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Experimental Simulation of Larger Quantum Circuits with Fewer Superconducting Qubits

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Although near-term quantum computing devices are still limited by the quantity and quality of qubits in the so-called NISQ era, quantum computational advantage has been experimentally demonstrated. Moreover, hybrid architectures of quantum and classical computing have become the main paradigm for exhibiting NISQ applications, where low-depth quantum circuits are repeatedly applied. In order to further scale up the problem size solvable by the NISQ devices, it is also possible to reduce the number of physical qubits by “cutting” the quantum circuit into different pieces. In this work, we experimentally demonstrated a circuit-cutting method for simulating quantum circuits involving many logical qubits, using only a few physical superconducting qubits. By exploiting the symmetry of linear-cluster states, we can estimate the effectiveness of circuit-cutting for simulating up to 33-qubit linear-cluster states, using at most 4 physical qubits for each subcircuit. Specifically, for the 12-qubit linear-cluster state, we found that the experimental fidelity bound can reach as much as 0.734, which is about 19% higher than a direct implementation on the same 12-qubit superconducting processor. Our results indicate that circuit-cutting represents a feasible approach of simulating quantum circuits using much fewer qubits, while achieving a much higher circuit fidelity.

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Introduction.—Quantum computing offers potential speedups over classical computing on many applications, such as factoring [1–3], unstructured search [4], and quantum simulation [5–7]. However, these applications require quantum computers to be fault tolerant, which is still out of reach of the current quantum technology. Instead, we have just entered the *noisy intermediate-scale quantum* (NISQ) era [8–10], meaning that the number of physical qubits are sizable in terms of the computational space, but they are error prone or noisy. Recent experimental demonstrations of quantum computation involves

about 50 to 60 qubits [11–14]. Although in terms of the memory size, they might have already exceeded the limits of classical supercomputers, the intractable noisy gates limits the depth of quantum circuits running on current quantum devices, which constitutes a major obstacle to finding practical applications. Therefore, it is of practical interest to solve large problems with smaller quantum devices, even with a tradeoff of using more classical resources.

This topic can be roughly categorized into two branches; one is at the algorithmic level, and the other is at the circuit

level. The former is to decompose a large problem into smaller subproblems, each of which is solved by a small quantum computer. Examples include quantizing classical divide-and-conquer algorithms to solve combinatorial optimization problems [15,16], and Fujii *et al.*'s deep variational quantum eigensolver framework [17], which is suitable for simulating physical systems when interactions between subsystems are weak. Partially quantizing a tensor network may also fall into this category [18,19].

The circuit-level schemes intends to decompose a large quantum circuit into smaller pieces, implement each piece independently, and finally use classical computers to combine the results. For example, Bravyi *et al.* [20] discussed methods of using classical postprocessing to add virtual qubits for sparse circuits and Pauli-based computation. Mitarai and Fujii [21] proposed a method to add virtual two-qubit gates, which means that a remote two-qubit gate can be simulated by a quasiprobability decomposition of local single-qubit gates, thus cutting the large quantum circuit. Their work is for general quantum circuits and has been extended in a recent work [22] to allow decomposing nonlocal quantum channels into local ones. On the other hand, using the language of tensor network, Peng *et al.* [23] proposed a tomographylike circuit-cutting scheme, which is endowed with a rigorous analysis of the required quantum and classical resources to simulate general quantum circuits. The circuit-cutting scheme is further analyzed and improved in later works [24–27].

In this work, we experimentally implement a tomographylike circuit-cutting scheme to simulate large linear-cluster states. The purpose of the experiment is to demonstrate the applicability and the actual performance of the circuit-cutting scheme in a real experimental platform. Because of the symmetry in linear-cluster states, we only need to run subcircuits with at most 4 superconducting qubits and the simulated linear-cluster states scale up to 33 qubits. To analyze the performance, we use the stabilizer technique [28,29] to estimate the fidelity lower bound. Then, it is compared with the fidelity bound obtained in a previous work that prepared the 12-qubit state directly [30]. The circuit-cutting scheme achieves a fidelity bound 0.734 in the 12-qubit case, which is about 19% higher than the previous experiment. Our experimental result shows the promise that the circuit-cutting scheme might become a standard tool in NISQ applications.

Cutting large quantum circuits.—The basic idea is to cut a qubit wire and then simulate the propagation of quantum information by classical means. We illustrate this with a toy example in Fig. 1(a). First, observe that at the time slice of the cutting point (the red cross), the reduced density matrix of the first two qubits can be decomposed as,

$$\rho^{ab} = \frac{1}{2} \sum_{j=0}^3 \text{Tr}_b(\rho^{ab} \sigma_j^b) \otimes \sigma_j^a. \quad (1)$$

where we use superscripts to indicate the qubit labels and $\sigma_j \in \{I, X, Y, Z\}$. Each Pauli operator can be further decomposed into its eigenstates, e.g., $Z = |0\rangle\langle 0| - |1\rangle\langle 1|$. Note that the identity operator can be written as $I = |0\rangle\langle 0| + |1\rangle\langle 1|$, which can be combined with Z [24]. Then, we have,

$$\rho^{ab} = \sum_{i=1}^6 c_i \text{Tr}_b(\rho^{ab} O_i^b) \otimes \rho_i^a, \quad (2)$$

where the c_i , O_i , and $\rho_i := |\psi_i\rangle\langle\psi_i|$ are listed in Fig. 1(b). The partial trace operation in Eq. (2) can be interpreted as measuring O_i in the qubit b of subcircuit 1, and then ρ_i is prepared and passed as input to subcircuit 2.

Suppose that we are interested in measuring the expectation of $X \otimes Z \otimes X$ of the 3-qubit circuit, denoted as $\langle XZX \rangle$. In subcircuit 1, one needs to collect the expectation values of $X \otimes O_i$, defined by $E_i^{(1)} = \text{Tr}(\rho^{ab} X \otimes O_i)$. In subcircuit 2, one needs to collect the expectation values of $Z \otimes X$, denoted as $E_i^{(2)}$, from circuits with varying initial state $|\psi_i\rangle$ in the first qubit [see Fig. 1(a)]. Then, according to Eq. (2), $\langle XZX \rangle$ can be recovered by [23]

$$\langle XZX \rangle = \sum_{i=1}^6 c_i E_i^{(1)} E_i^{(2)}. \quad (3)$$

This circuit-cutting procedure works for any observable in the form $A \otimes B$, where A is an observable of the qubit a and B is an observable of the qubits b and c . We remark that the combination of expectation values is achieved with a classical computer. In this process, we do not create a 3-qubit entangled state; instead, the 3-qubit state is

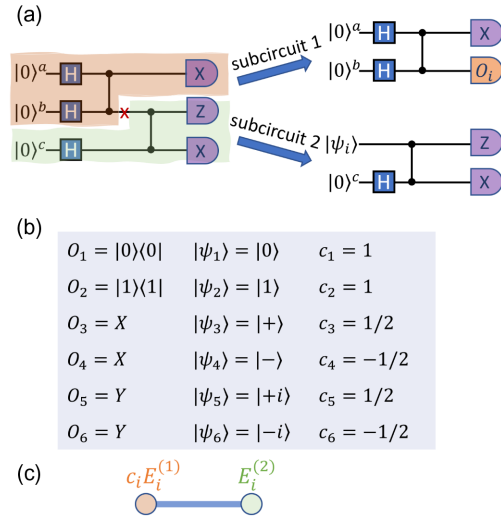


FIG. 1. (a) Example illustrating the circuit-cutting scheme. The quantum circuit on the left is cut at the red cross, and partitioned into two subcircuits. (b) The list of c_i , O_i and ρ_i for Eq. (2). (c) The tensor network representing the summation of Eq. (3).

simulated by a hybrid scheme of a 2-qubit quantum computer and a classical computer.

For more general and larger quantum circuits, one can apply this cutting scheme iteratively to multiple cutting points, to partition the whole circuit into several *disconnected* pieces of subcircuits. By running the subcircuits *independently*, and classically combining the subcircuit expectations with appropriate coefficients, one obtains expectations from the large quantum circuits. Moreover, those disconnected subcircuits can be viewed as nodes in a tensor network [23]. For example, the corresponding tensor network for Fig. 1(a) is a line with two nodes, and the edge has bond dimension 6, corresponding to the 6 terms in Eq. (3). The coefficient c_i can be absorbed into the node representing $E_i^{(1)}$ or $E_i^{(2)}$. Then, one can use tensor-network contraction to perform the combination to obtain quantities of the large circuit, with classical running time exponential in the treewidth of the tensor network [23].

To summarize, the protocol is as follows. (a) Identify appropriate cutting points to partition the large circuit into disconnected subcircuits. (b) Obtain the subcircuit expectations by enumerating the possible choices of $|\psi_i\rangle$ and O_i . (c) Construct a tensor network from these subcircuit expectations and coefficients c_i . (d) Contract the tensor network to obtain the expectation value with respect to the large circuit.

Linear-cluster states.—Cluster states are a family of highly entangled states, which can be used to achieve measurement-based quantum computation [31,32]. The linear-cluster state is a specific example of cluster states, where all qubits are aligned in one dimension. Explicitly, a linear-cluster state with n qubits can be expressed as

$$|\text{LC}_n\rangle = \left(\prod_{i=1}^{n-1} \text{CZ}^{i,i+1} \right) |+\rangle^{\otimes n}, \quad (4)$$

where the superscripts in the CZ gates indicate the qubits that they act on.

In this work, we experimentally simulate a 12-qubit linear-cluster state, with 4 qubits of a superconducting quantum processor, which is the same processor as in Ref. [30]. As in Fig. 2, there are 3 cutting points on the 12-qubit circuit, partitioning it into 4 subcircuits. The first 3 subcircuits are all in the form of subcircuit 1, while the last subcircuit is in the form of subcircuit 2. Note that the sequence of CZ gates on the left of Fig. 2 is chosen such that pieces 1–3 can be represented by the same subcircuit 1.

To compare the performance of the circuit cutting scheme with that of running the 12-qubit circuit directly, we need to estimate their fidelities. We follow the approach in Ref. [30], which uses techniques from entanglement detection in the stabilizer formalism [28,29]. Let $s_1 = X_1 Z_2$, $s_n = Z_{n-1} X_n$ and $s_i = Z_{i-1} X_i Z_{i+1}$ for $i \neq 1$ or n . It can be shown that a linear-cluster state is a stabilizer

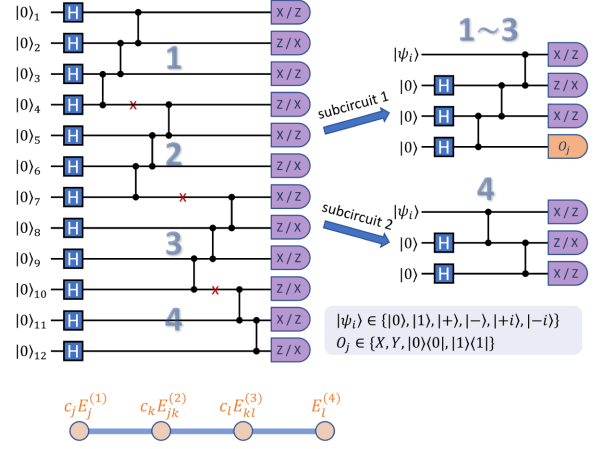


FIG. 2. Circuit-cutting scheme for the linear-cluster state. *Left.* A 12-qubit linear-cluster state, which is cut into 4 pieces. *Right.* The 12-qubit linear-cluster state can be simulated by combining measurement data from these two types of subcircuits. *Bottom.* The tensor network representing the classical combination of subcircuits.

state with a stabilizer group spanned by $\{s_1, \dots, s_n\}$, i.e., $s_i |\text{LC}_n\rangle = |\text{LC}_n\rangle$ for $i = 1, \dots, n$. Let

$$\text{ODD}_n := \prod_{i \text{ odd}} \frac{1 + s_i}{2}, \quad \text{EVEN}_n := \prod_{i \text{ even}} \frac{1 + s_i}{2}. \quad (5)$$

For a linear-cluster state, one has $|\text{LC}_n\rangle\langle\text{LC}_n| \geq \text{ODD}_n + \text{EVEN}_n - I$ [28] (Theorem 6). Therefore, for an unknown quantum state ρ , its fidelity relative to the linear-cluster state is lower bounded by

$$\text{Tr}(\rho |\text{LC}_n\rangle\langle\text{LC}_n|) \geq \text{Tr}(\rho \text{ODD}_n) + \text{Tr}(\rho \text{EVEN}_n) - 1, \quad (6)$$

which can be estimated by measuring ODD_n and EVEN_n . Observe that every term in the expansion of ODD_n can be measured in the basis $XZZZ \dots$, while every term in the expansion of EVEN_n can be measured in the basis $ZXZX \dots$. Therefore, to estimate the fidelity, one only needs to perform measurements in two bases. For simplicity, we will refer to them as XZ measurement and ZX measurement, respectively.

Below, we illustrate how to simulate the 12-qubit linear-cluster state with the circuit-cutting scheme. Suppose we want to obtain the expectation value $\langle P^{(1)} \otimes P^{(2)} \otimes P^{(3)} \otimes P^{(4)} \rangle$, where $P^{(i)}$ can be any 3-qubit observable of the i th 3-qubit group. In Fig. 2, we denote the final state of subcircuit 1 and 2 as $|\Phi_{1,i}\rangle$ and $|\Phi_{2,i}\rangle$, respectively, where the index i indicates one of the 6 states $|\psi_i\rangle$ in the first qubit. Then, define

$$\begin{aligned} E_j^{(1)} &:= \langle \Phi_{1,3} | P^{(1)} \otimes O_j | \Phi_{1,3} \rangle & E_{jk}^{(2)} &:= \langle \Phi_{1,j} | P^{(2)} \otimes O_k | \Phi_{1,j} \rangle \\ E_{kl}^{(3)} &:= \langle \Phi_{1,k} | P^{(3)} \otimes O_l | \Phi_{1,k} \rangle & E_l^{(4)} &:= \langle \Phi_{2,l} | P^{(4)} | \Phi_{2,l} \rangle \end{aligned} \quad (7)$$

to be the subcircuit expectations from pieces 1–4. According to the circuit-cutting scheme, we have

$$\langle P^{(1)} \otimes P^{(2)} \otimes P^{(3)} \otimes P^{(4)} \rangle = \sum_{j,k,l=1}^6 c_j c_k c_l E_j^{(1)} E_{jk}^{(2)} E_{kl}^{(3)} E_l^{(4)}, \quad (8)$$

where the coefficients c_i 's are shown in Fig. 1(b). Again, this summation can be viewed as tensor network contraction as in the bottom of Fig. 2.

The experimental procedure for estimating the fidelity is as follows. (a) Identify the observables in the expansion of ODD_{12} and EVEN_{12} . (b) For each observable, define $P^{(i)}$ for $i = 1, 2, 3, 4$. Measure $E_j^{(1)}$, $E_{jk}^{(2)}$, $E_{kl}^{(3)}$, and $E_l^{(4)}$, and use Eq. (8) to obtain the expectation value of that observable. (c) Calculate the fidelity lower bound according to Eq. (6). Note that this procedure can be easily generalized to larger linear-cluster states.

The expectation values in Eq. (8) can be obtained from subcircuits in Fig. 2. For subcircuit 1, we need to prepare the circuits with 6 different $|\psi_i\rangle$. The measurement bases for the first three qubits are XZX and ZXZ , and for the last qubit are X , Y or Z ; the expectation value of $|0\rangle\langle 0|$ or $|1\rangle\langle 1|$ can be obtained from Z measurement. Therefore, we need to implement $6 \times 2 \times 3 = 36$ different circuits in the form of subcircuit 1. As for subcircuit 2, similar argument shows that we need to implement 12 different circuits. Thus, a total of 48 subcircuits needs to be run [33].

Experiment.—To verify the feasibility and evaluate the actual performance of the scheme in the experiment, we run the subcircuits in Fig. 2 on a 12-qubit superconducting quantum processor. As shown in Fig. 3(a), the qubits are arranged in a one-dimensional chain. Each qubit has two control lines to provide full control of the qubit: a microwave XY control line to drive excitations between $|0\rangle$ and $|1\rangle$, and a magnetic flux bias line to tune the qubit resonance frequency. As the near-neighbor qubits are capacitively coupled, the fast adiabatic CZ gates [34,35] can be applied. The measurements of qubit are done through dispersively coupling to a readout resonator. We choose four adjacent qubits from a 12-qubit superconducting quantum processor to implement the experiments. The average performance of the chosen qubits are $T_1 \approx 36.1 \mu\text{s}$, $T_2^* \approx 4.3 \mu\text{s}$, single-qubit gate fidelity $\approx 99.93\%$ and CZ gate fidelity $\approx 98.5\%$. More detailed data are shown in the Supplemental Material [33].

All the experimental results are processed using the transition matrix error mitigation (TMEM) method [36,37], to suppress the readout noise. However, negative entries may appear in the probability distributions of the subcircuits after the TMEM. To make these distributions physical, we first transform those probability distributions into diagonal operators, and then use the maximum likelihood method to find a density operator that is the closest

to them [38,39]. The final distributions of the subcircuits are then extracted from these density operators. Before and after the experiment of circuit cutting, additional quantum state tomography on the final state of the circuit is performed to evaluate the performance of the experiments. The average fidelity of the 36 subcircuits in the form of subcircuit 1 is 0.944, and the average fidelity of the 12 subcircuits in the form of subcircuit 2 is 0.955, showing the high quality of the experiments.

As a warm-up, we show how to estimate the fidelity lower bounds for the 4-qubit and 3-qubit linear-cluster states. Note that if we take $|\psi_i\rangle = |+\rangle$ for the subcircuits, then they correspond to a 4-qubit and 3-qubit linear-cluster state, respectively. The corresponding probability distributions from the XZ measurement are shown in Fig. 3(b), which also includes the theoretical distributions for a comparison; see Supplemental Material [33] for the probability distributions from ZX measurement. From these distributions, one can obtain the expectations of terms in the ODD and EVEN operators of the 4-qubit and 3-qubit

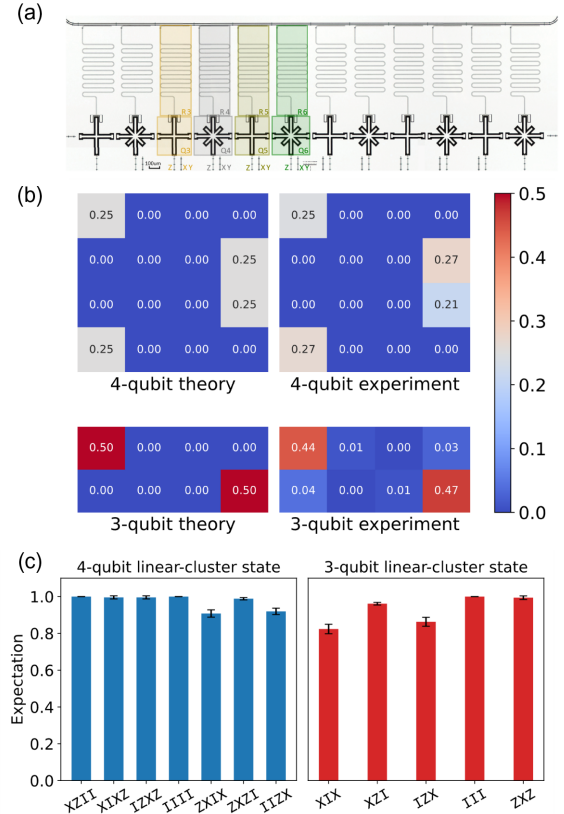


FIG. 3. (a) Schematic of the 12-qubit superconducting processor, where we used Q3 to Q6 for the circuit-cutting experiment. (b) The output distributions from XZ measurement of the 4-qubit and 3-qubit linear-cluster states; see Supplemental Material [33] for the labeling of each cell. (c) Expectation from XZ and ZX measurements of the 4-qubit and 3-qubit linear-cluster states. Ideal values are one. The error bars are due to the repeated experiments.

LC states, as shown in Fig. 3(c). The fidelity lower bound then follows from these expectations according to Eq. (6), which is 0.952 and 0.909 for the 4-qubit and 3-qubit LC states, respectively [33]. These bounds match the average fidelity (of all 36 circuits for subcircuit 1 and 12 circuits for subcircuit 2) from quantum state tomography.

We now turn our discussion to simulating large linear-cluster state with the circuit-cutting scheme. With the measurement data from the subcircuits, one can simulate larger linear-cluster states, and the fidelity bounds can be derived with similar procedures. Figure 4(a) presents the expectations of terms in ODD_{12} for the 12-qubit state obtained by the circuit-cutting scheme and a direct implementation, which is an analog of Fig. 3(c). Those expectations are from the XZ measurement and there are 2^6 expectations in total. The blue bars are reconstructed from the circuit-cutting scheme, while the orange bars are from the experimental data in Ref. [30]. Each bar corresponds to one specific terms in the expansion of ODD_{12} and the ideal value is one; we leave the concrete labelling and similar data for EVEN_{12} in Supplemental Material [33]. We remark that the distributions from the 12-qubit experiment are also processed with the same procedure (the fidelity bound after processing is 0.615), i.e., TMEM followed by a maximum likelihood method, for a fair comparison. From these expectations, we can similarly use Eq. (6) to estimate the fidelity. The estimated fidelity bound from the circuit-cutting scheme is 0.734, about 19% higher than that from the experiment of Ref. [30]. Note that the experiment of Ref. [30] implemented CZ gates in parallel, which will incur more severe crosstalk errors compared to our current

implementation, where CZ gates are applied individually (one for each layer; see the right of Fig. 2). Moreover, smaller circuits are easier to calibrate and control. Therefore, the circuit-cutting experiment achieves a better fidelity bound than Ref. [30]. The tradeoff for the circuit-cutting scheme is the increase of both the quantum and the classical running time [33].

Moreover, the symmetry in linear-cluster states allows us to reuse the measurement data from subcircuit 1 to simulate larger linear-cluster states, at a cost of increasing overhead in classical postprocessing. Specifically, we need to add more internal nodes to the tensor network in Fig. 2, to represent larger circuits (5 nodes for 15 qubits, 6 nodes for 18 qubits and so on). This allows us to simulate linear-cluster states of size $6 + 3k$, where k is a positive integer. We need to contract a longer chain to obtain one expectations of the large circuit, and there will be more expectations to be computed in order to obtain the fidelity lower bound. The obtained fidelity bound is expected to decay as the number of qubits increases, since the error accumulates in the classical postprocessing. The fidelity decay and classical postprocessing time are shown in Fig. 4(b). Here, the classical postprocessing is done on a conventional laptop, and the processing time shows the running time of the program for calculating the fidelity lower bound of larger circuits [40].

Discussion.—In this work, we experimentally demonstrate a circuit-cutting scheme and simulate larger linear-cluster state with size scaling up to 33 qubits, using at most 4 qubits. In the case of 12 qubits, we achieve a higher fidelity compared to that of a previous work that prepared the 12-qubit state directly [30], giving supportive evidence to the applicability of the circuit-cutting scheme.

Simulating large quantum circuits with small quantum devices is a promising direction in the NISQ era. Currently, there exist several circuit-cutting schemes [20–23]; it is necessary to further perform experimental benchmarking on these schemes, in order to evaluate their applicability in practice. On the other hand, although circuit-cutting schemes provide systematic methods to cutting quantum circuits into smaller pieces, to the best of our knowledge, there is no general method for determining the optimal cutting points. Therefore, we believe that the potential of circuit-cutting has not yet been fully explored.

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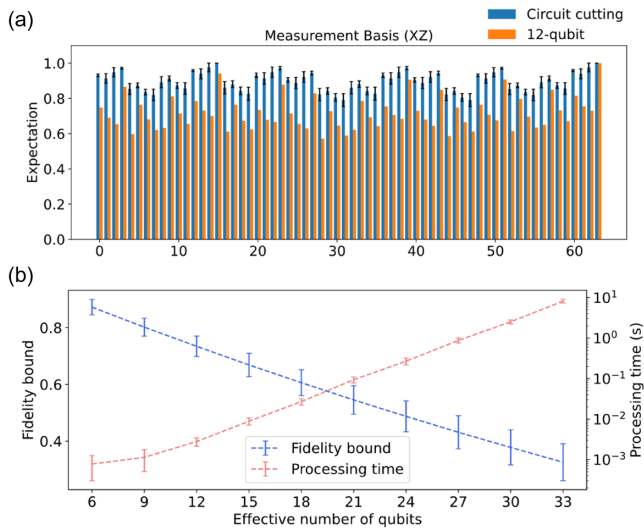


FIG. 4. (a) Comparison of the expectations with XZ measurement obtained from the 12-qubit circuit [30] (orange) and from the circuit-cutting scheme (blue). (b) Fidelity bound (blue) and processing time (red) for simulating larger linear-cluster state using the same experimental data. The error bars are due to repeated experiments.

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