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Quantification and control of disruption propagation in multi-level public transport networks

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ABSTRACT

Due to the multi-level nature of public transport networks, disruption impacts may spill-over beyond the primary effects occurring at the disrupted network level. During a public transport disruption, it is therefore important to quantify and control the disruption impacts for the total public transport network, instead of delimiting the analysis of their impacts to the public transport network level where this particular disruption occurs. We propose a modelling framework to quantify disruption impact propagation from the train network to the urban tram or bus network. This framework combines an optimisation-based train rescheduling model and a simulation-based dynamic public transport assignment model in an iterative procedure. The iterative process allows devising train schedules that take into account their impact on passenger flow re-distribution and related delays. Our study results in a framework which can improve public transport contingency plans on a strategic and tactical level in response to short- to medium-lasting public transport disruptions, by incorporating how the passenger impact of a train network disruption propagates to the urban network level. Furthermore, this framework allows for a more complete quantification of disruption costs, including their spilled-over impacts, retrospectively. We illustrate the successful implementation of our framework to a multi-level case study network in the Netherlands.

Introduction

Relevance of quantifying disruption propagation

Quantifying and minimising the impacts of public transport (PT) disruptions is important from the perspectives of both service users and service providers. PT disruptions can negatively affect passengers' nominal and perceived journey time as a result of longer in-vehicle times, additional transfers, and longer waiting times in case of missed connections. More severe crowding levels on remaining services also increase perceived in-vehicle times and can potentially result in an increase in the number of passengers being denied boarding (see for example Hörcher et al., 2017; Tirachini et al., 2017; Yap et al.,

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2018a). Over a longer time horizon PT disruptions can influence the mode choice of travellers, reducing the PT share in the modal split and reducing revenues for the PT service provider (Yap et al., 2018b). For a PT operator to provide an attractive and a competitive public transport service to passengers, it is thus of utmost importance to understand and limit the impacts of PT disruptions. In line with Oliveira et al. (2016), in this study we define a disruption as a change in system performance caused by distinctive incidents or events (such as a signal failure). This is in contrast to disturbances, which are typically defined as changes in system performance caused by stochastic demand or supply fluctuations (such as variability in passenger volumes or running times). A perturbation - any change from normal system performance (Ghosh and Lee, 2000) - can thus pertain either to a disturbance or a disruption.

An integrated PT network consists of different functional network levels - such as the (inter)national train network level, the regional train network level, and the urban tram and bus network level - which are hierarchically connected to each other. In this study, we use the term *multi-level network* to refer to the entire PT network consisting of these different network levels. As disruption impacts can spill-over from one network level to another network level, it is important to quantify and mitigate the disruption impacts for the total multi-level PT network, instead of delimiting the considerations to the disruption impact for the PT network level where this particular disruption occurs. The impact of a PT disruption on a certain network level can propagate to a lower network level in two different ways: via primary and secondary effects. First, a primary effect relates to the direct impact of a disruption on journeys of passengers who travel over the different network levels during one journey. For example, a passenger travelling on the regional train network level might miss the scheduled connection to the urban tram network level, due to a delayed train arrival at the transfer stop following a disruption on this train network level. Second, a secondary effect is experienced by passengers travelling on a lower PT network level who are affected indirectly by a disruption on a higher network level. For example, a disruption on the regional train network level might result in several delayed trains arriving almost simultaneously at the transfer stop. Consequently, this results in a sudden increase in transfer volume from the regional train network towards the urban network level, thus increasing crowding levels in the first urban trips serving this transfer location. Passengers making a journey only at the urban network level using one of these trips will experience higher levels of discomfort due to crowding, caused by a disruption on another network level. Such secondary effects have been found by Malandri et al. (2018), where impacts of simulated disruptions were observed at service segments located more than 10–15 km away from the location the disruption originated.

The abovementioned examples illustrate that disruption impacts do not stop at the border of the network level on which the disruption occurs, as often assumed, but can propagate to another network level as well. For a full understanding of the impact of a disruption and how to potentially mitigate its impact, one should therefore consider the impact a disruption may have on the multi-level PT network as a whole, including its propagation. Meanwhile, it is also important to consider the role other PT network levels can play in mitigating disruption impacts by increasing network robustness (as for example studied by Jenelius and Cats, 2015; Yap et al., 2018c).

Literature review

Studies focusing on quantifying and mitigating PT disruption impacts can broadly be classified as optimisation-based or simulation-based approaches. Several optimisation-based approaches propose a mathematical programming framework to determine the optimal vehicle holding time to regulate PT services or to synchronise services for transferring passengers. For example, Delgado et al. (2009) and Delgado et al. (2012) test vehicle holding, potentially combined with setting boarding limits to regulate bus services on a PT corridor with a deterministic mathematical programming model. Sanchez-Martinez et al. (2016) formulate a deterministic holding control model which incorporates dynamic running times and demand. Hadas and Ceder (2010) develop a dynamic programming model which minimises passengers' total travel time by synchronising PT services in an optimal way. Optimisation-based approaches are also applied to synchronise last train services during the late evening, for example for subway networks (e.g. Kang et al., 2015) or for bi-level networks to synchronise last urban trains to enable transfers from feeder high-speed railway lines (Long et al. 2020). Optimisation-based approaches are also commonly used to solve the railway traffic rescheduling problem, in case disruptions occur on the railway network. Selected examples of the extensive research performed in this area are Törnquist Krasemann (2012) proposing a greedy algorithm for train rescheduling, D'Ariano et al. (2007) using a branch-and-bound algorithm and Corman et al. (2010) testing a tabu search algorithm. For a comprehensive literature overview of algorithms proposed for real-time railway rescheduling, we refer the reader to Cacchiani et al. (2014).

Simulation-based approaches on the other hand are used for disruption impact quantification and for testing rule-based strategies for disruption management. For example, Cats and Jenelius use the dynamic agent-based PT assignment model BusMezzo to quantify the robustness value of spare capacity (Cats and Jenelius, 2016), the value of real-time information provision (Cats and Jenelius, 2014), and the impact of partial link closures (Cats and Jenelius, 2018) for high frequent urban PT networks. Leng et al. (2018) and Paulsen et al. (2018) use MATsim as agent-based simulation software to predict passenger delay impacts from rail disruptions in the metropolitan areas of Zürich and Copenhagen, respectively. Younan and Wilson (2010) develop a rule-based controller to support a real-time holding decision between two connecting bus routes based on expected impact on passengers' net travel time. Daganzo and Anderson (2016) use simulation to test a rule-based holding control strategy for transfer synchronisation between metro and bus, whilst Laskaris et al. (2018) use a simulation-based dynamic PT assignment model to test a multiline holding control strategy for transit corridors, applied to a selection of bus lines in Stockholm, Sweden. Gavriilidou and Cats (2019) propose a rule-based holding controller for urban PT services which considers capacity constraints and on-board crowding levels using a dynamic PT assignment model. For an extensive literature review on holding control strategies we refer to Gavriilidou and Cats (2019).

Few studies have adopted and applied a simulation-based optimisation approach for disruption impact quantification and mitigation. One example is Shakibayifar et al. (2017), who use a simulation-based optimisation model with the objective of minimising total train delay times during train disruptions. Schmaranzer et al. (2019) combine a discrete event simulation model and metaheuristic optimisation model to optimise headways for urban PT systems.

The review of related research studies illustrates that optimisation-based approaches are typically applied for disruption management on the train network level; whereas simulation-based approaches are primarily used for quantifying disruption impacts and testing disruption management strategies for the urban PT network level or for metropolitan PT networks where it is important to account for passenger flow re-distribution. Optimisation-based approaches, often using microscopic or mesoscopic models, result in (an approximation of) optimal rescheduling, retiming and rerouting of train services in response to a disruption. As the PT rescheduling problem for larger, real-world PT networks is considered NP-hard (Desaulniers and Hickman, 2007), these optimisation-based approaches typically account only for limited stochasticity in PT demand and supply. These studies predominantly employ deterministic mathematical programming models and generally do not consider stochastic passenger route choice over the PT network, stochastic demand patterns, or stochasticity related to vehicle running times or dwell times. Dynamic interactions between demand and supply, such as bunching, are typically not considered. Simulation-based methods, often using agent-based mesoscopic PT models, are able to capture dynamics in PT demand and supply and their interactions. For example, these models can consider stochastic running times, flow-dependent dwell times and stochastic and dynamic passenger route choice when being confronted with a disruption. These methods allow for testing rule-based control strategies or for testing the impact of several disruption scenarios, albeit without resulting in optimal disruption control strategies.

In recent years, different studies in the field of railway rescheduling have acknowledged that passenger route choice and impact are often not sufficiently accounted for when using traditional optimisation-based train rescheduling models. In response to this, some studies have proposed methods which account more explicitly for passenger delay impacts in disruption management. For example, Dollevoet et al. (2014) propose an iterative optimisation framework for delay management and train rescheduling. Yin et al. (2016) develop an optimisation-based rescheduling model for metro networks in an energy efficient manner, but explicitly incorporate time-dependent passenger arrival rates at different stations in their model. Zhu and Goverde (2019) weight train rescheduling decisions according to the time-dependent volumes, while Van der Hurk et al. (2018) adopt an iterative framework which combines a rolling stock and passenger advice optimisation model with passenger simulation. Their aim is to minimise passenger disruption impacts on the train network, whilst being able to reflect that passenger behaviour might not follow advice. Corman et al. (2017) and Ghaemi et al. (2018) combine a train rescheduling model with a passenger routing model to improve the passenger perspective in disruption management. Other studies to disruption management incorporate the passenger component using simulation-based optimisation (e.g. Altazin et al., 2020), stochastic dynamic programming (e.g. Schön and König, 2018) or by combining an optimisation model with passenger flow control strategies (e.g. Liu et al., 2020). Binder et al. (2017) propose an integer linear program to solve the multi-objective railway rescheduling problem, which integrates passenger rerouting and train rescheduling. Methods to integrate passenger assignment with railway optimisation can also be found in the planning phase when developing optimal timetables (e.g. Canca et al., 2016; Schmidt and Schöbel, 2015) or as part of delay management (e.g. Dollevoet et al., 2011; Corman, 2020). Our research builds on these works by combining passenger assignment and train rescheduling for multi-level PT networks, so that delay propagations to other PT network levels can be included and quantified in rescheduling decisions.

Railway networks do experience less stochasticity than urban PT networks on average, as train running times are not influenced by interactions with cars, cyclists and pedestrians. Furthermore, the lower network density of train networks reduces the route choice alternatives passengers realistically have. This results in deterministic route choice assumptions being less problematic for train networks, compared to relatively high-density urban PT networks which offer route redundancy. Moreover, the typically lower train frequencies combined with the prevention of early departures from most train stations do reduce the dynamic interaction between demand and supply which can result in bunching, as often observed for urban PT services. Due to the more complex interaction between PT demand and supply on the urban PT level, simulation-based dynamic assignment models are often necessary for sufficiently realistic predictions of the impact of disruptions and disruption management strategies when considering larger, real-world urban PT networks.

Quantifying and controlling the effects of a train network disruption beyond merely the train network poses two methodological challenges. First, a disruption on the train network level is typically solved by an optimisation-based train rescheduling model resulting in an updated train timetable. The extent to which a train disruption propagates to the urban network is thus a function of the train rescheduling optimisation model. This entails that the train optimisation model needs to be considered, when quantifying propagated disruption impacts to the urban network - typically using a simulation model - adequately. Second, this train rescheduling is based on the characteristics of the train network level only, and does not consider the impact of this rescheduling strategy on disruption propagation to the urban PT network. Passenger trips on the urban level can be subject to control strategies in response to this updated train timetable afterwards. This however implies that urban network control strategies in response to a train network disruption are performed in a sequential way, where first services on the train network level are optimised for this network level only, after which services on the urban network level can only be controlled taken the train network rescheduling as a given. This sequential approach may yield sub-optimal rescheduling solutions, as the disruption impacts are not considered for the integrated multi-level PT network simultaneously. Incorporating the impact of train network disruption management on the urban PT network level however requires considering the stochasticity and dynamics of the urban PT network. These dynamics are difficult to incorporate in an optimisation-based rescheduling model while still maintaining acceptable computation times.

Research contribution

In this study, we develop an iterative methodology to quantify the impact of a disruption occurring on the train network for the PT network as a whole by accounting for delay propagation, i.e. cross-network spill-over effects. Our objective is to develop a framework which can improve the development of PT contingency plans on a strategic and tactical level in response to short- to medium-lasting PT disruptions and to quantify disruption costs retrospectively, by incorporating how the passenger impact of a train network disruption propagates to the urban network level (see Fig. 1). Given our research objective, it is necessary to test train rescheduling strategies obtained from an optimisation-based method, and to assess the impact of each strategy on the integrated PT network including the urban level, calling for a simulation-based evaluation approach. We therefore propose a simulation-based optimisation framework to quantify disruption impact propagation from the train network to the urban network level. Using our proposed methodology, we test how different train rescheduling strategies can be used to mitigate disruption propagation to the urban network level. In our study, we only control train trips to mitigate disruption propagation: controlling urban PT trips subsequently (e.g. by applying holding control strategies tailored for the disruption conditions) falls outside our research scope. The main contribution of our work is the development of a method to quantify the impact of a train network disruption on other PT network levels, as this is an important research gap as identified in Section 1.2. To this end, we use two different models as our study input: an optimisation-based train rescheduling model and a simulation-based PT assignment model. By combining these two individual models, we are able to answer new research questions in relation to quantifying disruption propagation. As this study focuses on integrating a train optimisation model and PT simulation model, rather than developing or improving these individual models, we deem it appropriate to use an established train optimisation model and PT assignment model of which the individual performance is known to be good for their individual purposes. This gives us more confidence in our obtained results to be accurate when integrating these components into our proposed framework. Our framework can be used to improve contingency plans of PT agencies, in particular for more frequently occurring disruptions, to evaluate the impact of different strategies on passengers on the entire, multi-level PT network. As such, it can reduce total passenger disruption impacts and thus contribute to improved PT network robustness.

The main contributions of our study are the following:

- Development of a methodology to quantify disruption impact propagation from the train network to the urban PT network level.
- Apply a simulation-based and an optimisation-based approach iteratively into one modelling framework to predict disruption propagation impacts.

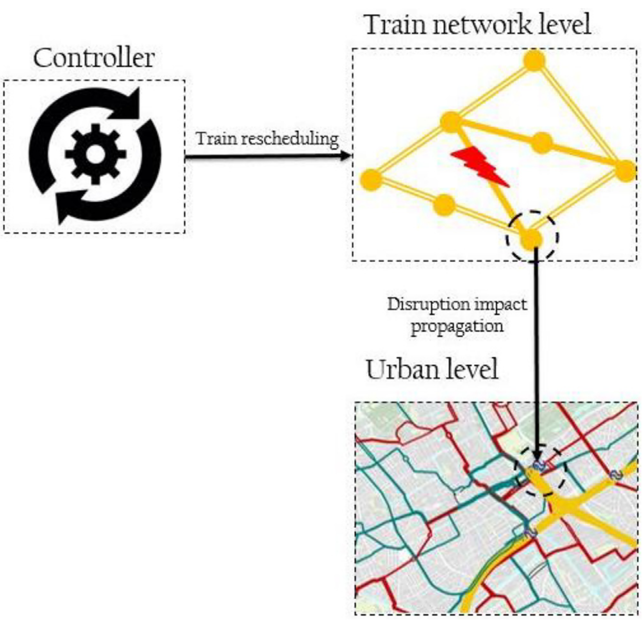

Fig. 1. Illustration propagation of train network disruption to urban network.

- Evaluation of the impact of different train rescheduling strategies on controlling disruption impacts for the multi-level PT network.

The remainder of this paper is structured as follows. Section 2 discusses our proposed modelling methodology to quantify and control disruption propagation. We apply this methodology to a real-world case study in The Hague, the Netherlands, which is introduced in Section 3. Results of this case study application are discussed in Section 4. Section 5 provides conclusions and recommendations for future research directions.

Methodology

This section discusses our proposed methodology to quantify and control disruption impact propagation over the multi-level PT network. Section 2.1 and Section 2.2 introduce the dynamic PT assignment model and the train rescheduling model, respectively, that we employ in this modelling framework. Section 2.3 describes our proposed modelling framework in detail. Table 1 first introduces the notations used throughout this section.

Dynamic PT assignment model

This section shortly discusses the properties of the dynamic PT assignment model employed in this study. Appendix A and Cats et al. (2016) provide more details on the properties of this model. We use a mesoscopic, simulation-based dynamic PT assignment model to represent the multi-level PT network. The train network level, as well as the urban tram and bus network level, are represented in this model using a directed graph $G(S,A)$ with S being the set of all stops and train stations and A the set of links. The train network level and urban network level are represented by subgraphs $G^t(S^t,A^t)$ and $G^u(S^u,A^u)$ respectively, with $G^t \in G$ and $G^u \in G$. Passenger demand n^{od} is defined from each origin stop $o \in S$ to each destination stop $d \in S$. In terms of granularity each node corresponds to a PT stop, and each link is the direct connection between two stops $s \in S$. These links typically represent a PT connection between two adjacent stops, whereas they represent a walk connection between stops located close to each other, for example within a single PT hub. We use an agent-based simulation model to mimic the emerging order from interactions among numerous vehicles and passengers. To be able to reflect stochastic demand patterns due to day-to-day variation, the arrival rate of passengers at the origin stop for each OD pair is assumed to follow a Poisson distribution. The arrival rate parameter of the Poisson distribution can typically be estimated from Automated Fare Collection (AFC) data.

The set of PT lines is denoted by L, with $|L|$ representing the total number of lines. Each line $l \in L$ is defined by a sequence of stops $l = \{s_{l,1}, s_{l,2}..s_{l,j}\}$ with $F = \{f_1, f_2..f_j\}$ denoting the set of scheduled trips on this line. The scheduled headway of a line is denoted byh_l, which can be time-dependent. The total time $t_{l,f}$ it takes a vehicle to complete trip f of line l equals the summation of all running times $t^r_{s_{l,f}}$ from stop $s_l$to stop s_{l+1} and dwell times $t^{dw}_{s_{l,f}}$ at each stops_l, as expressed by Eq.1. Running times $t^r_{s_{l,f}}$ can be assumed deterministic, using the scheduled times obtained from the timetable, or can be stochastic. In our study, we use deterministic minimum running times for the train network, as these running times are relatively stable given the limited interactions with other traffic. For urban tram and bus lines, we fit a lognormal or log-logistic distribution to the empirical Automated Vehicle Location (AVL) data, to capture the predominantly stochastic running times within an urban environment. The dwell times $t^{dw}_{s_{l,f}}$ for each trip $f \in F$ at each stop $s \in S$ depend on the number of boarding and alighting passengers $n^{board}_{s_{l,f}}$ and $n^{alight}_{s_{l,f}}$. The departure time of a trip $t^{dep}_{s_{l,f}}$ depends on the arrival time at that stop $t^{arr}_{s_{l,f}}$ and the required dwell time$t^{dw}_{s_{l,f}}$ (Eq.2). In case a stop is a holding point $s \in S^h$ and a schedule-based holding control regime is employed, the departure time can never be earlier than the scheduled departure time from that specific stop$t^{dep}_{s_{l,f}}$ (Eq.3). For urban PT networks, a selected number of stops are usually holding points, whereas all train network stations are holding points as passenger trains are generally not able to depart ahead of schedule from any station.

$$t_{l,f} = \sum_{s_{l,1}}^{s_{j-1}} t^r_{s_{l,f}} + \sum_{s_{l,1}}^{s_{j-1}} t^{dw}_{s_{l,f}} \; \forall f \in F, l \in L \tag{1}$$

$$t^{dep}_{s_{l,f}} = t^{arr}_{s_{l,f}} + t^{dw}_{s_{l,f}} \; \forall f \in F, l \in L, s \notin S^h \tag{2}$$

$$t^{dep}_{s_{l,f}} = \max\left(t^{arr}_{s_{l,f}} + t^{dw}_{s_{l,f}}, t^{dep}_{s_{l,f}}\right) \; \forall f \in F, l \in L, s \in S^h \tag{3}$$

The number of boarding and alighting passengers is obtained from a successive number of choices each individual passenger makes during the journey. At each stop a passenger can make a boarding decision to board a certain trip or to wait, or make a connection decision to walk to another PT stop. When boarded a certain trip, a passenger can make an alighting decision at each downstream stop whether to alight from this vehicle or to stay on-board. These decisions can be made en-route

Table 1
List with sets and indices, variables and parameters.

Sets and indices	
s, S	public transport stop as node of graph G, set of stops
a, A	edge of graph G, set of links
l, L	unidirectional public transport line, set of lines
f, F	public transport trip, set of trips
o, O	public transport stop representing origin node of G, set of origin nodes
d, D	public transport stop representing destination node of G, set of destination nodes
i, T	index for train trip, set of all train trips
j, B	index for rail infrastructure segment of train network, set of segments
k, E	index for time slot request event by train for a rail infrastructure segment, set of events
p, P	index for track for each train infrastructure segment, set
dw	index for dwell time
h	index for holding stop
r	index for running time
s	index for scenario
t	index for regional train network level
u	index for urban public transport network level
ivt	index for in-vehicle time
wkt	index for walking time
wtt	index for waiting time
$wtt - d$	index for waiting time due to denied boarding
$on - board$	index for passengers on-board a public transport trip
$alight$	index for alighting passengers
$board$	index for boarding passengers
tf	index for transferring passengers
arr	index for trip arrival
dep	index for trip departure
Variables	
h	scheduled headway of a public transport line
n	number of passengers
q	binary variable indicating if an event uses a certain track
r	binary variable indicating if an event occurs before another event
s	binary variable indicating if an event is rescheduled to occur after another event
t	Time
v	generalised passenger journey cost
w	train arrival time deviation
z	train delay
Parameters	
α	weight for penalising track changes
β	weight for alighting passengers in transfer-alighting based delay minimisation strategy
γ	weight for transferring passengers in transfer-alighting based delay minimisation strategy
ε	weights for passenger perception coefficients of travel time components
ζ	threshold convergence criterion 1
η	threshold convergence criterion 2
μ	track or platform initially intended to be used by an event

and in a stochastic and dynamic way if the expected utility of a certain choice changes during a journey, for example in response to high crowding levels or to information provided about a downstream disruption. The model considers in-vehicle time, walking time, waiting time (regular waiting time as well as waiting time caused by denied boarding in case of crowding) and the number of transfers, weighted by the corresponding coefficients. Whilst the model specification in this study assumes fixed (average) walking speeds between different stops or platforms, it is possible to specify a walking time distribution instead. In case station (over)crowding is expected to result in passengers missing their connecting service, one could specify a walking speed distribution to incorporate the passenger delay impact which might result from this. A single non-equilibrium assignment procedure without day-to-day learning is applied.

Train rescheduling model

This section shortly discusses the mesoscopic optimisation model used for train rescheduling in response to a train network disruption. The optimisation is formulated as mixed integer linear programming (MILP) problem and details of this model can be found in Appendix B and in Törnquist and Persson (2007). This model only represents the train network level$G^t \in G$, which is a subset of the total multi-level PT network G considered in this study. In this model, individual train trips are represented. Each node corresponds to a train station or infrastructure junction, such as a movable bridge or track merging; each separate track between two nodes is represented by an individual link. In this train rescheduling model, G^t is represented with a higher granularity than in the dynamic PT assignment model to allow for optimal rescheduling of each individual train trip in case of a disruption. Let T represent the set of all train trips in the selected train network level and let B denote the set of segments that defines the rail infrastructure for the train network level. E denotes the set of events, where an event can be seen as a time slot request by a train for a specific network segment. The index i is associated with a specific train service in the set T (i.e.$i \in T$), while the index j is associated with a specific network segment ($j \in B$), and index k is associated with an event ($k \in E$). An event is associated with a combination of a network segment and a train service. The set $K_i \subseteq E$ is an ordered set of events for each train trip$i$, while $L_j \subseteq E$ is an ordered set of events for each network segment$j$. Each segment j in Bhas a number of parallel tracks, with each track indicated by$p \in P_j$.

This train rescheduling model focuses primarily on train delay minimisation but allows for weighting the delay of different trains based on the number of passengers on-board the trains, in order to adopt a more passenger-oriented approach in the train delay minimisation. It should however be noted that passengers and their dynamic route choice are not explicitly modelled here, since incorporating demand- and supply-related stochastics and dynamics of both network levels of a real-world PT network is computationally expensive (as addressed in Section 1). The objective function of this model in its most basic form is therefore the minimisation of the sum of all delays z for all train trips. For our proposed iterative modelling framework, we have formulated new objectives functions (presented in Section 2.3) to better incorporate the passenger perspective and the additional delays for passengers which may result from missing their connections to the urban transport services.

Modelling framework

The main contribution of this study is the development of a simulation-based optimisation modelling framework as a methodology for quantifying and controlling the propagation of a train network disruption to the urban PT network. This modelling framework combines two different models: a simulation-based dynamic PT assignment model (Section 2.1) and an optimisation-based train rescheduling model (Section 2.2). The train rescheduling model is required, as the disruption propagation of a train network disruption to the urban network is a function of the train rescheduling strategy being applied to train services in response to a disruption. Using a dynamic PT assignment model which represents the entire multi-level PT network enables the quantification of the direct and propagated impact of a train network disruption for the PT network as a whole, including the dynamic and stochastic demand and supply characteristics particularly relevant for the urban network level.

Passenger train rescheduling strategies

To control train network disruption propagation to the urban network, different rescheduling strategies applied to the train network are tested. We consider a strategy as a certain intervention – in this study a train rescheduling intervention – aimed at reducing the passenger impact of a certain disruption scenario. A scenario is here defined as any new reality resulting from a certain disruption. A scenario is external and cannot be influenced as such, whilst strategies aim to mitigate the consequences of this scenario. Although passengers are not explicitly modelled within the train rescheduling model, it is possible to incorporate different passenger-oriented rescheduling strategies by adding different weights to the delays of different trains in the objective function based on the number of passengers in each train. This gives, for example, more importance to a delay of a busy train compared to a delayed train which is less busy. In our study, we test four different train weights, which are aimed to control the propagation of train disruption impacts to the urban PT network. These four different weights result in four alternative objective functions applied to the train rescheduling model, taking the objective function as introduced by Eq. B1 in Appendix B, as a base. We test the following train rescheduling strategies (S1-S4):

- Passenger based delay minimisation (Eq.4) (S1): minimise train delays larger than two minutes $z_{i,k}^{+2}$, where each train is weighted by the expected number of passengers leaving the corresponding train at each stop $n_{i,k}^{alight,t} + n_{i,k}^{tf,t\rightarrow u}$ (both alighting and transferring passengers).
- Transfer based delay minimisation (Eq.5) (S2): minimise train delays larger than two minutes $z_{i,k}^{+2}$, where each train is weighted by the expected number of transferring passengers from the train network level to the urban PT network level $n_{i,k}^{tf,t\rightarrow u}$. This implies that trains are only weighted according to the number of transferring passengers to the urban level.
- Transfer-time based delay minimisation (Eq.6) (S3): minimise train delays larger than two minutes $z_{i,k}^{+2}$, where each train is weighted by the number of transferring passengers to the urban PT network level multiplied with the headway of the urban PT service where is transferred to $n_{i,k}^{tf,t\rightarrow u} \cdot h_l^u$. This reflects the expected passenger waiting time for transferring passengers in case a scheduled transfer from train to urban network level would be missed due to a delayed train arrival at the transfer stop.
- Weighted transfer-alighting based delay minimisation (Eq.7) (S4): minimise train delays larger than two minutes $z_{i,k}^{+2}$, where each train is weighted based on the number of alighting passengers $n_{i,k}^{alight,t}$and the number of transferring passengers from train to urban PT network level $n_{i,k}^{tf,t\rightarrow u}$, with different weights β and γ respectively being applied to the two pas-

senger segments. This reflects that the impact of a delayed train arrival can potentially be more severe for transferring passengers, when a connection to the urban network level would be missed, than for alighting passengers reaching their final destination. This typically results in a higher weight $\gamma > \beta$ being applied to the number of transferring passengers.

The resulting objective functions corresponding to the different train rescheduling strategies are shown in Eq. 4–7. The model permits for example trains to run faster than scheduled and to run ahead of schedule at certain stretches between stations (i.e. arriving before the scheduled arrival time) in order to enable trains to catch-up from delays and make way for other trains quicker. The departure time from stations is however not permitted to start before the original scheduled departure time. The model also permits trains to change tracks and platforms at stations as well as to overtake and meet at other locations than initially planned, if that leads to a reduction of knock-on delays. Hence, in order to ensure that only such beneficial 'delay-reducing' rescheduling decisions are adopted, those need to be associated with a smaller penalty corresponding to e.g. one minute delay. Therefore, in addition to delay minimisation, the objective functions also minimise all arrival time deviations and track changes. The objective is to minimise train delays larger than two minutes$z_{i,k}^{+2}$, weighted by a passenger component depending on the rescheduling strategy, arrival time deviations$w_{i,k}$ and track changes. $q_{i,k,p}$ is the binary decision variable related to the use of track $p \in P_j$ by an event k, as defined in Appendix B. The parameter $\mu_{i,k}^{train}$ specifies the track or platform that was initially intended to be used by event k belonging to train trip$i$. The parameter α specifies the weight used for penalising track changes. If the time-related variable values are given in e.g. seconds, the value of α needs to be set quite high in order to balance the trade-off between reducing train delays and keeping the timetable intact as much as possible with respect to the planned routes of the trains through/within the stations.

$$minimise \sum_{i \in T} \sum_{k \in K_i} \left[\left(n_{i,k}^{alight,t} + n_{i,k}^{tf,t \to u} \right) \cdot z_{i,k}^{+2} + w_{i,k} \right] + \sum_{i \in T} \sum_{k \in K_i} \sum\nolimits_{p \in P_j : p \neq \mu_{i,k}^{train}} \alpha \cdot q_{i,k,p} \tag{4}$$

$$minimise \sum_{i \in T} \sum_{k \in K_i} \left[n_{i,k}^{tf,t \to u} \cdot z_{i,k}^{+2} + w_{i,k} \right] + \sum_{i \in T} \sum_{k \in K_i} \sum\nolimits_{p \in P_j : p \neq \mu_{i,k}^{train}} \alpha \cdot q_{i,k,p} \tag{5}$$

$$minimise \sum_{i \in T} \sum_{k \in K_i} \left[\left(n_{i,k}^{tf,t \to u} \cdot h_l^u \right) \cdot z_{i,k}^{+2} + w_{i,k} \right] + \sum_{i \in T} \sum_{k \in K_i} \sum\nolimits_{p \in P_j : p \neq \mu_{i,k}^{train}} \alpha \cdot q_{i,k,p} \tag{6}$$

$$minimise \sum_{i \in T} \sum_{k \in K_i} \left[\left(\beta \cdot n_{i,k}^{alight,t} + \gamma \cdot n_{i,k}^{tf,t \to u} \right) \cdot z_{i,k}^{+2} + w_{i,k} \right] + \sum_{i \in T} \sum_{k \in K_i} \sum\nolimits_{p \in P_j : p \neq \mu_{i,k}^{train}} \alpha \cdot q_{i,k,p} \tag{7}$$

The following train rescheduling actions are permitted in this model:

- Retiming: changing the departure and arrival times, while respecting the initial earliest departure time and minimum dwell times at commercial stops and running times of the trains.
- Reordering: permitting shift of train order and overtaking to neighbouring stations, while respecting the safety constraints in the network.
- Local rerouting: allowing change of track and platform assignment at train stations.

Our model does not incorporate (full or partial) train cancellations, global rerouting (i.e. rerouting trains via a completely different route) or the supply of rail-replacement bus services in the optimisation. This has implications for the type and magnitude of disruptions this model can be applied to. Our study focuses on disruptions which do not result in the complete blockage of certain rail infrastructure. For example, one can think of vehicle or infrastructure related disruptions (e.g. a signal failure, a faulty train) which result in delays, but which do not result in the complete unavailability of a certain infrastructure segment. When infrastructure becomes unavailable, train cancellations, short-turning or (global) train rerouting are measures commonly applied. Additionally, our method focuses primarily on unplanned disruptions with a short to medium-long duration (up to a couple of hours). For planned disruptions and for long-lasting unplanned disruptions - for example a disruption which lasts for multiple days - supply of rail-replacement buses can be expected. In these cases, a wider demand response than only rerouting can be expected as well, as passengers might also change their mode choice, destination choice or trip frequency choice.

Iterative modelling framework

The iterative modelling framework we propose in this study is shown in Fig. 2. This framework consists of three modelling steps, which needs to be performed to adequately quantify the disruption propagation impact subject to different train rescheduling strategies.

The first step is the model initialisation, where the dynamic PT assignment model is used to assign the total PT passenger demand n (for train and urban network level) over the multi-level PT network for a scenario without disruption s_0. This results in passenger route choice over the total multi-level PT network in case there would be no disruption, yet subject to recurrent service variations. Based on this, passengers' generalised travel costs v_{s_0}in the steady-state condition can be

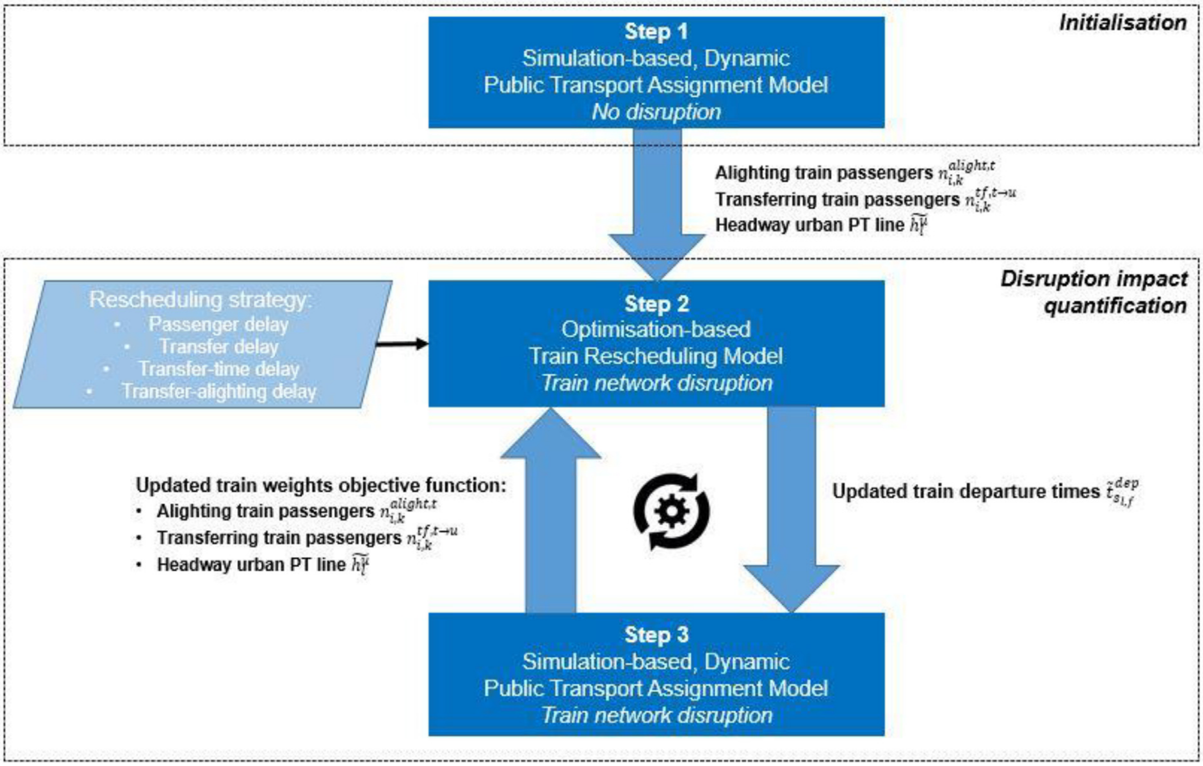

Fig. 2. Iterative modelling framework.

computed by multiplying the different travel time components (walking time t^{wkt}, waiting time t^{wtt}, in-vehicle time t^{ivt}, waiting time due to denied boarding t^{wtt-d}, number of transfers n^{tf}) by their corresponding weights ε reflecting the passenger perception of each component, after which the sum of this is multiplied by the value of time (VoT) (Eq. 8). This implies that we do not only consider the disruption impact on the nominal passenger journey time, but also on the perceived passenger journey time.

$$v_{s_0}^{od} = \left(\varepsilon_{ivt} \cdot t^{ivt,od} + \varepsilon_{wkt} \cdot t^{wkt,od} + \varepsilon_{wtt} \cdot t^{wtt,od} + \varepsilon_{wtt-d} \cdot t^{wtt-d,od} + \varepsilon_{tf} \cdot n^{tf,od}\right) * VoT \tag{8}$$

Second, the train rescheduling model is applied to perform an optimal train rescheduling for a given train network disruption s_i. The train rescheduling model requires the number of alighting passengers from each train trip at each train station $n_{i,k}^{alight,t}$ as input, as well as the number of transferring passengers $n_{i,k}^{tf,t\to u}$ from the train network level to the urban network level. Depending on the rescheduling strategy applied in this model, the scheduled headway of each urban PT lineh_l^u where passengers transfer to is also required as input (Eq.6). These three variables are outputs from the assignment process of the dynamic PT assignment model and are fed into the train rescheduling model as input for the objective function of a certain train rescheduling strategy. A train network disruption is coded as input for the train rescheduling model, after which the train rescheduling problem is solved for a selected rescheduling strategy. As output, the train rescheduling model provides an updated train timetable with rescheduled train departure times $t_{s_{l,f}}^{dep}$ (which are based on the decision variables values for $x_{i,k}^{begin}$in the MILP model presented in Appendix B). The scheduled train departure times for each station in the undisrupted case are equal for the train trips modelled in the train rescheduling model and in the dynamic PT assignment model. Due to the difference in granularity between the two models, only updated departure times from commercial train stations are fed back into the dynamic PT assignment model, i.e. departure times from other timetable time points such as movable bridges are not fed back from the train rescheduling model to the PT assignment model.

Third, the dynamic PT assignment model is applied again for each rescheduling strategy applied in the train rescheduling model. The updated train departure times from the train rescheduling model in response to the modelled disruption on the train network are used as input. Using this updated train timetable, the total PT demand is re-assigned over the multi-level PT network, based upon which the generalised travel costs v_{s_i}can be computed (Eq.8). Due to the stochastic nature of the dynamic PT assignment model, multiple replications of this model are required in both step 1 and step 3 of the model sequence. The number of replications required within one iteration is calculated using the convergence criterion based on the procedure detailed in Dowling et al. (2004), such that the allowable percentage error does not exceed 5%. The generalised travel costs are then averaged over the number of replications.

Next, step 2 and step 3 are repeated iteratively. This is of relevance as the re-assigned train passenger flows in the dynamic assignment model can update the number of alighting and transferring train passengers used as weights in the objective function of the train rescheduling model, which can affect the train rescheduling results as a consequence. This iterative process between step 2 and step 3 terminates when convergence is reached. We define two different convergence criteria in this study, of which at least one needs to be satisfied to consider results of the total model sequence as converged. The first criterion compares the generalised journey costs v_{s_i} for the total multi-level network between two iterations, as can be computed from the outputs of the PT assignment model (Eq.9). When the difference in generalised costs of Δv_{s_i} - as average over the multiple replications within each iteration - between two subsequent iterations j and $j-1$ is smaller than a predefined threshold ζ, convergence is reached after j iterations. In our study we use a strict convergence criterion of 0.5% for ζ. The second criterion compares the passenger volume assigned for each train trip $i \in T$ on each track segment between two iterations. If at least 95% of the train segment passenger volumes in the model differs by less than a predefined threshold η from the volumes in the previous iteration, convergence is reached (Eq.10). The abovementioned value of 95% is obtained from standard modelling guidelines applied in the United Kingdom for (e.g.) model validation, where it is suggested that across screenlines and cordons the difference in passenger volumes should, in 95% of the cases, be less than a certain threshold (TAG, 2020). The value of the threshold depends on the spatial level of analysis. When comparing across screenlines, TAG typically recommends a threshold of 15% for η, whereas a higher threshold of 25% is allowed when considering individual links or services (as in our study). In our study we opted for a stricter threshold of 10% for η, even though we are comparing passenger volumes for individual train services and links. Please note that η and ζ have no dimension and are expressed as percentages (see Eq.9 and Eq.10). This implies that the convergence performance will depend on the temporal discretisation and/or absolute link volume, as low absolute demand changes may result in large relative changes. In TAG (2020) it is therefore recommended to exclude links from the comparison where the hourly flow is less than 150 passengers, which we adopted here as well. We use two convergence criteria in this study, as this relates to the two models used. If the total assignment results between two iterations do not differ more thanζ, the PT assignment model results can be considered stable (first convergence criterion). If the train passenger volumes used as input for the train rescheduling model do not differ more thanη from the previous iteration, it indicates that the updated train departure times resulting from the train rescheduling model will be stable. As these are used to update the PT assignment model, consequently the results of the assignment model will be stable as well (second convergence criterion). Hence, satisfying one of these criteria is sufficient to consider the model results as converged, based on which the solution minimising total generalised travel costs can be selected.

$$\Delta v_{s_i} = \sum_{o \in S} \sum_{d \in S} \left(v_{s_i}^{od,j} - v_{s_i}^{od,j-1} \right) / \sum_{o \in S} \sum_{d \in S} \left(v_{s_i}^{od,j-1} \right) \tag{9}$$

$$\Delta n_{i,k}^{on-board,t} = \left(n_{i,k}^{on-board,t,j} - n_{i,k}^{on-board,t,j-1} \right) / n_{i,k}^{on-board,t,j-1} \forall i \in T, k \in E \tag{10}$$

Eq.11 quantifies the total passenger disruption impactΔv, expressed as generalised passenger delay costs. The generalised journey costs resulting from disruption s_i after convergencev_{s_i} are compared with these costs when there is no disruptionv_{s_0}. We distinguish between journeys with their origin and destination at the train network level or urban network level, which results in four different passenger segments. This enables the quantification of the impact of a train network disruption on this disrupted network level, as well as the spill-over impacts due to propagation to the urban PT network level. The disruption impact of a train network disruption on the disrupted train network level Δv^t relates to the increase in generalised travel costs for passengers starting and terminating their journey at the train network level G^t. The disruption propagation to the urban network level Δv^u relates to the additional generalised journey costs for passengers with their journey starting and/or terminating at the urban network level G^u. Δv^t and Δv^u can be computed by using Eq.11 for the relevant subset of passenger segments.

$$\Delta v = \sum_{o \in t,u} \sum_{d \in t,u} \left(v_{s_i}^{od} - v_{s_0}^{od} \right) \tag{11}$$

Case study

This section discusses the case study for which our methodology is applied. Section 3.1 introduces the case study network of The Hague, the Netherlands. Subsequently, Section 3.2 describes the tested disruption scenario.

Case study network

We apply our methodology to the multi-level public transport network of The Hague, the Netherlands. The Hague is the third largest city in the Netherlands, located in the main economic area of the Netherlands called the Randstad in the western part of the country. The population size of the city is over 500,000 inhabitants. The urban agglomeration of The Hague including its surrounding cities covers an area of 405 sq.km with more than 1 million inhabitants.

The case study multi-level PT network encompasses the complete urban PT network of The Hague consisting of 12 tram lines and 8 bus lines, and all train services calling at The Hague as depicted in Fig. 3. The tram and bus lines are operated by HTM, the urban public transport operator of The Hague. Two tram lines are light rail lines connecting the main city of The Hague with the satellite city of Zoetermeer. The other 10 tram lines function on the urban network level providing connections between different areas within The Hague and neighbouring municipalities. The eight considered bus lines all belong to the urban concession area of HTM in The Hague. The case study network consists of 498 bus, tram and light rail stops. All train services from/to the directions Leiden, Gouda and Delft starting at, terminating at, or serving one of the train stations of The Hague are incorporated in our case study. Both intercity train services, serving only larger cities, and local train services stopping at all stations are simulated. The train network is cordoned at the stations Leiden, Gouda and Delft Zuid, meaning that these stations are modelled as gate nodes for the parts of train services extending beyond the boundaries of the case study network. The cordoned train network consists of 16 stations, of which 10 stations allow passengers to transfer between the (inter)regional train network level and the urban tram and bus network of The Hague.

The passenger demand is obtained from Automated Fare Collection (AFC) data from 20 working days between 5 March and 30 March 2018. For the urban tram and bus network in The Hague, a distance based fare system applies where passengers are required to tap in and tap out at in-vehicle devices for each journey leg. This means that each complete AFC transaction consists of a tap in time, stop, line and vehicle ID, as well as a tap out time and stop (see also Van Oort et al., 2017). The dataset consists of 6.48 million AFC transactions solely for the urban tram and bus network, equating $\approx$325,000 AFC transactions per average working day made on the urban PT network. 29,271 AFC transactions (0.5%) were incomplete due to an error in the AFC system and removed from the dataset. Due to the on-board tap in and tap out devices, destination inference is not required for complete AFC transactions. In case of an incomplete AFC transaction where a passenger (un)deliberately did not tap out, a trip chaining algorithm is applied to infer the most plausible tap out stop (Munizaga and Palma, 2012). If there is only one AFC transaction made by a certain card ID on the day of the incomplete transaction, or if no candidate alighting stop is found within a plausible walking distance of 400 Euclidean metres from the next registered boarding stop, no destination inference is performed. Consequently, another 43,427 (0.7%) AFC transactions were removed from the dataset. For all remaining 6.39 million AFC transactions on the urban PT network, a transfer inference algorithm is applied to con-

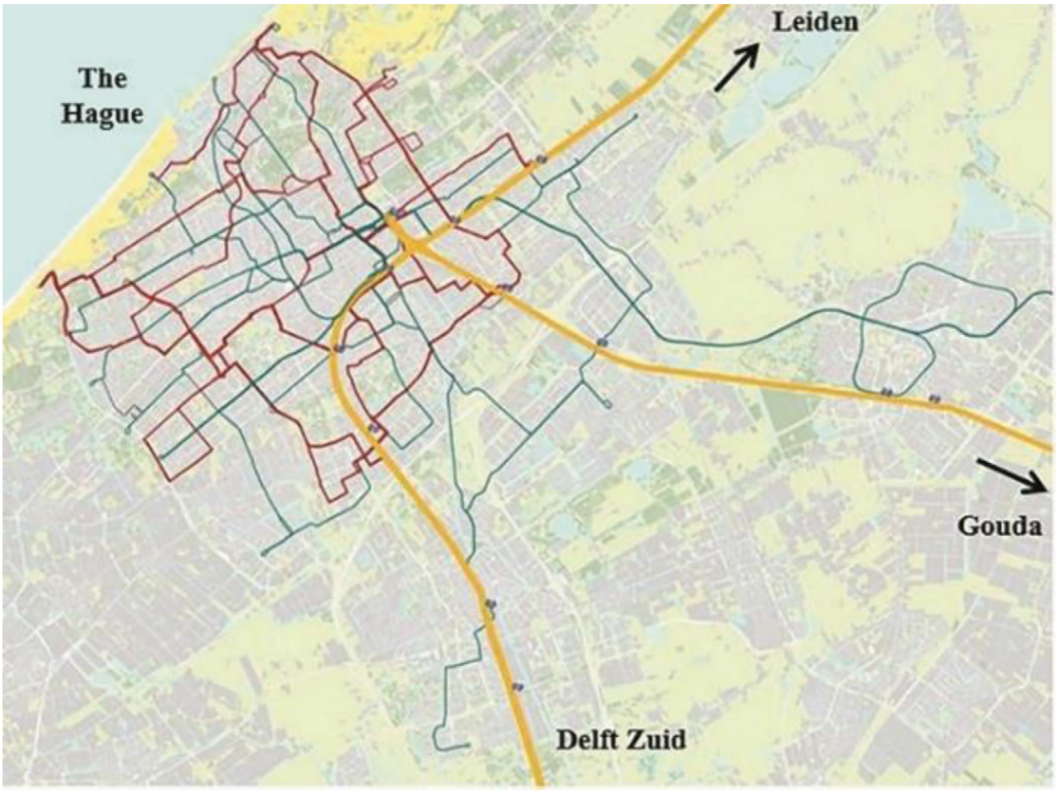

Fig. 3. Case study public transport network (yellow: train services / green: tram and light rail services / red: bus services).

struct stop-to-stop journeys based on Gordon et al. (2013) and Yap et al. (2017), thereby using both the AFC and AVL (open) data corresponding to this 20 working days period.

To construct a multi-level stop-to-stop OD matrix, the OD matrix generated solely for the urban PT network is amended based on information about transfers between the train and urban PT network. As the train and urban PT network are operated by different PT operators, AFC systems of these network levels are generally not linked together. Therefore, no direct multi-level OD matrix is available. However, the relative distribution of transferring passengers between intercity and local train services of the three case study train corridors (directions Leiden, Gouda or Delft), and the urban PT network was provided to us for each multi-level transfer location in The Hague. These transfer flows are distributed proportionally over the different urban PT stops as origins and destinations, thereby replacing the multi-level transfer location as origin/destination for the original urban PT journey. This results in an OD matrix for the total multi-level PT network. It should be noted that this complete OD matrix could alternatively be obtained from a strategic transport model rather than using direct empirical data, depending on data availability for the considered case study area.

In our case study, we focus on the disruption impacts for AM peak journeys with starting time between 7-9AM. Alongside simulating PT demand and supply between 7–9 AM, demand and supply are also simulated between 6-7AM and 9-10AM as warm-up and cooling-down period. This is necessary to make sure all passengers starting their journey between 7-9AM have PT supply available at all locations to start and finish their journey. It is also necessary to reflect crowding levels in PT services adequately by incorporating passengers starting their journey outside the AM peak, who affect crowding levels of passengers who started their journey within the AM peak. After applying the abovementioned transfer inference algorithm, there are about 104,000 journeys simulated for the multi-level PT network starting between 7-9AM in total. About 55,000 journeys (52%: marked blue in Fig. 4) start and/or end at the urban PT network level and can potentially benefit from our iterative approach, whilst approximately 49,000 journeys (48%) are only using the train PT network level.

Disruption scenario

We illustrate our proposed modelling framework by applying it to a disruption scenario. We would like to emphasise that this disruption case study only serves an illustrative purpose to demonstrate that our method can be applied successfully to real-world PT networks to quantify disruption propagation impacts. Given the variety of disruption types, lengths and locations, the disruption costs and performance of different rescheduling strategies always depend on the specific location, duration and time a specific disruption starts. For this scenario we quantify how the impact of a train network disruption propagates to the urban PT network level, after applying optimised rescheduling and control strategies to train services on the disrupted (inter)regional train network. We simulate an infrastructure failure - such as a signal failure or switch failure - at a certain (fixed) location, resulting in lower speeds and thus delays for all passing trains. The disruption is simulated just before Leiden for all inbound trains towards The Hague coming from Schiphol Airport (see Fig. 5). In this figure, the four most important transfer stations between the train network and the urban PT network are indicated (The Hague Central, The Hague HS, The Hague Laan van NOI and Delft). Potential disruption propagation from train to urban network occurs mainly via these stations. The disruption is simulated to last from 6AM to 9AM during the simulation period. The simulation hour from 9AM to 10AM is used for service recovery. It is assumed that all trains passing this disruption location between 6-9AM obtain a random delay drawn from a normal distribution with an average delay of 15 minutes and a standard deviation of 5 minutes. As our case study only serves an illustrational purpose to show how one can quantify propagated disruption impacts to other PT network levels, we only solved this for one instance. It is however recommended to perform multiple draws from this distribution to test the stability of the model outcomes when one wants to implement contingency plans in practice.

In our experiments we use BusMezzo as dynamic PT assignment model (Cats et al., 2010). The optimisation model for train rescheduling is implemented in Java and solved using Gurobi version 6.5.1. The exchange of inputs and outputs

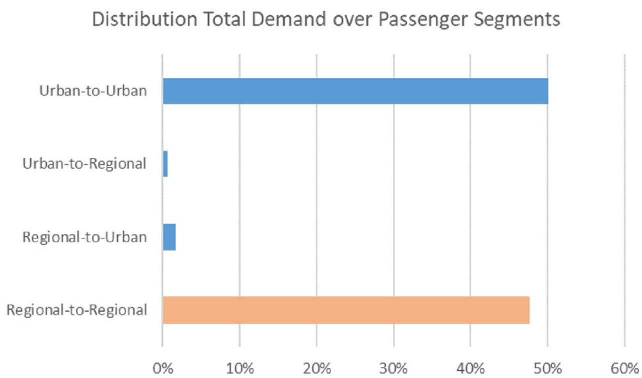

Fig. 4. Distribution of total AM peak (7–9 h) demand over the four distinguished passenger segments.

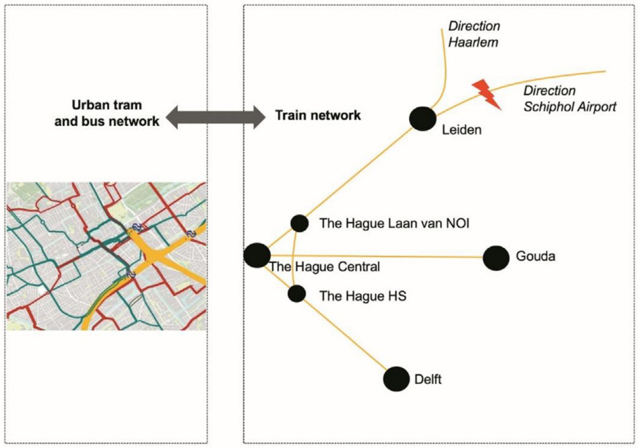

Fig. 5. PT network with disruption before Leiden affecting multiple trains from direction Schiphol Airport. *The yellow lines in the figure left correspond to the train network as shown in the figure right.*

between the two models is performed automatically using a model integration tool built in Java (Obrenovic, 2019). We test the four different train rescheduling strategies as outlined in Section 2.3. Table 2 provides an overview of the parameter values used for our case study. The coefficients of the travel time components of the generalised cost function (Eq.8) are obtained from a Revealed Preference study performed by Yap et al. (2018a) utilising smart card data records. The Value of Time is in line with values typically applied in the Netherlands. We use ζ=|0.005| and η=|0.10| as thresholds for our convergence criteria. This entails that convergence is reached if the passenger journey costs for the total network do not change more than 0.5% between two iterations, or if for at least 95% of all train segments the passenger load does not change more than 10% between two iterations.

Results and discussion

This section discusses train rescheduling results (Section 4.1), disruption impact results (Section 4.2) and case study implications (Section 4.3). For discussion of our case study results, we refer to the four different train rescheduling strategies as follows: S1 refers to total passenger based train delay minimisation, S2 to transferring passenger based train delay minimisation, S3 to transfer-time based train delay minimisation, and S4 to weighted alighting-transferring passenger based train delay minimisation.

Train rescheduling results

Based on the convergence criteria we adopted in this study, strategies S1, S2, S3 and S4 require 7, 3, 5 and 5 iterations, respectively, to reach convergence. For each iteration of the dynamic PT assignment model, 15 replications were required to capture the stochasticity in PT demand and supply for this case study. One replication of the dynamic PT assignment model takes about 3 minutes on a regular Dell Core i7 laptop, whilst solving the train rescheduling problem requires 5–10 minutes. Therefore, one complete iteration of both the train rescheduling model (5–10 minutes) and PT assignment model (3*15 = 45 minutes) requires 50–55 minutes. Please note that aforementioned values are computed on a regular single threaded laptop without any parallelisation being applied. Computation times could be reduced substantially if multiple processors would be used. However, the abovementioned computation times imply that our framework is best suitable to apply in the strategic / tactical planning phase to test contingency plans, and in the evaluation phase to assess the disruption impacts of past disruptions. For this framework to be applied in real-time control decisions, it would require a further reduction of computation

Table 2
Parameter values for case study.

Parameter	Parameter function
α = 60	Weight for penalising track changes
β = 1	Weight for alighting passengers
γ = 3	Weight for transferring passengers
ε_{ivt} = 1 / ε_{wkt}=1.58 / ε_{wtt}=1.58 / ε_{wtt-d}=3.5 / ε_{tf}=4.8	Coefficients in generalised travel cost function
ζ=|0.005|	Threshold for first convergence criterion
η=|0.10|	Threshold for second convergence criterion
VoT=€9 / hour	Value of Time

times even when considering potential parallelisation, as decisions will typically be required within 1–2 minutes after the occurrence of a disruption.

The results of the train rescheduling model show different updated timetables in response to the disruption, when different control strategies are applied. An example of the visualisation of the updated train timetable resulting from the rescheduling process is shown in a time–space diagram in Fig. 6. In Table 3, some high level statistics are shown for the different train rescheduling strategies. It can be seen that in strategy S1 most trains are rescheduled, but that average arrival delays at the final station of these trains are smaller than for the other strategies. Strategy S2 shows the opposite effect: the least trains are rescheduled, at cost of the largest arrival delay. Strategies S3 and S4 both sit between S1 and S4 in this regard. Fig. 7 provides the arrival delay of the affected train trips which arrive at The Hague Central, which is a terminal station for all train services and the most important transfer location between train and urban network level for our case study. It can be seen that the most severe delayed trains suffer from ≈22 minutes delay when arriving at the destination. For a couple of train trips, the different rescheduling strategies result in the same arrival delay at The Hague Central. Overall, we see that control strategies which incorporate the number of alighting (non-transferring) passengers result in a more similar train rescheduling: the results of strategy S1 and strategy S4 are similar for most trains. On the other hand, control strategies which are only based on the number of transferring passengers (strategies S2 and S3) result in comparable rescheduling decisions as well. In case relatively large passenger volumes transfer from a certain train to an urban tram or bus with a relatively low frequency, this train gets prioritised in strategy S3 compared to strategy S2, as can be seen for trains *2,246,001* and *2118001*. Compared to strategy S2, strategy S3 results in higher train arrival delays for trains with fewer passengers interchanging to urban lines with relatively low frequencies (e.g. trains *2,446,001* and *2263001*). When comparing strategies S1 and S4 on the one hand, and strategies S2 and S3 on the other hand, we conclude that the transfer(-time) based strategies S2 and S3 result in fewer trains arriving late at The Hague Central, with the average arrival delay being slightly smaller than for strategies S1 and S4. The (weighted) passenger based strategies S1 and S4 tend to distribute the delays over more trains, resulting in delays for a larger number of trains. This confirms that strategies which only incorporate transferring passenger volumes in the weighted train delay minimisation, tend to result in fewer trains arriving delayed at the important transfer stations.

Disruption impact results

The results from the dynamic PT assignment model allow for quantifying the disruption impact on the disrupted train network level Δv^t and the spilled-over disruption propagation to the urban PT network level Δv^u. Table 4 provides the monetised disruption impact in Euros between 7-9AM (left), and the relative share of disruption impact costs on the disrupted network level and the spilled-over urban network level. These values result from applying Eq.11, in which generalised costs are computed for the disrupted and undisrupted scenario using Eq.8. Depending on the rescheduling strategy being applied, the propagated disruption costs make up 5–15% of the total passenger disruption costs. For this case study, our results thus show that neglecting disruption propagation to the urban network results in an underestimation of 5–15% of the total disruption costs for passengers. The delayed train arrivals caused by this disruption influence journeys starting at the train network and terminating at the urban network (and in the opposite direction) due to potential missed connections or prolonged waiting times. In addition, the shifted train arrival trains can result in less uniform transfer volumes to the urban PT trips, thereby resulting in higher average crowding levels for urban PT trips. This can have a negative impact on journeys entirely made on the urban network as well.

When comparing the different control strategies, one can see that disruption propagation costs differ substantially between the strategies. When rescheduling strategy S1 is applied, the forecast propagated disruption costs are €3,100.

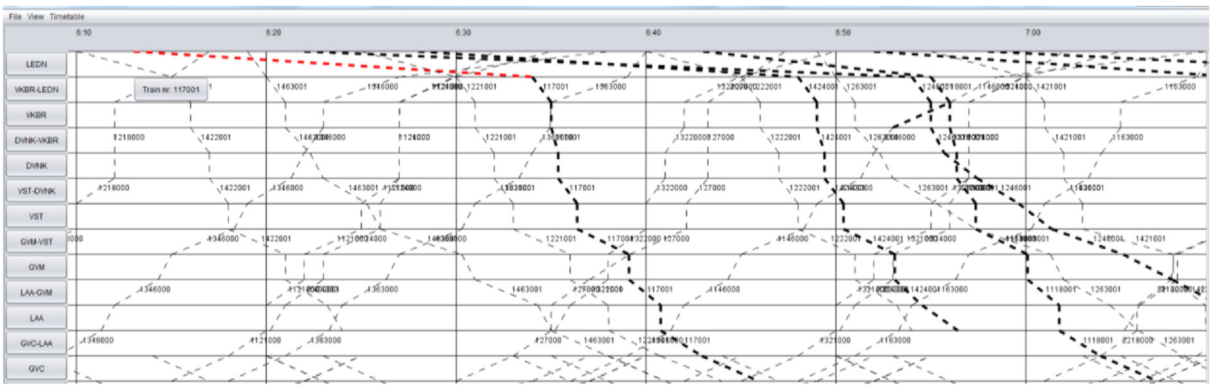

Fig. 6. Illustration time–space diagram for updated train timetable after rescheduling. *Bold, dashed lines represent rescheduled trains, whereas the red dashed line shows the first train being rescheduled.*

Table 3
Comparison train rescheduling results between strategies.

	Number of rescheduled trains	Average arrival delay of rescheduled trains (min)
Strategy S1 – passenger	42	6.5
Strategy S2 – transfer	39	7.6
Strategy S3 – transfer time	40	6.8
Strategy S4 – alighting-transfer	41	7.2

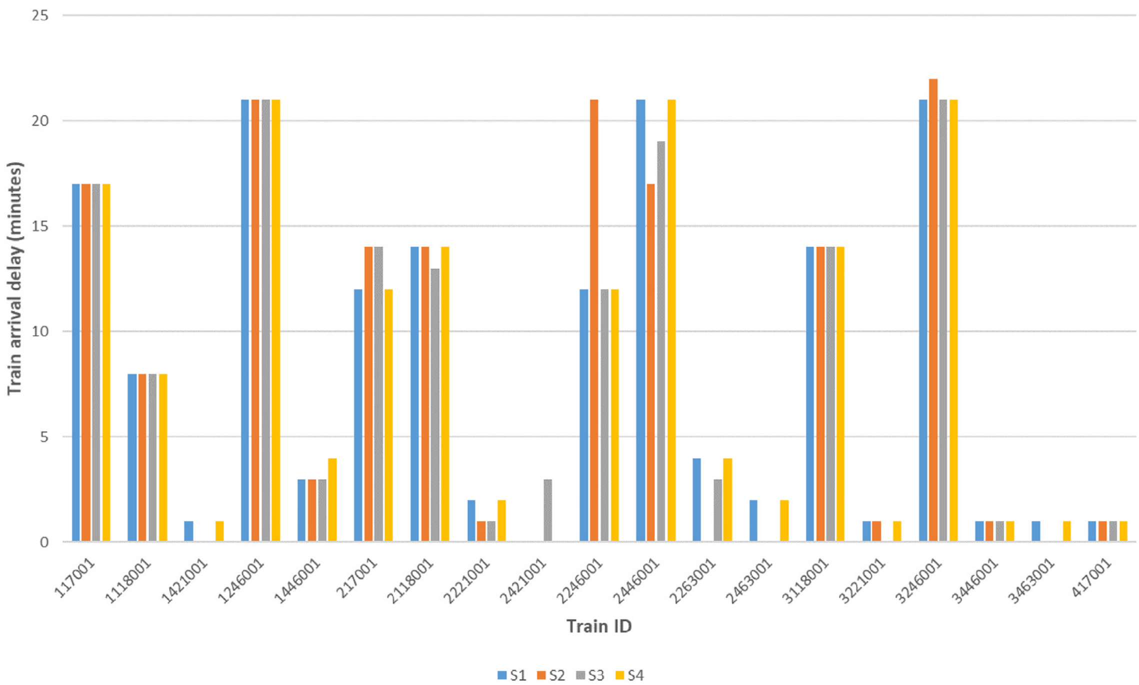

Fig. 7. Train arrival delay at The Hague Central for different train rescheduling strategies.

Rescheduling strategies S2-S4, which all add relatively more importance to transferring passengers to the urban network compared to alighting passengers during the train rescheduling, are all able to reduce the spilled-over disruption impact to the urban network. Strategies S2 and S4 result in spilled-over disruption impacts of €2,200-€2,500, whilst a further reduction in disruption propagation is forecast for strategy S3 (€1,600). When the headway of the urban PT service where passengers transferring to is incorporated (strategy S3), the propagated impact can be further reduced compared to strategy S2 which only considers the number of transferring passengers. This can be explained by the potential longer waiting times inflicted on passengers if passengers would miss a transfer to an urban PT route with a relatively low service frequency. In this specific case study, strategies S2 and S4 seem to provide beneficial impacts on both the train and urban network level, resulting in lower total disruption costs than strategies S1 and S3. Strategy S4, which does consider both alighting and transferring passengers, results in the lowest total disruption costs. These results suggest that for this case study putting more emphasis on transferring passengers during the train rescheduling process can result in more optimal rescheduling results from a total network perspective.

Table 4
Disruption impact for different train rescheduling strategies (7-9AM).

	Disruption impact (Euro)			Share of disruption impact	
	Disrupted level (train)	Spilled-over level (urban)	Total	Disrupted level (train)	Spilled-over level (urban)
Strategy S1 – passenger	€ 29,817	€ 3,070	€ 32,886	91%	9%
Strategy S2 - transfer	€ 23,554	€ 2,543	€ 26,097	90%	10%
Strategy S3 - transfer time	€ 31,335	€ 1,600	€ 32,935	95%	5%
Strategy S4 - alighting-transfer	€ 12,115	€ 2,163	€ 14,278	85%	15%

All values are computed by comparing the total generalised costs for passengers between the disrupted and undisrupted scenario (Eq.11). The generalised costs per scenario result from the perceived passenger delay monetised using a Value-of-Time (Eq.8)

Fig. 8 shows the total and propagated disruption impacts for the four passenger segments distinguished in this research, depending on whether the passenger journey starts and/or ends at the regional train network or urban network level. Plot (a) shows the total passenger journey costs from each strategy and confirms that most disruption impacts apply to journeys made exclusively using the train network (dark blue). Plots (b), (c) and (d) zoom in to journeys which start and/or end at the urban network, thus effectively excluding the regional-regional journeys as shown in plot (a). These plots show that the propagated disruption impact primarily affects journeys entirely made on the urban network, and journeys from urban to train network. Based on total journey costs (plot b), nominal journey time (plot c) and additional waiting time (plot d), we can see that strategy S3 is most effective in reducing delay propagation to the urban network, whilst strategy S4 realises the lowest total network delay when also incorporating the impact on the disrupted level (plot a). Strategies S3 and S4 both outperform strategies S1 and S2 in terms of reducing spilled-over delay propagation. Strategy S4 primarily reduces delay propagation for the urban-urban segment, which is the smallest across the four strategies (plot b). However, this reduction comes at the expense of increasing the journey costs for the urban-regional segment, particularly in comparison to strategy S3.

Discussion

The main contribution of the proposed method is that it enables to assess the dependencies between the different PT network layers and their control mechanisms. Compared to existing models used to predict or control disruption impacts for either the train network, or the urban PT network, the main beneficiary of the proposed modelling framework is that it can be quantified how a disruption spills over from one network level to another. In addition, this framework allows quantifying how the impacts of control interventions directed at one network level can influence the extent that a disruption propagates over the integrated PT network. Hence, this method provides most added value for PT systems where there are relatively strong interdependencies between different network levels, for example caused by multiple transfer locations between different networks or a relatively high share of multi-level passenger journeys. In the event of relatively interdependent network levels, the benefit of incorporating disruption propagation in control decisions has the potential to outweigh the costs of adopting this more extended, but also computationally more expensive modelling framework. This modelling framework is based on the principle that control interventions are taken in the benefit of passengers from an overall passenger perspective, thereby considering the disruption impacts on passengers for the entire PT network. Implementation of such integrated control strategies can be relatively straightforward if the same service provider operates PT services

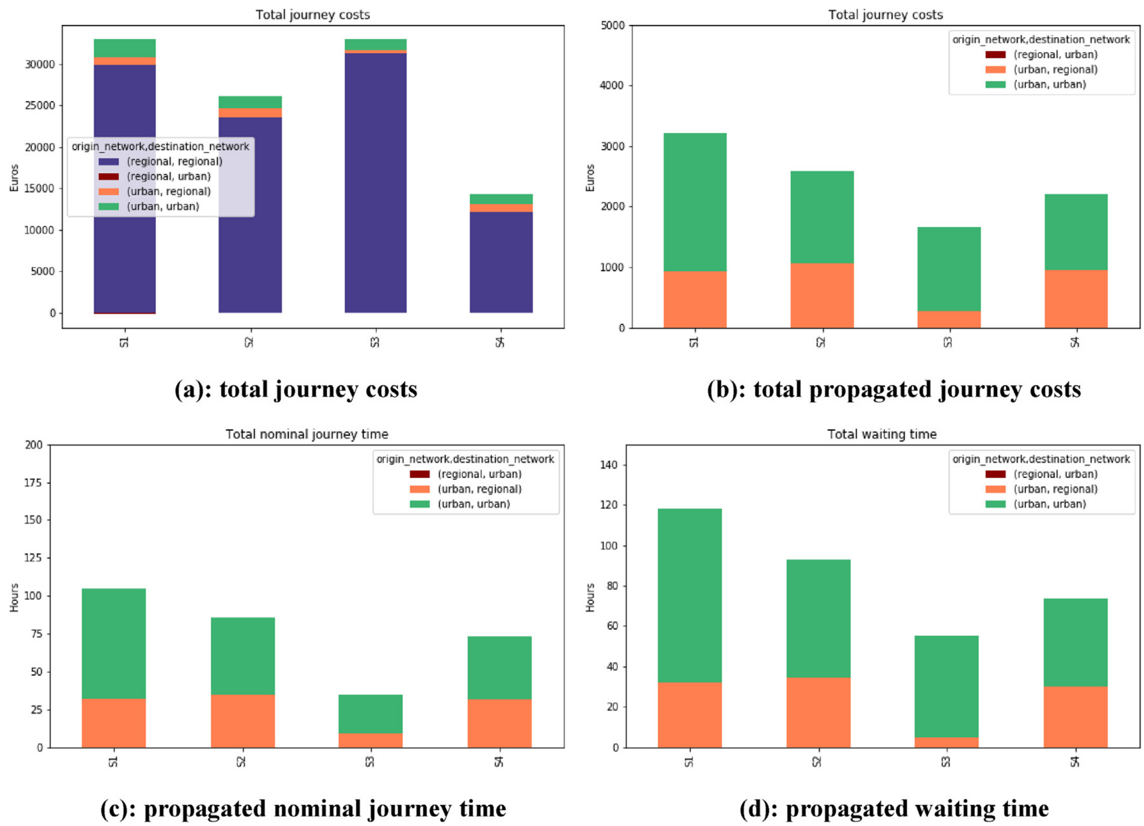

Fig. 8. Total and propagated passenger delay impact for different passenger segments.

on the different network levels. When different PT operating companies are involved, implementation can potentially become more challenging when commercial aspects come into play. In these cases, there is a role for the overarching transport authority to incentivise train operating companies to apply control strategies which are most beneficial from an integrated network perspective, rather than for their subnetwork only, for example via contractual agreements or compensation schemes.

As mentioned earlier in Section 2.3, the train rescheduling model as used in this study applies particularly to the development of contingency plans for disruptions with a short to medium-long duration in which a rail infrastructure segment is not entirely blocked, given that global rerouting of trains, supply of rail-replacement bus services and wider passenger demand responses are not captured. Most value can be expected when these contingency plans are developed for disruption types which occur relatively frequently, or for locations which are frequently exposed to certain disruptions. Rail-replacement buses could however be incorporated in the dynamic PT assignment model, based on which train rescheduling could be optimised using the availability of certain bus-bridging routes as input. Moreover, generalised journey costs resulting from the dynamic PT assignment model could be used to assess the expected reduction in PT demand levels during certain disruption types, for example by adopting an elasticity-based approach (e.g. Yap et al., 2018b) or by coupling this assignment model to a variable demand model.

Our study results can be applied when developing contingency plans at a strategic and tactical level, which describe in which way train rescheduling can be performed from a passenger perspective, thereby including propagating impacts to other PT networks. Our proposed framework enables testing and optimising train rescheduling for different disruption types, with different disruption durations at different locations on the train network. This can support controllers in their decisions which trains to prioritise in case of different types of disruptions. Based on historical data about expected demand levels during different times of the day, it is possible to develop contingency plans for different times of the day. Given the required computation times of our modelling framework, we deem our method most suitable to apply for the development of contingency plans during the planning phase. For our framework to be used during real-time rescheduling, it is necessary to reduce computation times to only a few minutes in total. As mentioned in Section 4.1, these could be reduced substantially if parallel computing would be applied. In addition, it is possible to reduce computation times by applying our model in a non-iterative manner. Although this clearly comes at cost of accuracy, it is possible to run only one model sequence from Step 1 to Step 3, without further iterating between Step 2 and Step 3 (see Fig. 2). In Fig. 9, we set out a comparison in performance between the first and final iteration of each of the tested strategies to assess the impact on the solution quality. On average for the different strategies tested, it can be shown that reducing the computation times with up to factor 7 (by eliminating the need for the required seven iterations for strategy S1) comes at a cost of the solution quality being 32% less than using the converged solution. This provides insights into the trade-off between accuracy and computation time one could make, when applying this method in a real-time application rather than for contingency planning purposes.

For our case study, we assumed a disruption duration of 3 hours in our modelling work. However, due to uncertainty regarding the disruption duration, modelled and real-world disruption impacts might differ. Setting an appropriate disruption duration upfront is a non-trivial task, which depends on the risk strategy (often implicitly) adopted by the PT operator in practice. The modelled duration is not necessarily limited to the expected or average disruption duration obtained from historical data, but can take on any value from the duration distribution function. In case of a more optimistic, risk-taking strategy where a short disruption duration is assumed, there is a risk that passengers need to be informed twice when the disruption proves to last longer than expected and additional trains need to be rescheduled. On the other hand, adopting a more pessimistic, risk-averse strategy assuming a long duration might risk trains being rescheduled longer than necessary. We refer to the work of Ghaemi et al. (2018), which illustrates the passenger impacts when adopting different modelled disruption lengths. For our study, this entails that any duration from the disruption length distribution might be selected, depending on the strategy of the particular PT operator. Estimates for disruption length can be derived from historical data as proposed by Yap et al. (2018c).

Conclusions

In this research we propose a methodology to quantify the propagation of the impact of a train network disruption to the urban PT network level, as well as the extent to which it is potentially mitigated when applying different train rescheduling control strategies. We propose a modelling framework which consists of a dynamic PT assignment model and an optimisation-based train rescheduling model in an iterative process. We incorporate the number of transferring passengers to the urban network level in the optimisation process by weighting train delays accordingly. This allows the train rescheduling model to incorporate potential disruption propagation to the urban PT network level when determining which trains to prioritise for retiming, reordering or rerouting.

We tested our modelling framework successfully to a multi-modal case study in the Netherlands. This illustrates how our framework can contribute to the development of improved contingency plans in anticipation of short- to medium-lasting PT disruptions, by considering spill-over disruption impacts to other PT network levels. In addition, our framework can be used to quantify the PT disruption costs in a more complete and accurate way when evaluating the impact of past disruptions. For this specific case study disruption, our findings illustrate that adding more importance to transferring passengers compared to alighting passengers in the objective function of the train rescheduling model can considerably reduce propagation of pas-

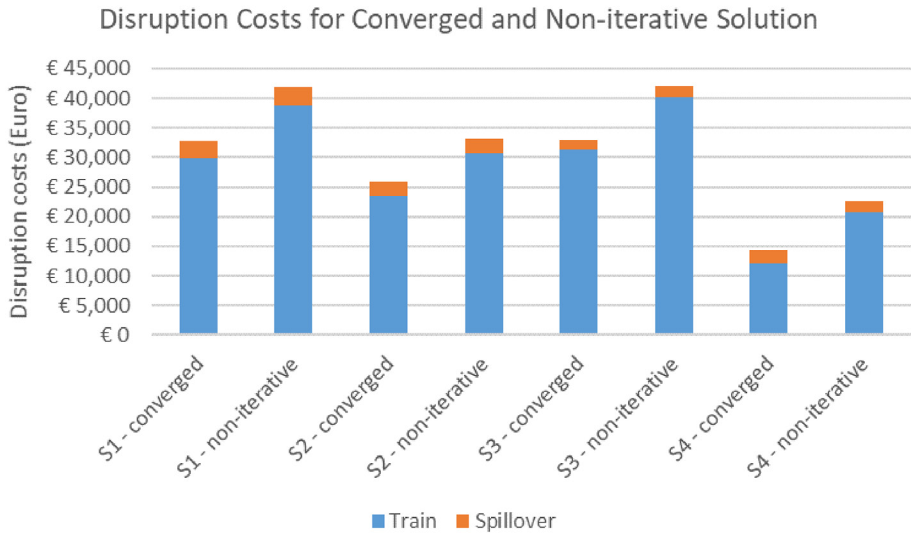

Fig. 9. Comparison of disruption costs between converged and non-iterative solution.

senger delays to the urban network level, whilst in some cases also reducing passenger delays at the train network level. For our case study, this suggests that weighting alighting and transferring passengers equally may yield suboptimal (i.e. Pareto inefficient) rescheduling from a total passenger perspective.

Based on our findings we recommend train network managers to consider how control decisions can result in train disruption impacts propagating to the urban PT network with which they interface. Whilst operators in practice often only consider the trips and passengers on the part of the network they are assigned to monitor and control, our research offers evidence that it can be made beneficial for all passenger groups to consider the wider PT network in control decisions. This can potentially reduce the disruption impact for passengers on the network level where the disruption occurs, as well as for passengers travelling on another PT network level. For a successful implementation of these overarching control strategies, attention should be paid to removing practical barriers between parties resulting from the institutional context of the PT system. These barriers can for example rise due to conflicting (control) interests of different operators, conflicting contractual agreements with the transport authority, or due to commercial interests conflicting with sharing required data.

We formulate several recommendations for future research directions. The main purpose of our case study application is illustrating that our modelling framework can be used to quantify propagated disruption impacts at other PT network levels for large, real-world public transport networks. For future research we recommend testing the disruption (propagation) impacts for more disruptions, locations and time periods, and exploring the use of different weights for control strategy S4 using our proposed modelling framework. This can provide a more systematic insight into the relation between different train control strategies and their impact on controlling disruption propagation, and hence provide more generalizable conclusions based on the case study outcomes. In addition, in this research only the passenger costs that are associated with a disruption are quantified. Other disruption costs for the service provider, such as crew-related costs, rescheduling costs or reduced revenues, are not calculated in this study as our focus is on the passenger disruption impacts. Notwithstanding, calculating the disruption costs for the PT service provider is a relevant topic we recommend to incorporate in future research. Moreover, it is recommended to determine a selection of stations to be subject to these control interventions, in order to make the problem manageable. A method as proposed by Yap et al. (2019), which adopts a data-driven approach to identify the most important stations for control, can potentially be applied for this purpose. At last, in our study we only consider train trips to be subject to control interventions to mitigate disruption propagation. It should however be mentioned that additional control can be applied to urban PT trips to further alleviate disruption propagation impacts, for example by holding an urban trip to enable delayed passengers to make their connection. This would require quantifying the trade-of between passengers on-board and waiting downstream for the urban service (experiencing longer in-vehicle or waiting times), and transferring passengers (saving additional wait time for the next service) to determine the optimal holding strategy (see for example Gavriilidou and Cats 2019). For future research, we therefore recommend exploring how control of train trips and urban PT trips can be applied simultaneously to further mitigate and encapsulate the impacts of disruptions.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgements

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Appendix A

This appendix provides details of the dynamic, simulation-based PT assignment model as used in our study. Table A1 lists the sets and indices, variables and parameters used for this model.

PT supply dynamics

Each line $l \in L$ is defined by a sequence of stops $l = \{s_{l,1}, s_{l,2}..s_{l,j}\}$ with $F = \{f_1, f_2.f_j\}$ denoting the set of scheduled trips on line $l \in L$. The total time $t_{l,f}$ it takes a vehicle to complete trip f of line l equals the summation of all running times $t^r_{s_{l,f}}$ from stop $s_l$to stop s_{l+1} and dwell times $t^{dw}_{s_{l,f}}$ at each stops$_l$, as expressed by Eq.A1. Running times $t^r_{s_{l,f}}$ can be deterministic, using the scheduled times from the timetable, or stochastic.

$$t_{l,f} = \sum_{s_{l,1}}^{s_{j-1}} t^r_{s_{l,f}} + \sum_{s_{l,1}}^{s_{j-1}} t^{dw}_{s_{l,f}} \quad \forall f \in F, l \in L \tag{A1}$$

The dwell times $t^{dw}_{s_{l,k}}$ for each trip $f \in F$ at each stop $s \in S$ depend on the number of boarding and alighting passengers $n^{board}_{s_{l,f}}$ and $n^{alight}_{s_{l,f}}$. The flow-dependent dwell time function used in this study assumes a linear relation between the number of boarding and alighting passengers and the required dwell time, whilst the model also allows for adding a non-linear effect of on-board crowding on dwell times based on Weidman (1994). As crowding levels for our case study network are relatively

Table A1
List with sets and indices, variables and parameters for dynamic PT assignment model.

Sets and indices	
s, S	public transport stop as node of graph G, set of stops
a, A	edge of graph G, set of links
l, L	unidirectional public transport line, set of lines
f, F	public transport trip, set of trips
o, O	public transport stop representing origin node of G, set of origin nodes
d, D	public transport stop representing destination node of G, set of destination nodes
g, G	passenger route choice action, set of actions
dw	index for dwell time
h	index for holding stop
r	index for running time
s	index for scenario
t	index for regional train network level
u	index for urban public transport network level
ivt	index for in-vehicle time
wkt	index for walking time
wtt	index for waiting time
$wtt - d$	index for waiting time due to denied boarding
$on - board$	index for passengers on-board a public transport trip
$alight$	index for alighting passengers
$board$	index for boarding passengers
tf	index for transferring passengers
arr	index for trip arrival
dep	index for trip departure
Variables	
h	scheduled headway of a public transport line
n	number of passengers
t	time
v	generalised passenger journey cost
Parameters	
δ	dwell time coefficient
ε	weights for passenger perception coefficients of travel time components

low, even when subject to the disruption types we consider, we deem using a simple linear function sufficient and beneficial in terms of computation times. For PT networks or disruption types where severe crowding occurs, the use of a dwell time function with non-linear crowding effect is however recommended. In case separate doors of a vehicle are used for boarding and alighting, the dwell time depends on the maximum of the number of boarding and alighting passengers, multiplied by the related dwell time coefficient δ which reflects the required boarding or alighting time per passenger (Eq.A2). When all doors are used for both boarding and alighting, the dwell time is calculated using Eq.A3. The dwell time function is calibrated for different vehicle types (e.g. high-floor trams, low-floor trams and buses) by executing a regression analysis predicting the realised dwell times based on the boarding and alighting volumes obtained from AFC and AVL data. A separate dwell time function is calibrated for each vehicle type. If different bus vehicle types (e.g. buses with a different number of doors or with different boarding regimes) would be used for different lines, different coefficients need to be calibrated. Similarly, if a tram line would be operated by longer trams (for example, two coupled tram carriages), separate coefficients need to be estimated for this line due to the different number of total doors available for boarding and alighting. For our case study, the calibrated dwell time constant δ_0 equals 20.4 / 16.2 / 19.8 seconds; boarding coefficient δ_1 equals 0.188 / 0.178 / 0.313; and alighting coefficient δ_2 equals 0.218 / 0.119 / 0.177 seconds for high floor trams, low floor trams and buses, respectively.

$$t^{dw}_{s_{l,f}} = \delta_0 + \max\left(\delta_1 \cdot n^{board}_{s_{l,f}}, \delta_2 \cdot n^{alight}_{s_{l,f}}\right)\ \forall f \in F, l \in L, s \in S \tag{A2}$$

$$t^{dw}_{s_{l,f}} = \delta_0 + \delta_1 \cdot n^{board}_{s_{l,f}} + \delta_2 \cdot n^{alight}_{s_{l,f}}\quad \forall f \in F, l \in L, s \in S \tag{A3}$$

The departure time of a trip $t^{dep}_{s_{l,f}}$ depends on the arrival time at that stop $t^{arr}_{s_{l,f}}$ and the required dwell time$t^{dw}_{s_{l,f}}$ (Eq.A4). In case a stop is a holding point $s \in S^h$and a schedule-based holding control regime is employed, the departure time can never be earlier than the scheduled departure time from that specific stop$t^{\overline{dep}}_{s_{l,f}}$ (Eq.A5). For urban PT networks, a select number of stops are usually holding points, whereas all train network stations are holding points as trains are generally not able to depart ahead of schedule from a station.

$$t^{dep}_{s_{l,f}} = t^{arr}_{s_{l,f}} + t^{dw}_{s_{l,f}}\quad \forall f \in F, l \in L, s \notin S^h \tag{A4}$$

$$t^{dep}_{s_{l,f}} = \max\left(t^{arr}_{s_{l,f}} + t^{dw}_{s_{l,f}}, t^{\overline{dep}}_{s_{l,f}}\right)\quad \forall f \in F, l \in L, s \in S^h \tag{A5}$$

PT demand dynamics

The number of boarding and alighting passengers is obtained from a successive number of choices each individual passenger makes during the journey. At each stop a passenger can make a boarding decision to board a certain trip or to wait, or make a connection decision to walk to another PT stop. When boarded a certain trip, a passenger can make an alighting decision at each downstream stop whether to alight from this vehicle or to stay on-board. These decisions can be made en-route and in a stochastic and dynamic way if the expected utility of a certain choice changes during a journey, for example in response to high crowding levels or to information provided about a downstream disruption. These successive decisions are based on the expected utility of a path v_g corresponding to a certain action g as logsum over the path set $A_g \in A_{od}$ associated with this action (Eq.A6). The probability of passenger n choosing this action g is calculated using a multinomial logit (MNL) model (Eq.A7), which results in stochastic route choice over the network. The structural part of the utility function is calculated based on the sum product of the expected values of the different travel time attributes and the weights of the corresponding coefficients. The model considers in-vehicle time (nominal and perceived in-vehicle time caused by crowding), walking time, waiting time (regular waiting time as well as waiting time caused by denied boarding in case of crowding) and the number of transfers. For different travel time components, different coefficients are used reflecting the perceived time by passengers, as well as a fixed transfer penalty for each transfer. To alleviate potential violations of the IIA assumption of the MNL model, common stops and lines are merged into hyper-paths. A single non-equilibrium assignment procedure without day-to-day learning is applied.

$$v_{n,g} = ln \sum_{a \in A_g} e^{v_{n,a}}\ \forall n \in N, g \in G \tag{A6}$$

$$p_{n,g} = \frac{e^{v_{n,g}}}{\sum_{g \in G} e^{v_{n,g}}}\ \forall n \in N, g \in G \tag{A7}$$

Appendix B

In this appendix, the mixed integer linear programming (MILP) formulation of the optimisation-based train rescheduling model is provided.

Objective function and decision variables

Let T represent the set of all train trips on the selected train network level and let B denote the set of segments that defines the rail infrastructure for the train network level. E denotes the set of events, where an event can be seen as a time slot request by a train for a specific network segment. The index i is associated with a specific transport service in the set T (i.e.$i \in T$), while the index j is associated with a specific network segment ($j \in B$), and index k is associated with an event ($k \in E$). An event is associated with a combination of a network segment and a transport service. The set $K_i \subseteq E$ is an ordered set of events for each transport service$i$, while $L_j \subseteq E$ is an ordered set of events for each network segment$j$. Each segment j in Bhas a number of parallel tracks, with each track indicated by $p \in P_j$. Each track requires a separation in time between subsequent events (i.e. the minimum time required between one train leaving the track and the next train entering the same track). The latter is reflected by δ_j^m for the minimum time between trains driving in the opposite direction, and by δ_j^f for trains following each other in the same direction.

The objective function of the train rescheduling model in its most basic form is the minimisation of the sum of all delays (larger than a certain threshold value) for all train trips (Eq.B1). The decision variables reflecting the retiming, reordering, and track allocation decisions to be made during the train rescheduling are reflected by Eq.B2-B5 below.

$$minimise \sum_{i \in T} \sum_{k \in K_i} \left(z_{i,k}^{+\tau} \right) \tag{B1}$$

$$q_{i,k,p} = \begin{cases} 1, \text{ if event } k \text{ uses track } p, k \in K_i, k \in L_j,\ i \in T,\ p \in P_j,\ j \in B \\ 0, \text{ otherwise} \end{cases} \tag{B2}$$

$$r_{k,\widehat{k}} = \begin{cases} 1, \text{ if event } k \text{ occurs before event } \widehat{k},\ k \in L_j,\ j \in B : \mathrm{k} < \widehat{k} \\ 0, \text{ otherwise} \end{cases} \tag{B3}$$

$$s_{k,\widehat{k}} = \begin{cases} 1, \text{ if event } k \text{ is rescheduled to occur after event } \widehat{k},\ k \in L_j,\ j \in B : \mathrm{k} < \widehat{k} \\ 0, \text{ otherwise} \end{cases} \tag{B4}$$

$$x_{i,k}^{begin}, x_{i,k}^{end}, z_{i,k}^{+\tau}, w_{i,k} \geq 0, k \in K_i, i \in T \tag{B5}$$

Constraints

The optimisation is subject to several constraints related to the timing and sequence of events and the capacity and safety limitations of the infrastructure. We introduce the following notations specifically related to the model constraints. The variables $x_{i,k}^{begin}$ and $x_{i,k}^{end}$ reflect the arrival time at a segment for a specific train, and the departure time from this segment, respectively. The initially scheduled start and end time of each event are reflected by $b_{i,k}^{initial}$ and $e_{i,k}^{initial}$, whilst a disruption is modelled by changing the start and end time of selected events to $b_{i,k}^{static}$ and $e_{i,k}^{static}$. The minimum running time of each trip for each segment $d_{i,k}$ is provided as model input. The constraints reflected by Eq.B6-B13 are related to train restrictions. Each event of a specific train trip needs to be followed directly by the next event of this trip (Eq.B6). Events which started before the disruption starts, but are not finished yet when the disruption start, should start as planned (Eq.B7-B8). The duration of each event for a certain segment should at least be equal to the minimum running time required for this segment (Eq.B9), whilst events are not allowed to start before their original scheduled departure time (Eq.B10). In Eq.B11, the train delay $z_{i,k}$ exceeding threshold τ minutes is calculated for each event. Eq.B12-B13 compute the time deviation of each event.

$$x_{i,k}^{end} = x_{i,k+1}^{begin}, k \in K_i, i \in T : k \neq |K_i| \tag{B6}$$

$$x_{i,k}^{begin} = b_{i,k}^{static}, k \in K_i, i \in T : b_k^{static} > 0 \tag{B7}$$

$$x_{i,k}^{end} = e_{i,k}^{static}, k \in K_i, i \in T : e_k^{static} > 0 \tag{B8}$$

$$x_{i,k}^{end} \geq x_{i,k}^{begin} + d_{i,k}, k \in K_i, i \in T \tag{B9}$$

$$x_{i,k}^{begin} \geq b_{i,k}^{initial}, k \in K_i, i \in T \tag{B10}$$

$$x_{i,k}^{end} - e_{i,k}^{initial} - \tau \leq z_{i,k}^{+\tau}, k \in K_i, i \in T \tag{B11}$$

$$x_{i,k}^{end} - e_{i,k}^{initial} \leq w_{i,k}, k \in K_i, i \in T \tag{B12}$$

$$e_{i,k}^{initial} - x_{i,k}^{end} \leq w_{i,k}, k \in K_i, i \in T \tag{B13}$$

The constraints as formulated in Eq.B14-B20 concern the permitted interactions between trains, given the capacity limitations of the infrastructure (including safety restrictions). First, each event must use exactly one track per segment (Eq. B14). Eq.B15-B19 make sure that if two events using the same track within a segment, this can only occur if the first event has finished and the minimum required time δ_j^m or δ_j^f has passed (depending whether these subsequent trains are running in the same or opposite direction). o_k refers here to the point of origin of event k, which enables determining whether two subsequent events are using a segment in the same or in opposite direction. M is a large positive constant. Eq.B20 guarantees that an event k cannot be scheduled both before and after event $\widehat{k}$.

$$\sum_{p \in P_j} q_{i,k,p} = 1, k \in K_i k \in L_j, i \in T, p \in P_j, j \in B \tag{B14}$$

$$q_{i,k,p} + q_{\widehat{i},\widehat{k},p} - 1 \leq r_{k,\widehat{k}} + s_{k,\widehat{k}},$$

$$k, \widehat{k} \in L_j, k \in K_{i,} \widehat{k} \in K_{\widehat{i},} p \in P_j, j \in B, i, \widehat{i} \in T : k < \widehat{k} \tag{B15}$$

$$x_{\widehat{i},\widehat{k}}^{begin} - x_{i,k}^{end} \geq \delta_j^m r_{k,\widehat{k}} - M\left(1 - r_{k,\widehat{k}}\right),$$

$$k, \widehat{k} \in L_j, k \in K_{i,} \widehat{k} \in K_{\widehat{i},} p \in P_j, j \in B, i, \widehat{i} \in T : k < \widehat{k}, o_{\widehat{k}} \neq o_k \tag{B16}$$

$$x_{\widehat{i},\widehat{k}}^{begin} - x_{i,k}^{end} \geq \delta_j^f r_{k,\widehat{k}} - M\left(1 - r_{k,\widehat{k}}\right),$$

$$k, \widehat{k} \in L_j, k \in K_{i,} \widehat{k} \in K_{\widehat{i},} p \in P_j, j \in B, i, \widehat{i} \in T : k < \widehat{k}, o_{\widehat{k}} = o_k \tag{B17}$$

$$x_{i,k}^{begin} - x_{\widehat{i},\widehat{k}}^{end} \geq \delta_j^m s_{k,\widehat{k}} - M\left(1 - s_{k,\widehat{k}}\right),$$

$$k, \widehat{k} \in L_j, k \in K_{i,} \widehat{k} \in K_{\widehat{i},} p \in P_j, j \in B, i, \widehat{i} \in T : k < \widehat{k}, o_{\widehat{k}} \neq o_k \tag{B18}$$

$$x_{i,k}^{begin} - x_{\widehat{i},\widehat{k}}^{end} \geq \delta_j^f s_{k,\widehat{k}} - M\left(1 - s_{k,\widehat{k}}\right),$$

$$k, \widehat{k} \in L_j, k \in K_{i,} \widehat{k} \in K_{\widehat{i},} p \in P_j, j \in B, i, \widehat{i} \in T : k < \widehat{k}, o_{\widehat{k}} = o_k \tag{B19}$$

$$r_{k,\widehat{k}} + s_{k,\widehat{k}} \leq 1, k, \widehat{k} \in L_j, j \in B : k < \widehat{k} \tag{B20}$$

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