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## A Volumetric Method of Moments for Integrated Lens Antennas

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*Abstract*—A Volumetric Method of Moments accelerated by means of iterative solvers combined with FFT matrix-vector products has been presented. The method allows analyzing small lens antennas with their integrated feeds efficiently since different structures and geometries can be studied with the same discretization, just by adding the material characteristics in postprocessing.

### I. INTRODUCTION

NTEGRATED lens antennas have gained popularity in the last decade in the field of wireless communications [1], [2],

astronomical instrumentation [3], and security screening [4]. Small-size lenses, i.e., having a diameter  $\approx 1\lambda_0 - 2\lambda_0$ , have started being used as elements for coherent arrays [1], or as the core lens in core-shell lens structures [2]. However, techniques such as the PO/FO [5] are inaccurate when the lens diameter is not electrically large therefore computationally demanding fullwave simulations are the only strategy to simulate such structures. This contribution aims to develop a full-wave Volumetric Method of Moments (V-MoM) suitable for the analysis of lens antennas and their feeds.

### II. MATHEMATICAL FORMULATION

By resorting to the volume equivalence theorem [6], it is possible to define equivalent currents  $\vec{J}_{eq}$  as  $\vec{J}_{eq} \equiv j\omega\varepsilon_0(\varepsilon_r - 1)\vec{E}$  (with  $\vec{E}$  being the total electric field), which radiate in free space and replace the scatterer material. The integral volume equation can be written as

$$\vec{E}^{i}(\vec{r}) = \frac{\vec{J}_{eq}(\vec{r})}{j\omega\varepsilon_{0}(\varepsilon_{r}-1)} - \iiint_{\mathcal{V}} G^{fs}(\vec{r},\vec{r}') \cdot \vec{J}_{eq}(\vec{r}')d\vec{r}', \quad (1)$$

where  $\vec{E}^i$  and  $G^{fs}$  are the incident field and the free-space Green's function, respectively. By discretizing the problem with a hexahedral grid of edge  $\Delta$ , it is possible to expand the currents as

$$\vec{J}_{\rm eq}(\vec{r}) \approx \sum_{n=1}^{N} i_n \vec{b}_n(\vec{r}), \qquad (2)$$

Where  $\vec{b}_n(\vec{r})$  are the basis functions defined as

$$\vec{b}_n(\vec{r}) = \frac{1}{\Delta^2} \operatorname{rect}\left(\frac{\vec{r} - \vec{r}_n}{\Delta}\right) \hat{p}_n, \text{ with } \hat{p}_n \in \{\hat{x}, \hat{y}, \hat{z}\}.$$
 (3)

By using the Galerkin's method on eq. (1), one obtains

$$\langle \vec{E}^{i}, \vec{b}_{m} \rangle = \sum_{n=1}^{N} \left[ \frac{\langle \vec{b}_{n}, \vec{b}_{m} \rangle}{j \omega \varepsilon_{0} (\varepsilon_{r} - 1)} - \langle \iiint_{\mathcal{V}} G^{\text{fs}} \cdot \vec{b}_{n} d\vec{r}', \vec{b}_{m} \rangle \right] i_{n},$$

$$(4)$$

where the notation  $\langle \dots, \dots \rangle$  stands for the inner product operator.



Fig. 1. Depiction of the Volume Equivalence Theorem, with the original problem on the left and the equivalent problem on the right.

By defining the entries of the matrix impedance  $Z^{mat}$  and  $Z^{rad}$  as

$$[\mathbf{Z}^{\text{mat}}]_{m,n} = \frac{\langle \vec{b}_n, \vec{b}_m \rangle}{j\omega\varepsilon_0(\varepsilon_r - 1)},$$
$$\mathbf{Z}^{\text{rad}}]_{m,n} = -\langle \iiint_{\mathcal{V}} G^{\text{fs}} \cdot \vec{b}_n d\vec{r}', \vec{b}_m \rangle,$$

And the forcing term  $\boldsymbol{v}$  as

$$[\mathbf{v}]_m = \langle \vec{E}^i, b_m \rangle,$$
  
the resulting linear system is written as  
$$\mathbf{v} = (\mathbf{Z}^{\text{mat}} + \mathbf{Z}^{\text{rad}}) \cdot \mathbf{i}$$
(5)

 $v = (Z^{mat} + Z^{rad}) \cdot i.$  (5) While  $Z^{mat}$  is calculated analytically, thanks to the orthogonality of the basis functions, and considers the presence of the specific material and geometry, the evaluation of  $Z^{rad}$  is computationally intensive but it happens in free space. The power of the volumetric formulation stands in the fact that different geometries can be studied with the same discretization for which  $Z^{rad}$  is precomputed, and the specific material characteristics are added in post-processing with  $Z^{mat}$ .

In the presented numerical solution, the linear system solution is obtained by means of the Conjugate Gradient (CG) method, for which  $n_s$  steps are required to reach the convergence. Acceleration techniques such as the ones proposed in [7] are applied for the solution. Thanks to the uniform sampling used for the discretization and to the space invariance of the free-space Green's function,  $\mathbf{Z}^{rad}$  is a Block Toeplitz-Toeplitz Block (BTTB) matrix. Due to the symmetries associated with this feature, the memory requirements for  $Z^{rad}$ are scaled down from  $N^2$  (whole matrix storage) to 2N - 1, allowing to handle large-scale problems. Moreover, the convolutional properties of BTTB matrices allow the calculation of fast matrix-vector products with the aid of the Fast Fourier Transform (FFT) in the CG solution. Combining the CG with the FFT improves the computational complexity from  $\mathcal{O}(N^3)$  of a direct solver to  $n_s \mathcal{O}(N \log_2 N)$ .

## **III. RESULTS DISCUSSION**

The method has been validated by studying the field scattered by a dielectric sphere having a relative permittivity  $\varepsilon_r = 2.34$ and diameter  $D = \lambda_0$ , when illuminated by a plane wave

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Fig. 2. Electric field inside a dielectric sphere of diameter  $D = \lambda_0$  and relative permittivity of  $\varepsilon_r = 2.34$ , when a plane wave is impinging from broadside.

impinging from broadside. Basis functions having side  $\Delta = \lambda_0 / 30$  are used in the V-MoM discretization. The results are shown in Fig. 2, showing a good agreement between the V-MoM and a commercial full-wave solver [8].

The application scenario is shown in Fig. 3(a), where a semihemispherical lens with diameter  $D = 2\lambda_0$  made of dielectric with permittivity  $\varepsilon_r = 2.34$  is fed by a half-wavelength dipole having a square cross-section of  $\lambda_0/20$  and feeding gap  $\lambda_0/10$ . The basis functions used to discretize the problem have edge  $\Delta = \lambda_0/20$ . The input impedance of the stand-alone dipole and the input impedance of the same dipole when used to illuminate the lens are shown in Figs. 3(b) and 3(c), respectively. Some differences between the V-MoM and [8] are present in modelling the lens and the dipole. In the V-MoM, the dipole is represented with only a basis function in the cross-section, and it is excited with a constant excitation on the entire gap volume, a feature that cannot be obtained with [8]. Moreover, the curved lens surface is not accurately represented with the voxel discretization of the V-MoM. Nevertheless, the agreement between the V-MoM and [8] is excellent for the input impedance of Figs. 3(b) and 3(c), and for the field distribution inside the lens when fed by the dipole at f = 294 GHz.

The lens and the dipole need different levels of discretization: a sampling in the order of  $\lambda_d/10$  is sufficient to characterize the field inside the lens, a much finer discretization is needed to represent the thin metallization of a realistic dipole. Therefore, with the same mesh size the simulation of the lens-dipole ensemble would require an immense number of unknowns. It is our goal to decouple the simulation of the thin metallization from the one of the dielectric lens, by simulating the lens fed by an auxiliary dipole coarsely discretized and then to add in postprocessing the analytical correction for the different metal.

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**Fig. 3.** (a) Hemispheric dielectric lens fed by an half-wave dipole, (b) input impedance of the dipole in free-space, (c) input impedance of the dipole when exciting the lens, (d) electric field distribution inside the lens, and (e) electric field pattern at 1 m when fed by the dipole at f = 294 GHz.

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