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A pilot study with webxr as an accessible tool**

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**DOI**

[10.5821/conference-9788412322262.1218](https://doi.org/10.5821/conference-9788412322262.1218)

**Publication date**

2022

**Document Version**

Final published version

**Published in**

Proceedings of the 50th Annual Conference Universitat Politècnica de Catalunya · BarcelonaTech (UPC)

**Citation (APA)**

Wong, L. Y. J., Bayoumy, S., Freeke, A., & Cabo, A. J. (2022). Augmented reality for learning mathematics: A pilot study with webxr as an accessible tool. In H.-M. Järvinen, S. Silvestre, A. Llorens, & B. Nagy (Eds.), *Proceedings of the 50th Annual Conference Universitat Politècnica de Catalunya · BarcelonaTech (UPC)* (pp. 1805-1813) <https://doi.org/10.5821/conference-9788412322262.1218>

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ISBN: 978-84-123222-6-2

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Printed by: Artes Gráficas Torres S.L., Huelva 9, 08940 Cornellà de Llobregat, Spain

*The manuscript was closed on 30 November 2022.*



## **AUGMENTED REALITY FOR LEARNING MATHEMATICS: A PILOT STUDY WITH WEBXR AS AN ACCESSIBLE TOOL**

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**Conference Key Areas:** *Mathematics at the heart of Engineering.*

**Keywords:** *service mathematics, augmented reality, student engagement, motivation, technology-enhanced learning*

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## ABSTRACT

One of the concerns in service mathematics courses, such as calculus for engineering, is students' interest in these studies. Research suggests that engineering undergraduates' lack of awareness about the importance of mathematics for their study success and for their careers contributes to their low motivation for mathematics. An approach to increasing student motivation is to take advantage of technological tools to provide students with more engaging learning experiences. Recent studies showed that augmented reality (AR) enhances student engagement, motivation, and knowledge retention. However, implementing AR can be challenging since it can be quite costly and technically complex. The current paper describes a case study in which an AR application was designed and developed using WebXR, in the context of a service mathematics course for teaching calculus. The AR content involves drawing of level curves and the visualization of a volcano and the flow of lava to support students' learning of directional derivatives. A pilot study was conducted to examine engineering undergraduates' perceptions of using AR for learning mathematics. Results show that students perceived using AR for learning math as enjoyable and motivating. Students reported that AR content adds value to their classes by making the mathematical concepts clearer and helping them apply what they have learned to real life. However, the AR content did not work well on all mobile phones and all versions of web browsers. Lessons learned from the design and development of AR using WebXR as well as recommendations for future studies are discussed in this paper.

## 1 INTRODUCTION

### 1.1 Service Mathematics in Higher Engineering Education

Mathematics is often service-taught by mathematicians to students who are not majoring in mathematics as a foundation for science, technology, and engineering in higher education. Research shows that engineering students' motivation to study mathematics is low and many perform poorly in service mathematics courses, despite the importance of mathematics for academic success and professional practice [1]. One of the reasons is that many engineering students do not perceive mathematics as their primary interest [2]. This leads to procrastination of tasks for learning. Another reason is that students are not aware of the importance of mathematics as the foundation of the subjects in their field of study, nor are they familiar with how mathematics is applied in their subject of interest [3].

At the Delft University of Technology, the Program of Innovation in Mathematics Education (PRIME, <https://www.tudelft.nl/en/eemcs/the-faculty/departments/applied-mathematics/education/prime>) was introduced in 2014 to redesign service mathematics courses. The goals of PRIME are to improve study results, enhance the connection between mathematics and





engineering, and increase students' active participation and motivation. Over the years, a number of innovative practices, such as pre-lecture videos, interactive quizzes and online exercises, have been introduced. To better meet the needs of engineering students, PRIME continues to examine ways in which various technologies (e.g., augmented reality) can be leveraged to enhance the teaching and learning of mathematics in higher engineering education.

### **1.2 Using AR to Support Learning of Mathematics**

The technology of Augmented reality (AR) gives users the opportunity to see digital information superimposed onto the real world, and hence, offers a novel experience and interactivity with the learning content. Research on AR in education has been rising over the last 25 years and one of the trending topics is the use of mobile devices for deploying AR learning experiences [4]. In a recent systematic review on the use of AR for learning mathematics [5], AR was shown to be an effective tool for learning mathematics by increasing confidence and understanding, enhancing visualization, and making learning interactive across all educational levels. To date, research examining the effect of AR on mathematics learning in the context of higher education is relatively scarce.

In [6], a combination of 3D tools (i.e., augmented reality, virtual environments, and 3D printing) was used in the process of mathematics teaching and learning to enhance spatial visualization and orientation skills. The study showed that students in the experimental group who took the multivariable calculus course under the new methodology scored seven points higher than those in the control group. There was a 14% drop in the experimental group's failure rate. Furthermore, the new methodology helps students to improve the way they described objects in space using mathematical language. In another study [7], AR content was integrated into the Mathematical Analysis class for electrotechnical engineering. Students in the study perceived the AR tool as a way to better understand and access knowledge. Moreover, they were curious and interested in the prospects of the software.

More recently, a "Vector AR3" app was developed as a means to shift from a theoretical approach to teaching math to an approach that focuses more on the application of the learning content [8]. While the app has won the International eLearning award 2020, the impact of the app on teaching and learning has yet to be evaluated due to Covid-19. The studies described in this section suggest the potential of AR to enhance mathematics teaching and learning. In order to reap the potential benefits of AR for mathematics education at universities, it is important to explore ways to ease the use of AR for the growing student population.

### **1.3 Current Study: Using WebXR to**

Pitfalls of AR include accessibility and scalability. To address these pitfalls, the current study employs WebXR for the AR tool. Unlike mobile



applications that require different codes for android and ios, WebXR allows the AR tool to be embedded into the learning management systems and can be deployed as a web application and accessed through a mobile browser [9]. Moreover, students will not be required to download an application through an app store via Google or Apple, bypassing the procedure of downloading and logging in. To our knowledge, very few studies have examined WebXR as an AR tool for teaching and learning math. The aim of the current study is to examine the feasibility of using WebXR to develop an AR tool for learning mathematics. The study consists of two parts: 1) designing and developing the AR tool, and 2) examining the usability of the AR tool.

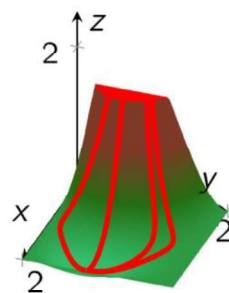
## 2 DESIGN AND DEVELOPMENT

### 2.1 Designing the AR tool

When designing the AR tool, meetings were conducted between the developer, math lecturer, course coordinator, and educational researcher. During the meetings, the team members brainstormed ideas for the math topics and ways in which the AR tool can be implemented. The math lecturer proposed a math application question that was already covered in the existing math lecture as a task for the AR tool. Figure 1 illustrates the math application question. Two design features were proposed to deliver the math application in the AR tool:

- 1) Drawing level curves that will be transformed into a 3D volcano.
- 2) Visualizing lava flow as a connection to directional derivative.

#### Application: Predicting lava flows



Suppose we have a volcano and lava flows out at the top.

Where will the lava flow?

At any point the lava will follow the direction of steepest descent.

The directional derivative should be the opposite of the gradient vector in that point.



Figure 1. Mathematics application taught in the course

### 2.2 Developing the AR tool

Figure 2 illustrates the technical infrastructure of the AR tool. A web application that has been developed through a 3D JavaScript library, ThreeJs, was employed. This library enables developers to create 3D web applications using WebGL API, which supports WebXR to create AR content. The AR tool features an empty drawing screen where students can draw the outline and level curves of the 3D volcano by using touch input. The first shape that the student draws is an outline of how the 3D volcano should be shaped in general. This outline

shape is used as a ground level to build the volcano geometry. To recreate an organic volcano formation from an outline, multiple points are scattered randomly inside this outline and Delaunay triangulation [10] is performed to create a triangulated mesh. From this point on, students can draw multiple level curves on top of the triangulated mesh which is used to calculate how the mesh should deform in height. The final step of generating a 3D volcano is by starting the AR view, where the tool will calculate the final shape of the 3D volcano by combining all drawn level curves and taking account of a set of parameters; Height, noise, crater size and crater depth. With AR view started, the 3D volcano is displayed in AR and following the camera movement. The volcano can be placed on any surface by pressing on the screen. With the volcano in place, the student can touch any volcano surface to start a lava flow trail. Gradient descent is used to calculate the path of the lava and is then projected on the volcano. With these features in place, the application is packaged and hosted online to be accessed through a browser.

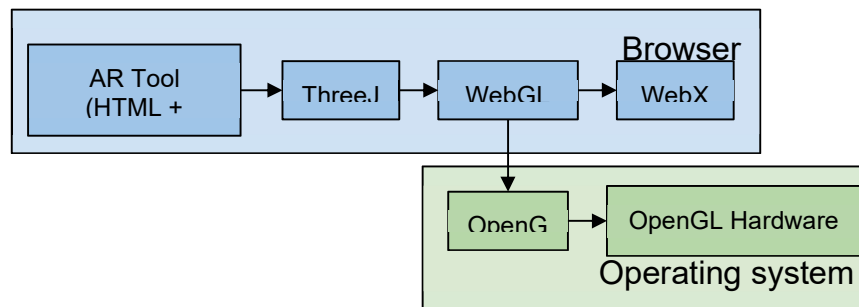


Figure 2. The technical infrastructure of the AR tool

### 3 PILOT STUDY

#### 3.1 Method

The aim of the pilot study was to examine how students perceive the utility of the AR tool for learning math. Students taking Calculus I in Applied Earth Sciences were invited to take part in the pilot study. Nine students (four males and five females) participated in the pilot study. The average age of the students was 18.6 years old. Students were directed to a web link (<https://sbayoumy.github.io/threejs-test/>) that enabled them to use the AR tool on their mobile phones. In addition, students were provided with a set of questions to aid their exploration of the AR tool. Figure 3 illustrates an example of a volcano drawn and visualized using the AR tool.

After exploring the AR tool, students were asked to complete a survey consisting of 14 Likert-scale questions and three open-ended questions. The Likert-scale questions included 5 items measuring *student satisfaction* with the AR tool, 3 items measuring *engagement in learning* concerning whether the AR tool promotes active learning, 3 items measuring *enhancement of learning* addressing the question of whether the AR tool helps to enhance their understanding, and 3 items measuring the *extension of learning* concerning

whether the AR tool helps to extend the learning of math to other contexts. The survey items were adapted from [11], [12], and [13]. Students were asked to respond to each item on a 5-point Likert scale from 1 (strong disagree) to 5 (strongly agree). The open-ended questions were used to gather students' perspectives on what they liked or disliked about the AR tool and how the AR tool supported their learning.

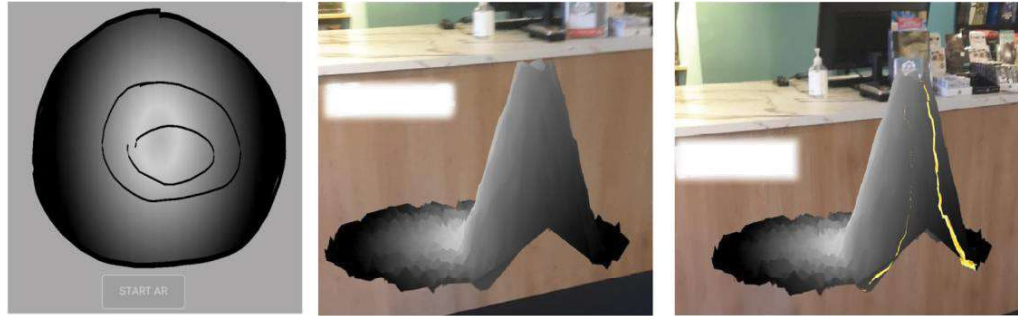


Figure 3. An example of a volcano drawn and visualized using the AR tool

### 3.2 Results

Table 1 provides an overview of students' satisfaction ratings of the AR tool. The results show that students generally agree that using the AR tool was enjoyable and entertaining. As for user-friendliness, students' ratings varied. This reflected the instability of the AR tool and the challenges of making the AR tool work on all mobile devices. Diverse ratings were obtained on the two items measuring satisfaction with the AR tool as a study tool and overall satisfaction with the AR tool.

Table 1. Students' satisfaction rating of the AR tool

	Strongly disagree	disagree	neutral	agree	Strongly agree
I was satisfied with the AR application as a study tool.	1	3	4	1	-
I found the AR application entertaining for me.	-	-	3	4	3
I enjoyed using the AR application.	-	-	1	7	1
I found the AR application user-friendly.	1	1	1	3	2
Overall, I am satisfied with the AR application.	-	3	1	3	1

Mean scores and standard deviations were obtained for engagement ( $M=3.79$ ;  $SD=.67$ ), enhancement ( $M=3.79$ ;  $SD=.56$ ), and extension of learning ( $M=3.93$ ;  $SD=.49$ ). The mean scores suggest that students were generally positive about how the AR tool can support their learning. For engagement in



learning, almost all of the students (8 out of 9) agree or strongly agree that the AR tool motivates them to start learning and causes a passive to active behavioural shift. There was less agreement regarding whether the AR tool allows them to focus on the task with less distraction.

For enhancement of learning, students felt rather neutral about whether the AR tool allows them to demonstrate their understanding of learning better than other tools. Students mostly agree that the AR tool allows them to develop a better understanding of the learning content and makes it easier to understand concepts. For extension of learning, three-quarters of the students (6 out of 8) agree or strongly agree that the AR tool creates opportunities for them to transfer learning to other contexts, creates a bridge between mathematics and engineering, and allows them to develop skills that they can use for their program of study.

When asked about what they liked most about using the AR tool and to what extent the AR tool contributes to their learning, students mentioned that the AR tool makes learning fun and helps them to apply what they have learned to a real-life context. They also found it helpful to be able to have a visualization of the mathematical functions learned in class. A student stated that being able to see how the math is being applied makes learning motivating. In addition, a student reasoned that if the AR tool can be implemented in a good way in the math exercise classes, it might give students an incentive to attend the classes. Some of the quotes from the students are shared below.

*“It is very interactive. You can make your own volcano and then get your own equation.”*

*“It’s fun to use and I understand the concept of level curves better with it, it helps visualize the whole thing.”*

Through the survey, we also received feedback on what students disliked about the AR tool. The AR tool did not seem to work very well and did not run on different models of mobile phones. Students also experienced difficulties with the drawing tool. A student mentioned that the drawing tool was slow even when the wifi is good and another student was not able to get the drawing tool to work. The problems that students encountered when using the current AR tool could have influenced their satisfaction and engagement in learning. This can be reflected in the quote below.

*“It doesn’t work very well yet, which makes it frustrating to use. This would make me distracted from my learning.”*

#### **4 DISCUSSIONS AND CONCLUSION**

The results of the pilot study show that the AR tool has the potential to enhance learning of math for applications and facilitate student engagement. Students were positive about using the AR tool to learn math. However, challenges remain with the technical deployment and design of the AR tool. As identified in



a scoping review of augmented reality in K-12 and higher education [13], technical thresholds and design considerations are major limitations in studies examining the effect of AR on learning. Technical malfunctions and problems may lead to student frustrations when using the app and reduce learning effectiveness.

Developing the AR tool with WebXR is a compromise for the lack of support of the WebXR API in a set of browser versions. However, the current state of the WebXR API and its features are yet to be fully supported on existing browsers or standardized in the WebKit platform (see <https://caniuse.com/webx> for a list of supported browsers). Therefore, developing cross-platform web applications involve risks around having to implement specific solutions for different browsers and devices. It is crucial for future iterations of the AR tool to take the technical thresholds into account and explore ways to mitigate these risks.

One of the strengths of the current study is the participation of the math instructors in the design of the AR learning content. Therefore, the AR tool is complementary to the learning content in the ongoing mathematics course. As demonstrated by the results of the pilot study, students perceived the learning content as helpful, particularly the visualization of the math concepts that were taught in class. Estapa and Nadolny [15] reasoned that the benefits to manipulate, visualize and use authentic contexts make AR a natural fit for mathematics instruction. Despite the small sample size in the pilot study, the results suggest that implementing AR in the math course could potentially have a positive impact on student motivation. Given that learning performance was not measured in the pilot study, it is not clear if and how the AR tool increases math knowledge. To better understand the effects of AR on learning, there is a need to use multiple metrics (both quantitative and qualitative on motivation and learning effectiveness) [16]. Also, future research with a bigger sample size and using randomized controlled trials is needed to further investigate the effect of scalable AR tools on learning mathematics, student engagement, and motivation.

In conclusion, our study demonstrates the benefits of involving teachers and students in the design and development of an AR tool using WebXR for teaching and learning math. The math instructor plays a critical role in ensuring that the content of the AR tool is meaningful for the students. Similarly, students' perceptions and feedback that we have gathered in the pilot study will inform the future iterations of the current AR tool.

## 5 ACKNOWLEDGMENTS

The authors would like to thank Dennis den Ouden-van der Horst and Beryl van Gelderen for the mathematics input in the design of the AR solution and implementation of the study, Winnie Dao for data collection, Job Vlak for initiating the educational innovation, and the students who participated in the pilot study.

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