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Low Complexity Single-Snapshot DoA Estimation via Bayesian Compressive Sensing

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***Abstract*—The problem of single-snapshot direction of arrival (DoA) estimation with antenna arrays has been considered. A sectorized approach based on Bayesian Compressive Sensing (BCS) has been proposed. In this method, the angular space is discretized, defining many non-overlapping small grids which cover the desired large angular space. First, a BCS estimation is run in each of the sectors to estimate the DoA of the signals. Then, a second stage is performed to correct the inconsistencies at the edges due to signal leaking between sectors. The performance of the method has been analyzed via extensive Monte-Carlo simulations in which the number of targets, their Radar Cross Section (RCS), and their location have been varied in a large extent, and the targets were observed by a Frequency Modulated Continuous Wave (FMCW) radar with an 86-element Uniform Linear Array (ULA). The results are compared with state-of-the-art methods in terms of estimation accuracy and resolution. Moreover, an analysis of the computational time, critical for many real-time applications, is presented, which shows a reduction of 20 times in the computational time compared with the standard BCS. Finally, the method has also been validated using experimental data collected with a commercial automotive radar.**

***Index Terms*—Bayesian compressive sensing (BCS), direction-of-arrival estimation (DoA), antenna arrays.**

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I. Introduction

The estimation of the Direction of Arrival (DoA) of signals impinging on an antenna array is a widely explored topic, and it is essential in fields such as wireless communications and remote sensing. In general, the accuracy and the resolution of the DoA estimation are directly related to the array aperture. For a uniform linear array (ULA), the performance is related to the number of array elements. However, in many applications such as automotive radar, the size of the system is a critical constraint [1]. For this reason, efficient methods for DoA estimation with a limited number of elements are needed.

Many algorithms have been developed in the last decades, where the most commonly used ones are the multiple signal classification (MUSIC) or the signal estimation parameter via rotational invariance technique (ESPRIT). However, one of the main drawbacks of these algorithms is the need for the a priori knowledge of the number of signals impinging on the array, and the need for multiple snapshots of the same scene for an accurate covariance matrix estimation. From an automotive radar perspective, both constraints present a problem. First, the number of targets (i.e., signals) present on the scene is unknown and stochastic. Second, targets can move at high velocities; thus, coherently processing multiple snapshots is not always possible. In this single-snapshot scenario, the estimation of the inverse of the noise covariance matrix cannot be computed since the estimate is rank deficient.

In recent years, a new family of approaches in DoA estimation has been explored, which exploits the fact that the signals are intrinsically sparse in the spatial domain, i.e., only a few signals or targets are present in the scene. These methods use the compressive sensing (CS) theory [2]–[4]. However, the main drawback of these methods is that the sensing matrix they use must satisfy the restricted isometry property (RIP) [5] to guarantee the correct estimations. In practice, the sensing matrix for the DoA estimation problem depends on the physical array topology, and the satisfaction of the RIP is usually not achieved. Hence, a new family of algorithms has been developed, which cast the deterministic problem in a Bayesian probabilistic framework, leading to the so-called Bayesian compressive sensing (BCS) [6]. BCS is usually solved efficiently with the relevance vector machine (RVM) and has been proved effective in DoA estimation, using both the standard BCS and the multi-task BCS (MT-BCS) [7] by processing multiple snapshots. In any CS-based DoA estimation algorithm, the estimation performance depends on how fine the space discretization is done: the finer the grid, the better the performance, until a point where the sensing matrix starts to be coherent, and the performance degrades. However, increasing the search space also greatly impacts the computational cost of the algorithms, since most of them have an exponential computational complexity with respect to the grid size. Thus, different strategies have been proposed to overcome this limitation. For example, in [8], the authors propose a multi-scaling approach, where a coarse grid is defined first. Then, by using the estimation uncertainty provided by BCS, a refined grid is formed in the regions of interest.

Many applications require a very fast DoA estimation; for example, current automotive radars provide a whole radar cube (range, Doppler, and angle estimation) every 50ms. The current trend in automotive radars is to include more and more antennas to increase the angular resolution; thus, the discretization grid for DoA estimation needs to become finer. Moreover, the DoA estimation must be implemented in embedded hardware with low computational resources. For this reason, the computational complexity of the DoA algorithms should not be overlooked, and new low-complexity methods are needed to deal with the increasing number of antennas.

This paper proposes a low complexity DoA estimation method based on BCS by using multiple non-overlapping small grids. The proposed method is verified using simulated and measured data, analyzing the estimation accuracy, the resolution capabilities, and the computational cost. Moreover, the performance is compared with different state-of-the-art methods, namely Fourier beamformer, single snapshot MUSIC algorithm [9], and the BCS implementation from [10].

The rest of the paper is organized as follows. In Section II, the mathematical formulation of the problem is provided, the BCS theory is briefly introduced, and the proposed method is presented. In Section III, the performance of the proposed method and state-of-the-art methods are analyzed using 7000 scenes simulated in a Monte Carlo approach with varying targets' characteristics, and a case study with experimentally measured data is presented. Finally, conclusions are presented in Section IV.

II. Mathematical Formulation

A. Signal Model

The basic principle of DoA estimation relies on the extra distance a signal must travel to reach different elements in the antenna array. Consider K narrowband signals impinging on a ULA with $n = 1, \ldots, N$ elements with d spacing between them. Without loss of generality, the single snapshot baseband received signal can be written in matrix form as:

$$y = \tilde{\mathrm{A}}(\theta)\tilde{x} + e, \quad (1)$$

where $y \in \mathbb{C}^{N\times 1}$ is the complex sample vector, $x \in \mathbb{C}^{K\times 1}$ is a vector that accounts for all channel losses, antenna radiation pattern and target radar cross section (RCS), $e \in \mathbb{C}^{N\times 1}$ is the additive complex Gaussian noise, and $\tilde{\mathrm{A}}(\theta) \in \mathbb{C}^{N\times K}$ is the measurement matrix formed by the virtual steering vectors pointing to K targets as:

$$\tilde{\mathrm{A}}(\theta) = [v(\theta_1), ..., v(\theta_K)], \quad (2)$$

$$v(\theta_k) = [1, e^{j\frac{2\pi d}{\lambda}\sin\theta_k}, ..., e^{j\frac{2\pi d}{\lambda}(N-1)\sin\theta_k}], \quad (3)$$

being λ the signal wavelength, and θ_k the DoA corresponding to the k^{th} signal and the variable to be estimated. In classical signal processing, $\tilde{\mathrm{A}}$ and $\tilde{x}$ are unknowns, and the goal is to estimate them with the information contained in the measurement y. Note that (2) is nonlinear, since the unknowns θ_k are present in the exponential terms of the steering vectors. However, this problem can be tackled from another perspective, discretizing the angular space in a grid of M cells and assuming the DoA of the signals lie on the grid. Thus, a new measurement matrix $\mathrm{A} \in \mathbb{C}^{N\times M}$ can be formed with M steering vectors pointing to the grid, the unknowns in this case being the coefficients of the sparse vector x. Knowing that only a few signals are present in the scene (i.e., $M \gg K$) and that the problem is now linear, it can be solved with CS techniques. Thus, a sparse estimate of x can be found solving the following constrained optimization problem:

$$\min_x ||x||_1 \quad s.t. \quad \mathrm{A}x = y. \quad (4)$$

Different CS-beamforming algorithms have been developed to solve (4), and in [11], the performance of many of them is analyzed in terms of estimation accuracy, detection, and resolution capabilities. However, in order to guarantee reliable estimations, the measurement matrix A must satisfy the restricted isometry property (RIP). In this case, A is defined by the array topology; thus, the fulfillment of RIP is not always guaranteed. To overcome this limitation, some researchers have proposed BCS-based DoA estimation.

B. Bayesian Compressive Sensing

It is not the aim of this paper to present a comprehensive explanation of the underlying theory of BCS, and a mathematical formulation of BCS-based DoA principles can be found in [10]. The core concept relies on a new formulation of the problem from a probabilistic approach, where the full posterior density function for x is sought. To do this, a hierarchical sparseness-promoting prior is imposed on x, and the posterior is found by means of the relevance vector machine (RVM). Following [10], the BCS estimate can be written as:

$$\hat{x}_{\text{BCS}} = \frac{1}{\sigma^2}\left(\frac{\hat{\mathrm{A}}(\theta)^T\hat{\mathrm{A}}(\theta)}{\sigma^2} + diag(\alpha)\right)^{-1}\hat{\mathrm{A}}(\theta)^T\hat{y}, \quad (5)$$

being $\hat{\mathrm{A}}(\theta), \hat{x}_{\text{BCS}}, \hat{y}$ the commonly used real and imaginary expansions of $A(\theta), x_{\text{BCS}}, y$ to yield a real-valued problem suitable for BCS [12]. Notice that two hyperparameters, σ^2 and α, have to be estimated before obtaining x_{BCS}, usually by means of an expectation maximization algorithm such as:

$$\mathcal{L}(\sigma^2, \alpha) = -\frac{1}{2}[2M\log 2\pi + \log|C_{\text{BCS}}| + \hat{y}^T C_{\text{BCS}}^{-1}\hat{y}], \quad (6)$$

where $C_{\text{BCS}} = \sigma^2 I + \hat{\mathrm{A}}(\theta)diag(\alpha)^{-1}\hat{\mathrm{A}}(\theta)^T$.

In [7] a sequential method is derived from the fast RVM algorithm [13]. This algorithm works in a constructive manner, adding, deleting or re-estimating a relevant vector in each iteration. Thus, the complexity of the algorithm is related to the number of relevant vectors m (i.e., the number of targets present in the scene), and it is proved to be $\mathcal{O}(Mm^2)$. This algorithm is the one used in the rest of the paper.

C. Sectorized BCS

Automotive radars need to perform many DoA estimations per second, usually in relative low resource hardware. Thus, very low complexity algorithms are required without reducing the performance in angular resolution. This work proposes an adaptation of the fast BCS developed in [7], where the entire angular space is divided into smaller sections, and independent BCS estimations are performed. Then, a common stage is used to combine the estimations per sector into a global DoA estimate, where signal leakage between sectors is resolved.

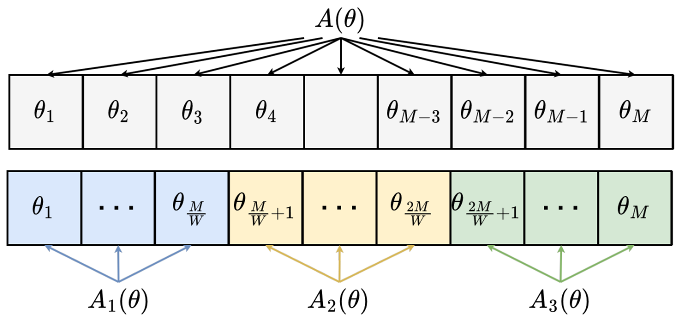

Fig. 1. Schematic of the DoA angle discretization. In the upper part, a large grid covers the M possible DoAs in a conventional approach. In the lower part, three non-overlapping smaller grids cover the same M possible DoAs with the proposed approach.

The first step of the proposed approach is to build W new sensing matrices A_w, being W the number of sectors covering the same angular space. A visual representation of these sensing matrices can be seen in Fig. 1. Then, W BCS algorithms are run in parallel, and the output of each of them is concatenated to form a single output. The reduction in computation cost in this step comes from two main reasons. First, because of the quadratic dependency of the complexity with the number of targets and knowing that the sum of squares is less than the square of sums. Thus:

$$\mathcal{O}(m_1^2+m_2^2+...+m_W^2) < \mathcal{O}((m_1+m_2+...+m_W)^2), \quad (7)$$

where m_i is the number of targets in sector i. Second, due to the sparse nature of the scene (i.e., there are only a few targets in each range-Doppler cell), many sectors will contain no targets, leading to a speed-up in the processing. However, signals cannot be physically separated in sectors, and thus, leakage between sectors may happen when the DoA to be estimated are close to the edge between sectors. As an example of this phenomenon, Fig. 2 and Fig. 3 are presented. In this case, the discretization grid is 0.5°, and the sectors are defined such as the last cell of one of them is located at -0.5°, and the first cell of the adjacent one is at 0°. Two cases are analyzed, where the targets are placed on-grid and off-grid (i.e., at 0° and at -0.2°). As can be seen in the upper plot of both figures, many targets (i.e., relevance vectors) are generated due to the signal leaking. Thus, a correction stage is needed to compensate for this effect.

After the W BCS estimations have been performed, a new BCS algorithm is run but in this case using the full $\mathrm{A}(\theta)$. However, only those relevance vectors generated from the previous stage are used as candidate basis vectors. This implies that only the conditions in step 7 and 8 of the algorithm in [13] need to be evaluated, to asses if a relevance vector should be removed. Therefore, the computational cost of this step is very small. The middle plots of Fig. 2 and Fig. 3 show the DoA estimation after the proposed correction step, while the lower plots show the estimation with the standard BCS for comparison. As it can be seen, both estimations are very close, but with the proposed method being much faster.

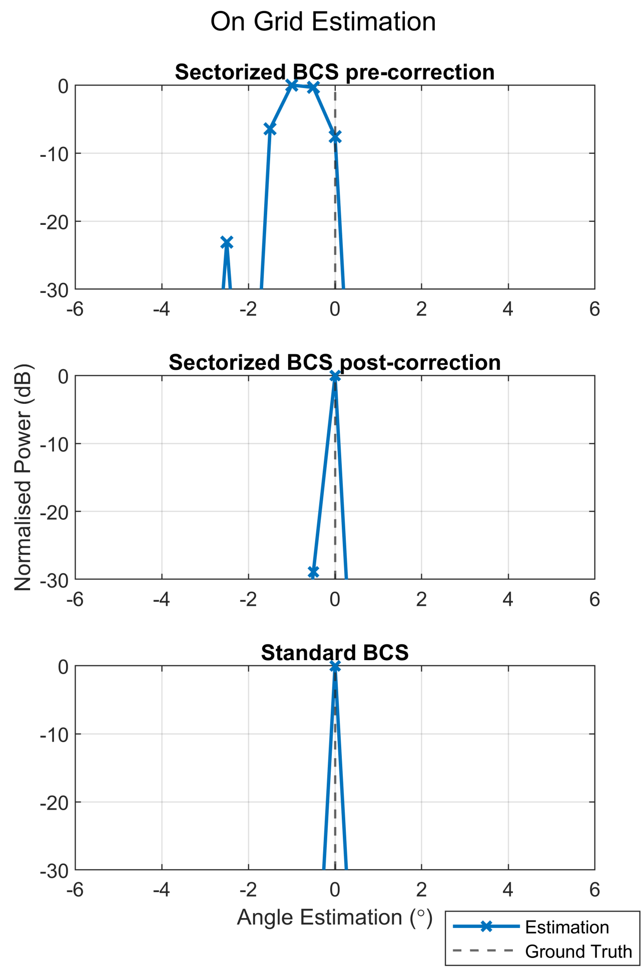

Fig. 2. DoA estimation for the on-grid case at the boundary between sectors. On the upper plot, the estimation before the correction stage. The middle plot shows the result after the correction stage, and the lower plot the standard BCS for comparison.

It is important to note that, unlike for BCS, this approach cannot be applied to most of the DoA estimation methods since their computational complexity is related with the grid size (e.g., the complexity of an FFT is $\mathcal{O}(MlogM)$ while the complexity of the MUSIC algorithm is $\mathcal{O}(M^3)$). The next section presents an analysis to quantify the reduction in computational complexity, as well as some performance analysis.

III. Results

A. Simulation Results

To evaluate the performance of the proposed approach, a ULA with $\lambda/2$ separation between elements has been simulated, with isotropic radiation patterns for the $y > 0$ half-plane. Seven thousand scenes have been generated in a Monte Carlo fashion, where a different number of point targets

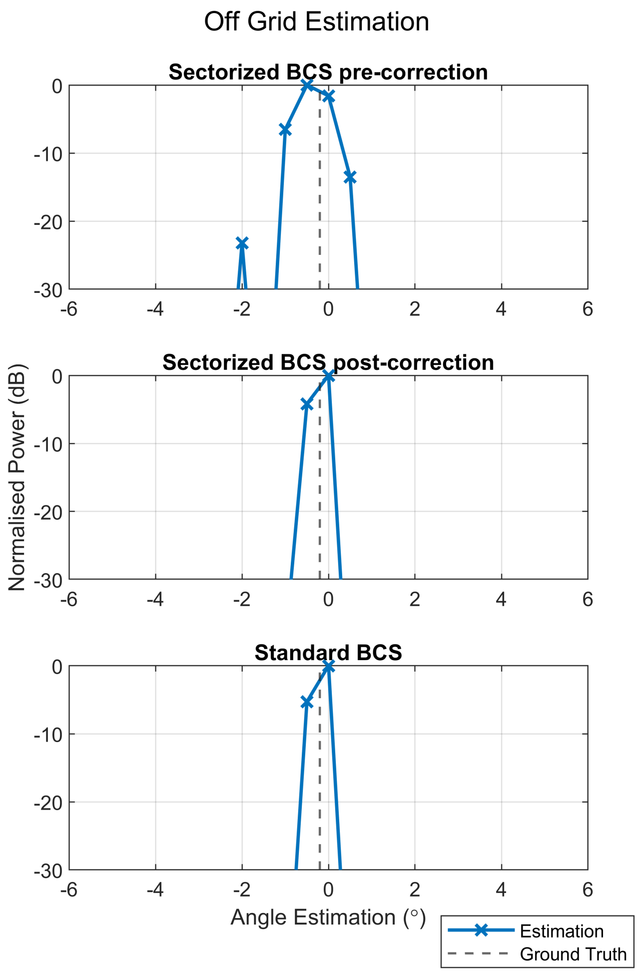

Fig. 3. DoA estimation for the off-grid case at the boundary between sectors. On the upper plot, the estimation before the correction stage. The middle plot shows the result after the correction stage, and the lower plot the standard BCS for comparison.

TABLE I
MONTE CARLO SIMULATION PARAMETERS

Number of scenes	7000
Number of targets per scene	$X \sim \mathcal{U}(1, 10)$
Position of the targets [o]	$X \sim \mathcal{U}(-70, 70)$
RCS of each target [dBsm]	$X \sim \mathcal{U}(0, 10)$
SNR in the scene [dB]	$X \sim (-5, 0, 5, 10, 15, 20, 25)$
Number elements in array	86

have been placed at different azimuth angles. The RCS and location of these targets have been sampled from a uniform distribution. Table I summarizes the parameters of the Monte Carlo simulation.

First, an evaluation of the estimation accuracy has been performed. To this end, several DoA estimators are compared:

1) The single-snapshot MUSIC (SS-MUSIC) estimator from [9] with a grid size of 0.5°.
2) The conventional Fourier beamforming with a 0.5° grid.
3) The standard BCS with a grid size of 0.5° [7].

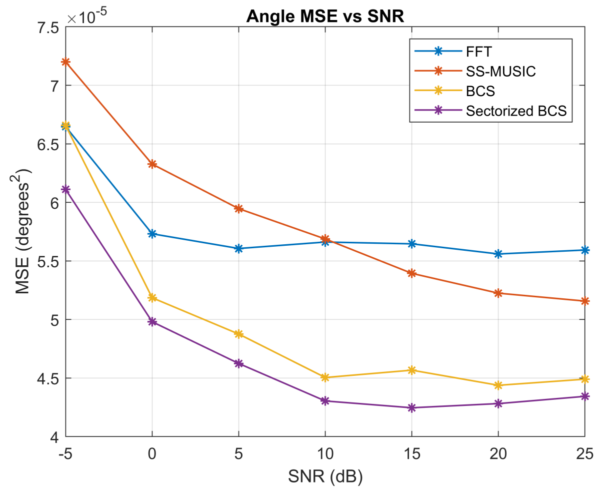

Fig. 4. MSE for the different DoA estimators as a function of the SNR.

4) The proposed approach, sectorized BCS with $W = 10$ sectors covering 18° each, with a grid size of 0.5°.

Fig. 4 shows the mean square error (MSE) for the four different estimators. As it can be seen, the sectorized BCS slightly outperforms the classical BCS, while the SS-MUSIC and the Fourier beamformer have the worst performance.

In addition to the MSE, it is important to analyze the resolution capabilities of the proposed method. To this end, the same approach as in [14] is used, where the following random inequality is defined as:

$$\gamma(\theta_1, \theta_2) = \frac{1}{2}[\hat{x}(\theta_1) + \hat{x}(\theta_2)] - \hat{x}(\frac{\theta_1 + \theta_2}{2}) > 0. \quad (8)$$

With this, two signals are said to be resolvable if the inequality holds, and to be irresolvable otherwise. Therefore, the probability of resolution can be written as a binary decision problem as:

$$P_{res} = Pr\{\gamma > 0\}. \quad (9)$$

Fig. 5 shows the probability of resolution for the DoA estimators mentioned above as a function of the true angular separation of the targets. As it can be seen, the proposed approach follows the same trend as the standard BCS, with a very similar performance. Moreover, it can be seen how the BCS-based methods have much higher P_{res} than the SS-MUSIC and the Fourier beamformer. Furthermore, in Fig. 6, the probability of resolution for different SNRs can be seen, where the methods behave similarly to the previous result.

From the results presented in this section it is clear that the proposed sectorized BCS has similar estimation accuracy and resolution capabilities with respect to the standard BCS, overperforming the Fourier beamformer and the Single Snapshot MUSIC algorithm. However, as explained in the previous section, the more considerable improvement is in the computational cost of the method. To analyze this improvement, the

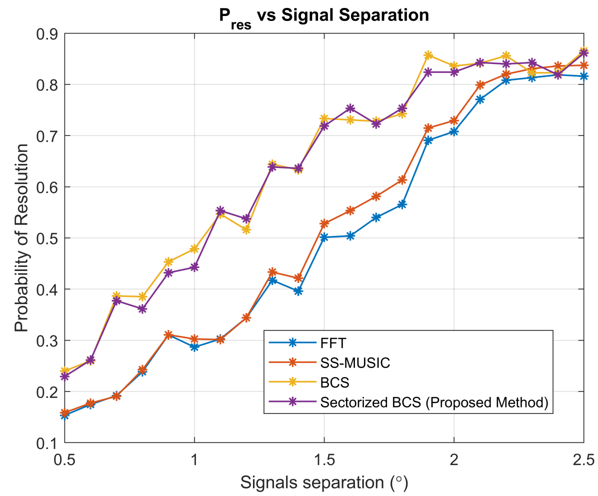

Fig. 5. Probability of resolution of the DoA estimators as a function of the true angular separation of targets.

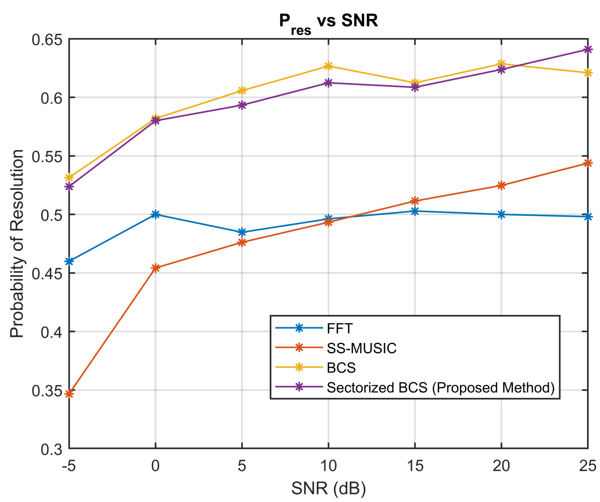

Fig. 6. Probability of resolution of the DoA estimators as a function of the SNR.

TABLE II
COMPUTATIONAL TIME FOR DIFFERENT DOA ESTIMATORS

Method	Time (ms)
SS-MUSIC [9]	16.3
FFT	0.035
BCS [10]	101.5
Sectorized BCS (Proposed Aproach)	5.0

average computational time over the 7000 scenes for each DoA estimator has been calculated, and results are shown in Table II. As it can be seen, the proposed method can perform the DoA estimation in only 5ms, which is approximately 20 times faster than the standard BCS (68% of the time is spent in the first step of the method, and 28% on the following correction stage).

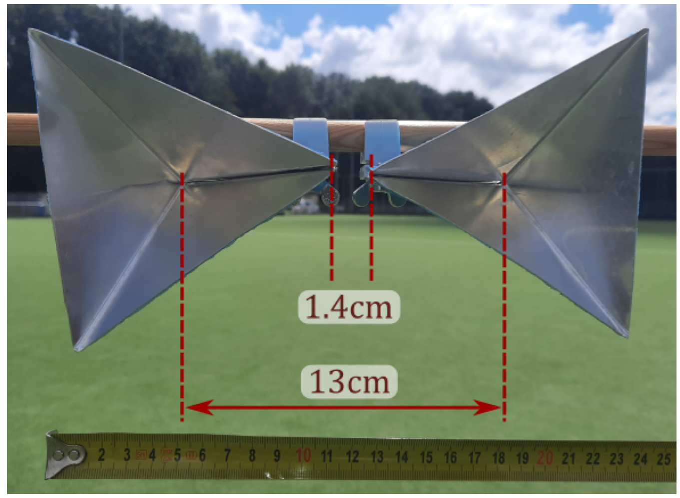

Fig. 7. Placement of the corner reflectors with 13cm between their centers, leading to 1.3° separation at 5.6m distance from the radar.

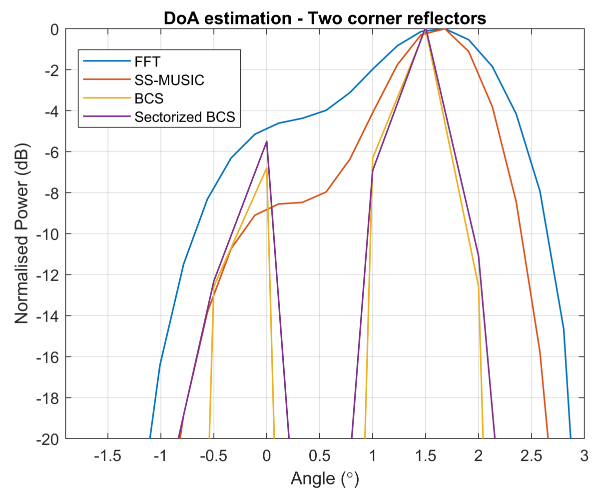

Fig. 8. DoA estimation results of two corner reflectors with 1.3° separation between them. The proposed sectorized BCS as well as the standard BCS can resolve the two targets. The FFT beamformer and SS-MUSIC only show one peak.

B. Experimental Results

A field experiment has been carried out with a 77GHz FMCW radar with an 86 virtual ULA to evaluate the performance of the proposed method with experimental data. Two corner reflectors have been placed at a range of 5.6m with a distance of 13cm (equivalent to approximately 1.3°) between the centers, as shown in Fig. 7. A single snapshot has been captured, and the same estimators mentioned in the previous section have been applied. As seen in Fig. 8, both BCS-based methods can resolve two peaks, while the FFT and SS-MUSIC fail to do so. Notice that the targets are placed in the boundary of two sectors, so the example shown is the most challenging case for the proposed method. The estimation is as good as the standard BCS, but with significantly less computational complexity.

Finally, it is important to mention that the estimation accuracy cannot be evaluated due to the lack of an accurate ground truth. The experimental results are shown only as an example, and future work will perform a statistical analysis with additional data.

IV. Conclusions

This paper presents a low-complexity single-snapshot DoA estimation method that uses a sectorized approach to exploit the advantages of Bayesian Compressive Sensing. The performance of the proposed method has been analyzed in terms of estimation accuracy, resolution capabilities, and computational cost. To this end, a Monte Carlo simulation has been performed with 7000 runs, simulating different scenarios with a varying number of targets with different characteristics. Results prove that the proposed method can perform as well as the standard BCS in terms of accuracy and resolution but with a significant reduction of 20 times the computational cost. Moreover, the method's performance has been verified experimentally using commercially available radar, proving that the proposed method can work well with measured data.

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