An Approach for High-Angular Resolution Implementation in Moving Automotive MIMO Radar

Yuan, Sen; Fioranelli, Francesco; Yarovoy, Alexander

**Important note**
To cite this publication, please use the final published version (if applicable).
Please check the document version above.

# An Approach for High-Angular Resolution Implementation in Moving Automotive MIMO Radar

Sen Yuan, Francesco Fioranelli, Alexander Yarovoy

MS3 Group, Department of Microelectronics, Faculty of EEMCS, Delft University of Technology, The Netherlands
{S.Yuan-3, F.Fioranelli, A.Yarovoy}@tudelft.nl

*Abstract* — **A method exploiting the movement of the vehicle to boost the cross-range resolution of automotive radar by forming a larger virtual array is proposed. Initial simulated results show that the proposed method with the traditional Digital beamforming (DBF) algorithm can separate targets that cannot be otherwise recognized by the traditional MIMO approach. Furthermore, the proposed approach does not require prior knowledge of the number of targets, and can solve the MUSIC rank deficiency problem because of its larger virtual planar antenna.**

*Keywords* — **MIMO array, high resolution, multiple signal classification (MUSIC), automotive radar.**

## I. INTRODUCTION

Important requirements for automotive radar are high resolution in range, Doppler and angle [1], low hardware cost, and small size. With the advent of high-bandwidth and high-frequency FMCW radar, the resolution of range and velocity increased dramatically. To obtain high cross-range resolution in angle, large aperture antenna arrays are usually implemented via phased array [2], synthetic aperture radar (SAR) [3], or multiple-input-multiple-output (MIMO) array techniques [4]. Phased arrays typically use numerous antennas to form a large aperture having a narrow beamwidth. However,they are usually not an economical option for civilian applications. The SAR techniques form a large effective (i.e., virtual) aperture array by moving a small antenna or array which reduce the number of antennas required for imaging, thus providing a cost-effective solution for high-resolution imaging applications but work efficiently only for side-looking applications. MIMO radar technology exploits spatial diversity of transmit and receive antenna arrays, which has received considerable attention from automotive radar manufacturers. This can achieve a high angular resolution with relatively small numbers of antennas. For that ability, it has been exploited in the current-generation automotive radar for advanced driver assistance systems (ADAS), as well as in next-generation high-resolution imaging radar for autonomous driving.

However, the practical implementation on vehicles requires limited size of the radar to be feasible, hence limiting the number of MIMO antennas. In automotive MIMO radar with a virtual uniform linear array (ULA), angle finding can be done with digital beamforming (DBF) [5] by performing FFTs on snapshots taken across the array elements. However, DBF is not a high-resolution angle-finding method. Higher angular resolution can be

achieved with Minimum Variance Distortionless Response (MVDR) [6], subspace-based methods, such as multiple signal classification (MUSIC) and estimation of signal parameters via rational invariance techniques (ESPRIT) [7], sparse sensing-based methods [8], using the movement of radar [9], and the iterative adaptive approach (IAA) [10]. Those are proposed to break the Rayleigh limits under current configuration by signal processing or information theory. The performance of subspace-based angle-finding methods relies on accurate estimation of the array covariance matrix with multiple snapshots, which is challenging in highly nonstationary automotive radar scenarios. In such a context, spatial smoothing [11] is applied for introducing virtual snapshots for array covariance-matrix estimation. While sparse sensing-based methods and IAA angle estimates are based on a single snapshot, which is important for snapshot-limited automotive radar, they have a much higher computational cost, limiting their real-time use.

In this article, rather than focusing on those high-resolution angle finding methods, an approach based on the combination of SAR and MIMO technology is proposed. By adding the movement of the radar together with the spatial diversity of the transmit and receive antenna, a robust method to increase the virtual antenna to boost the angular resolution for side-looking automotive MIMO radar is developed. The proposed method can separate close targets that are overlapped in traditional MIMO approaches using DBF technique. Additionally, the proposed method can solve MUSIC rank deficiency problem and its difficulties to obtain the prior number of targets.

The rest of the article is organized as follows. In Section II, the formation of virtual aperture array of MIMO antenna is analyzed, and the geometry relationship of the radar movement is demonstrated using the $2 * 2$ MIMO ULA. The simulation results and some analysis of the effects of different parameters such as vehicle velocity and acceleration are provided in Section III. Finally, conclusions are drawn in Section IV.

## II. PROPOSED METHOD

### A. Proposed formation of Virtual Antenna

In automotive MIMO radar with $M_t$ transmit and $M_r$ receive antennas, a virtual ULA of $M_t M_r$ elements can be synthesized with inter-element spacing equal to $d$.

Here, a generic 1-D MIMO array placed on the y direction is considered, with the x-axis pointing toward the illuminated scene and assuming no movement in the z axis. The radar

where the MIMO array is installed is moving along the y axis, with objects located in the far-field of the MIMO array.

The position of the $i$-th transmit antenna at time $t$ is $\mathbf{T}x_i(t) = (x_{\mathbf{T}i}(t), y_{\mathbf{T}i}(t), 0)$. It is worth mentioning that $t$ in this article refers to slow time, which equals to $t = mt_m$, where $t_m$ is the chirp duration. The position of the $j$-th receive antenna at time $t$ is $\mathbf{R}x_i(t) = (x_{\mathbf{R}i}(t), y_{\mathbf{R}i}(t), 0)$. The position of a target in the field of view is $\mathbf{T_o} = (x_\mathbf{o}, y_\mathbf{o}, 0)$. The range between the target and $i$-th transmit antenna and $j$-th receive antenna at time $t$ would be $D_{i,j}(t)$, as in equation (1)

$$D_{i,j}(t) = |\mathbf{T}x_i(t) - \mathbf{T_o}| + |\mathbf{R}x_j(t) - \mathbf{T_o}| \qquad (1)$$

A property of the virtual uniform linear array is that the range differences between targets and different receiver antenna pairs will be approximately equal to a constant with respect to the direction of arrival, and the distance between different receive antennas is $d$, which is equal to $d = \frac{\lambda}{2}$. The range differences are shown as in equations (2-3)

$$D_{i,j}(t) - D_{i,j+1}(t) \approx d\sin\theta \qquad (2)$$

$$D_{i+1,j}(t) - D_{i,j}(t) \approx M_\mathbf{r}d\sin\theta \qquad (3)$$

where $\theta$ is the angle of incidence. Thus the $M_\mathbf{r}M_\mathbf{t}$ virtual linear array can be formed.

With the movement of radar over time, the additional virtual antenna can be formed by

$$D_{i,j}(t_1) - D_{i,j}(t_0) \approx D_{i,j}(t_0) - D_{i,j+1}(t_0) \qquad (4)$$

where the $(j+1)$-th receive antenna can be generated with the movement of the radar.

To calculate (4), the time step $t_1$ can be found as in (5)

$$t_1 = t_0 + \lfloor \frac{d}{2M_\mathbf{t}t_m v_r} \rfloor tm \qquad (5)$$

with $v_r$ the radar speed, and $\lfloor \rfloor$ the rounding operation.

*B. The geometry relationship of the radar movement*

Using 2 transmit and 2 receive antenna MIMO array as an example shown in Fig.1 with solid line, the virtual aperture array can be formed with 4 antennas with phase relationships as shown in Table 1.
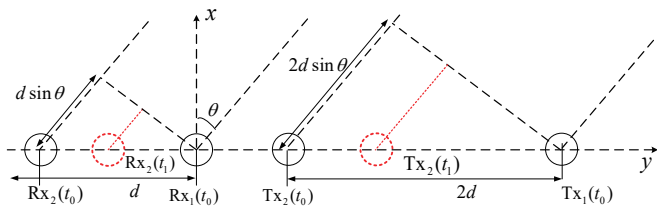


Fig. 1. The geometrical configuration of MIMO approach, where the black solid line shows the MIMO antennas at time $t_0$ and the red dotted line shows the antennas moved after time $t_1$

Table 1. The phase relationship of different $2*2$ MIMO antenna pairs

| Phases | 0 | $d\sin\theta$ | $2d\sin\theta$ | $3d\sin\theta$ |
|---|---|---|---|---|
| Tx Rx pairs | $D_{1,1}(t_0)$ | $D_{1,2}(t_0)$ | $D_{2,1}(t_0)$ | $D_{2,2}(t_0)$ |

In Fig. 1, after moving from $t_0$ to $t_1$, the transmit and receive antenna in black solid line marked with $\mathbf{R}x_2$ and $\mathbf{T}x_2$, represented as $D_{2,2}$, will arrive at the positions marked with the red dotted lines. If the time step satisfies (4), the phase differences between the two antenna pair at time $t_0$ and $t_1$ will be the same as the different adjacent antenna pairs, for example $D_{2,1}$ and $D_{2,2}$, at time $t_0$. So by moving with different multiples of $t_1$ according to (5), the virtual aperture of $2*2$ MIMO can be expanded as shown in Table 2. In this way, the MIMO virtual aperture array is expanded three times, and this can be further expanded more times with the same approach.

Table 2. The phase relationship of additional antenna pairs generated by radar movement

| Phases | $-3d\sin\theta$ | $-2d\sin\theta$ | $-d\sin\theta$ |
|---|---|---|---|
| Tx Rx pairs | $D_{1,1}(3t_1)$ | $D_{1,1}(2t_1)$ | $D_{1,1}(t_1)$ |
| Phases | $4d\sin\theta$ | $5d\sin\theta$ | $6d\sin\theta$ |
| Tx Rx pairs | $D_{2,2}(t_1)$ | $D_{2,2}(2t_1)$ | $D_{2,2}(3t_1)$ |

III. SIMULATION RESULTS AND DISCUSSION

*A. Simulation parameters*

Our simulated test is based on $2*2$ MIMO as discussed in section II.A. All the simulation are performed in MATLAB, and some of the key parameters are shown in Table 3. To test the performance of the proposed idea, using the method of expanding the aperture shown in II.B, the virtual aperture is expanded to $2*2+6$. The MIMO antenna on the side-looking radar was located at the coordinate center, with targets placed at the same range bin of 10 meters to meet the Fraunhofer distance [12] and ensure targets are in the far-field.

Table 3. The chosen parameters for the simulation test

| | Parameters | Value |
|---|---|---|
| | Central Frequency (GHz) | 77 |
| | Slope (MHz/us) | 87.89 |
| tr | Sampling Rate (Msps) | 150 |
| | Bandwidth (MHz) | 150 |
| | Modulation | TDMA |

*B. Simulation result*

To evaluate the performance of the proposed method, several different tests are performed, including two targets, four targets for the rank deficiency problem of MUSIC, the influence of different parameters in (4), and some discussion on robustness to uncertain estimations, detailed as follows.

*1) Two targets*

The two targets are placed at -7 and 8 degrees according to the coordinate introduced in III-A. The result is shown in Fig.2, in which the traditional MIMO approach plus DBF algorithm cannot separate the closed targets. On the contrary, these can be separated by the traditional MIMO approach plus MUSIC algorithm and by our proposed method. The traditional MUSIC estimated the angle as -7.4 and 7.6, while the proposed method estimated -7.6 and 7.9.
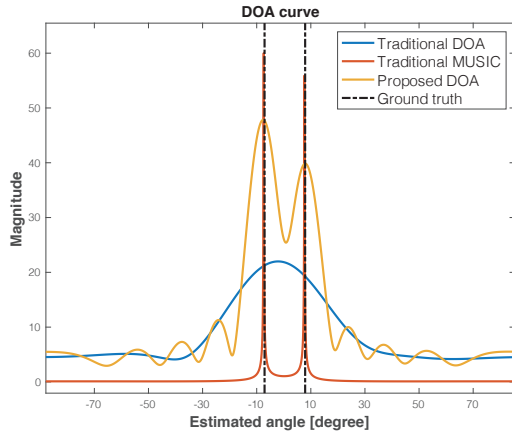
450

Fig. 2. The angle estimation of 2 closed targets for different methods

### 2) Four targets

The four targets are placed at $\pm 25$ and $\pm 10$ degrees, while the other settings are the same as those in III-B1. The MUSIC algorithm can only be performed when the number of targets in a certain range bin is known and less than the virtual array number $M_\mathbf{t}M_\mathbf{r}$, 2*2, which is related to the rank deficiency problem. From Fig. 3, it is shown that the proposed method can solve the existing problem of MUSIC, and separate the four targets successfully with -26.0, -10.0, 9.0 and 25.5, respectively.



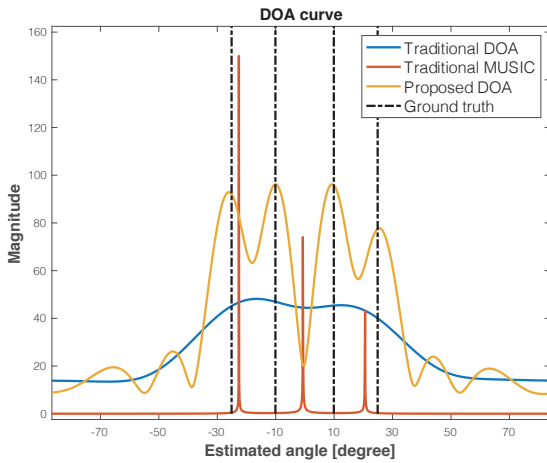Fig. 3. The angle estimation of 4 closed targets for different methods

## IV. DISCUSSION

### A. Effect of the different parameters

As seen in (5), the speed of radar and the chirp duration will influence the accuracy of the time tag-$t_1$ in (4-), thus affecting the coherency of the formed virtual aperture, as shown in Fig.4. The coherency of the signal depends on its time tag, where longer intervals of time tag mean more accurate coherency. That happens in the lower speed and shorter chirp duration, leading to higher angular estimation

performance, where the time tag will be more sensitive to the uncertainty and performance will deteriorate quickly.
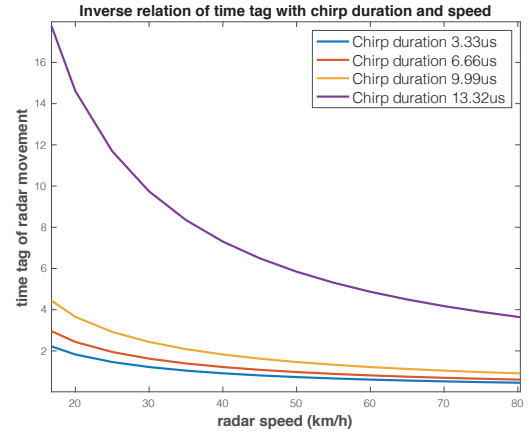


Fig. 4. The inverse relation between the time tag, $t_1$ in equation (5), and radar velocity for different chirp duration

In Fig.5, the influence of different combinations of radar speed and chirp duration is demonstrated. For a better visual comparison, all the results utilize the root mean square error (RMSE) between the ground truth and the estimated angle in (dB), as $20log_{10}(\theta_{truth} - \theta_{estimated})$. The estimation results of the proposed MUSIC are better than those of traditional MUSIC. Additionally, the angular estimation for shorter chirp duration yields higher performance, and so does lower speed, as discussed with respect to Fig.4.
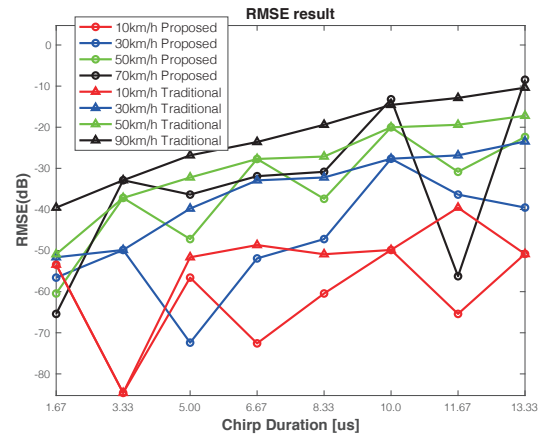


Fig. 5. The RMSE of the estimated angle as a function of radar speed and chirp duration

### B. Effect of uncertainty in parameters estimation

Typically, during the movement of radar, there are two types of uncertainty: one comes from the velocity measurement, another is the velocity fluctuation because of the uneven ground. When the velocity is measured by global positioning system (GPS), the mean and variance of measurement error will typically be $3.6mm/s$ and $1.8mm^2/s^2$, respectively, according to [13]. Considering the

451

mechanical model of typical vehiclesthe force of the road on a car will influence the acceleration directly. According to the human comfort model [14], the maximum acceleration for humans is $0.63 m/s^2$.
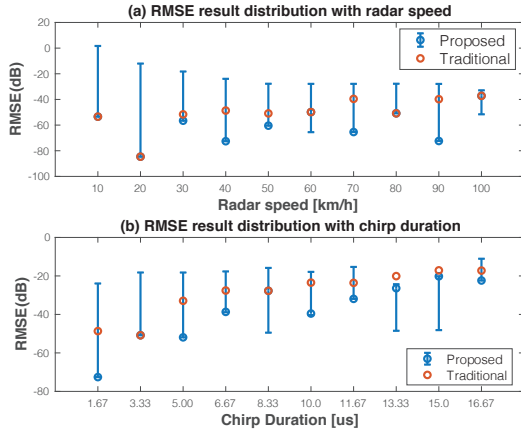


Fig. 6. The RMSE of the estimated angle: (a)radar speed; (b)chirp duration

In Fig.6, measurement errors from -0.6 $m/s$ to 0.6 $m/s$ are tested with different chirp duration and radar speed. The results of the proposed method plus MUSIC are presented in blue line with the result with no measurement error marked with blue circle. The red circle marker is the corresponding result of traditional MUSIC. It can be seen that the errors are comparable. From Fig.6a, it appears that the higher speed is more robust to measurement errors but yield higher error in the angular estimates, while the lower speed is more sensitive to measurement errors but yield better accuracy. Also, at a shorter chirp duration the angle is estimated more precisely but suffers from a larger variation of uncertainty error than longer chirps. This is in line with the deduction in Fig.4. Note that this test is more significant than the current GPS error, hence the proposed method can be implemented with current GPS.
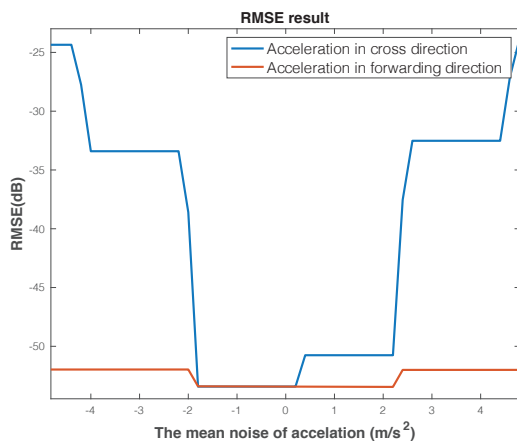


Fig. 7. The RMSE of the estimated angle with different acceleration

A similar test is performed for the acceleration in x and y direction. The radar speed and the chirp duration are 10 km/h and 1.67 us, respectively. Results in Fig. 7 show that the estimated angle is less sensitive to fluctuations in the forward direction (here y direction), than the in the x, cross-forward direction.

## V. CONCLUSION

In this paper, a novel method combining SAR and MIMO processing is proposed to boost angular resolution in automotive radar. Initial simulation results show that the proposed method can solve the issue of rank deficiency and the requirement of prior knowledge of the number of targets of MUSIC, considered as the benchmark for comparison. Future work will focus on validation of the method on more realistic simulations and experimental data.

## ACKNOWLEDGMENT

## REFERENCES

[1] S. M. Patole, M. Torlak, D. Wang, and M. Ali, "Automotive radars: A review of signal processing techniques," *IEEE Signal Processing Magazine*, vol. 34, no. 2, pp. 22–35, 2017.

[2] J. Wang, P. Aubry, and A. Yarovoy, "3-d short-range imaging with irregular mimo arrays using nufft-based range migration algorithm," *IEEE Transactions on Geoscience and Remote Sensing*, vol. 58, no. 7, pp. 4730–4742, 2020.

[3] Y. K. Chan and V. C. Koo, "An introduction to synthetic aperture radar (sar)," *Progress In Electromagnetics Research*, vol. 2, pp. 27–60, 2008.

[4] E. Biglieri, R. Calderbank, A. Constantinides, A. Goldsmith, A. Paulraj, and H. V. Poor, *MIMO wireless communications*. Cambridge university press, 2007.

[5] P. Barton, "Digital beam forming for radar," *IEE Proceedings F (Communications, Radar and Signal Processing)*, vol. 127, pp. 266–277(11), August 1980.

[6] J. Capon, "High-resolution frequency-wavenumber spectrum analysis," *Proceedings of the IEEE*, vol. 57, no. 8, pp. 1408–1418, 1969.

[7] R. Roy and T. Kailath, "Esprit-estimation of signal parameters via rotational invariance techniques," *IEEE Transactions on Acoustics, Speech, and Signal Processing*, vol. 37, no. 7, pp. 984–995, 1989.

[8] T. T. Cai and L. Wang, "Orthogonal matching pursuit for sparse signal recovery with noise," *IEEE Transactions on Information Theory*, vol. 57, no. 7, pp. 4680–4688, 2011.

[9] S. Xu and A. Yarovoy, "Joint features extraction for multiple moving targets using (ultra-)wideband fmcw signals in the presence of doppler ambiguity," *IEEE Transactions on Signal Processing*, vol. 68, pp. 6562–6577, 2020.

[10] W. Roberts, P. Stoica, J. Li, T. Yardibi, and F. A. Sadjadi, "Iterative adaptive approaches to mimo radar imaging," *IEEE Journal of Selected Topics in Signal Processing*, vol. 4, no. 1, pp. 5–20, 2010.

[11] T.-J. Shan, M. Wax, and T. Kailath, "On spatial smoothing for direction-of-arrival estimation of coherent signals," *IEEE Transactions on Acoustics, Speech, and Signal Processing*, vol. 33, no. 4, pp. 806–811, 1985.

[12] G. A. Tyler and B. J. Thompson, "Fraunhofer holography applied to particle size analysis a reassessment," *Optica Acta: International Journal of Optics*, vol. 23, no. 9, pp. 685–700, 1976.

[13] L. Serrano, D. Kim, R. B. Langley, K. Itani, and M. Ueno, "A gps velocity sensor: How accurate can it be? a first look," in *Proceedings of the 2004 National Technical Meeting of The Institute of Navigation*, 2004, pp. 875–885.

[14] Y. Du, Y. Li, and C. Liu, "A comfort-based driving planning method for autonomous driving," China Patent WO2 018 122 808A1, Jul. 05, 2018.