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USING BREMMER SERIES FOR MODELLING ELASTIC REflECTION RESPONSES IN 1.5D MEDIA

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Summary

We develop a seismic modelling scheme that construct wavefields in elastic 1.5D media by using the Zoeppritz equations and exploiting the mechanics of the Bremmer series. The modified modelling algorithm allows to construct the wavefield in an iterative manner and provides access to upgoing and downgoing wavefield components at all depth levels, which provides additional flexibility and makes the method suitable for various applications such as well-log analysis and modelling VSP responses. We conduct numerical experiments and benchmark results of the proposed modelling scheme with the results obtained by Kennett modelling. Similarity of the results allows to validate the modified modelling scheme.

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Introduction

Accurate elastic seismic modelling plays a crucial role in many steps in seismic processing and imaging workflows. For heterogeneous media, we usually resort to FD modelling methods (Virieux 1984) but e.g. when modelling data from a well-log, cheaper 1.5D methods are preferred. For accurate modelling elastic responses that includes the full Zoeppritz equations, Kennett (1979) proposed to simulate elastic reflection response by layer building approach and computing recursively reflection response via propagating it from bottom of the model to the surface (Aki and Richards 2002). On the other side, most of the imaging and inversion algorithms simulate the wavefield using the original source locations, which maybe not always at the surface. Such approach can be more applicable for imaging/inversion by handling acquisition geometries and possibility to implement adjoint modelling required for computing model updates. Therefore, we are motivated to utilize a modeling algorithm that also exploits Zoeppritz reflection and transmission operators, but that is based on different wavefield propagation mechanics that allows to model separately upgoing and downgoing wavefields and at different depth locations. We use modelling based on the Bremmer series (Bremmer 1951), in which every term represents additional order of scattering. Such modification allows us to compute up/downgoing P and S waves at every subsurface depth level, but also handle various acquisition geometries, model ghost effects, include mode conversions and iteratively accumulate orders of scattering, etc. Although using the Bremmer series for modelling seismic responses is not new (Corones 1975; Wapenaar 1996; Hoop 1996), based on the work of (Berkhout 2014) on Full Wavefield modelling, we feel that this method has not been embedded to a large extend in our community and we would re-initiate the value of such approach in this paper. The structure of this paper is as follows. In the theory section we will first describe the Kennett approach and then discuss differences associated with Bremmer series. In the numerical example section the wavefield computer by Bremmer Series and Kennett modelling are compared.

Theory

As we will compare the modelling algorithm with the elastic Kennett modelling scheme, a brief description for the latter methodology is discussed first. Using Zoeppritz equations, elastic reflection $\mathbf{R}_n = [R_{pp}, R_{ps}; R_{sp}, R_{ss}]$ and transmission $\mathbf{T}_n = [T_{pp}, T_{ps}; T_{sp}, T_{ss}]$ operators are defined in the frequency-wavenumber domain at every depth level n of the subsurface model. Using propagation, or phase-shift, operators $\mathbf{W} = [e^{-ik_z(V_p)\Delta z}, e^{-ik_z(V_s)\Delta z}; e^{-ik_z(V_p)\Delta z}, e^{-ik_z(V_s)\Delta z}]$ the reflection response is recursively built up from the deepest depth level to the surface:

$$\hat{\mathbf{R}}_n^{\cup} = \mathbf{W}_{n-1}\mathbf{R}_n^{\cup}\mathbf{W}_{n-1} \tag{1}$$

$$\mathbf{R}_{n-1}^{\cup} = \mathbf{R}_n^{\cup} + \mathbf{T}_n^{-}[\mathbf{I} + \mathbf{R}_n^{\cap}\hat{\mathbf{R}}_n^{\cup}]^{-1}\mathbf{T}_n^{+}. \tag{2}$$

The Bremmer modelling can be described as iterative modelling of sequential recomputation of downgoing and upgoing wavefields. In the elastic case we have P and pseudo-S downgoing wavefields:

$$\mathbf{D}_n = \sum_{m<n} \mathbf{W}_m \delta\mathbf{S}_m^{+}, \quad \mathbf{U}_n = \sum_{m>n} \mathbf{W}_m \delta\mathbf{S}_m^{-}, \tag{3}$$

where $\mathbf{D} = [\mathbf{P}^+; \mathbf{S}^+]$ is a matrix that includes downgoing P- and S-wavefields and $\mathbf{U} = [\mathbf{P}^-; \mathbf{S}^-]$ contains upgoing version of these wavefields. The scattering term $\delta\mathbf{S}^{\pm} = [\delta\mathbf{S}_p^{\pm}; \delta\mathbf{S}_s^{\pm}]$ includes reflection and transmission operators at the specific depth level. In more detail, these operators are defined as:

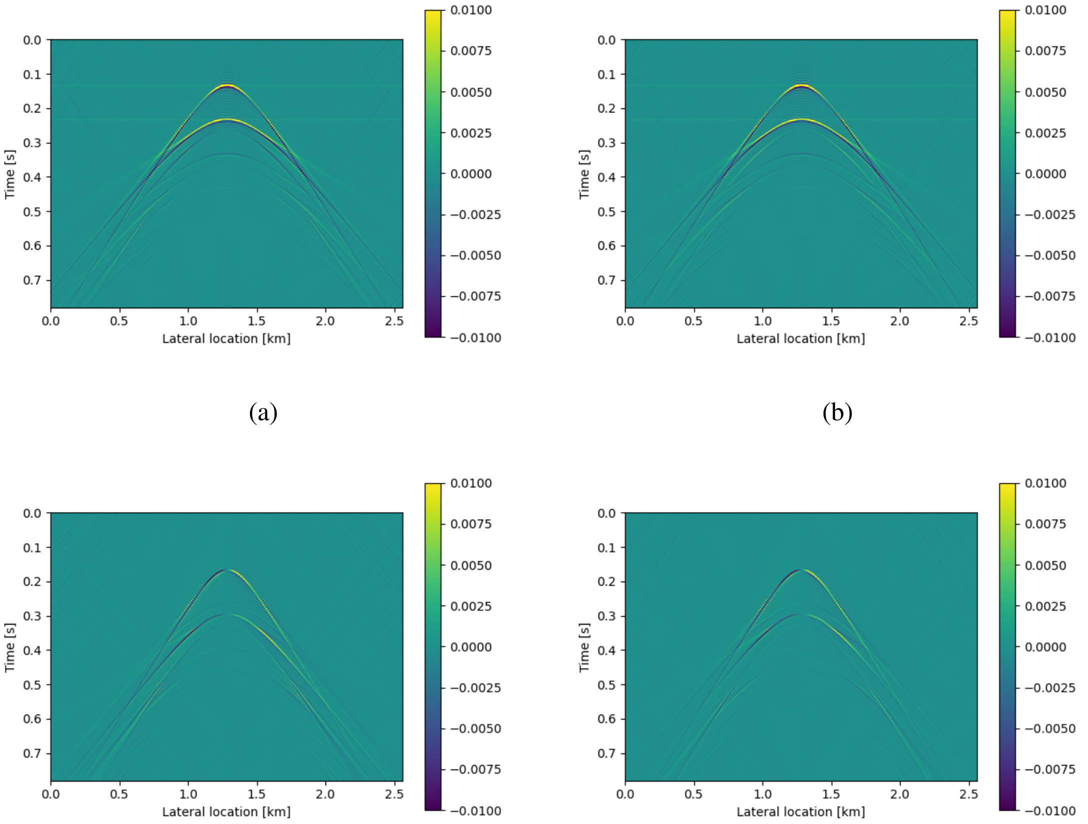
$$\begin{aligned}
\delta\mathbf{S}_p^{+} &= \mathbf{T}_{pp}^{+}\mathbf{P}_{i-1}^{+} + \mathbf{R}_{pp}^{\cap}\mathbf{P}_{i-1}^{-} + \mathbf{T}_{sp}^{+}\mathbf{S}_{i-1}^{+} + \mathbf{R}_{sp}^{\cap}\mathbf{S}_{i-1}^{-},\\
\delta\mathbf{S}_s^{+} &= \mathbf{T}_{ss}^{+}\mathbf{S}_{i-1}^{+} + \mathbf{R}_{ss}^{\cap}\mathbf{S}_{i-1}^{-} + \mathbf{T}_{ps}^{+}\mathbf{P}_{i-1}^{+} + \mathbf{R}_{ps}^{\cap}\mathbf{P}_{i-1}^{-},\\
\delta\mathbf{S}_p^{-} &= \mathbf{T}_{pp}^{-}\mathbf{P}_{i-1}^{-} + \mathbf{R}_{pp}^{\cup}\mathbf{P}_{i-1}^{+} + \mathbf{T}_{sp}^{-}\mathbf{S}_{i-1}^{-} + \mathbf{R}_{sp}^{\cup}\mathbf{S}_{i-1}^{+},\\
\delta\mathbf{S}_s^{-} &= \mathbf{T}_{ss}^{-}\mathbf{S}_{i-1}^{-} + \mathbf{R}_{ss}^{\cup}\mathbf{S}_{i-1}^{+} + \mathbf{T}_{ps}^{-}\mathbf{P}_{i-1}^{-} + \mathbf{R}_{ps}^{\cup}\mathbf{P}_{i-1}^{+}.
\end{aligned} \tag{4}$$

This modelling methodology in a multidimensional sense is also known as full wavefield modelling and was proposed by Berkhout (2014) by using reflection and transmission operators, estimated by inversion. It was proposed to include converted waves using Shuey approximation by Hoogerbrugge and Verschuur (2020). In this work, we attempt to use reflection and transmission operators using Zoeppritz equations without approximation, but operating in the 1.5D subsurface.

Examples

We compare modelling examples from two methods on the following 1.5D model. There are 3 interfaces at $z = [0, 100, 200]m$ with layers defined by $V_p = [1500, 2000, 4000]m/s$, $V_s = [1000, 1250, 2000]m/s$ and $\rho = [1000, 2000, 3000]kg/m^3$. The vertical and horizontal sampling is 5m, time sampling is 0.004s.

Figure 1 demonstrates comparison of wavefields computed by different approaches. Figure 1a shows upgoing P wavefield computed by Bremmer series, whereas Figure 1b demonstrates the PP components computed by Kennett modelling. Similarly, Figure 1c shows upgoing S wavefield using Bremmer series and Figure 1d shows the PS wavefield. It is visible that wavefields are very similar which assures accuracy of the proposed modelling approach.



(a) (b)

(c) (d)

Figure 1: Modelling example of elastic response using. (a) Bremmer series – P component, (b) PP components from Kennett modelling. (c) Bremmer series – S component and (d) PS component from Kennett modelling.

Further we demonstrate the ability of the Bremmer series modelling to compute separately wavefields propagating in the upgoing and downgoing directions inside the medium, which is not possible using the Kennett methodology. Figure 2a shows a VSP-type wavefield of the downgoing P-wavefield computed at the first iteration, where receivers are in the vertical 'borehole'. Note that this wavefield includes only the direct wave. Figure 2b shows such downgoing P- wavefield, but computed after 5 modelling iterations, where we now also observe downgoing internal multiples. In similar manner we demonstrate single-scattered upgoing P wavefields in Figure 2c and its multi-scattered version in Figure 2d. Figure 2e shows the downgoing S wavefield at the first modelling iteration and Figure 2f displays the multi-scattered version. Finally, Figures 2g,h show single-scatted and multi-scattered upgoing S wavefield, respectively. Note that because the virtual 'borehole' is located 50m away from source location, we do observe wave conversions.

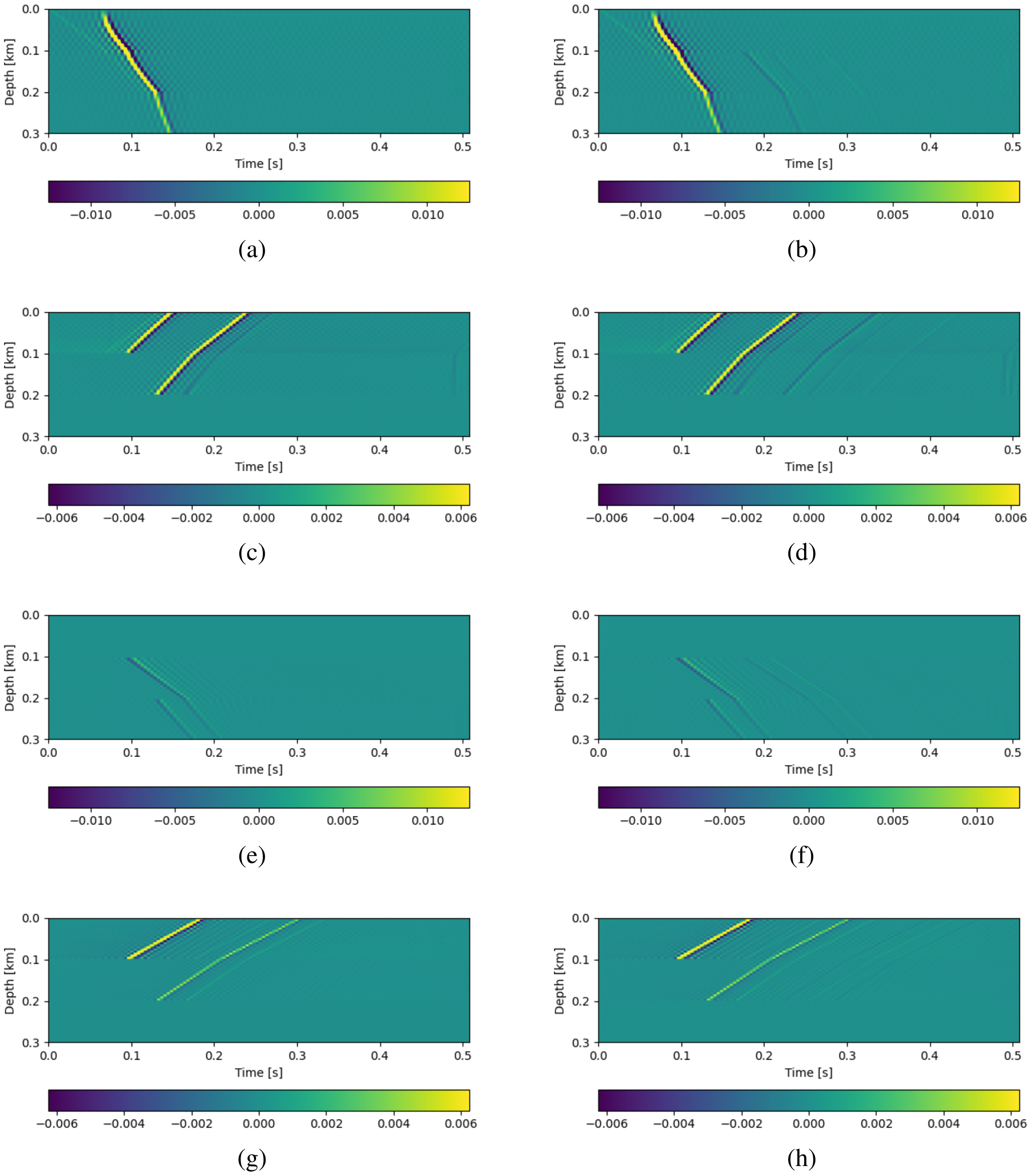


Figure 2: Modelling example of elastic response using the Bremmer series. (a) Downgoing P component under single-scattering assumption, (b) downgoing P component under multi-scattering assumption, (c) upgoing P component under single-scattering assumption, (d) upgoing P component under multi-scattering assumption, (e) downgoing S component under single-scattering assumption, (f) downgoing S component under multi-scattering assumption, (g) upgoing S component under single-scattering assumption, (h) upgoing S component under multi-scattering assumption.

Conclusions

We have implemented elastic 1.5D modelling scheme based on the Bremmer series using the Zoeppritz equations. Numerical tests validate accuracy of the proposed modelling scheme – obtained wavefields are very similar to the reflection response obtained by Kennett modelling. The proposed algorithm has several flexibility advantages over Kennett modelling, as it provides access to upgoing/downgoing components of the wavefields at any depth level and can handle various acquisition geometries, which also makes the method more appealing for applications such as well-log analysis.

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References

Aki, Keiiti, and Paul G Richards. 2002. *Quantitative Seismology*.

Berkhout, A. J. 2014. "Review Paper: An Outlook on the Future of Seismic Imaging, Part I: Forward and Reverse Modelling." *Geophysical Prospecting* 62 (5): 911–30.

Bremmer, H. 1951. "The WKB Approximation as the First Term of a Geometric-Optical Series." *Communications on Pure and Applied Mathematics* 4 (1): 105–15.

Corones, J. 1975. "Bremmer Series That Correct Parabolic Approximations." *Journal of Mathematical Analysis and Applications* 50 (2): 361–72.

Hoogerbrugge, L, and E Verschuur. 2020. "Including Converted Waves in Full Wavefield Migration." In *82nd Eage Annual Conference & Exhibition*, 2020:1–5. 1. European Association of Geoscientists & Engineers.

Hoop, Maarten V de. 1996. "Generalization of the Bremmer Coupling Series." *Journal of Mathematical Physics* 37 (7): 3246–82.

Kennett, BLN. 1979. "Theoretical Reflection Seismograms for Elastic Media." *Geophysical Prospecting* 27 (2): 301–21.

Virieux, Jean. 1984. "SH-Wave Propagation in Heterogeneous Media: Velocity-Stress Finite-Difference Method." *Geophysics* 49 (11): 1933–42.

Wapenaar, CPA. 1996. "One-Way Representations of Seismic Data." *Geophysical Journal International* 127 (1): 178–88.