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Performance Analysis of the Wind Field Estimation for a Very Fast Scanning Weather Radar

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***Abstract*—The performance and limitations of the Doppler processing of the scattered signals from extended meteorological objects (precipitation) are analysed in the case of radar with fast azimuthal scanning. The classical method of the Discrete Fourier Transform (DFT) has been applied to simulated weather radar signals to estimate the Doppler velocity spectrum and characterise it with the mean Doppler velocity and the Doppler spectrum width. The accuracy and resolution of these estimations have been analysed as a function of the scanning radar rotation speed. Finally, the performances of the 2D wind field retrieval are analysed in relation to the accuracy and resolution of Doppler spectra estimations. The wind field retrieval has been done using the classical velocity azimuthal display (VAD) retrieval technique that gives an overall/average estimate of the wind field over an observation region. A few possible approaches for improving the accuracy and resolution of a fast scanning weather radar Doppler signal processing are proposed and analysed based on simulated scanning radar data.**

***Index Terms*—Weather radars, Doppler processing, DFT, VAD**

I. Introduction

Nowadays, very fast scanning phased array radars are used at airports to detect and track drones and birds. Such radars have multiple antenna channels for electronic beamforming in the elevation plane in combination with mechanical scan in azimuth. Due to the need for a fast update rate, such radars require a high-speed rotation for the azimuthal scan, up to 60 rotations per minute (rpm). Such an update rate provides a possibility to use the scan-to-scan range-azimuth-height migration for the reliable estimation of the 3D motion of the point targets of interest.

Modern demands for improving the situational awareness in airports and urban regions formulated the interest to extend the capabilities of such radars' with new functions - to observe and estimate the intensity of precipitations, to retrieve the 3D wind field with high temporal and spatial resolution. The last task requires the coherent Doppler signal processing for the meteorological objects' Doppler velocity spectrum estimation. The performance of such processing is limited by the negative effect of fast radar antenna rotation, limiting the radar's dwell time (T_d) - the time the radar spends in one specific azimuthal sector of space. This time on target is directly proportional to the radar's beamwidth (BW) and inversely proportional to the scanning speed of the radar. The applicability of Doppler processing becomes questionable with such dependency between the dwell time and the radar scan speed because Discrete Fourier transform (DFT) technique requires at least 16 samples to be coherently integrated for Doppler velocity spectrum estimation for weather applications [1]. This is because the Doppler spectrum for weather targets has a specific continuous shape, in general, Gaussian or skewed-Gaussian in nature [2]. In meteorological applications, such spectra are characterised by two Doppler spectral moments - the mean Doppler velocity and the Doppler spectrum width- that must be estimated with high accuracy.

In this paper, the performance of DFT algorithm is analysed with respect to the scanning speed Ω of the radar and the retrieval of Doppler moments such as the mean Doppler velocity μ and the Doppler spectrum width σ. Finally, we analyse the performances of the wind field retrieval from the Doppler spectra observations. For the wind speed and direction estimation, we use the velocity azimuthal display (VAD) retrieval method [3] that requires the knowledge of the mean Doppler velocity at various observation directions. This method assumes that the wind field is homogeneous all over the space around the radar, and, therefore, it estimates the overall wind field in space. This paper evaluates the performance of VAD technique as a function of the radar's scanning speed Ω.

The paper is structured as follows. In section II, the weather radar echo signal model is explained that is used for the simulation. Section III includes the results and analysis of the measured with a scanning radar Doppler spectra with respect to the azimuth angle with the help of the DFT at different scanning speeds. Section IV includes the performance analysis of the Doppler moments retrieval. Section V presents a few proposed techniques to improve the Doppler moments retrieval. Section VI shows how the errors/accuracy of the Doppler processing for different radar scanning velocities influence the retrieval accuracy of 2D wind field. Finally, the conclusions are presented in section VII.

II. Weather Radar Signal Model

The weather radar signal model used in this paper is an adapted version of the radar echo model described in [4]. We assume that a radar observes the surrounding space using a horizontally scanning antenna. This space is homogeneously filled with the same precipitation type. The spatial variation

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of the precipitation remains constant during the time of observation by the radar. There are many discrete scatterers (e.g. raindrops) present in every resolution volume of the radar. If we consider that number as N, the model of radar echo from such resolution volume can be represented as

$$s(k) = \sum_{i=1}^{N} A_k^{(i)} \exp{[j\psi_k^{(i)}]}, \quad (1)$$

where $s(k)$ is the received signal at the time k, $A_k^{(i)}$ and $\psi_k^{(i)}$ are the amplitude and the phase of the echo signal from one specific scatterer i, N is the number of scatterers in the resolution volume. The phase $\psi^{(k)}$ is defined as,

$$\psi^{(i)} = 2\pi D^{(i)}/\lambda, \quad (2)$$

where $D^{(i)}$ is the two-way distance of the scatterer from the radar, λ is the operating wavelength of the radar. The positions of the scatterers change based on their velocities in time. It can be represented as,

$$\mathbf{X}^{(i)}(k) = \mathbf{X}^{(i)}(k-1) + \mathbf{V}^{(i)}(k-1)\Delta T; \quad (3)$$

where k is the time index, and ΔT is the time step known as the radar's pulse repetition time (PRT). The term $X^{(i)}$ is the position of the scatterer i in Cartesian coordinates $[x, y]$ and $V^{(i)}$ is the velocity of the same scatterer in the same Cartesian coordinate system $[u, v]$. As we assume the radar antenna to be horizontally aligned, the vertical fall velocity of the scatterers is not considered. Therefore, the model remains two dimensional, with velocities of the scatterers having two components in space and defined by the horizontal wind. The amplitudes $A^{(i)}$ are considered to be the same for all particles and, without loss of generality, set to unity.

The temporal model of the measured radar signal can be defined as

$$z(k) = \sum_{i=0}^{N} A_k^{(i)} \exp\left(-j\frac{4\pi}{\lambda}R_k^{(i)}\right) + n(k)\ , \quad (4)$$

$$R_k^{(i)} = \sqrt{\left(x_k^i + u_{k-1}^i\Delta T - x_0\right)^2 + \left(y_k^i + v_{k-1}^i\Delta T - y_0\right)^2}$$

where $n(k)$ is a complex white Gaussian noise $\mathcal{N}(\mu = 0, \sigma_n^2)$ with zero mean and a standard deviation of σ_n. The vector $[x_0, y_0]$ describes the coordinates of the radar position. The coordinates $[x_k^i, y_k^i]$ describe the position of scatterer i in space at the time step k. To simulate the Gaussian shape of the rain's Doppler spectrum, it can be assumed that two orthogonal components $[u_i, v_i]$ of the horizontal velocities of the scatterers $i \in [1, N]$ are random variables with Gaussian distributions $\mathcal{N}(U_{i,mean}, \sigma_{i,U}^2)$ and $\mathcal{N}(V_{i,mean}, \sigma_{i,V}^2)$. Here $U_{i,mean}$ and $V_{i,mean}$ are the mean components of the velocity of the scatterers i inside the resolution volume, and the terms $\sigma_{i,U}$ and $\sigma_{i,V}$ are standard deviations of the velocity components of the same scatterers i. Each of these mean and standard deviation variables is also of the length N.

For the analysis in this paper, the radar parameters that are used for simulations are presented in Table I.

TABLE I: Radar Parameters

Parameter	Description	Value
$f_{central}$	Central Frequency	$10[GHz]$
$\lambda_{central}$	Central Wavelength	$3[cm]$
v_{amb}	Maximum Unambiguous velocity	$7.5[m.sec^{-1}]$
Δr	Range resolution	$50[m]$
ΔT	Pulse repetition time	$1[ms]$
$\Delta\phi$	Angular resolution	$1°$

III. The Simulation of the Doppler Spectrum of Precipitation with Respect to Azimuth

The time-domain model of the measured radar signal (4) provides a possibility to generate the timeline of scatterers from precipitation echo signals with arbitrary duration for any specific point of coordinate $[x, y]$. At the same time, the scanning in horizontal plane radar produces the timeline of echo signals in a polar coordinate system. Let's assume that the simulated radar is placed inside the completely homogeneous volume of precipitation moving with the wind. The range from the radar is fixed in this analysis. This has been done to exclude SNR as a variable parameter in the model. As the range is fixed, the focus is only on the Doppler spectrum along the azimuth. It allows us to study the Doppler spectrum as a function of the difference between the observation angle (radar beam) and the actual wind field direction. For such analysis in this study, we are using the Discrete Fourier Transform (DFT) of N_s number of sequential time-domain echo signals. The resulting Doppler spectrum has to be formed on the Doppler frequency axes around the mean *radial velocity*, which is defined by the relative orientation of the radar beam with respect to the wind direction:

$$v_r = u \cdot \cos\theta \cos\phi + v \cdot \cos\theta \sin\phi, \quad (5)$$

where $[u, v]$ are the horizontal wind velocity components along x and y axes, θ and ϕ are the elevation and azimuth angles, respectively. In this study $\theta = 0$.

After conversion from Doppler frequencies to velocities, we can write the estimated Doppler spectrum using DFT as [5]

$$S[v_i] = \frac{1}{N_s}\sum_{k=0}^{N_s-1} z(k) \exp\left[-j\frac{4\pi}{\lambda}v_i\frac{k}{N_s}\right] \quad (6)$$

where N_s is the transform length for the DFT operation on the echo signals and v_i is the velocity points. This approach is performed in sectors of azimuth, i.e, space covered by one unit of beamwidth. The term v_i represents the velocity points in the analysis.

A few examples of the behaviour of the Doppler spectrum as a function of the observation azimuth angle for various scanning speed is shown in Figure 1. These plots are simulated for the same values of the mean velocity $\mu = 5m/s$ and the standard deviation $\sigma = 1m/s$ of the horizontal wind, for the raw received signal with $SNR = 30dB$. The elevation of radar antenna is $\theta = 0°$. The wind direction is $\phi_{target} = 0°$. During the simulation the number of scatterers in each resolution volume is considered as $N = 200$.

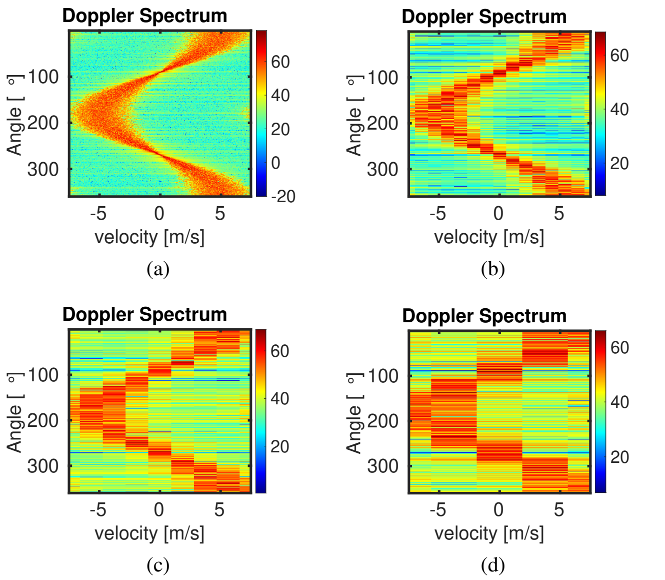

Fig. 1: An observed during azimuthal scan Doppler Spectrum for (a) $\Omega = 1\ rpm$; (b) $\Omega = 20\ rpm$; (c) $\Omega = 40\ rpm$; (d) $\Omega = 60\ rpm$

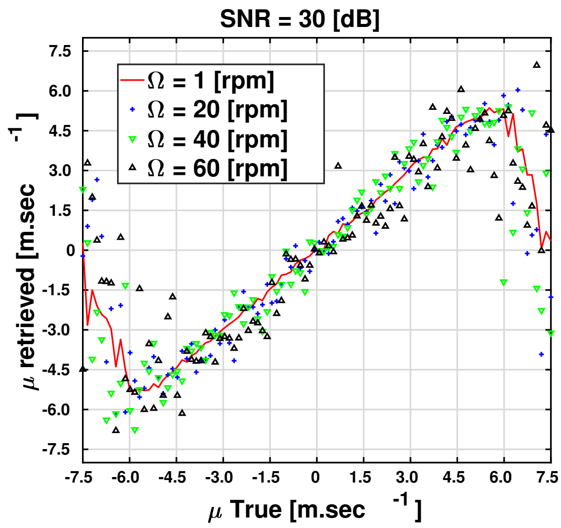

Fig. 2: The estimated values of the mean Doppler velocity versus their true values, for various radar scanning speeds, at the azimuth direction $\phi = \phi_{wind}$.

The degradation of the Doppler spectrum resolution observed in Figure 1 with the increase of the radar scanning speed is related to the decreasing number of profiles or pulses (Same as the transform length N_s) that are received from the same radar volume. This number is a function of the radar antenna azimuthal beam width and the scanning speed:

$$N_s = T_d/\Delta T \tag{7}$$

where T_d is the dwell time (the time on target) - the time that the radar spends in one beamwidth sector:

$$T_d = BW/\left(\Omega \cdot 2\pi/60\right) \tag{8}$$

where BW is the radar antenna's azimuthal beamwidth in radian, and Ω is the scanning speed of the radar in rotations per minute (rpm).

Both the mean and the standard deviation of Doppler velocity follow a cosine dependence concerning the azimuth angle in the Doppler spectrum. With higher speeds, the value of N_s decreases and it can be observed from Figure 1 that the spectrum suffers from the broadening - the resolution of the Doppler DFT becomes compatible and even wider than the real Doppler spectrum width. Suppose the ground truth of the velocity is chosen to be closer to the maximum unambiguous velocity of the radar. In that case, the Doppler spectrum folds itself, especially at higher scanning speeds ($40[rpm]$ and $60[rpm]$).

IV. Accuracy of the Doppler Moments Estimation

The Doppler moments can be found by the following equations [5], the total power or zeroth moment of Doppler (P_T),

$$P_T = \int_{-v_{amb}}^{+v_{amb}} |S(v)|^2\, dv \tag{9}$$

where dv is the Doppler velocity resolution.

The mean Doppler velocity μ can be found as,

$$\mu = \frac{1}{P_T}\int_{-v_{amb}}^{+v_{amb}} v\,|S(v)|^2\, dv \tag{10}$$

The Doppler spectrum width can be calculated by the following relation,

$$\sigma = \sqrt{\int_{-v_{amb}}^{+v_{amb}} \frac{1}{P_T}[v-\mu]^2\,|S(v)|^2\, dv} \tag{11}$$

Numerically, these integrals are implemented with summations over the Doppler velocity grid, which can be represented when N_s is odd as

$$v = [-v_{amb}, v_{amb}, N_s] \tag{12}$$

and, in case when N_s is even, as

$$v = [-v_{amb}, v_{amb} - dv, N_s] \tag{13}$$

The dependencies of the estimated values of the mean Doppler velocity and the Doppler spectrum width versus their true values are shown in Figures 2 and 3, respectively, for various rotational speeds. These calculations are performed for a radar orientation that is the same as the wind direction in azimuth ($\phi = \phi_{wind}$), for the case of $SNR = 30dB$ before the Doppler processing, and for the true value of the wind velocity fluctuations' standard deviation $\sigma_{true} = 1m/s$.

It can be observed from Figures 2 and 3 that the mean Doppler velocity retrieval suffers from folding errors towards the maximum unambiguous velocity. Folding errors increase with an increase in the scanning speed. It is also observed that as rotation speed increases, the bias in the estimate of the Doppler spectrum width increases. This is because the true Doppler spectrum width becomes smaller than the Doppler

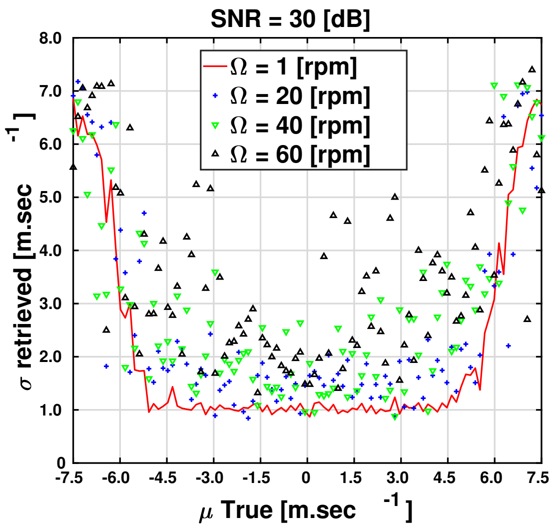

Fig. 3: The estimated values of the Doppler spectrum width versus their true values, for various radar scanning speeds, at the azimuth direction $\phi = \phi_{wind}$.

resolution. For example, at the rotation speed of $60rpm$, the Doppler resolution is equals to $\Delta v = 3.75m \cdot sec^{-1}$, which is a few times wider than the true spectrum width $\sigma_{true} = 1m \cdot sec^{-1}$ of the simulated rain. Therefore, in the case of a very small number of analysed radar profiles, the DFT-based approach for estimating the Doppler spectrum parameters cannot accurately resolve the Doppler spectrum width that is narrower than the resolution of the Doppler spectrum. As a result, there is a positive bias in the estimation of the Doppler spectrum width for very fast scanning radars.

V. Techniques to Improve Doppler Velocity Retrieval

There are two approaches studied in this section to improve Doppler velocity retrieval.

A. Improving Doppler velocity retrieval with multiple rotations with constant ground truth

In this section, retrieval of multiple rotations is used to compute the Doppler moments. There is a time gap between two consecutive rotations with multiple rotations; the radar received signals can not be coherently integrated. Therefore, the statistical analysis is done only with retrieved Doppler moments. This analysis assumes that the SNR before the processing (in the signal model) is constant. Still, the noise associated with each received signal is random and not correlated in every rotation. The analysis has been carried out with 32 rotations for scanning speeds of $1\ rpm, 20\ rpm, 40\ rpm$ and $60\ rpm$. For $1\ rpm$ scanning speed, assuming that the ground truth remains consistent for 32 rotations is not very physical. Still, for example, $60\ rpm$ scanning speed, assuming the truth to be consistent for 32 rotations, is more physical with normal precipitation events. The retrieval results are then averaged for the final estimation. The averaged estimates of the mean Doppler and Doppler spectrum width is shown in Figures 4 and 5.

It can be observed that the mean Doppler velocity estimate has been improved by incorporating retrieval of multiple rotations. However, the Doppler spectrum still suffers from a bias error with increased scanning speed. This is because the resolution for the Doppler spectrum for each rotation is not improved by this averaging.

B. Improving Doppler velocity resolution by compromising angular resolution

In the above analysis, we found that the estimation of Doppler spectrum width doesn't improve with averaging over range axis or averaging over multiple rotations at $60\ rpm$ over a beamwidth of $1°$. This is because the Doppler moments retrieval with only five samples in time gives rise to

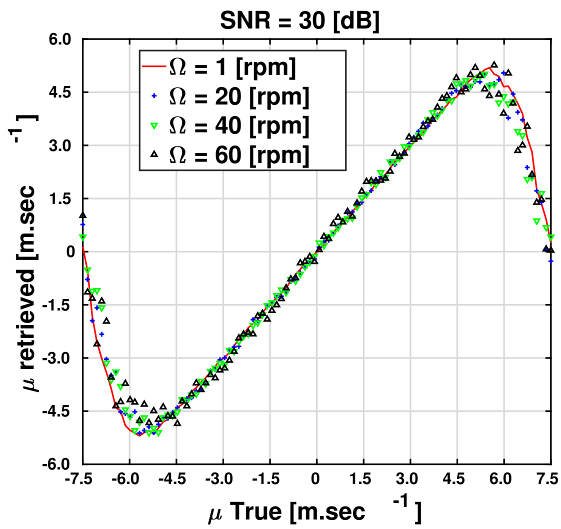

Fig. 4: Mean of mean Doppler estimate for 32 scans.

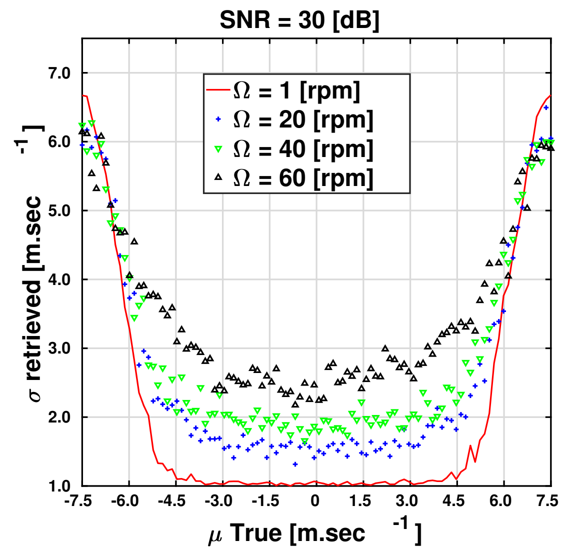

Fig. 5: Mean of Doppler spectrum width estimate for 32 scans.

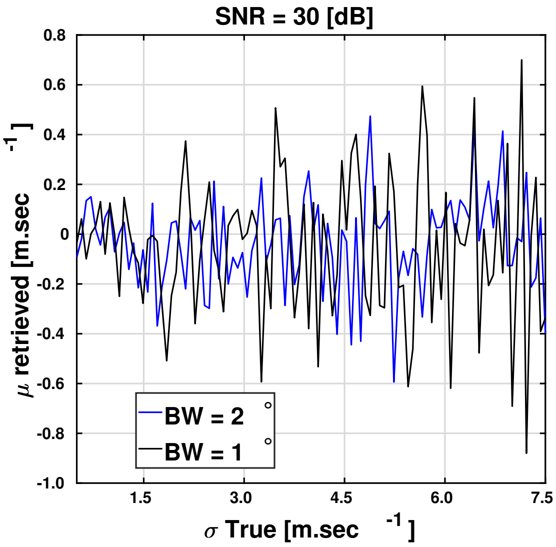

Fig. 6: Effect of integration of two beamwidth sectors with twice the pulse repetition in the estimation of mean Doppler velocity. $\mu_{True} = 0[m.sec^{-1}]$, $v_{amb,1BW} = 7.5[m.sec^{-1}]$, $v_{amb,2BW} = 3.75[m.sec^{-1}]$, $dv_{1BW} = 3.75[m.sec^{-1}]$, $dv_{1BW} = 1.875[m.sec^{-1}]$.

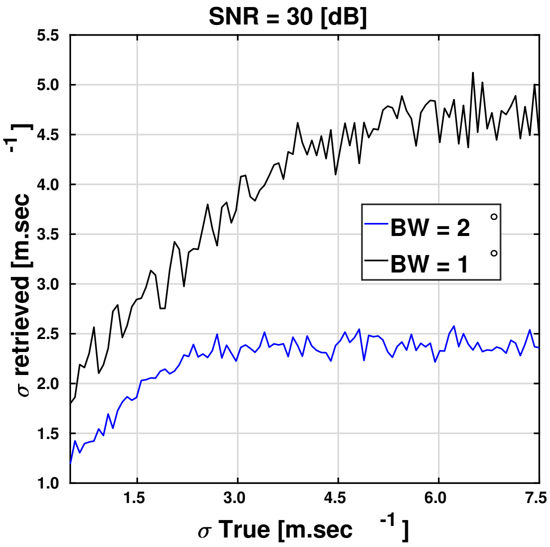

Fig. 7: Effect of integration of two beamwidth sectors with twice the pulse repetition in the estimation of Doppler spectrum width. The parameters are similar to the Figure 6

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a poor velocity resolution. This approach aims to improve the Doppler velocity resolution with the same number of pulses that are received from one angular resolution cell. For example, at a scanning speed of $60[rpm]$, the number of pulses per beamwidth is $N_s = 5$. This approach aims to improve the Doppler velocity resolution with only five pulses. One possible solution could be to consider pulses over a broader angular region instead of only one beamwidth sector but take the same amount of pulses (in this case, at $\Omega = 60\ rpm$, it is $N_s = 5$). In this way, the pulse repetition time is increased by a factor of the number of beam widths to be integrated. This, by extension, improves the Doppler velocity resolution by the same factor. It can be argued that the Doppler processing can be done with more pulses because the number of pulses that are received from n_{BW} number of beamwidths is $5n_{BW}$ at a scanning speed of 60 rpm. However, this approach explores how the Doppler resolution is improved with only five samples at a scanning speed of $60rpm$. Hence, the trade-off of considering multiple beamwidths is discussed.

$$dv = \frac{\lambda}{4(n_{BW}PRT).N_s}.$$

Here, n_{BW} is the number of beamwidths to be integrated. This technique comes with its shortcomings. By increasing the pulse repetition time, we decrease the maximum unambiguous velocity by the same factor

$$v_{amb} = \frac{\lambda}{4(n_{BW}PRT)}.$$

Some folding correction techniques can compensate for the folding in the Doppler moments retrieval. The results for the mean Doppler velocity and Doppler spectrum width are shown in Figure 6 and 7 concerning the true spectrum width at a fixed range and averaged over 32 rotations. The comparison is made at two different angular resolutions, one at the usual beamwidth $1°$ and one at two beamwidths $2°$. In the latter case, the Doppler spectrum width estimation is better up to around a true spectrum width of 2 m/s. Beyond that point, the retrieval saturates due to a lower maximum unambiguous velocity of 3.75 m/s. This is not the case in the former simulation, when only one beamwidth is considered. The saturation occurs at around a true spectrum width of 4 m/s. However, in this case, below a true spectrum width of around 2 m/s, the estimated Doppler spectrum width suffers from errors due to a poor Doppler resolution of $\Delta v = 3.75\ m/s$. It can be concluded that, below a true spectrum width of around 2 m/s, there is an improvement in the estimation of Doppler spectrum width if a bigger angular region is considered for integration. It suffers from a relatively poor angular resolution—furthermore, the farther the range, the wider the physical area.

$$dA = r \cdot dr \cdot d\phi \tag{14}$$

Therefore, an increase in angular resolution value (poorer resolution) $d\phi$ increases the physical area of analysis.

VI. Wind Speed Retrieval with a Scanning Radar

In this paper, we analyse the wind field retrievals only from the velocity azimuthal display (VAD) measurements using the technique [3] that provides an overall estimate of the wind field. The radial Doppler velocity can be written in terms of the 2D wind field as

$$\mu(\phi) = a_0 + \sum_{k=1}^{2} a_k \cos(k\phi) + b_k \sin(k\phi) \tag{15}$$

The constant term a_0 in (15) gives the magnitude of divergent wind field, the first harmonic's coefficients a_1 and b_1 characterise the average speed and direction of the homogeneous wind component. The coefficients of the second harmonic a_2 and b_2 gives an idea about the non-linear deformation of the wind field.

The analysis is done in a linear wind field, the wind field direction over the rim of observation is considered constant. In ideal conditions with an precise estimate of mean Doppler velocity, for this kind of wind field, only the coefficients of the first harmonic in equation (15) should be non-zero, and the rest of the coefficients should be zero. However, the Doppler mean velocity retrieval will have errors at higher rotation speeds and with wider Doppler spectrum width. The following Table II shows the values of the coefficients mentioned in equation (15) for different scanning speeds of the radar for a pure translational motion of wind blowing at $\phi_{wind} = 45°$ with a horizontal wind speed of $V_h = 6m/s$ ($u = |V_h|\cos(\phi_{wind}) = 4.24\ m/s$, $v = |V_h|\sin(\phi_{wind}) = 4.24\ m/s$). The spectrum width in space (azimuth) is $1m/s$.

From the Table II, it is seen that with an increase in scanning speed, the errors in a_0, a_2 and b_2 are increasing, indicating that despite ground truth, the retrieved wind field has a divergent as well as a deformation component. However, it is seen that the coefficients of the second harmonic have a maximum error up to $5.3\ cm/sec$ only and can be neglected. Therefore, even at the highest scanning speed, it can be concluded that the retrieved wind velocity has only one dominant harmonics, which relates to the pure translational motion. The constant term a_0 measures the divergence of the wind field, the measured wind speed component independent of the azimuth angle. The error in a_0 estimation also increases with the rotation speed, approaching the value of $14.6\ cm/sec$ at the rotation rate $60\ rpm$, which can be already considered for the application-based analysis.

VII. Conclusions

The paper presents the analysis of the Doppler spectrum estimation performances using the DFT algorithm in case of the observation of extended meteorological objects (precipitation) with a fast scanning in azimuth radar. It has been demonstrated that the estimated Doppler spectrum suffers from errors when the scanning speed is increased and the related time on target is decreased using simulated weather radar signals.

Two techniques to improve the Doppler processing accuracy and resolution limitations coming from the short time on target have been proposed and analysed. Both of those assume that the true velocity of the scatterers does not change during a certain observation period. Within the first method, the mean Doppler velocity estimation accuracy can be improved by averaging the estimated mean Doppler velocities for multiple sequential radar scans. Unfortunately, the Doppler spectrum width estimation does not improve at higher scanning speeds even after such scan averaging. It is related to the fact that the Doppler width of the precipitation spectrum in analysed cases is comparable or even less than the Doppler processing resolution. To overcome this limitation and improve the accuracy of the Doppler spectrum width estimation, we also propose a simple approach that improves the resolution of the Doppler processing by decreasing the Doppler velocity ambiguity for the same number of integrated signals. It can be easily done by processing only every second signal in the received sequence. However, it comes with the cost of a reduced angular resolution and unambiguous Doppler velocity for the radar.

Finally, the 2D wind field retrieved with the classical VAD technique is analysed at different scanning speeds of the radar. For this analysis, the wind field was simulated as a linear wind with a fixed direction at an azimuth of $\phi_{wind} = 45°$. The retrieval algorithm gives a possibility to estimate from the VAD angular pattern the wind's azimuthal harmonics, which relate not only to the mean wind speed and direction, but also wind field's divergence and non-linear deformations. It has been shown that the errors in these harmonics increase with an increase in the scanning speed. The degree to which the mean linear components of the wind velocity are underestimated is proportional to the radar scan speed.

The results can be used to design the processing chains for meteorological object detection and their velocity estimation and design multi-function radars with very fast scanning (very high scan update rate).

TABLE II: 2D Wind Fourier Series Coefficients estimations at different scanning speeds Ω

$\Omega\ [rpm]$	$a_0\ [m/s]$	$a_1\ [m/s]$	$b_1\ [m/s]$	$a_2\ [m/s]$	$b_2\ [m/s]$
Truth	0	4.2426	4.2426	0	0
1	−0.007	4.276	4.247	0.004	0.0003
20	0.040	3.967	3.970	−0.025	0.024
40	−0.101	3.863	3.837	−0.090	−0.052
60	−0.146	3.442	3.507	−0.053	−0.043

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