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ORIGINAL ARTICLE

Nonadditive best-worst method: Incorporating criteria interaction using the Choquet integral

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ABSTRACT
The best-worst method (BWM) is a multicriteria decision-making (MCDM) method to derive the relative importance (weight) of a set of criteria used to evaluate a set of alternatives. Several models (e.g., nonlinear, linear, Bayesian, and multiplicative) have been developed to find the weights based on the provided pairwise comparisons, conducted among the criteria, by the decision-maker(s)/expert(s). The existing BWM models, however, do not handle interactions that might exist between the criteria encountered in a decision problem. In this study, a nonadditive BWM is developed that considers possible interactions between the criteria. To this end, we use the Choquet integral, one of the most widely accepted techniques, to incorporate criteria interactions. A nonlinear optimization model is introduced to minimize the maximum deviation of the obtained weights from the provided pairwise comparisons, considering the information about the interactions between the criteria. We then introduce a linear variant of the nonadditive BWM and discuss its property compared to the nonlinear model. The applicability of the proposed approach is demonstrated through a real-world case study of a battery-powered electric vehicle (BEV) selection problem.

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1. Introduction

In a multicriteria decision-making (MCDM) problem, the relative importance (weight) of criteria plays an important role in evaluating, selecting, sorting, and ranking alternatives. In recent decades, several methods have been developed to infer criteria weights using the preferences of a decision-maker/expert, such as the analytic hierarchy process (AHP) (Saaty, 1977), the analytic network process (ANP) (Saaty, 1990), the simple multiattribute rating technique (SMART) (Edwards, 1977), the Swing method (von Winterfeldt & Edwards, 1986), and Tradeoff (Keeney & Raiffa, 1976). The focus of this study is the best-worst method (BWM), which was proposed by Rezaei (2015). It is a pairwise comparison-based method that requires a decision-maker/expert to conduct pairwise comparisons between the criteria considering two reference points (*best:* the most important criterion, and *worst:* the least important criterion). This structure has several advantages including higher data efficiency, higher consistency, mitigation of anchoring bias (Rezaei, 2020, 2022), and equalising bias (Rezaei et al., 2022).

Since its introduction, several extended versions of the method have been introduced in the literature (Brunelli & Rezaei, 2019; Kheybari et al., 2021; Rezaei, 2016; Safarzadeh et al., 2018). In the original BWM (Rezaei, 2015), the criteria weights are obtained by means of a nonlinear optimization model, which might result in multiple optimal solutions. Rezaei (2016) introduced a linear BWM model, which maintains the main idea of the original BWM and reduces the computational complexity. With the aid of the same idea, a multiplicative model of BWM was presented (Brunelli & Rezaei, 2019). To handle multicriteria group decision-making problem, a Bayesian BWM was introduced, and further, a credal ranking method was adopted to yield the ranking of criteria weights described by probability distributions (Mohammadi & Rezaei, 2020). In addition, Safarzadeh et al. (2018) presented a group BWM using two optimization models, which considered the weights of experts, discourse power, and backward feedback for criteria weights.

The BWM has been integrated with several other MCDM methods including Technique for Order Preference by Similarity to an Ideal Solution (TOPSIS) (Askarifar et al., 2018), VlseKriterijuska Optimizacija I Komoromisno Resenje (VIKOR) (Gupta, 2018), and TODIM (an acronym in Portuguese of interactive and multipleattribute decision making) method (Nie et al., 2022). The original BWM and its extensions have been used widely, such as for supplier selection (Lajimi et al., 2021),

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hybrid vehicle engine selection (Hafezalkotob et al., 2020), project management (Liu et al., 2021), technology acceptance and return management in apparel e-commerce (Kalpoe, 2020), and initial water rights allocation (Xu et al., 2021). For more applications, refer to Mi et al. (2019).

The original BWM and all its extensions implicitly assume no interdependence between criteria. Nevertheless, in some practical decision-making problems, preferential independence among criteria may be debatable, i.e., criteria are often interdependent, redundant, or complementary (Grabisch, 1996; Keeney & Raiffa, 1976), which cannot be effectively managed by existing BWM models. For example, a customer would like to purchase a car and she/he needs to consider multiple criteria including price, maximum power, and oil consumption. In this case, these three criteria are often related, price and maximum power positively interact, and price and oil consumption negatively interact. This implies that the combined weight of price and maximum power might be greater than the sum of weights of these two criteria considered individually and that the combined weight of price and oil consumption might be smaller than the sum of weights when considered separately. Criteria interactions, i.e., nonadditive relations, affect criteria relative importance. Namely, the relative importance of a criterion is a function of the capacity of criterion alone as well as the capacity of all its interactions with the other criteria.

To handle the phenomenon of criteria interaction, many nonadditive integral-based methodologies have emerged, such as the Choquet integral (Choquet, 1953) and the Sugeno integral (Sugeno, 1974). The Choquet integral is regarded as an effective and widespread approach to manage criteria interactions and has been integrated with other MCDM methods such as the robust ordinal regression (Arcidiacono et al., 2020) and stochastic multiobjective acceptability analysis (SMAA) (Zhao et al., 2022). It is often modelled by fuzzy measures considering weights of criteria alone and their subsets. To make BWM handle the criteria interactions, this study introduces a novel BWM-based method, called the nonadditive BWM that incorporates criteria interactions described by Shapely value (Shapley, 1953) employing a 2-additive fuzzy measure (Grabisch, 1997). To determine criteria relative importance, a nonlinear optimization model is formulated based on input pairwise comparisons and relationships among criteria interactions. To rank the alternatives, the Choquet integral (Choquet, 1953) is adopted to evaluate their performances. Hence, the contributions of this study are summarised as follows:

1. A novel criterion weight elicitation method, i.e., nonadditive BWM, is developed to handle criteria interaction. Based on the optimal criteria weights and the criteria interaction degrees, alternatives are ranked using Choquet integral. The developed model is nonlinear and could result in multiple optimal solutions.
2. To acquire a unique optimal solution, i.e., criteria weights and criteria interaction degrees, a linear model of nonadditive BWM is developed.
3. Both nonlinear and linear nonadditive BWM are applied to a real-world problem to illustrate the feasibility and effectiveness of the models. The results are also compared to the ones obtained by existing BWM models.

The remainder of this paper is organised as follows. Section 2 briefly introduces the basics of the original BWM. Section 3 provides an overview of the basic knowledge of the Choquet integral and its advantages. Section 4 presents a nonadditive BWM to obtain the optimal criteria relative importance, alongside the Choquet integral to consider interactions among the criteria. A linear model of nonadditive BWM is proposed in Section 5. Section 6 demonstrates a real-world application for a new-energy vehicle selection problem and provides the comparative analysis to illustrate the effectiveness of our proposal. Finally, Section 7 presents research conclusions and points out some future directions.

2. Best-worst method

The BWM is a simple and practical approach with a strong theoretical background for solving MCDM problems. One of the main advantages of BWM is the use of two criteria references (the most important: best, and the least important: worst) to make two pairwise comparison vectors. These two vectors are used as input for an optimization model, which can effectively reduce the number of pairwise comparisons. In what follows, we introduce the main steps of the original BWM (Rezaei, 2015).

Step 1. A criteria set $C = \{c_j, j = 1, 2, \ldots, n\}$ is provided by the decision-maker/expert.

Step 2. The best (e.g., the most important) criterion, c_B, and the worst (e.g., the least important) criterion, c_W, are identified from the set C by the decision-maker/expert.

Step 3. The decision-maker/expert conducts pairwise comparisons to compare the best criterion to other criteria using a number from 1 to 9, where 1 means “equally important” and 9 means “extremely more important”. Namely, a Best-to-Others vector

Table 1. Consistency index (*CI*) in original BWM (Rezaei, 2015).

a_{BW}	1	2	3	4	5	6	7	8	9
CI	0.00	0.44	1.00	1.63	2.30	3.00	3.73	4.47	5.23

$BO = (a_{B1}, a_{B2}, ..., a_{Bn})$ needs to be provided by the decision-maker/expert.

Step 4. Similarly, the decision-maker/expert gives the pairwise comparisons for other criteria to the worst criterion using a number from 1 to 9, denoted as an Others-to-Worst vector $OW = (a_{1W}, a_{2W}, ..., a_{nW})^T$.

Step 5. Based on the collected pairwise comparison vectors, a model is established to minimize the maximum absolute differences $\left|\frac{w_B}{w_j} - a_{Bj}\right|$ and $\left|\frac{w_j}{w_W} - a_{jW}\right|$ for $j = 1, 2, ..., n$, where the criteria weights need to satisfy sum-to-one and nonnegativity constraints as follows:

$$\begin{cases} \min \quad \max\limits_j \left\{\left|\frac{w_B}{w_j} - a_{Bj}\right|, \left|\frac{w_j}{w_W} - a_{jW}\right|\right\} \\ s.t. \qquad \begin{cases} \sum_{j=1}^n w_j = 1; \\ w_j \geq 0, j = 1, 2, ..., n. \end{cases} \end{cases} \tag{1}$$

Model (1) can be transformed into the following equivalent model:

$$\begin{cases} \min \xi \\ s.t. \begin{cases} \left|\frac{w_B}{w_j} - a_{Bj}\right| \leq \xi, j = 1, 2, ..., n; \\ \left|\frac{w_j}{w_W} - a_{jW}\right| \leq \xi, j = 1, 2, ..., n; \\ \sum_{j=1}^n w_j = 1; \\ w_j \geq 0, j = 1, 2, ..., n. \end{cases} \end{cases} \tag{2}$$

Solving model (2) results in the optimal $\xi = \xi^*$. The model generates a unique solution for fully consistent pairwise comparison systems or problems with fewer than four criteria regardless of the consistency of the pairwise comparisons.

To check the consistency of the provided pairwise comparisons, two types of consistency ratios are developed: the input-based consistency ratio and the output-based consistency ratio. The former provides immediate feedback for the consistency of the given pairwise comparisons and the latter measures the veracity of the obtained optimal weights and the input pairwise comparisons.

The input-based consistency ratio CR^I (Liang et al., 2020) is defined by

$$CR^I = \max_j CR_j^I, \tag{3}$$

where CR_j^I shows the consistency level under criterion c_j using

$$CR_j^I = \begin{cases} \frac{|a_{Bj} \times a_{jW} - a_{BW}|}{(a_{BW})^2 - a_{BW}}, & a_{BW} > 1; \\ 0, & a_{BW} = 1. \end{cases} \tag{4}$$

The output-based consistency ratio CR^O (Rezaei, 2015) is defined as

$$CR^O = \frac{\xi^*}{CI}, \tag{5}$$

where the fixed values of CI for different values of a_{BW} are shown in Table 1.

The smaller CR^I and CR^O are, the higher the consistency is. For checking the acceptability of the consistency of the pairwise comparison system, we can use the thresholds for input-based consistency ratio or output-based consistency ratio, respectively, from Tables 3 and 4 of Liang et al. (2020).

3. Basic definitions and advantages of the Choquet integral

In this part, we first introduce some basic knowledge concerning the Choquet integral and then discuss the advantages of the Choquet integral when criteria interactions exist in MCDM problems.

3.1. Basic definitions of the Choquet integral

Consider an MCDM problem where we have q alternatives $a_p, p = 1, 2, ..., q$, n criteria $c_j, j = 1, 2, ..., n$, and the score of the alternative a_p under the criterion c_j is $g_j(a_p)$. The decision-maker employs the personalised value function to convert the evaluation score $g_j(a_p)$ into the normalized value $v(g_j(a_p))$. The weights of the criteria are $w_j, j = 1, 2, ..., n$, where $w_j \geq 0$ and $\sum_{j=1}^n w_j = 1$. The goal is to select an alternative that has the best overall performance. Usually, an additive value function is adopted to make a choice, where $V(a_p)$ describes the overall value of a_p as follows:

$$V(a_p) = \sum_{j=1}^n w_j v(g_j(a_p)). \tag{6}$$

The additive value function performs well when the criteria are mutually preferentially independent. However, in some practical problems, criteria interactions might exist, implying that an assumption of additivity is not always true (Keeney & Raiffa, 1976; Wakker, 1989). To cope with criteria interaction, the Choquet integral is introduced.

Definition 1. (Choquet, 1953) For a criteria set $C = \{c_j, j = 1, 2, ..., n\}$, a capacity is defined as a set function $\mu : 2^C \to [0, 1]$ on the power set 2^C, where the following properties should be satisfied for the set of all subsets of C :

i. (Boundary) $\mu(\emptyset) = 0$ and $\mu(C) = 1$;
ii. (Monotonicity) $S \subseteq T \subseteq C,\ \ \mu(S) \leq \mu(T)$.

For convenience, the Möbius representation is often employed as an equivalent transformation (Grabisch et al., 2000). The Möbius representation of the capacity μ on 2^C is the function $m : 2^C \rightarrow \mathbb{R}$, for all $T \subseteq C$, $\mu(T) = \sum_{S \in T} m(S)$ and the Möbius representation can be obtained by $m(T) = \sum_{S \in T} (-1)^{|T|-|S|} \mu(S)$, *for all* $T \subseteq C$.

Definition 2. (Choquet, 1953) The Choquet integral-based performance of the alternative $a_p, p = 1, 2, \ldots, q$ is defined as follows:

$$C_\mu\left(a_p\right) = \sum_{j=1}^{n} \left[\left(v(g_{(j)}(a_p)) - v(g_{(j-1)}(a_p)) \right) \mu(\{(c_j), \ldots, (c_n)\}) \right] \tag{7}$$

$$= \sum_{T \in C} m(T) \min_{c_j \in T} v(g_j(a_p)),$$

where $g_{(j)}(a_p)$ is the score of the alternative a_p under the criterion c_j, $v(g_{(j)}(a_p))$ is the normalized value of $g_{(j)}(a_p)$, the subscript $(\cdot)$ represents the criteria order such that $v(g_{(0)}(a_p)) \leq v(g_{(1)}(a_p)) \leq \cdots \leq v(g_{(n)}(a_p))$ and $v(g_{(0)}(a_p)) = 0$.

In general, when using the Choquet integral, $2^C - 2$ parameters $\mu(T)$, $T \subseteq C$ need to be determined, except $T = \emptyset$ $(\mu(\emptyset) = 0)$ and $T = C$ $(\mu(C) = 1)$. As specifying of all parameters is cognitively difficult, the k-additive capacity is defined (Grabisch, 1997). If $m(T) = 0$ for all $T \subseteq C$, such that $|T| > k$ and $\exists T \subseteq C$, $|T| = k$, $m(T) \neq 0$, then the capacity is k-additive. Indeed, the 2-additive capacity is enough to describe the criteria interaction. The 2-additive capacity, μ can be described by the Möbius representation (Rota, 1964):

$$\mu_j = m_j, c_j \in C; \tag{8}$$

$$\mu_{ij} = m_i + m_j + m_{ij}, c_i, c_j \in C \text{ and } i \neq j; \tag{9}$$

$$\mu(T) = \sum_{c_i \in T} m_i + \sum_{c_i, c_j \in T} m_{ij}$$

$$= \sum_{c_i, c_j \in T} \mu_{ij} - (|T| - 2) \sum_{c_i \in T} m_i, T \subseteq C \text{ and } |T| > 2; \tag{10}$$

where $\mu(\{c_j\})$ is simplified as μ_j, $\mu(\{c_i, c_j\})$ is simplified as μ_{ij}, $m(\{c_j\})$ is denoted as m_j and $m(\{c_i, c_j\})$ is written as m_{ij} for convenience.

Thus, the Choquet integral-based performance of alternative a_p based on the 2-additive capacity can be given by

$$C_\mu\left(a_p\right) = \sum_{c_j \in C} m_j v\left(g_{(j)}\left(a_p\right)\right) + \sum_{c_i, c_j \in C} m_{ij}\, min\{v(g_{(i)}(a_p)), v(g_{(j)}(a_p))\}. \tag{11}$$

The properties of a capacity can be equivalently described by the Möbius representation:

i. (Boundary) $m(\emptyset) = 0$ and $\sum_{c_j \in C} m_j + \sum_{c_i, c_j \in C} m_{ij} = 1$;
ii. (Monotonicity) $m_j + \sum_{c_i \in T} m_{ij} \geq 0$, $\forall j = 1, 2, \ldots, n$ and $T \subseteq C\ \{c_j\}, T \neq \emptyset$; where $m_j \geq 0$, $j = 1, 2, \ldots, n$.

3.2. Advantages of the Choquet integral

To illustrate the relative importance of criteria interaction, we provide a practical example to reveal the limitations of inappropriately assuming independence among criteria.

Example 1. Consider a car selection problem where a customer would like to make a choice from four qualified cars a_1, a_2, a_3, and a_4, based on three criteria, c_1 : price (in ten hundred dollars), c_2 : engine performance (weak, great, or perfect), and c_3 : oil consumption (litre per 100 km). The value of each alternative under each criterion is supposed according to its performance as shown in Table 2. Assuming that this customer adopts the additive value function $\sum_{j=1}^{3} w_j v(g_j(a_p))$ to give the priorities, where $g_j(a_p)$ represents the score of a_p under the criterion c_j, with its normalized value of $v(g_j(a_p))$, $w_j \in [0, 1]$ is the weight of criterion c_j and $\sum_{j=1}^{3} w_j = 1$.

Suppose that based on the preferences of the decision-maker and considering the interactions between the criteria we found $w_1 = 0.692, w_2 = 0.231, w_3 = 0.077$, $\mu_1 = 0.757$, $\mu_2 = 0.295$, $\mu_3 = 0.021$, $\mu_{12} = 0.868$, $\mu_{13} = 0.834$, $\mu_{23} = 0.372$, and $\mu_{123} = 1$ (for more details see Example 2). Using the additive value function in Equation (6) and the Choquet integral in Equation (7), the performances of the four considered cars are given in Table 2. We observe that a_2 is superior to a_1 when using the additive value function, but a_1 is superior to a_2 when using the Choquet integral. Indeed, for two cheaper cars, a_1 and a_2, a_1 is preferred to a_2 because a_1 has lower oil consumption. Hence, the result might be counter-intuitive when the decision-maker adopts the additive value function.

Table 2. The features of considered cars.

Cars	c_1	c_2	c_3	$v(g_1(a_k))$	$v(g_2(a_k))$	$v(g_3(a_k))$	Additive value function	Choquet integral
a_1	20	Great	5	0.9	0.5	0.75	0.796	0.822
a_2	20	Perfect	7	0.9	0.75	0.50	0.835	0.803
a_3	50	Great	5	0.5	0.5	0.75	0.519	0.505
a_4	50	Perfect	7	0.5	0.75	0.50	0.558	0.574

Table 3. BO and OW pairwise comparison vectors.

BO	c_1	c_2	c_3
Best criterion: c_1	1	3	9
OW		Worst criterion: c_3	
c_1		9	
c_2		3	
c_3		1	

The additive value function is based on the assumption that criteria are mutually preferentially independent. This assumption means that linear compensation relationships exist among the criteria. In other words, if alternative a_p is worse than a_f under the criterion c_i, then, it can be proportionally offset by a_f being superior to a_p under another criterion c_j. Obviously, it is difficult to distinguish these two alternatives, and it even causes contradictions in some situations. In Example 1, c_1 and c_2 are redundant, c_1 and c_3 are synergistic, and c_2 and c_3 are synergistic. The combined weights of c_1 and c_2 together should be less than the sum of their weights alone, and the combined weights of c_1 and c_3 and that of c_2 and c_3 should be greater than the sum of their corresponding weights alone. Therefore, it is necessary to analyse the criteria interaction to determine the relative importance of the criteria and further evaluate the performance of alternatives in MCDM problems.

4. Nonadditive BWM

In this section, to derive the relative importance of the criteria considering their potential interdependence, a new version of BWM is developed. The steps are as follows:

Step 1. A criteria set, $C = \{c_j, j = 1, 2, ..., n\}$, is provided by the decision-maker/expert.

Step 2. The most important/preferred criterion, *best*, c_B, and the least important/preferred criterion, *worst*, c_W, are selected and pairwise comparisons are conducted using scores from 1 to 9 by the decision-maker/expert. The preferences for the best criterion over other criteria are denoted in a Best-to-Others vector, $BO = (a_{B1}, a_{B2}, ..., a_{Bn})$. Similarly, the preferences for other criteria over the worst criterion are denoted in an Others-to-Worst vector, $OW = (a_{1W}, a_{2W}, ..., a_{nW})^T$.

Step 3. The criteria interactions are provided by the decision-maker/expert. In this study, the 2-additive fuzzy measure is adopted to portray criteria interactions, which not only retains the nonadditive characteristics but also has low computational complexity. In practice, the decision-maker provides the qualitative assessments for the relationships among the criteria, such as positive, negative, no interaction, or unknown, which can be given using the criteria interaction matrix, $D = (t_{ij})_{n\times n}$ as follows:

$$D = \begin{array}{c} \\ c_1 \\ c_2 \\ \vdots \\ c_n \end{array} \begin{array}{c} \begin{array}{cccc} c_1 & c_2 & \cdots & c_n \end{array} \\ \begin{bmatrix} / & t_{12} & \cdots & t_{1n} \\ t_{21} & / & \cdots & t_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ t_{n1} & t_{n2} & \cdots & / \end{bmatrix} \end{array}, \quad (12)$$

where $t_{ij} \in \{+, -, \Delta, \text{ʋ}\}$, $i \neq j, i, j = 1, 2, ..., n$ is employed to give the redundant and synergistic relationships among criteria, "$+$" means two criteria interact positively, "$-$" means negative interaction between the two criteria, Δ means no interaction and ʋ means the decision-maker is unknown to the interaction relationship. $t_{ij} = t_{ji}$, $i \neq j, i, j = 1, 2, ..., n$, i.e., T is constructed as a symmetric matrix.

Without loss of generality, in this step, we use the 2-additive fuzzy measure to model criteria interactions. For each pair of criteria $c_i, c_j \in C, i \neq j$, $i, j = 1, 2, ..., n$, the degree of criteria interaction $I_{ij}, i \neq j, i, j = 1, 2, ..., n$ is positive, negative, null (zero) or belonging to $[-1, 1]$. Therefore, the degree of criteria interaction I_{ij} satisfies

$$\begin{cases} I_{ij} > 0, & if \quad t_{ij} = +; \\ I_{ij} < 0, & if \quad t_{ij} = -; \\ I_{ij} = 0, & if \quad t_{ij} = \Delta; \\ I_{ij} \in [-1, 1], & if \quad t_{ij} = \text{ʋ}. \end{cases} \quad (13)$$

Owing that $I_{ij} = I_{ji}, i \neq j, i, j = 1, 2, ..., n$, we only consider I_{ij}, $i < j; i, j = 1, 2, ..., n$.

The criteria interaction I_{ij} is equivalent to the Möbius representation m_{ij} :

$$I_{ij} = m_{ij}, \quad i \neq j, i, j = 1, 2, ..., n. \quad (14)$$

Then the relative importance of the criterion c_j, i.e., I_j, can be formulated by means of the Shapely value (Shapley, 1953) for the 2-additive fuzzy measure as follows:

$$I_j = m_j + \sum\nolimits_{c_i \in C \ \{c_j\}} \frac{m_{ij}}{2}, \quad j = 1, 2, ..., n. \quad (15)$$

Step 5. The relative importance of the criteria is obtained such that the maximum absolute difference between the provided pairwise comparisons and their associated weight ratios $\left|\frac{I_B}{I_j} - a_{Bj}\right|$ and

$\left|\frac{I_j}{I_W}-a_{jW}\right|$ for all j are minimized as follows:

$$\begin{cases} \min\max\limits_{j}\left\{\left|\frac{I_B}{I_j}-a_{Bj}\right|,\left|\frac{I_j}{I_W}-a_{jW}\right|\right\} \\ s.t.\begin{cases} m(\emptyset)=0;\sum\limits_{c_j\in C}m_j+\sum\limits_{c_i,c_j\in C}m_{ij}=1; \\ m_j+\sum\limits_{c_i\in T}m_{ij}\geq 0, i,j=1,2,\ldots,n \text{ and } T\subseteq C\backslash\{c_j\}, T\neq\emptyset; \\ I_{ij}=m_{ij}, i\neq j, i,j=1,2,\ldots,n; \\ I_j=m_j+\sum\limits_{c_j\in C\backslash\{c_i\}}\frac{m_{ij}}{2}, i\neq j, i,j=1,2,\ldots,n; \\ I_{ij}>0, \text{if } t_{ij}=+, i<j, i,j=1,2,\ldots,n; \\ I_{ij}<0, \text{if } t_{ij}=-, i<j, i,j=1,2,\ldots,n; \\ I_{ij}=0, \text{if } t_{ij}=\Delta, i<j, i,j=1,2,\ldots,n; \\ I_{ij}\in[-1,1], \text{if } t_{ij}=\text{u}', i<j, i,j=1,2,\ldots,n. \end{cases} \end{cases} \tag{16}$$

Model (16) can be equivalently transferred into model (17):

$$\begin{cases} \min\xi \\ s.t.\begin{cases} m(\emptyset)=0;\sum\limits_{c_j\in C}m_j+\sum\limits_{c_i,c_j\in C}m_{ij}=1; \\ m_j+\sum\limits_{c_i\in T}m_{ij}\geq 0, i,j=1,2,\ldots,n \text{ and } T\subseteq C\ \{c_j\}, T\neq\emptyset; \\ I_{ij}=m_{ij}, i\neq j, i,j=1,2,\ldots,n; \\ I_j=m_j+\sum\limits_{c_i\in C\ \{c_j\}}\frac{m_{ij}}{2}, i\neq j, i,j=1,2,\ldots,n; \\ \left|\frac{I_B}{I_j}-a_{Bj}\right|\leq\xi, j=1,2,\ldots,n; \\ \left|\frac{I_j}{I_W}-a_{jW}\right|\leq\xi, j=1,2,\ldots,n; \\ I_{ij}>0, \text{if } t_{ij}=+, i<j, i,j=1,2,\ldots,n; \\ I_{ij}<0, \text{if } t_{ij}=-, i<j, i,j=1,2,\ldots,n; \\ I_{ij}=0, \text{if } t_{ij}=\Delta, i<j, i,j=1,2,\ldots,n; \\ I_{ij}\in[-1,1], \text{if } t_{ij}=\text{u}', i<j, i,j=1,2,\ldots,n. \end{cases} \end{cases} \tag{17}$$

Similar to Rezaei (2016), we could show that model (17) (excluding the last four constraints of criteria interactions) has always a solution. For instance, $I_j=m_j=\frac{1}{n}, I_{ij}=m_{ij}=0, i,j=1,2,\ldots,n,\quad i\neq j,$ and $\xi=a_{BW}-1$ is a feasible solution. Considering the last four constraints of model (17) (the criteria interaction constraints), it is possible that the model leads to infeasibility. In that case, one can employ the method suggested by Mousseau et al. (2003) to determine the minimal set of preference information which can be the basis for modification for the decision-maker to remove the incompatibility among the criteria interaction constraints. The CI values (see Table 1) are the maximum values of the ξ (Rezaei, 2015), which demonstrates that the objective function of model (17) is bounded. According to Weierstrass' Theorem (Borwein & Lewis, 2005), model (17) must exist at least one optimal solution.

Model (17), in case of compatibility among the criteria interaction constraints, like the original BWM, may result in a unique optimal solution or multiple optimal solutions. If model (17) has a unique optimal solution, the decision-maker obtains the optimal criteria relative importance and criteria interaction degrees. Otherwise, the decision-maker can select an arbitrary solution from the set of optimal solutions provided that the sum of the selected weights is 1.

Owing that the input pairwise comparisons may not satisfy the consistency requirement, we need to check the acceptability of the consistency ratio. Nonadditive BWM uses input-based consistency ratio to judge whether input information meets the thresholds that are provided by Liang et al. (2020). We could also check the output consistency ratio using Equation (5) with the thresholds reported in Liang et al. (2020).

In what follows, we provide Example 2 as an illustration.

Example 2. Considering Example 1, the decision-maker provides pairwise comparisons shown in Table 3. The criteria interaction matrix $D=(t_{ij})_{3\times 3}$ is given by virtue of Equation (12) as follows:

$$D=\begin{matrix} & c_1 & c_2 & c_3 \\ c_1 \\ c_2 \\ c_3 \end{matrix}\begin{bmatrix} / & - & + \\ - & / & + \\ + & + & / \end{bmatrix}.$$

Based on the given matrix D, we have the criteria interaction relationships as follows:

- Price (c_1) and engine performance (c_2) have negative interaction ($t_{12}=-$);
- Price (c_1) and oil consumption per 100 km (c_3) have positive interaction ($t_{13}=+$);
- Engine performance (c_2) and oil consumption per 100 km (c_3) have positive interaction ($t_{23}=+$).

The proposed optimization model in Equation (17) is employed to derive the criteria relative importance and the car rankings. According to the two pairwise comparison vectors in Table 3 provided by the decision-maker, we establish the following optimization model:

$$\begin{cases} \min\xi \\ s.t.\begin{cases} m_1+m_2+m_3+m_{12}+m_{13}+m_{23}=1; \\ m_1+m_{12}+m_{13}\geq 0; \\ m_2+m_{12}+m_{23}\geq 0; \\ m_3+m_{13}+m_{23}\geq 0; \\ I_1=m_1+0.5m_{12}+0.5m_{13}; \\ I_2=m_2+0.5m_{12}+0.5m_{23}; \\ I_3=m_3+0.5m_{13}+0.5m_{23}; \\ \left|\frac{I_1}{I_2}-3\right|\leq\xi;\left|\frac{I_1}{I_3}-9\right|\leq\xi;\left|\frac{I_2}{I_3}-3\right|\leq\xi; \\ I_{12}<0; I_{13}<0; I_{23}>0; \\ I_{12}=m_{12}; I_{13}=m_{13}; I_{23}=m_{23}. \end{cases} \end{cases} \tag{18}$$

Solving this model, we have $\xi^* = 0$, $I_1 = 0.692$, $I_2 = 0.231$, $I_3 = 0.077$, $m_1 = 0.757$, $m_2 = 0.295$, $m_3 = 0.021$, $m_{12} = -0.185$, $m_{13} = 0.056$, $m_{23} = 0.055$. Based on the relationships between the fuzzy measure and the Möbius representation, the fuzzy measure is obtained by virtue of Equations (8)–(10) as $\mu_1 = 0.755$, $\mu_2 = 0.295$, $\mu_3 = 0.021$, $\mu_{12} = 0.868$, $\mu_{13} = 0.834$, $\mu_{23} = 0.372$, $\mu_{123} = 1$. Hence, the performance of four cars is calculated using the Choquet integral shown in Table 2. This decision result is in line with the preferences of the decision-maker, a_1 is superior to a_2 and a_4 is superior to a_3, which verifies the effectiveness of our method.

In an MCDM problem, supposing that there are an alternative set, $A = \{a_p, p = 1, 2, ..., q\}$, and criteria set, $C = \{c_j, j = 1, 2, ..., n\}$, the performance of the alternative a_p under the criterion c_j is denoted as $g_j(a_p)$. Considering that different criteria have various measurement scales, we need to normalize the original performance. There are various approaches to do this, such as using value functions (Keeney & Raiffa, 1976, Rezaei, 2018), or normalization formulas, such as the following commonly used one:

$$v(g_j(a_p)) = \begin{cases} \dfrac{g_j(a_p) - l_j}{h_j - l_j}, & \text{if } c_j \text{ is } a \text{ benefit criterion;} \\ \dfrac{h_j - g_j(a_p)}{h_j - l_j}, & \text{if } c_j \text{ is } a \text{ cost criterion;} \end{cases} \tag{19}$$

where $v(g_j(a_p))$ is the normalized value of $g_j(a_p)$. h_j and l_j, respectively, show the predefined maximum and minimum scores under the criterion c_j across the whole set of alternatives given by the decision-maker to ensure $v(g_j(a_p)) \in [0, 1]$.

The Choquet integral of a_p can be obtained by Equation (11). The ranking of alternatives can be given by comparing $C_\mu(a_p), p = 1, 2, ..., q$ and the larger $C_\mu(a_p)$ means the more preferred alternative a_p. Notably, if there exists a unique solution for criteria weights and criteria interaction degrees, then the final ranking of alternatives can be obtained in virtue of this solution. If there exist multiple solutions, then we can derive the priority of alternatives using Monte Carlo Approach (Mohammadi & Rezaei, 2021) or SMAA in the optimal space (Lahdelma et al., 1998).

5. A Linear model of nonadditive BWM

As discussed in Section 4, the proposed nonadditive BWM may result in a unique optimal solution or multiple optimal solutions. In this section, we present a model which results in a unique solution. The model minimizes the maximum absolute differences $|I_B - a_{Bj}I_j|$ and $|I_j - a_{jW}I_W|$ for all j as follows:

$$\begin{cases} \min \max_j \left\{ |I_B - a_{Bj}I_j|, |I_j - a_{jW}I_W| \right\} \\ s.t. \begin{cases} m(\emptyset) = 0; \sum_{c_j \in C} m_j + \sum_{c_i, c_j \in C} m_{ij} = 1; \\ m_j + \sum_{c_i \in T} m_{ij} \geq 0, i, j = 1, 2, ..., n \text{ and } T \subseteq C \setminus \{c_j\}, T \neq \emptyset; \\ I_{ij} = m_{ij}, i \neq j, i, j = 1, 2, ..., n; \\ I_j = m_j + \sum_{c_j \in C \setminus \{c_i\}} \dfrac{m_{ij}}{2}, i \neq j, i, j = 1, 2, ..., n; \\ I_{ij} > 0, \text{if } t_{ij} = +, i < j, i, j = 1, 2, ..., n; \\ I_{ij} < 0, \text{if } t_{ij} = -, i < j, i, j = 1, 2, ..., n; \\ I_{ij} = 0, \text{if } t_{ij} = \Delta, i < j, i, j = 1, 2, ..., n; \\ I_{ij} \in [-1, 1], \text{if } t_{ij} = \text{u'}, i < j, i, j = 1, 2, ..., n. \end{cases} \end{cases} \tag{20}$$

Model (20) can be equivalently transferred into a linear model (21):

$$\begin{cases} \min \xi \\ s.t. \begin{cases} m(\emptyset) = 0; \sum_{c_j \in C} m_j + \sum_{c_i, c_j \in C} m_{ij} = 1; \\ m_j + \sum_{c_i \in T} m_{ij} \geq 0, i, j = 1, 2, ..., n \text{ and } T \subseteq C\ \{c_j\}, T \neq \emptyset; \\ I_{ij} = m_{ij}, i \neq j, i, j = 1, 2, ..., n; \\ I_j = m_j + \sum_{c_i \in C\ \{c_j\}} \dfrac{m_{ij}}{2}, i \neq j, i, j = 1, 2, ..., n; \\ I_B - a_{Bj}I_j \leq \xi, j = 1, 2, ..., n; I_B - a_{Bj}I_j \geq -\xi, j = 1, 2, ..., n; \\ I_j - a_{jW}I_W \leq \xi, j = 1, 2, ..., n; I_j - a_{jW}I_W \geq -\xi, j = 1, 2, ..., n; \\ I_{ij} > 0, \text{if } t_{ij} = +, i < j, i, j = 1, 2, ..., n; \\ I_{ij} < 0, \text{if } t_{ij} = -, i < j, i, j = 1, 2, ..., n; \\ I_{ij} = 0, \text{if } t_{ij} = \Delta, i < j, i, j = 1, 2, ..., n; \\ I_{ij} \in [-1, 1], \text{if } t_{ij} = \text{u'}, i < j, i, j = 1, 2, ..., n. \end{cases} \end{cases} \tag{21}$$

It is easy to demonstrate that the coefficient matrix and its augmented matrix of model Equation (21) are both full rank. Hence, the proposed linear nonadditive BWM has a unique optimal solution. Solving the model using a standard optimization package, we obtain the optimal criteria weights and the criteria interaction degrees. If the decision-maker wants to rank alternatives, then the performance of alternatives can be evaluated using Equation (11) on the basis of the calculated result.

6. A real-world application

In this section, the empirical application of the nonadditive BWM is given using the purchase of new energy vehicles. We first provide the research background and then illustrate the considered criteria and data gathering. Following the steps in the proposed methodology, the main computational results are provided. Furthermore, comparative analysis is implemented to verify the practicality and feasibility of the proposed approach.

6.1. Criteria determination and data collection

Currently, to reduce oil consumption and carbon dioxide emissions, the trend for worldwide vehicle

Table 4. Identified dimensions, criteria, and explanation.

Dimensions	Criteria	Explanation
Economic performance	Price/thousand RMB (c_1)	Manufacturer's suggested retail price (MSRP) is a price standard set by automobile manufacturers to avoid price competition among dealers and reduce service level
	100 km power consumption/kwh (c_2)	The amount of electric energy consumed by electric vehicles driving 100 kms
Automobile performance	Slow charging time/h (c_3)	The charging interface of the AC charging pile, the wand that converts AC into DC through the charger inside the car, and then the completed charging after battery input
	Maximum power of the electric motor/kw (c_4)	The maximum power at which it can operate normally for a long time under rated voltage
	Pure electric range of MIIT/km (c_5)	The longest driving range of BEV or PHEV under battery energy and comprehensive working conditions
	Max speed/km/h (c_6)	The speed a car can reach on a good level road surface is the speed at which the driving resistance and driving force are balanced on a flat road with no wind
	Battery capacity/kwh (c_7)	The amount of electricity released by the battery under certain conditions (discharge rate, temperature, termination voltage, etc.)
Service availability	Warranty kilometres/10 thousand km (c_8)	Vehicle warranty means that all kinds of parts are replaced free of charge when it is confirmed that they are not damaged by human beings but by quality problems

Table 5. Set of new-energy vehicles and their values for the considered criteria.

Number	Name	Type	Manufacturer	c_1	c_2	c_3	c_4	c_5	c_6	c_7	c_8
a_1	Ciimo X-NV 2020 (Dian che)	SSUV	DONGFENG HONDA	17.98	14.09	9	120	401	140	53.6	12
a_2	Encino 2020 Top (Yue xiang)	SSUV	BEIJING- HYUNDAI	19.88	13.8	10.5	150	500	170	64.2	10
a_3	E-HS3 2019	CSUV	FAW-HONGQI	22.58	16	12	114	407	160	52.5	10
a_4	Audi-Q2L e-tron 2019 (Zhi xiang)	SSUV	FAW-AUDI	23.73	13.9	17	100	265	150	39.7	10
a_5	Chevrolet-Changxun 2020(Xing yu)	CC	SHANGHAI GM	17.99	13.1	8	110	410	150	52.5	10
a_6	Xpeng G3 2020 520i (Zu xiang)	CSUV	XPENG	19.98	14.1	5.5	145	520	170	66.5	10
a_7	Volkswagen 2020 (Chi Pro)	CSUV	FAW-VOLKSWAGEN	16.88	13.6	5	100	270	150	40	12
a_8	Aion V2020 80 max	CSUV	GAC NE	23.96	14.8	9.5	135	600	175	80	15
a_9	GE3 2020 530 (Zun xiang)	SSUV	GAC NE	18.08	13.6	8	132	410	165	48.39	10
a_{10}	Buick-Velite 6 2020 (Zhi xiang) Plus	CC	SHANGHAI GM	18.98	13.1	8	110	410	150	52.5	10
a_{11}	Leapmoter S01 2019 380 Pro	SC	LEAPMOTOR	13.99	11.9	6	125	305	135	35.6	12
a_{12}	Aiways U5 2019 Pro	MSUV	AIWAYS	24.79	13.8	10.5	140	503	160	65	15
a_{13}	E-Golf 2018	CC	VOLKSWAGEN IMPORT	24.08	13.6	5	100	255	150	35.8	12
a_{14}	Bora 2020 (Shang) Pro	CC	FAW-VOLKSWAGEN	14.68	13.6	5	100	270	150	40	12
a_{15}	Toyota C-HR EV2020 (Zun gui tian chuang)	SSUV	GAC TOYOTA	24.98	13.1	6.5	150	400	160	54.3	10
a_{16}	Territory EV2019 (Xing ling)	CSUV	JMC	22.3	14.9	6.8	120	360	150	49.14	10
a_{17}	Leopaard CS9 2019 (Feng shang)	SSUV	LEOPAARD	20.58	14.56	8	90	360	130	50.38	10
a_{18}	Honda VE-1 2020 (Hao hua)	SSUV	GAC HONDA	17.98	14	9	120	401	140	53.6	10
a_{19}	Auchan X7 EV2020 (Ling hang) 405	CSUV	CHANGAN	17.99	14.9	9	150	405	160	59.9	12
a_{20}	Peuguot e2008 2020 3 D (Zhen shang)	SSUV	DONGFENG MOTOR	18.8	14.5	8	120	360	150	45.24	10
a_{21}	Venucia T60EV 2020 AI (Qi jian)	SSUV	DONGFENG MOTOR	18.18	14.7	6.8	120	442	125	60.7	12
a_{22}	Neta 2020 520 U	CSUV	HOZON MOTOR	19.98	14.6	10	150	500	155	68	12

technology development is towards saving energy and protecting the environment. For this purpose, many countries are developing energy-saving and new-energy vehicles that mainly include two types: battery-powered electric vehicles (BEVs) and plug-in hybrid electric vehicles (PHEVs). According to the research of global electric vehicle (EV) market, Canalys estimates that 6.5 million electric vehicles (EVs) were sold in 2021 all over the world (see https://www.canalys.com/newsroom/global-electric-vehicle-market-2021 for more detail). Meanwhile, over 3.2 million EVs were sold in Mainland China in 2021, which accounts for 50 percent of global EVs sales, 2 million more than in 2020. From data reported by Intelligent Electric Vehicle Professional Committee of China Electronics Chamber of Commerce (see http://www.cecc.org.cn/car for more detail), in 2021, 2.36 million BEVs were sold, which occupies 11.37% of new energy vehicles sales. Therefore, there is a trend towards selecting a BEV for an ordinary resident. To actively respond to country's call and satisfy the life demands, a customer (decision-maker) plans to purchase a BEV considering a budget of 150–300 thousand RMB. Because the number of passengers who will use the selected car is greater than 4 and less than 7, this customer wants to select a BEV from these five types: compact car (CC), small car (SC), compact sports utility vehicle (CSUV), small sports utility vehicle (SSUV), and medium sports utility vehicle (MSUV). According to the consumer satisfaction report about China's new energy vehicles in 2020 and customers' purchase intention studies, eight criteria are measured over three dimensions as shown in Table 4. The customer would like to consider the full list of criteria for the assessment. From each manufacturer, the values for 22 considered BEVs (data from https://www.maiche.com) are collected: economic data, automobile performance data, and service availability data, which are specified in Table 5.

To better analyze the performance of these new-energy cars, a customer uses a professional consultation about the relationships among criteria in this field. The criteria interaction matrix is shown as follows:

$$D = \begin{array}{c} \\ c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \\ c_7 \\ c_8 \end{array} \begin{array}{c} \begin{array}{cccccccc} c_1 & c_2 & c_3 & c_4 & c_5 & c_6 & c_7 & c_8 \end{array} \\ \begin{bmatrix} / & + & u' & - & - & - & - & \Delta \\ + & / & \Delta & + & + & \Delta & \Delta & \Delta \\ u' & \Delta & / & \Delta & \Delta & \Delta & - & \Delta \\ - & + & \Delta & / & - & - & \Delta & \Delta \\ - & + & \Delta & - & / & \Delta & - & \Delta \\ - & \Delta & \Delta & - & \Delta & / & \Delta & \Delta \\ - & \Delta & - & \Delta & - & \Delta & / & \Delta \\ \Delta & \Delta & \Delta & \Delta & \Delta & \Delta & \Delta & / \end{bmatrix} \end{array}.$$

6.2. Criteria relative importance

Based on the gathered information given in Section 6.1, the decision-maker adopts the nonadditive BWM and its linear model to make a selection. First, the reference levels under each criterion are given in Table 6, without loss of generality, $v(l_j) = 0$ and $v(h_j) = 1$. Then, the original data are transformed into normalized values using Equation (19).

Following the steps of the nonadditive BWM, the decision-maker first determines the best criterion c_1 and the worst criterion c_8 based on personal preferences. Then, pairwise comparisons are provided based on these two reference criteria shown in Table 7. For the given input pairwise comparisons, the input-based consistency ratio is computed as 0.0694 using Equations (3) and (4), which is less than its associated threshold of 0.3620 (see Table 3 in Liang et al. (2020) for the combination of 8 criteria and 9 scale). Hence, the consistency of input pairwise comparisons satisfies the requirement. Based on Equation (17), we establish the following optimization model:

$$\left\{\begin{array}{l} \min \xi \\ s.t. \left\{\begin{array}{l} m(\emptyset) = 0; \sum\limits_{c_j \in C} m_j + \sum\limits_{c_i, c_j \in C} m_{ij} = 1; \\ m_j + \sum\limits_{c_i \in T} m_{ij} \geq 0, i,j = 1,2,\ldots,8 \text{ and } T \subseteq C\backslash\{c_j\}, T \neq \emptyset; \\ I_{ij} = m_{ij}, i \neq j, i,j = 1,2,\ldots,8; I_j = m_j + \sum\limits_{c_j \in C\backslash\{c_i\}} \frac{m_{ij}}{2}, i,j = 1,2,\ldots,8; \\ \left|\frac{I_1}{I_2} - 3\right| \leq \xi; \left|\frac{I_1}{I_3} - 4\right| \leq \xi; \left|\frac{I_1}{I_4} - 7\right| \leq \xi; \left|\frac{I_1}{I_5} - 3\right| \leq \xi; \\ \left|\frac{I_1}{I_6} - 6\right| \leq \xi; \left|\frac{I_1}{I_7} - 3\right| \leq \xi; \left|\frac{I_1}{I_8} - 9\right| \leq \xi; \left|\frac{I_2}{I_8} - 3\right| \leq \xi; \\ \left|\frac{I_3}{I_8} - 3\right| \leq \xi; \left|\frac{I_4}{I_8} - 2\right| \leq \xi; \left|\frac{I_5}{I_8} - 2\right| \leq \xi; \left|\frac{I_6}{I_8} - 2\right| \leq \xi; \left|\frac{I_7}{I_8} - 3\right| \leq \xi; \\ I_{12} > 0; I_{13} \in [-1,1]; I_{14} < 0; I_{15} < 0; I_{16} < 0; I_{17} < 0; I_{18} = 0; \\ I_{23} = 0; I_{24} > 0; I_{25} > 0; I_{26} = 0; I_{27} = 0; I_{28} = 0; I_{34} = 0; \\ I_{35} = 0; I_{36} = 0; I_{37} < 0; I_{38} = 0; I_{45} < 0; I_{46} < 0; I_{47} = 0; \\ I_{48} = 0; I_{56} = 0; I_{57} < 0; I_{58} = 0; I_{67} = 0; I_{68} = 0; I_{78} = 0. \end{array}\right. \end{array}\right.$$

Solving this model, we obtain the optimal value $\xi^* = 0.5714$, and further the output-based

Table 6. Reference levels for considered criteria.

Reference level	c_1	c_2	c_3	c_4	c_5	c_6	c_7	c_8
l_j	10	10	4	80	240	120	25	8
h_j	30	17	18	160	635	190	85	17

Table 7. Pairwise comparison vectors of best-to-others and others-to-worst.

Pairwise comparisons	c_1	c_2	c_3	c_4	c_5	c_6	c_7	c_8
Best criterion c_1	1	2	4	7	3	6	3	9
Worst criterion c_8	9	3	3	2	2	2	3	1

consistency ratio is 0.1093, which is less than its associated threshold of 0.4587 (see Table 4 in Liang et al. (2020)). Hence, the pairwise comparisons are sufficiently consistent. Owing that the optimal solutions are not unique, we choose a Pareto optimality, namely $I_1 = 0.3766$, $I_2 = 0.123$, $I_3 = 0.1087$, $I_4 = 0.0586$, $I_5 = 0.1054$, $I_6 = 0.0637$, $I_7 = 0.123$, $I_8 = 0.041$, $I_{12} = 0.0518$, $I_{13} = -0.2118$, $I_{14} = -0.0607$, $I_{15} = -0.0781$, $I_{16} = -0.0464$, $I_{17} = -0.0809$, $I_{18} = 0$, $I_{23} = 0$, $I_{24} = 0.0853$, $I_{25} = 0.0685$, $I_{26} = 0$, $I_{27} = 0$, $I_{28} = 0$, $I_{34} = 0$, $I_{35} = 0$, $I_{36} = 0$, $I_{37} = -0.0027$, $I_{38} = 0$, $I_{45} = -0.0482$, $I_{46} = -0.0339$, $I_{47} = 0$, $I_{48} = 0$, $I_{56} = 0$, $I_{57} = -0.0591$, $I_{58} = 0$, $I_{67} = 0$, $I_{68} = 0$, $I_{78} = 0$, $m_1 = 0.5897$, $m_2 = 0.0203$, $m_3 = 0.2158$, $m_4 = 0.0873$, $m_5 = 0.1639$, $m_6 = 0.1039$, $m_7 = 0.1943$, $m_8 = 0.0410$. Based on the feasible space, we employ SMAA to derive the ranking of alternatives using 1000 random optimal solutions, and the result is shown in Figure 1. The ranking of alternatives is $a_{11} \succ a_{14} \succ a_7 \succ a_{13} \succ a_5 \succ a_{17} \succ a_{10} \succ a_{21} \succ a_{20} \succ a_9 \succ a_{18} \succ a_1 \succ a_{16} \succ a_4 \succ a_{19} \succ a_{15} \succ a_6 \succ a_2 \succ a_{22} \succ a_3 \succ a_{12} \succ a_8$. Consequently, the final new-energy purchase is a_{11} Leapmoter S01 2019 380 Pro.

Using the linear nonadditive BWM, we obtain the optimal solution as $I_1 = 0.3645$, $I_2 = 0.1308$, $I_3 = 0.0981$, $I_4 = 0.0561$, $I_5 = 0.1122$, $I_6 = 0.0654$, $I_7 = 0.1308$, $I_8 = 0.0421$, $I_{12} = 0.0435$, $I_{13} = 0.1340$, $I_{14} = -0.0787$, $I_{15} = -0.1257$, $I_{16} = -0.0563$, $I_{17} = -0.0521$, $I_{18} = 0$, $I_{23} = 0$, $I_{24} = 0.1109$, $I_{25} = 0.0709$, $I_{26} = 0$, $I_{27} = 0$, $I_{28} = 0$, $I_{34} = 0$, $I_{35} = 0$, $I_{36} = 0$, $I_{37} = -0.1009$, $I_{38} = 0$, $I_{45} = -0.0516$, $I_{46} = -0.0253$, $I_{47} = 0$, $I_{48} = 0$, $I_{56} = 0$, $I_{57} = -0.0315$, $I_{58} = 0$, $I_{67} = 0$, $I_{68} = 0$, $I_{78} = 0$, $m_1 = 0.4322$, $m_2 = 0.0182$, $m_3 = 0.0816$, $m_4 = 0.0784$, $m_5 = 0.1811$, $m_6 = 0.1062$, $m_7 = 0.2231$, $m_8 = 0.0421$. The ranking of alternatives using this linear model is $a_{11} \succ a_{14} \succ a_7 \succ a_5 \succ a_{13} \succ a_{18} \succ a_{10} \succ a_9 \succ a_{17} \succ a_1 \succ a_{21} \succ a_{20} \succ a_4 \succ a_{19} \succ a_{16} \succ a_6 \succ a_2 \succ a_{22} \succ a_{15} \succ a_3 \succ a_{12} \succ a_8$. From the computational results of nonadditive BWM and its linear model, we can find that the difference of criteria weights is no more than 0.0267 and the ranking difference of alternatives is no more than 3. Based on both nonlinear and linear models, the optimal alternative is a_{11}.

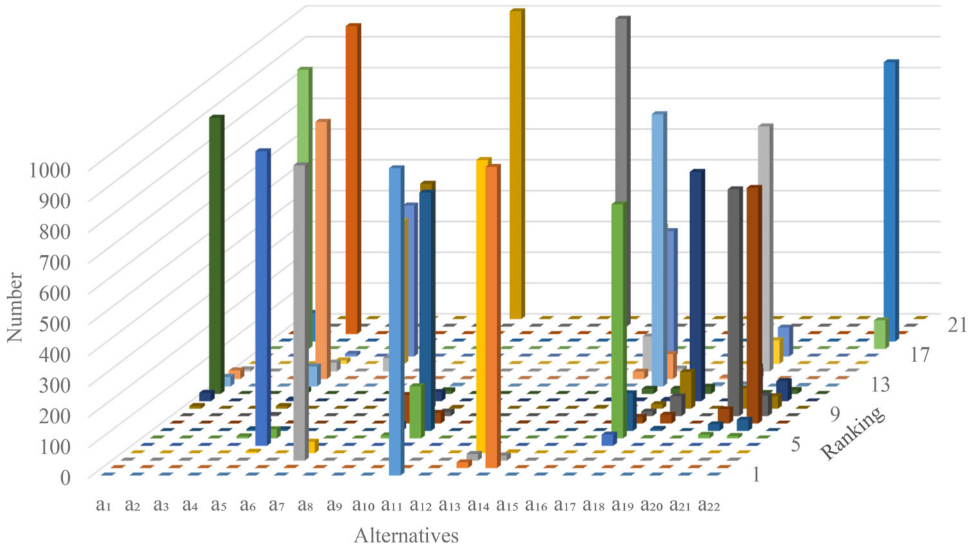

Figure 1. Ranking of alternatives in the optimal solution space.

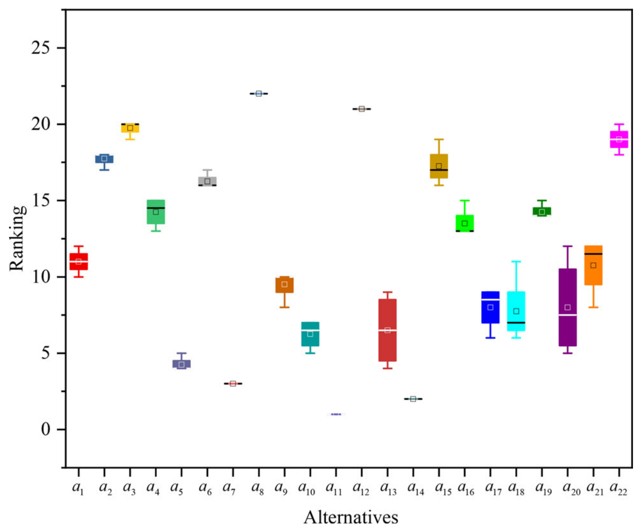

Figure 2. Comparison of ranking of alternatives using different methods.

6.3. Comparative analysis

To verify the effectiveness of nonlinear and linear nonadditive BWM, we compare them with the two existing variants of BWM: nonlinear BWM (Rezaei, 2015), and linear BWM (Rezaei, 2016), to obtain the weights of criteria and further employ additive value function to prioritise alternatives. In nonlinear BWM (Rezaei, 2015), without loss of generality, the centers of the optimal weight intervals of criteria are regarded as the final representative weights of criteria. Solving the same problem in subsection 5.2 with these four methods, we find that the criteria weights obtained by different methods have little difference. Specifically, the maximum difference is 0.0267 for criterion c_2 and the differences for criteria c_1, c_3, c_5, c_6, c_8 are less than 0.02.

Furthermore, to observe the ranking variation of alternatives obtained by various methods, the comparative results are provided in Figure 2. We find that the best and worst alternatives are identical in these methods, i.e., a_{11} and a_8. There are some identical rankings for a_7, a_8, a_{12}, a_{14} and little ranking differences for other alternatives. We can find that a_{20} have the maximum ranking difference, i.e., a_{20} ranks 12th in linear nonadditive BWM and linear BWM, 5th in existing additive BWM-based methods. This is because the alternative a_{20} has poor performance in terms of criteria c_1 and c_2, and these two criteria have larger weights and c_1 has negative interactions with c_3, c_4, c_5, c_6, c_7. Besides, a_{13} has big ranking difference, i.e., a_{13} ranks 4th in nonadditive BWM and 9th using nonlinear BWM. The reason why the proposed method leads to higher ranking is that a_{13} has good performances under the criteria c_3, c_4, c_5, and c_7, which brings about big contribution in negative interaction for c_3 and c_7, c_4 and c_5, c_5 and c_7. Additionally, the ranking differences of other alternatives are no more than 3. The comparative result manifests that the nonadditive BWM and the linear nonadditive BWM are effective to derive the criteria weights and further derives the ranking of alternatives using the Choquet integral.

7. Conclusions and future research

In this paper, we present a nonadditive BWM incorporating criteria interaction. The nonadditive BWM is developed using a nonlinear programming model, which yields the optimal criteria relative importance and degrees of criteria interaction. To rank the alternatives, the Choquet integral is adopted to aggregate the performance of alternatives and the result of nonadditive BWM. To obtain the unique optimal solution, we provide a linear model of nonadditive BWM. Eventually, the proposed nonadditive BWM and its linear model are applied to a new-energy vehicle selection problem to illustrate its effectiveness and practicality. Comparative analysis is conducted to illustrate the advantages of the proposed nonadditive BWM and linear nonadditive BWM.

The nonadditive BWM is a significant extension of the original BWM, where criteria interactions are considered. While the extended BWM in our study could be used for selection and ranking problems, developing an extension for sorting problems could be considered as an interesting direction for future research.

Disclosure statement

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