologies such as KBE and exploiting the recent developments in MDAO. These advanced design methods, MDO and KBE, are key enablers for the application of computational similitude theory and must be harnessed to improve the quality and applicability of SFT.

A close examination of the literature also exposes the classical notion of using one sub-scale model to predict the complete flight behaviour of the prototype. To overcome this, we propose the development of a catalog of sub-scale models, whose individual responses can be superimposed to predict the overall prototype behaviour. Development of such a catalog of designs, testing all designs economically, and then combining the results of all the tests remains an open challenge whose solution will be the next breakthrough in the field of sub-scale flight testing.

CRediT authorship contribution statement

A. Raju Kulkarni: Conceptualization, Methodology, Investigation, Data curation, Writing – original draft, Writing – review & editing, Visualization. **G. La Rocca:** Conceptualization, Writing – review & editing, Supervision, Project administration. **L.L.M. Veldhuis:** Writing – review & editing, Supervision, Funding acquisition. **G. Eitelberg:** Writing – review & editing, Resources.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Appendix A. Example of classical similitude theory application

Example 1. A sub-scale model must be used to study the short period motion of a full-scale aircraft with 34 m span (b), 4.2 m mean aerodynamic chord (c) and 73 000 kg mass ($W_{full-Scale}$) flying at a velocity ($V_{full-scale}$) of 472 km/h and an altitude ($h_{full-scale}$) of 2300m. Two certification constraints[4] are considered in the design of sub-scale model as follows:

1. model must fly at an altitude (h_{model}) of 4000 m
2. model mass (W_{model}) should not exceed 100 kg

Based on this information, the test conditions (i.e., speed and altitude) and the size of the sub-scale model must be determined.

Step 1: Selection of relevant Parameters in similitude full-scale parameters:

$$W_{full-scale}, \quad V_{full-scale}, \quad c_{full-scale}, \quad h_{full-scale} \tag{13}$$

model parameters:

$$W_{model}, \quad V_{model}, \quad c_{model} \quad h_{model} \tag{14}$$

Step 2: Selection and application of scaling laws[5] Applying mass scaling and correcting for densities due to difference in test altitude gives the scaling factor λ:

$$\lambda^3 \frac{W_{full-scale}}{\rho_{full-scale}} = \frac{W_{model}}{\rho_{model}} \implies \lambda = 0.1176 \tag{15}$$

Applying geometric scaling gives the mean aerodynamic chord of the model:

$$c_{model} = \lambda * c_{full-scale} \implies c_{model} = 0.5 \text{ m} \tag{16}$$

Step 3: Selection and application of model laws[1] Applying Froude number scaling to ascertain the ratio of gravity forces to inertia forces of the model and prototype are similar [25]:

$$Fr_{model} = K_{Fr} * Fr_{full-scale} \tag{17}$$

where $K_{Fr} = 1$ (to ensure Froude number scaling) and

$$Fr = \frac{V}{\sqrt{gL}} \implies V_{model} = 44.44 \text{ m/s} \tag{18}$$

With the available capabilities of COTS components, 150 kg models have been flown at 50 m/s [63,124]. Thus, 44 m/s is a reasonable test velocity, provided the flight box is sufficiently large. In addition, for short period motion, Reynolds number scaling must also be satisfied [25] which is defined as

$$Re_{model} = K_{Re} * Re_{full-scale} \tag{19}$$

where, Reynolds number is given by formula

$$Re = \frac{\rho V L}{\mu} \tag{20}$$

Since all the variables in Eq. (20) are known, they are used to calculate K_{Re} which is the only unknown in Eq. (19). By combining Equations (16), (18), and (20), we get:

$$K_{Re} = 0.035 \tag{21}$$

conversely, if we impose

$$K_{Re} = 1, \tag{22}$$

$$k_{Fr} = 28.90 \tag{23}$$

Step 4: Evaluation

Clearly, Froude and Reynolds number similarity cannot be achieved simultaneously when used in conjunction with the certification requirements. In such cases, engineers typically choose to match a sub-set of original similitude criteria and attribute variations of results with respect to full-scale aircraft to those criteria for which similitude could not be matched.

In the case where no certification requirements are imposed, both Froude and Reynolds number similarity can be achieved by solving Eqs. (17)–(20). Fig. 24(a), (b) and (c) show the velocity, mean aerodynamic chord and span of the model calculated using Eqs. (17)–(20). The weight of the model is calculated using Eq. (15) (Fig. 24(d)). When no certification constraints are applied, the model size of the sub/super-scale model ranges from 70%–130% at varying altitudes and the weight varies between 83%–236% of the full-scale aircraft. Such large models could be as expensive as the full-scale aircraft, which defeats the purpose of using SFT (i.e., cost-effectiveness). Thus, even when no

[4] For this example, a representative value 100 kg is chosen, which is either a certified or specified category model depending on its span as per the categorization provided by the Dutch Government (Section 2.1.2.6) [73,74,176]. Certification authorities also require safe model operation proof [73,74,176], which is beyond the scope of classical similitude theory.

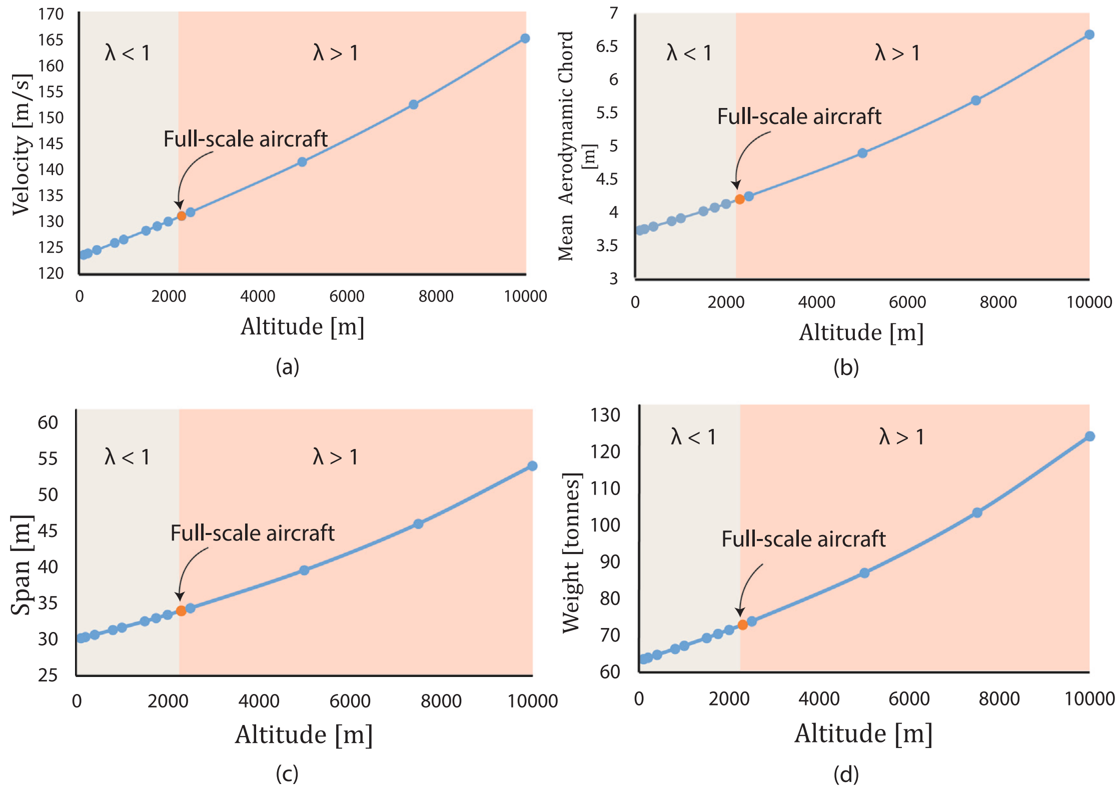


Fig. 24. The (a) velocity, (b) mean aerodynamic chord, (c) span and (d) weight of the model at different test altitudes for geometrically scaled models, determined using classical similitude theory without any certification constraints.

certification constraints are imposed, engineers limit the model weight and size to ensure that the cost of the test remains low.

Step 5: Upscaling the results

In case both model laws are satisfied simultaneously, the result of non-dimensional results of the sub-scale model tests are the same as the full-scale flight behaviour for the cases under consideration. For example, the coefficient of lift, drag and moments and aerodynamic derivatives such as the change in moment with respect to pitch rate of the model C_{M_q}, change in force in Z-direction with respect to pitch rate of the model C_{Z_q}, etc. would be the same.

Appendix B. Example of Governing equations based similitude theory application

Example 2. A sub-scale model must be designed such that its pressure distribution is similar to a 2-dimensional full-scale model, using governing equations. Where, the full-scale model has a span of length L and operates at Mach number 0.65. Furthermore, the sub-scale model must be tested at 0.3 Mach number. Unsteady effects and viscous effects may be ignored (if necessary) while ensuring similitude.

Since, similitude is established by matching the governing equations, the actual values of the flight conditions and the size of the model and the prototype are not important, as long as the governing equations used to establish similitude are the same for model and prototype. The equations shown in the remainder of the section are assumed to apply to both model and prototype.

[5] As described by Wolowicz et al. [25].

B.1. Iteration 1

Step 1: Selection of governing equation:

This problem can be best solved using Navier–Stokes equation. This equation is represented as follows: x-momentum equation:

$$\rho u\frac{\partial u}{\partial x} + \rho v\frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \frac{\partial}{\partial y}\left(\mu\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)\right) \tag{24}$$

y-momentum equation:

$$\rho u\frac{\partial v}{\partial x} + \rho v\frac{\partial v}{\partial y} = -\frac{\partial p}{\partial x} + \frac{\partial}{\partial x}\left(\mu\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)\right) \tag{25}$$

where, x is the x-coordinate, y is the y-coordinate, u is the x-component of the velocity, v is the y-component of the velocity, μ is the dynamic viscosity, ρ is the density, and p is the pressure.

Step 2: Normalization of governing equation:

The process of normalizing Navier–Stokes equation is shown in Eqs. (8) and (9). In order to ensure similitude between the model and the prototype, the coefficients of these equations must be equal for the model and the prototype (i.e., all three π-terms shown in Eq. (11)).

Step 3: Comparison of coefficients

Both Reynolds number and coefficient of pressure cannot be matched owing to differences in operating conditions. (Fig. 17)

Step 4: Transformation of the model

With the governing equation shown in Eqs. (8) and (9), transformation is not possible as long as viscous effects are a part of the equations (as shown in the example in Fig. 17). In other words, just changing the shape of the model will not be sufficient to make the coefficients of model and prototype equal.

Step 5: Application of approximation theory

Since the model cannot simultaneously match Reynolds number and the coefficient of pressure, the viscous effects are ignored to simplify the governing equations and to enable transformation.

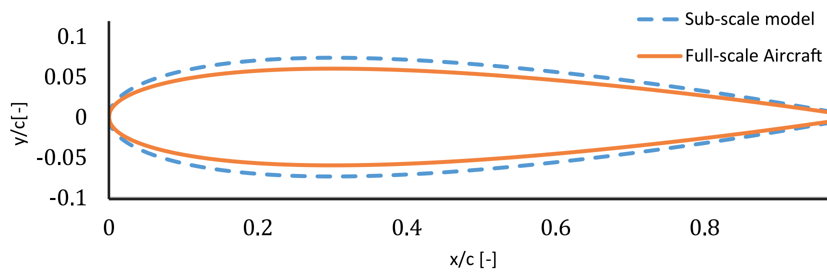


Fig. 25. The transformation of aerofoil shape necessary to maintain similar pressure distribution in inviscid flow between a sub-scale model tested at 0.3 Mach and the full-scale aircraft tested at 0.65 Mach.

B.2. Iteration 2

Step 1: Selection of governing equation

The simplifications lead to new governing equation as follows:

$$\frac{\partial^2\phi}{\partial x^2}+\frac{1}{1-M^2}\frac{\partial^2\phi}{\partial y^2}=0 \tag{26}$$

the boundary condition at $y=0$:

$$\frac{\partial\phi}{\partial y}=U_1\frac{\partial y}{\partial x} \tag{27}$$

the boundary condition at ∞:

$$\frac{\partial\phi}{\partial x}=\frac{\partial\phi}{\partial y}=0 \tag{28}$$

$$(c_p)_{body}=-\frac{2}{U_1}\frac{\partial\phi}{\partial x} \tag{29}$$

where, ϕ is the velocity potential, M is the Mach number, U_1 is the component of the velocity tangential to the body, C_p is the coefficient of pressure on the body and the remaining terms remain the same as described in Iteration 1.

Step 2: Normalization of governing equations

These governing equations are normalized as follows

$$\overline{\phi}=\frac{\phi}{U_1L},\qquad \overline{x}=\frac{x}{L},\qquad \overline{y}=\frac{y}{L} \tag{30}$$

This keeps the governing equation the same:

$$\frac{\partial^2\overline{\phi}}{\partial\overline{x}^2}+\frac{1}{1-M^2}\frac{\partial^2\overline{\phi}}{\partial\overline{y}^2}=0 \tag{31}$$

and the C_p value changes to

$$(c_p)_{body}=-2\frac{\partial\overline{\phi}}{\partial\overline{x}} \tag{32}$$

However the boundary condition changes to:

$$\frac{\partial\overline{\phi}}{\partial\overline{y}}=\frac{\partial y}{\partial x} \tag{33}$$

Step 3: Comparison of coefficients

With different Mach numbers of the model and the prototype, the coefficients of governing equation (i.e., Eq. (31)) cannot be equal for the model and the prototype. Thus, similitude cannot be established with the equations shown in Step 2.

Step 4: Transformation of the model

In order to absorb the Mach number terms into the governing equations, the y-axis can be transformed as follows:

$$y_1=\overline{y}\sqrt{1-M^2} \tag{34}$$

this makes the governing equation

$$\frac{\partial^2\overline{\phi}}{\partial\overline{x}^2}+\frac{\partial^2\overline{\phi}}{\partial {y_1}^2}=0 \tag{35}$$

which makes the boundary condition as follows

$$\frac{\partial\overline{\phi}}{\partial y_1}=\frac{1}{\sqrt{1-M^2}}\frac{\partial y}{\partial x} \tag{36}$$

assuming the dimensions of the model in y direction can be defined as a product of an arbitrary function f and a thickness scaling factor t, it is given as

$$\overline{y}=tf(\overline{x})\implies\frac{\partial y}{\partial x}=tf'(\overline{x}) \tag{37}$$

this changes the boundary conditions to

$$\frac{\partial\overline{\phi}}{\partial y_1}=\frac{t}{\sqrt{1-M^2}}f'(x) \tag{38}$$

Since $f(\overline{x})$ is purely a function of the shape the model, Eqs. (35) and (38) can be used to establish similitude (Appendix B.3). Furthermore, since the transformation is performed on y axis alone, it has no effect on $(c_p)_{body}$.

B.3. Iteration 3

Step 3: Comparison of coefficients

It is clear from Eq. (35) that the coefficients of the governing equations will always be equal because they are always equal to 1. However, at different operating Mach number, the coefficients in the boundary conditions shown in Eq. (38) will be different. If these coefficients, can be matched, the model and the prototype will be similar to one another. Thus the following conditions should be satisfied:

$$\frac{t_{Model}}{\sqrt{1-M^2_{Model}}}=\frac{t_{full-scale}}{\sqrt{1-M^2_{full-scale}}} \tag{39}$$

Step 6: Selection of model scaling

Substituting the values of the model and the full-scale flight speed from the example in Eq. (39), the thickness scaling factor of the model (t_{Model}) is 1.255. This is shown using a normalized aerofoil in Fig. 25. Thus, when the model is scaled geometrically, by a scaling factor (SF) in x-direction, it must be distorted in y-direction by a factor 1.255(SF) to ensure that the pressure distributions are the same for model and prototype. It is important to note that the relations shown in this example are not applicable to transonic flows because of singularity in Eq. (39). However, similar mathematical effort can be performed to develop scaling laws for inviscid flow in transonic conditions as shown by Kline [135].

References

[1] IATA, IATA - environment, 2017, 2017-09-28, URL: http://www.iata.org/whatwedo/environment/Pages/index.aspx.

[2] IATA, IATA - environmental policy, 2017, 2017-09-28, URL: http://www.iata.org/policy/environment/Pages/default.aspx.

[3] IATA, IATA - IATA forecasts passenger demand to double over 20 years, 2017, 2017-09-28, URL: http://www.iata.org/pressroom/pr/Pages/2016-10-18-02.aspx.

[4] IATA, 2036 Forecast reveals air passengers will nearly double to 7.8 billion, 2018, 2018-11-23, URL: https://www.iata.org/pressroom/pr/pages/2017-10-24-01.aspx.

[5] E. Torenbeek, Advanced Aircraft Design: Conceptual Design, Analysis and Optimization of Subsonic Civil Airplanes, John Wiley & Sons, 2013.

[6] E. Torenbeek, Synthesis of Subsonic Airplane Design: An Introduction To the Preliminary Design of Subsonic General Aviation and Transport Aircraft, with Emphasis on Layout, Aerodynamic Design, Propulsion and Performance, Springer Science & Business Media, 2013.

[7] D. Raymer, Aircraft Design: A Conceptual Approach, American Institute of Aeronautics and Astronautics, Inc. 2012.

[8] A. Frediani, The prandtl wing, in: Innovative Configurations and Advanced Concepts for Future Civil Transport Aircraft, in: Von Karman Institute for Fluid Dynamics: VKI Lecture Series, Von KÁrman Institute for Fluid Dynamics, Rhode St-GenÈse, 2005.

[9] R. Vos, M.F.M. Hoogreef, System-level assessment of tail-mounted propellers for regional aircraft, in: Proceedings of the 31st Congress of the International Council of the Aeronautical Sciences, 2018, https://repository.tudelft.nl/islandora/object/uuid.

[10] H. Kok, M. Voskuijl, M. van Tooren, Distributed propulsion featuring boundary layer ingestion engines for the blended wing body subsonic transport, in: 51st AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference 18th AIAA/ASME/AHS Adaptive Structures Conference 12th, 2010, p. 3064.

[11] B.J. Brelje, J.R.R.A. Martins, Electric, hybrid, and turboelectric fixed-wing aircraft: A review of concepts, models, and design approaches, Prog. Aerosp. Sci. 104 (2019) 1–19, http://dx.doi.org/10.1016/j.paerosci.2018.06.004, URL: http://www.sciencedirect.com/science/article/pii/S0376042118300356.

[12] J.R. Chambers, R.M. Hall, Historical review of uncommanded lateral-directional motions at transonic conditions, J. Aircr. 41 (3) (2019) 436–447, http://dx.doi.org/10.2514/1.4470, 2019-12-17, URL: https://arc.aiaa.org/doi/10.2514/1.4470.

[13] R. Hall, R. Biedron, D. Ball, D. Bogue, J. Chung, B. Green, M. Grismer, G. Brooks, J. Chambers, Computational methods for stability and control (COMSAC): The time has come, in: AIAA Atmospheric Flight Mechanics Conference and Exhibit, American Institute of Aeronautics and Astronautics, San Francisco, California, 2019, http://dx.doi.org/10.2514/6.2005-6121, 2019-12-17, URL: http://arc.aiaa.org/doi/10.2514/6.2005-6121.

[14] G. Murphy, Similitude in Engineering, Ronald Press Co. 1950.

[15] W. Wei, M. Hansen, Cost economics of aircraft size, J. Transp. Econ. Policy 37 (2) (2003) 279–296, URL: https://www.jstor.org/stable/20053934.

[16] D. Starkie, S. Ellis, The production economics of a very large civil aircraft, J. Air Transp. Manag. 2 (1) (2020) 11–16, http://dx.doi.org/10.1016/0969-6997(95)00020-C, 2020-09-08, URL: https://linkinghub.elsevier.com/retrieve/pii/096969979500020C.

[17] D.P. Bergmann, J. Denzel, A. Baden, L. Kugler, A. Strohmayer, Innovative scaled test platform e-genius-mod—Scaling methods and systems design, Aerospace 6 (2) (2019) 20.

[18] D. Kuehme, N.R. Alley, C. Phillips, B.R. Cogan, Flight test evaluation and system identification of the area-I prototype-technology-evaluation research aircraft (PTERA), in: AIAA Flight Testing Conference, 2014, p. 2577.

[19] C. Jouannet, D. Lundström, K. Amadori, P. Berry, Design of a very light jet and a dynamically scaled demonstrator, in: 46th AIAA Aerospace Sciences Meeting and Exhibit, 2008, pp. 7–10.

[20] T. Models, TJD models radio control trainer planes, 2020, 2020-11-19, URL: https://www.tjdmodels.com/electric-aircraft-trainer-c-12_29_76.html.

[21] A. Kurte, HST® impeller mit vollintegrierten motoren, 2020, 2020-11-19, URL: https://schuebeler-jets.de/de/?Itemid={107}.

[22] Model, Micro turbines for remote controlled model jets - model aircraft company, 2020, 2020-11-19, URL: http://modelaircraftcompany.com/newshop/en/18-micro-jet-turbines.

[23] Jetcat, Products - JetCat, 2020, 2020-10-09, URL: https://www.jetcat.de/en/products/.

[24] J. Chambers, Modeling Flight NASA Latest Version: The Role of Dynamically Scale Free Flight Models in Support of NASA Aerospace Programs, Vol. 3, Joseph Chambers, 2017, 2017-10-02.

[25] C.H. Wolowicz, J.S. Brown Jr., W.P. Gilbert, Similitude requirements and scaling relationships as applied to model testing, 1979.

[26] D.M. Bushnell, Scaling: wind tunnel to flight, Annual Review of Fluid Mechanics 38 (2006) 111–128.

[27] C.P. Coutinho, A.J. Baptista, J. Dias Rodrigues, Reduced scale models based on similitude theory: A review up to 2015, Eng. Struct. 119 (2016) 81–94, http://dx.doi.org/10.1016/j.engstruct.2016.04.016, URL: http://www.sciencedirect.com/science/article/pii/S0141029616301274.

[28] A. Casaburo, G. Petrone, F. Franco, S. De Rosa, A review of similitude methods for structural engineering, Appl. Mech. Rev. 71 (3) (2020) 030802, http://dx.doi.org/10.1115/1.4043787, 2020-01-27, URL: https://asmedigitalcollection.asme.org/appliedmechanicsreviews/article/doi/10.1115/1.4043787/725695/A-Review-of-Similitude-Methods-for-Structural.

[29] A. Sobron, D. Lundström, P. Krus, A review of current research in subscale flight testing and analysis of its main practical challenges, Aerospace 8 (3) (2021) 74.

[30] J.R.R.A. Martins, Perspectives on aerodynamic design optimization, in: AIAA Scitech 2020 Forum, American Institute of Aeronautics and Astronautics, 2020, http://dx.doi.org/10.2514/6.2020-0043, URL: https://arc.aiaa.org/doi/abs/10.2514/6.2020-0043.

[31] Y. Fan, W. Li, Review of far-field drag decomposition methods for aircraft design, J. Aircr. 56 (1) (2019) 11–21, http://dx.doi.org/10.2514/1.C034781.

[32] A. Viviani, A. Aprovitola, G. Pezzella, C. Rainone, CFD design capabilities for next generation high-speed aircraft, Acta Astronaut. (2020).

[33] D. Greenwell, A review of unsteady aerodynamic modelling for flight dynamics of manoeuvrable aircraft, in: AIAA Atmospheric Flight Mechanics Conference and Exhibit, 2004, p. 5276.

[34] M. Ghoreyshi, K.J. Badcock, A.D. Ronch, S. Marques, A. Swift, N. Ames, Framework for establishing limits of tabular aerodynamic models for flight dynamics analysis, J. Aircr. 48 (1) (2011) 42–55.

[35] J.P. Fielding, H. Smith, FLAVIIR an innovative university/industry research program for collaborative research and demonstration of UAV technologies, in: Proceedings of the 25th International Congress of the Aeronautical Sciences, ICAS, 2006.

[36] A. Yarf-Abbasi, J.P. Fielding, Design integration of the eclipse and demon demonstrator UAVs, in: Proceedings of the Seventh AIAA Aviation Technology, Integration and Operations Conference (ATIO), Belfast, UK, 2007, pp. 18–20.

[37] O.D. Dantsker, R. Mancuso, Flight data acquisition platform development, integration, and operation on small- to medium-sized unmanned aircraft, in: AIAA Scitech 2019 Forum, American Institute of Aeronautics and Astronautics, 2019, http://dx.doi.org/10.2514/6.2019-1262, URL: https://arc.aiaa.org/doi/10.2514/6.2019-1262.

[38] O.D. Dantsker, R. Mancuso, M.S. Selig, M. Caccamo, High-frequency sensor data acquisition system (SDAC) for flight control and aerodynamic data collection, in: 32nd AIAA Applied Aerodynamics Conference, 2014, p. 2565.

[39] A. Bergmann, A. Huebner, T. Loeser, Experimental and numerical research on the aerodynamics of unsteady moving aircraft, Prog. Aerosp. Sci. 44 (2) (2008) 121–137, http://dx.doi.org/10.1016/j.paerosci.2007.10.006, URL: http://www.sciencedirect.com/science/article/pii/S0376042107000838.

[40] D.D. Vicroy, T.D. Loeser, A. Schütte, Static and forced-oscillation tests of a generic unmanned combat air vehicle, J. Aircr. 49 (6) (2012) 1558–1583, http://dx.doi.org/10.2514/1.C031501, URL: https://arc.aiaa.org/doi/10.2514/1.C031501.

[41] JAXA, Aeroengine test facilities | test facilities | aeronautical technology directorate, 2021, 2021-05-31, URL: https://www.aero.jaxa.jp/eng/facilities/aeroengine/.

[42] J. Schijve, Fatigue of aircraft materials and structures, Int. J. Fatigue 16 (1) (1994) 21–32.

[43] DNW, DNW wind-tunnels, 2021, 2021-05-31, URL: https://www.dnw.aero//.

[44] W.R. Krüger, J. Dillinger, M.Y. Meddaikar, J. Lübker, M. Tang, T. Meier, M. Büwald, K.I. Soal, M. Pusch, T. Kier, Design and wind tunnel test of an actively controlled flexible wing, in: Proc. International Forum on Aeroelasticity and Strucutural Dynamics 2019, 2019, URL: https://elib.dlr.de/127970/.

[45] M. Novak, A.G. Davenport, Aeroelastic instability of prisms in turbulent flow, J. Eng. Mech. Div. 96 (1) (2021) 17–39, http://dx.doi.org/10.1061/JMCEA3.0001210, 2021-05-31, URL: https://ascelibrary.org/doi/abs/10.1061/JMCEA3.0001210.

[46] T. Sinnige, Aerodynamic and Aeroacoustic Interaction Effects for Tip-Mounted Propellers: An Experimental Study (Ph.D. Thesis), Delft University of Technology, 2018.

[47] M.B. Rivers, A. Dittberner, Experimental investigations of the NASA common research model, J. Aircr. 51 (4) (2014) 1183–1193, http://dx.doi.org/10.2514/1.C032626.

[48] D. Vicroy, T. Loeser, A. Schuette, SACCON dynamic wind tunnel tests at DNW-NWB and 14'x22' NASA larc, in: 28th AIAA Applied Aerodynamics Conference, American Institute of Aeronautics and Astronautics, Chicago, Illinois, 2010, http://dx.doi.org/10.2514/6.2010-4394, URL: http://arc.aiaa.org/doi/10.2514/6.2010-4394.

[49] A. Cartieri, Experimental investigations on the common research model at ONERA-S2MA, in: AIAA Scitech 2020 Forum, in: AIAA SciTech Forum, American Institute of Aeronautics and Astronautics, 2021, http://dx.doi.org/10.2514/6.2020-0779, URL: https://arc.aiaa.org/doi/10.2514/6.2020-0779.

[50] S. Gholizadeh, A review of non-destructive testing methods of composite materials, Procedia Struct. Integr. 1 (2016) 50–57.

[51] M.C.-Y. Niu, M. Niu, Airframe Structural Design: Practical Design Information and Data on Aircraft Structures, second ed., Adaso/Adastra Engineering Center, 2011.

[52] T.H.G. Megson, Aircraft Structures for Engineering Students, Butterworth-Heinemann, 2016.

[53] Safran, A specific range of test equipment, 2021, 2021-05-31, URL: https://www.safran-aero-boosters.com/test-facilities/specific-range-test-equipment.

[54] T.C.A. Stokkermans, Aerodynamics of propellers in interaction dominated flowfields: an application to novel aerospace vehicles, 2020, http://dx.doi.org/10.4233/uuid:46178824-bb80-4247-83f1-dc8a9ca7d8e3.

[55] L.L.M. Veldhuis, Propeller wing aerodynamic interference, 2005, https://repository.tudelft.nl/islandora/object/uuid.
[56] Y.-c. Fung, An Introduction to the Theory of Aeroelasticity, Wiley, 1955.
[57] A.R. Collar, The first fifty years of aeroelasticity, Aerospace (R. Aeronaut. Soc. J.) 5 (2) (1978) 12–20.
[58] C. Mertens, T. de Rojas Cordero, J. Sodja, A. Sciacchitano, B. van Oudheusden, Determination of collar's triangle of forces on a flexible wing based on particle tracking velocimetry measurements, in: AIAA Scitech 2021 Forum, in: AIAA SciTech Forum, American Institute of Aeronautics and Astronautics, 2021, http://dx.doi.org/10.2514/6.2021-0221, URL: https://arc.aiaa.org/doi/10.2514/6.2021-0221.
[59] M.E. Goldstein, Aeroacoustics, McGraw-Hill International Book Co. New York, 1976.
[60] J.F. Williams, Aeroacoustics, Annu. Rev. Fluid Mech. 9 (1) (1977) 447–468.
[61] C.K. Tam, Computational aeroacoustics: An overview of computational challenges and applications, Int. J. Comput. Fluid Dyn. 18 (6) (2004) 547–567.
[62] B. Owens, D. Cox, E. Morelli, Development of a low-cost sub-scale aircraft for flight research: The FASER project, in: 25th AIAA Aerodynamic Measurement Technology and Ground Testing Conference, American Institute of Aeronautics and Astronautics, San Francisco, California, 2019, http://dx.doi.org/10.2514/6.2006-3306, 2019-12-06, URL: http://arc.aiaa.org/doi/10.2514/6.2006-3306.
[63] D. Vicroy, X-48B blended wing body ground to flight correlation update, in: AIAA Aero Sciences Meeting January 4-7, 2011, 2011, p. 30.
[64] B. Owens, J. Brandon, M. Croom, M. Fremaux, G. Heim, D. Vicroy, Overview of dynamic test techniques for flight dynamics research at NASA larc, in: 25th AIAA Aerodynamic Measurement Technology and Ground Testing Conference, American Institute of Aeronautics and Astronautics, San Francisco, California, 2019, http://dx.doi.org/10.2514/6.2006-3146, 2019-11-13, URL: http://arc.aiaa.org/doi/10.2514/6.2006-3146.
[65] A.I. Neihouse, W.J. Klinar, S.H. Scher, Status of Spin Research for Recent Airplane Designs, Vol. 57, National Aeronautics and Space Administration, 1960.
[66] J.S. Bowman Jr., Free-spinning-tunnel investigation of gyroscopic effects of jet-engine rotating parts (or of rotating propellers) - NACA technical note, 1955.
[67] J.S. Browman Jr., F.M. Healy, Free-spinning-tunnel investigation of a 1/25-Scale model of the chance vought F8U-1P airplane - NACA technical note, 1959.
[68] A. Dorobantu, A. Murch, B. Mettler, G. Balas, System identification for small, low-cost, fixed-wing unmanned aircraft, J. Aircr. 50 (4) (2013) 1117–1130.
[69] A.T. Perry, T. Bretl, P.J. Ansell, System identification and dynamics modeling of a distributed electric propulsion aircraft, in: AIAA Aviation 2019 Forum, 2019, p. 3086.
[70] D. Fratello, M. Croom, L. Nguyen, C. Domack, Use of the updated NASA langley radio-controlled drop-model technique for high-alpha studies of the X-29A configuration, in: 14th Atmospheric Flight Mechanics Conference, 1987, p. 2559.
[71] M. Croom, H. Kenney, D. Murri, K. Lawson, Research on the F/A-18E/F using a 22 percent-dynamically-scaled drop model, in: Atmospheric Flight Mechanics Conference, 2000, p. 3913.
[72] E.C. Holleman, Summary of Flight Tests To Determine the Spin and Controllability Characteristics of a Remotely Piloted, Large-Scale (3/8) Fighter Airplane Model, National Aeronautics and Space Administration, 1976.
[73] EU, Commission implementing regulation (EU) 2019/947 of 24 may 2019 on the rules and procedures for the operation of unmanned aircraft (text with EEA relevance), 2019, 32019R0947URL: http://data.europa.eu/eli/reg_impl/2019/947/oj/eng.
[74] T.N. Government, Rules for flying drones, 2021, 2021-06-14, URL: https://business.gov.nl/regulation/drones/.
[75] W.E. Baker, P.S. Westine, F.T. Dodge, Similarity Methods in Engineering Dynamics: Theory and Practice of Scale Modeling, Spartan Books;[distributed by] Hayden Book Co., 1973.
[76] H.L. Langhaar, Dimensional Analysis and Theory of Models, Vol. 2, Wiley, New York, 1951.
[77] J.P. Slotnick, Integrated CFD validation experiments for prediction of turbulent separated flows for subsonic transport aircraft, in: NATO Science and Technology Organization, Meeting Proceedings RDP, STO-MP-AVT-307, 2019.
[78] S.H. Goldthorpe, K.F. Rossitto, D.C. Hyde, K.R. Krothapalli, X-48b blended wing body flight test performance of maximum sideslip and high to post stall angle-of-attack command tracking, in: AIAA Guidance, Navigation, and Control Conference, 2010, pp. 1–17.
[79] T.L. Jordan, J.V. Foster, R.M. Bailey, C.M. Belcastro, AirSTAR: A UAV platform for flight dynamics and control system testing, in: 25th AIAA Aerodynamic Measurement Technology and Ground Testing Conference, 2006, pp. 2006–3307.
[80] D.-Y. Kwak, K. Yoshida, H. Ishikawa, M. Noguchi, Flight test measurements of surface pressure on unmanned scaled supersonic experimental airplane, in: 24th AIAA Applied Aerodynamics Conference, Vol. 3483, 2006, p. 2006.
[81] E.M. Kraft, The air force digital thread/digital twin - life cycle integration and use of computational and experimental knowledge, in: 54th AIAA Aerospace Sciences Meeting, in: AIAA SciTech Forum, American Institute of Aeronautics and Astronautics, 2016, 2016-01-02, URL: http://arc.aiaa.org/doi/10.2514/6.2016-0897.
[82] J. Hileman, Z. Spakovszky, M. Drela, M. Sargeant, Airframe design for" silent aircraft", in: 45th AIAA Aerospace Sciences Meeting and Exhibit, 2007, p. 453.
[83] K.R. Moore, A. Ning, Distributed electric propulsion effects on existing aircraft through multidisciplinary optimization, in: 2018 AIAA/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference, 2018, p. 1652.
[84] V.N. Vatsa, J.C. Lin, L.G. Pack Melton, D.P. Lockard, R. Ferris, CFD and experimental data comparisons for conventional and AFC-enabled CRM high-lift configurations, in: AIAA Aviation Forum, 2020, p. 2939.
[85] E.N. Tinoco, An evaluation and recommendations for further CFD research based on the NASA common research model (CRM) analysis from the AIAA drag prediction workshop (DPW) series, 2019.
[86] A. Sobron, On Subscale Flight Testing: Applications in Aircraft Conceptual Design (Ph.D. Thesis), Linköping University Electronic Press, 2018.
[87] N. Matheny, G. Panageas, HiMAT aerodynamic design and flight test experience, in: 1st Flight Test Conference, American Institute of Aeronautics and Astronautics, Las Vegas,NV,U.S.A., 2020, http://dx.doi.org/10.2514/6.1981-2433, 2020-02-05, URL: http://arc.aiaa.org/doi/10.2514/6.1981-2433.
[88] J.L.L. Lockenour, RPRV research focus on himat, Astronaut. Aeronaut. (2020) 2020-02-05, URL: https://ntrs.nasa.gov/search.jsp?R=19760042755.
[89] D.A. Deets, HiMAT Flight Program: Test Results and Program Assessment Overview, Vol. 86725, Scientific and Technical Report, National Aeronautics and Space Administration, 1986.
[90] K.L. Petersen, Flight control systems development of highly maneuverable aircraft technology /himat/ vehicle, 2017, Aug. 20-22, 1979, 2017-10-11, URL: https://ntrs.nasa.gov/search.jsp?R=19790063865.
[91] U. Cho, K.L. Wood, H. Crawford, Novel empirical similarity method for the reliable product test with rapid prototypes, in: International Design Engineering Technical Conferences and Computers and Information in Engineering Conference (Vol. 80326, P. V002T02A029), American Society of Mechanical Engineers, 1998, p. 14.
[92] R.N. Chuk, V.J. Thomson, A comparison of rapid prototyping techniques used for wind tunnel model fabrication, Rapid Prototyp. J. 4 (4) (1998) 185–196, http://dx.doi.org/10.1108/13552549810239030.
[93] M. French, An application of structural optimization in wind tunnel model design, in: 31st Structures, Structural Dynamics and Materials Conference, American Institute of Aeronautics and Astronautics, Long Beach,CA,U.S.A., 2020, http://dx.doi.org/10.2514/6.1990-956, 2020-01-29, URL: http://arc.aiaa.org/doi/10.2514/6.1990-956.
[94] T. Duda, L.V. Raghavan, 3D metal printing technology, 17th IFAC Conference on International Stability, Technology and Culture TECIS 2016, IFAC-PapersOnLine 49 (29) (2016) 103–110, http://dx.doi.org/10.1016/j.ifacol.2016.11.111, URL: http://www.sciencedirect.com/science/article/pii/S2405896316325496.
[95] P. John, The flapless air vehicle integrated industrial research (FLAVIIR) programme in aeronautical engineering, Proc. Inst. Mech. Eng. G 224 (4) (2010) 355–363.
[96] R. Kurzweil, The Singularity Is Near: when Humans Transcend Biology, 2005, Penguin, Google-Books-ID, 9FtnppNpsT4C.
[97] Cardwave, SD cards: A trip down memory lane | cardwave, 2019, 2019-04-05, URL: https://www.cardwave.com/knowledge-hub/sd-cards-a-trip-down-memory-lane/.
[98] KOOFR, History and evolution of memory cards, 2021, 2021-06-09, URL: https://koofr.eu/blog/posts/history-and-evolution-of-memory-cards.
[99] R.A. Bunge, A.E. Alkurdi, E. Alfaris, I. Kroo, In-flight measurement of wing surface pressures on a small-scale UAV during stall/spin maneuvers, in: AIAA Flight Testing Conference, American Institute of Aeronautics and Astronautics, 2021, http://dx.doi.org/10.2514/6.2016-3652, 2021-06-08, URL: https://arc.aiaa.org/doi/10.2514/6.2016-3652.
[100] L. Meier, P. Tanskanen, F. Fraundorfer, M. Pollefeys, Pixhawk: A system for autonomous flight using onboard computer vision, in: 2011 IEEE International Conference on Robotics and Automation, IEEE, 2011, pp. 2992–2997.
[101] Urban air mobility | airbus, 2021, https://www.airbus.com/en/innovation/zero-emission/urban-air-mobility.
[102] Advancing aerial mobility: a national blueprint | the national academies press, 2021, https://www.nap.edu/download/25646.
[103] Delft University of Technology, DUUC aircraft with the innovative 'propulsive empennage' concept, 2020, 2020-04-16, URL: https://www.youtube.com/watch?v=VDbJBkcQBPI.
[104] K. Hameeteman, Unconventional propulsive empennage - future or fiction?: Stability and control analysis and the effect of scaling of the DUUC, 2017, https://repository.tudelft.nl/islandora/object/uuid.
[105] L.A. Walker, Flight Testing the X-36: The Test Pilots Perspective, Technical Report, 1997, URL: https://ntrs.nasa.gov/search.jsp?R=19970031950.
[106] M.A. Dornheim, Mcdonnell douglas rolls out X-36, AW & ST 25 (1996) 20–22.
[107] P. Woods, FLAVIIR–an integrated programme of research for UAVs, in: Proc. of the 3rd AIAA Flow Control Conference, San Francisco, 2017, pp. 5–7, 2017-10-02, URL: https://arc.aiaa.org/doi/pdfplus/10.2514/6.2006-3504.
[108] A. Yarf-Abbasi, A. Clarke, C.P. Lawson, J.P. Fielding, Design and development of the eclipse and demon demonstrator UAVs, in: Proceedings of the 26th Congress of International Council of Aeronautical Sciences (ICAS 2008), Canada, 2008.

[109] C. Jouannet, D. Lundström, K. Amadori, P. Berry, Design and flight testing of an ECO-sport aircraft, in: 48th AIAA Aerospace Sciences Meeting Including the New Horizons Forum and Aerospace Exposition, American Institute of Aeronautics and Astronautics, Orlando, Florida, 2010, http://dx.doi.org/10.2514/6.2010-1206, URL: http://arc.aiaa.org/doi/10.2514/6.2010-1206.

[110] W.J.M. Fredericks, Greased Lightning (GL-10) Flight Testing Campaign, Technical Report, 2017, URL: https://ntrs.nasa.gov/search.jsp?R=20170007194.

[111] P. Schmollgruber, J.-L. Gobert, P.-E. Gall, Z. Goraj, H.W. Jentink, A. Näs, R. Voit-Nitchmann, An innovative evaluation platform for new aircraft concepts, Aeronaut. J. 114 (1157) (2010) 451–456, http://dx.doi.org/10.1017/S0001924000003936, URL: https://www.cambridge.org/core/journals/aeronautical-journal/article/an-innovative-evaluation-platform-for-new-aircraft-concepts/F202F2A88561E91FFD66AE15685B64C7.

[112] Z. Goraj, K. Kitmann, R. Voit-Nitschmann, M. Szender, Design and integration of flexi-bird-a low cost sub-scale research aircraft for safety and environmental issues, in: 27th International Congress of the Aeronautical Sciences, Nice, 2010.

[113] O.D. Dantsker, M. Caccamo, M. Vahora, R. Mancuso, Flight & ground testing data set for an unmanned aircraft: Great planes avistar elite, in: AIAA Scitech 2020 Forum, American Institute of Aeronautics and Astronautics, 2020, http://dx.doi.org/10.2514/6.2020-0780, URL: https://arc.aiaa.org/doi/abs/10.2514/6.2020-0780.

[114] Airbus, The albatross is inspiring tomorrow's aircraft wings, 2020, Airbus, 2020-02-06, URL: https://www.airbus.com/newsroom/stories/the-albatross-is-inspiring-tomorrows-next-generation-of-aircraft-wings.html.

[115] Airbus, Airbus unveils 'maverick', its blended wing aircraft demonstrator, 2020, 2020-02-12, URL: https://www.gonewsindia.com/latest-news/technology/airbus-unveils-maverick-a-blended-wing-aircraft-demonstrator-8832.

[116] Y. Ma, W. Zhang, Y. Zhang, X. Zhang, Y. Zhong, Sizing method and sensitivity analysis for distributed electric propulsion aircraft, J. Aircr. (2020) 1–12, http://dx.doi.org/10.2514/1.C035581, URL: https://arc.aiaa.org/doi/10.2514/1.C035581.

[117] Delft University of Technology, Flying-v, 2020-10-30, URL: https://www.tudelft.nl/en/ae/flying-v/.

[118] M. Palermo, R. Vos, Experimental aerodynamic analysis of a 4.6%-scale flying-v subsonic transport, in: AIAA Scitech 2020 Forum, 2020, p. 2228.

[119] C. Courtin, R.J. Hansman, M. Drela, Flight test results of a subscale super-STOL aircraft, in: AIAA Scitech 2020 Forum, 2020, p. 0977.

[120] A. Gross, C. Pearman, R. Kremer, B. Napier, C. Gosla, A. Kurz, S. Mack, C. Brehm, B. Heine, A. Radi, B. Marovic, S. Retzko, N. Feindler, B. Zickler, H. Fasel, A. Osbrink, 1/5 scale model of aeromot 200s SuperXimango for scaled flight research, in: 26th AIAA Applied Aerodynamics Conference, American Institute of Aeronautics and Astronautics, Honolulu, Hawaii, 2019, http://dx.doi.org/10.2514/6.2008-6416, 2019-12-10, URL: http://arc.aiaa.org/doi/10.2514/6.2008-6416.

[121] O.D. Dantsker, G.K. Ananda, M.S. Selig, GA-UStar phase 1: Development and flight testing of the baseline upset and stall research aircraft, in: 35th AIAA Applied Aerodynamics Conference, American Institute of Aeronautics and Astronautics, Denver, Colorado, 2017, http://dx.doi.org/10.2514/6.2017-4078, URL: https://arc.aiaa.org/doi/10.2514/6.2017-4078.

[122] T. Jordan, W. Langford, C. Belcastro, J. Foster, G. Shah, G. Howland, R. Kidd, Development of a Dynamically Scaled Generic Transport Model Testbed for Flight Research Experiments, NASA Technical Report, 2004.

[123] C. Regan, In-flight stability analysis of the X-48b aircraft, in: AIAA Atmospheric Flight Mechanics Conference and Exhibit, 2008, p. 6571.

[124] T. Risch, G. Cosentino, C.D. Regan, M. Kisska, N. Princen, X-48b flight-test progress overview, in: 47th AIAA Aerospace Sciences Meeting Including the New Horizons Forum and Aerospace Exposition, 2009.

[125] K. Amadori, C. Jouannet, P. Berry, Development of a subscale flight testing platform for a generic future fighter, in: 27th International Congress of the Aeronautical Sciences-ICAS 2010, 2010.

[126] A. Sobron, On Subscale Flight Testing: Applications in Aircraft Conceptual Design, Vol. 1819, Linköping University Electronic Press, 2018.

[127] K. Pieper, A. Perry, P. Ansell, T. Bretl, Design and development of a dynamically, scaled distributed electric propulsion aircraft testbed, in: 2018 AIAA/IEEE Electric Aircraft Technologies Symposium (EATS), 2018, pp. 1–2, ISSN: null.

[128] D.P. Raymer, Dan Raymer's Simplified Aircraft Design for Homebuilders, first ed., Design Dimension Press, 2003.

[129] R.M. Hueschen, Development of the Transport Class Model (TCM) Aircraft Simulation From a Sub-Scale Generic Transport Model (GTM) Simulation, Technical Report, 2019, 2019-12-17, URL: https://ntrs.nasa.gov/search.jsp?R=20110014509.

[130] G. Hagler, William froude, in: Modeling Ships and Space Craft, Springer New York, New York, NY, 2020, pp. 109–134, http://dx.doi.org/10.1007/978-1-4614-4596-8_6, 2020-01-07, URL: http://link.springer.com/10.1007/978-1-4614-4596-8_6.

[131] D. Jackson, B. Launder, Osborne Reynolds and the publication of his papers on turbulent flow, Annu. Rev. Fluid Mech. 39 (1) (2020) 19–35, http://dx.doi.org/10.1146/annurev.fluid.39.050905.110241, 2020-01-07URL: https://doi.org/10.1146/annurev.fluid.39.050905.110241.

[132] O. Reynolds, IV. on the dynamical theory of incompressible viscous fluids and the determination of the criterion, Philos. Trans. R. Soc. Lond. (A.) 186 (2020) 123–164, http://dx.doi.org/10.1098/rsta.1895.0004, 2020-01-07, URL: https://royalsocietypublishing.org/doi/abs/10.1098/rsta.1895.0004.

[133] E. Buckingham, On physically similar systems; illustrations of the use of dimensional equations, Phys. Rev. 4 (4) (2020) 345–376, http://dx.doi.org/10.1103/PhysRev.4.345, 2020-01-07., URL: https://link.aps.org/doi/10.1103/PhysRev.4.345.

[134] L. Rayleigh, The principle of similitude, Nature 95 (1915) 66.

[135] S.J. Kline, Similitude and Approximation Theory, Springer Science & Business Media, 2012.

[136] E.O. Macagno, Historico-critical review of dimensional analysis, J. Franklin Inst. B 292 (6) (1971) 391–402, http://dx.doi.org/10.1016/0016-0032(71)90160-8, URL: http://www.sciencedirect.com/science/article/pii/0016003271901608.

[137] E. Szücs, Similitude and Modelling, Elsevier, 2012, Google-Books-ID, ORQ7AAAAQBAJ.

[138] M. Scherberg, R.V. Rhode, Mass distribution and performance of free flight models, 2019, 2019-12-16, URL: http://ntrs.nasa.gov/search.jsp?R=19930081026.

[139] A.I. Neihouse, P.W. Pepoon, Dynamic similitude between a model and a full-scale body for model investigation at full-scale mach number, 1950.

[140] L.I. Sedov, Similarity and Dimensional Methods in Mechanics, CRC Press, 2018.

[141] J.N. Goodier, W.T. Thomson, Applicability of Similarity Principles to Structural Models, Vol. 933, National Advisory Committee for Aeronautics, 1944.

[142] P.W. Bridgman, Dimensional Analysis, Yale University Press, 1922.

[143] J. Mas Colomer, N. Bartoli, T. Lefebvre, S. Dubreuil, J. Martins, E. Bénard, J. Morlier, Similarity maximization of a scaled aeroelastic flight demonstrator via multidisciplinary optimization, in: 58th AIAA/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference, 2017, p. 0573, 2017-02-14, URL: http://arc.aiaa.org/doi/pdf/10.2514/6.2017-0573.

[144] C. Adams, J. Bs, T. Melz, An experimental investigation of vibrating plates in similitude and the possibility to replicate the responses using sensitivity-based scaling laws, in: INTER-NOISE and NOISE-CON Congress and Conference Proceedings, Vol. 257, Institute of Noise Control Engineering, 2018, pp. 799–810.

[145] P. Pereira, L. Almeida, A. Suleman, V. Bond, R. Canfield, M. Blair, Aeroelastic scaling and optimization of a joined-wing aircraft concept, in: 48th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference, 2007, p. 1889.

[146] J. Wissmann, Dynamic stability of space vehicles. structural dynamics model testing(dimensional analysis, and similitude and scaling laws for structural dynamics model testing of spacecraft dynamic stability), 1968.

[147] M. French, An application of structural optimization in wind tunnel model design, in: 31st Structures, Structural Dynamics and Materials Conference, 1990, p. 956.

[148] A. Ricciardi, R. Canfield, M. Patil, N. Lindsley, Nonlinear aeroelastic scaling of a joined wing aircraft, in: 53rd AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics and Materials Conference 20th AIAA/ASME/AHS Adaptive Structures Conference 14th AIAA, 2012, p. 1454.

[149] J. Mas-Colomer, Aeroelastic Similarity of a Flight Demonstrator via Multidisciplinary Optimization (Ph.D. thesis), DOCTORAT DE L'UNIVERSITÉ DE TOULOUSE Délivré par : l'Institut Supérieur de l'Aéronautique et de l'Espace (ISAE), 2018, URL: https://hal.archives-ouvertes.fr/tel-02023612.

[150] Q. Ye, F. Avallone, D. Ragni, M. Choudhari, D. Casalino, Effect of surface roughness geometry on boundary-layer transition and far-field noise, AIAA Journal (2021) 1–13.

[151] P.S. Klebanoff, K.D. Tidstrom, Mechanism by which a two-dimensional roughness element induces boundary-layer transition, The Physics of Fluids 15 (7) (1972) 1173–1188.

[152] W. Saric, E. White, H. Reed, Boundary-layer receptivity to freestream disturbances and its role in transition, in: 30th fluid dynamics conference, 1999, p. 3788.

[153] A.G. Hansen, Similarity Analyses of Boundary Value Problems in Engineering(text on Similarity Solutions of Partial Differential Equations for Boundary Value Problems in Engineering), Englewood Cliffs, N. J., Prentice-HallL, Inc., 1964, p. 114, 1964.

[154] K. Pettersson, A. Rizzi, Aerodynamic scaling to free flight conditions: Past and present, Prog. Aerosp. Sci. 44 (4) (2018) 295–313, http://dx.doi.org/10.1016/j.paerosci.2008.03.002, 2018-06-06, URL: http://www.sciencedirect.com/science/article/pii/S0376042108000262.

[155] A. Raju Kulkarni, G. La Rocca, L.L. Veldhuis, Degree of similitude estimation for sub-scale flight testing, in: AIAA Scitech 2019 Forum, 2019, p. 1208.

[156] J.R. Richardson, E.M. Atkins, P.T. Kabamba, A.R. Girard, Scaling of airplane dynamic response to stochastic gusts, J. Aircr. 51 (5) (2019) 1554–1566, http://dx.doi.org/10.2514/1.C032410, 2019-10-17, URL: http://arc.aiaa.org/doi/10.2514/1.C032410.

[157] A.P. Ricciardi, R.A. Canfield, M.J. Patil, N. Lindsley, Nonlinear aeroelastic scaled-model design, J. Aircr. 53 (1) (2016) 20–32, http://dx.doi.org/10.2514/1.C033171, URL: http://arc.aiaa.org/doi/10.2514/1.C033171.

[158] A. Raju Kulkarni, C. Varriale, M. Voskuijl, G. La Rocca, L.L. Veldhuis, Assessment of sub-scale designs for scaled flight testing, in: AIAA Aviation 2019 Forum, 2019, p. 3089.

[159] O.D. Dantsker, S. Yu, M. Vahora, M. Caccamo, Flight testing automation to parameterize unmanned aircraft dynamics, in: AIAA Aviation 2019 Forum, 2019, p. 3230.

[160] W. Williams, UAV Handling Qualities... You Must Be Joking, Aerospace Sciences Corporation Pty. Ltd, 2003.

[161] W.H.J.J. Van Staveren, Analyses of aircraft responses to atmospheric turbulence, 2003, https://repository.tudelft.nl/islandora/object/uuid.

[162] P. Lovett, A. Ingram, C. Bancroft, Knowledge-based engineering for SMEs - a methodology, J. Mater Process. Technol. 107 (1–3) (2000) 384–389, http://dx.doi.org/10.1016/S0924-0136(00)00728-7.

[163] A.H. van der Laan, M. Van Tooren, Parametric modeling of movables for structural analysis, J. Aircr. 42 (6) (2005) 1605–1613, http://dx.doi.org/10.2514/1.9764, URL: https://arc.aiaa.org/doi/10.2514/1.9764.

[164] G. La Rocca, M. Van Tooren, Knowledge-based engineering to support aircraft multidisciplinary design and optimization, Proc. Inst. Mech. Eng. G 224 (9) (2010) 1041–1055, http://dx.doi.org/10.1243/09544100JAERO592.

[165] P.D. Ciampa, B. Nagel, Towards the 3rd generation MDO collaborative environment, in: 30th ICAS, 2016, pp. 1–12.

[166] N. Solutions, Optimus | noesis solutions | noesis solutions, 2020, 2020-04-18, URL: https://www.noesissolutions.com/our-products/optimus.

[167] ModelCenter, ModelCenter Integrate | Model Based Engineering Software, Phoenix Integration, 2020, 2020-04-18, URL: https://www.phoenix-int.com/product/modelcenter-integrate/.

[168] Esteco, Modefrontier, 2020, 2020-04-18, URL: https://www.esteco.com/modefrontier.

[169] S. Technology, Sigma technology. products, 2020, 2020-04-18, URL: http://iosotech.com/product.htm.

[170] OpenMDAO, OpenMDAO.org | an open-source framework for efficient multidisciplinary optimization, 2020, 2020-04-18, URL: https://openmdao.org/.

[171] DLR, DLR - institut für softwaretechnologie - RCE, 2020, 2020-04-18, URL: https://www.dlr.de/sc/desktopdefault.aspx/tabid-5625/9170_read-17513/.

[172] Dassault Systèmes, Isight & SIMULIA execution engine | dassault systèmes®, 2020, 2020-04-18, URL: https://www.3ds.com/products-services/simulia/products/isight-simulia-execution-engine/.

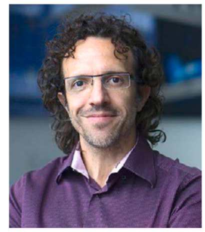
[173] Siemens, Toonaangevend leverancier van product lifecycle management en manufacturing operations management software, 2020, 2020-04-18, URL: https://www.plm.automation.siemens.com/country/nl-nl/index.html.

[174] M. Hoogreef, Advise, Formalize and Integrate MDO Architectures: A Methodology and Implementation, 2017, OCLC: 7792805507.

[175] I. van Gent, Agile MDAO systems: A graph-based methodology to enhance collaborative multidisciplinary design, 2019, http://dx.doi.org/10.4233/uuid:c42b30ba-2ba7-4fff-bf1c-f81f85e890af, URL:.

[176] ICAO, Manual on remotely piloted aircraft systems (RPAS), 1st ed.; international civil aviation organization: Montreal, QC, Canada, 2015., 2021, 2021-06-12, https://skybrary.aero/sites/default/files/bookshelf/4053.pdf.

ir. **Akshay Raju Kulkarni** completed his Bachelor of Engineering in Mechanical Engineering for Bangalore University, India and MSc in Aerospace engineering from TU Delft. He is currently a Ph.D. candidate at Delft University of Technology working on a European Union research project called Large Passenger Aircraft Design (Cleansky2). His Ph.D. focuses on developing a methodology to design representative sub-scale aircraft to support Sub-scale Flight Testing. In addition to sub-scale flight testing, his interests lie in design methods such as Knowledge Based Engineering and Multidisciplinary Design Optimization.



Dr.ir. **G. (Gianfranco) La Rocca** holds a MSc in Aerospace Engineering from the University of Pisa, Italy and a Ph.D. from Delft University of Technology in the Netherlands. Currently, he is associate professor in the section of Flight Performance and Propulsion of the same faculty where he pursued his doctoral title. He has developed and taught several BSc and MSc courses on Aircraft Design, Systems Engineering and advanced design methodologies, such as Knowledge Based Engineering (KBE) and Multidisciplinary Design Analysis and Optimization (MDAO). Dr. La Rocca's research activities and expertise include the development of design automation solutions to support the development of novel and more sustainable aircraft architectures and contribute to the digitalization of complex, knowledge intensive design processes. As from 2000, Dr. La Rocca has been involved in many national and European projects related to aircraft design and collaborative MDAO, including the DESCARTES Prize finalist project MOB and the ICAS 2018 Innovation Award project AGILE.

Prof.dr.ir. **Leo Veldhuis** holds an MSc and Ph.D. from Delft University of Technology. In 1987 he started as Assistant professor in the Aerodynamics group. Currently he is full professor and acts as head of the section Flight Performance and Propulsion (FPP) and at the same time is the Head of the Department AWEP (Aerodynamics, Wind Energy, Flight Performance and Propulsion. Leo is the former Head of the Wind Tunnel Laboratories at the Faculty of Aerospace Engineering. He has more than 30 years of experience in aircraft aerodynamics. His research interests are: aircraft aerodynamics, open rotors and propulsion integration, high lift systems, flow control and aircraft design. Leo teaches various courses in the field of aircraft aerodynamics. He has extensive experience in the cooperation and support in EU-funded research projects. His current research supports of the aircraft design capability of TUD with a special focus to the development of propeller integration, scaled flight testing capability and ground based testing of novel aircraft configurations.

Prof. Dr. **Georg Eitelberg** is a full professor at the section of Flight Performance and Propulsion in the Faculty of Aerospace Engineering at TU Delft. Prof. Eitelberg obtained his engineering qualifications (Dipl.-Ing. And Dr.-Ing.) from the Technical University of Karlsruhe (now renamed KIT). After working as a researcher at the DLR Göttingen, he joined the Faculty of the University of NSW as a lecturer in Mechanical Engineering, at the Australian Defense Force Academy. After 5 years of that he re-joined the DLR in Göttingen, starting up the High Enthalpy Tunnel HEG and carrying on in functions such as Section Head for Aerothermodynamics in the institute of Fluid Mechanics and as Head of the Wind tunnel Division of the DLR. In 1998 Georg Eitelberg relocated to the Netherlands to take over the position of Director of DNW, an organization responsible for operating the major wind tunnels in Germany and the Netherlands. In addition to his organizational responsibilities, he joined the faculty of the Aerospace Engineering at the TUD, to continue with his academic interests related to aircraft propulsion integration. Prof. Eitelberg has a wide scope of experience in experimental aerodynamics ranging from low speed civil transport applications to the hypersonic high energy flows such as in orbital re-entry from space, where chemical reactions can dominate the flow aspects related to stability and control of the air vehicles. He has been involved in numerous European and international research and development projects and has participated in many international working groups (AGARD, EUROMECH, etc.)