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DOI
10.1109/LSP.2023.3293468

Publication date
2023

Document Version
Final published version

Published in
IEEE Signal Processing Letters

Citation (APA)
Jin, Y., & Li, Z. (2023). Theoretical Framework for A Succinct Empirical Mode Decomposition. *IEEE Signal Processing Letters*, *30*, 888-892. https://doi.org/10.1109/LSP.2023.3293468

Theoretical Framework for A Succinct Empirical Mode Decomposition

Yang Jin and Zili Li

***Abstract*—Empirical mode decomposition (EMD) lacks a strong theoretical support although extensively applied. We propose a theoretical framework for a succinct EMD in this work, with the assumption of invariant extrema locations for one IMF extraction. We define the envelope mean filter (EMF) and prove that the filter matrix satisfies five properties. The sifting matrix is convergent to an idempotent matrix. An IMF is the projection of the input signal on the generalized eigenspace of the EMF matrix. An IMF is orthogonal to the residual signal, but different IMFs have no orthogonality. With numerical experiments on different signals, our framework achieves similar results to the classic EMD.**

***Index Terms*—Empirical mode decomposition, adaptive signal processing, time-varying filters, time-frequency analysis**

I. Introduction

EMPIRICAL mode decomposition (EMD) [1] is an adaptive time-frequency analysis technique for processing nonstationary and nonlinear signals. It has evolved in algorithms [2]–[5] and presented superior performances in extensive applications [6]–[8]. The intrinsic mode functions (IMFs) acquired within naturally determined bandwidths represent the oscillation modes and have physical meanings for Hilbert transform. An IMF should satisfy that: (1) the numbers of extrema and zero-crossings differ by 0 or 1, and (2) the local mean determined by envelopes is 0. For a discrete signal $y(t)$, Huang et al. [1] developed the following sifting process to extract an IMF:

1) Identify the local extrema of the input signal $h_i(t)$, where $h_0(t) = y(t)$;
2) Calculates the upper and lower envelopes using the extrema and then calculate the envelope mean $M(t)$;
3) Update $h_{i+1}(t) = h_i(t) - M(t)$;
4) Repeat the upper steps with $i = 0, 1, 2, ...$ until $h_{i+1}(t)$ becomes an IMF.

Although EMD is proposed on the basis of the Hilbert transform, its data-driven algorithm lacks strong theoretical support [9]–[11]. This is a significant issue of EMD. The primary research decomposing the fractional Gaussian noise [12] revealed the filter characteristic of EMD experimentally, with similar findings in [13]–[16]. Yang et al. [17] demonstrated that the cubic B-spline interpolation to formulate the local envelopes is a low-pass filter. Indeed, the cubic spline filter constitutes a time-varying signal processing system [18], and the bandwidth narrows with iteration [19]. An alternative approach, namely partial differential equation [20]–[22], for calculating the envelope-mean provides an analytical expression of EMD. However, these researches only concerned the expression of envelopes and altering the envelope approach [23]–[25] did not promote the theory further. To our best knowledge, a theoretical framework that interprets the sifting process and IMFs and proves multiple properties (e.g., convergence and orthogonality) of EMD remains to be developed.

In our work, we propose a theoretical framework for a succinct EMD. The difference with classic EMD is our assumption that the locations of the extrema in the time domain are invariant when extracting one IMF. The cubic spline interpolation is discussed and several properties of the envelope mean are proved. The convergence of the sifting process and the orthogonality of IMFs are investigated. In numerical experiments, we compare the decomposition results under our framework with those of classic EMD.

Manuscript received on

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II. Theory

A. Envelope mean

A time-varying filter bank consists of the decimator, filters and expander with their parameters altering with time [26]–[28]. The envelope mean calculated from cubic spline interpolation is demonstrated as a time-varying filter [18]. Indeed, there are multiple approaches (e.g., [5], [23]) to construct envelopes that present similar time-varying filter properties, and thus we define the envelope-mean filter (EMF) for EMD.

Definition 1: For a time series $(t_m,\, y_m)$, $m = 1, 2, ..., N$, the signal envelopes are calculated by interpolating with all maxima $(t_{u_k},\, y_{u_k})$ $(k = 1, 2, ..., N_u)$ and minima $(t_{v_l},\, y_{v_l})$ $(l = 1, 2, ..., N_v)$. The EMF is a time-varying filter by averaging the envelopes and the filter matrix Q should satisfy the following properties:

1.1 Only the columns u_k and v_l contain non-zero values;
1.2 The summation of each row equals 1;
1.3 All entries $q_{i,j}$, $i, j = 1, 2, ..., N$ are dependent on t_m, u_k and v_l, and independent on any y_m;
1.4 The geometric multiplicity of eigenvalue 0 is at least $N - N_u - N_v + 1$;
1.5 All eigenvalues $\lambda \in [0, 1]$.

The cubic spline envelopes construct such an EMF, and the proof requires the expression of matrix Q. First, we consider the upper envelopes using the maxima. Extracting the maxima $(t_{u_k},\, y_{u_k})$ from the signal $(t_m,\, y_m)$ is a time-varying multirate decimator [17], [18]. The general expression of natural cubic spline interpolation [29]–[31] on $(t_{u_k},\, y_{u_k})$ is

$$S_k(t) = \frac{t - t_{u_k}}{\Delta t_{u_k}} y_{u_{k+1}} + (\frac{(t - t_{u_k})^3}{6\Delta t_{u_k}} - \frac{\Delta t_{u_k}(t - t_{u_k})}{6}) z_{u_{k+1}} + \frac{t_{u_{k+1}} - t}{\Delta t_{u_k}} y_{u_k} + (\frac{(t_{u_{k+1}} - t)^3}{6\Delta t_{u_k}} - \frac{\Delta t_{u_k}(t_{u_{k+1}} - t)}{6}) z_{u_k} \tag{1}$$

$$\frac{\Delta t_{u_{k-1}}}{6} z_{u_{k-1}} + \frac{\Delta t_{u_{k-1}} + \Delta t_{u_k}}{3} z_{u_k} + \frac{\Delta t_{u_k}}{6} z_{u_{k+1}} = \frac{1}{\Delta t_{u_{k-1}}} y_{u_{k-1}} - (\frac{1}{\Delta t_{u_{k-1}}} + \frac{1}{\Delta t_{u_k}}) y_{u_k} + \frac{1}{\Delta t_{u_k}} y_{u_{k+1}} \tag{2}$$

where, $S_k(t)$ is the interpolating function with $t_{u_k} \leq t \leq t_{u_{k+1}}$, z_{u_k} are the coefficients, $\Delta t_{u_k} = t_{u_{k+1}} - t_{u_k}$, and $z_{u_1} = z_{u_{N_u}} = 0$. Considering vectors $Y_u = \{y_{u_k} \mid k = 1, 2, ..., N_u\}$, $Z_u = \{z_{u_k} \mid k = 1, 2, ..., N_u\}$ and $S = \{S_k t_m \mid m = 1, 2, ..., N\}$, Equations (1) and (2) are rewritten in the matrix form.

$$\begin{aligned} S^\dagger &= \widehat{A_1} Y_u^\dagger + \widehat{B} Z_u^\dagger \\ C Z_u^\dagger &= D Y_u^\dagger \end{aligned} \tag{3}$$

where $\dagger$ represents the transpose of matrices or vectors. $\widehat{A_1}$ (Eq. (4)) is a $N \times N_u$ matrix with entries $\hat{a}_{m,k} = \frac{t_{u_{k+1}} - t_m}{\Delta t_{u_k}}$, $\hat{a}_{m,k+1} = \frac{t_m - t_{u_k}}{\Delta t_{u_k}}$ ($m \in [0, u_{k+1}]$, $k = 1$; $m \in [u_k, u_{k+1}]$, $k = 2, ..., N_u - 2$; $m \in [u_k, N]$, $k = N_u - 1$) and others equaling 0. $\widehat{B}$ is a $N \times N_u$ matrix with entries $\hat{b}_{m,k} = \frac{(t_{u_{k+1}} - t_m)^3}{6\Delta t_{u_k}} - \frac{\Delta t_{u_k}(t_{u_{k+1}} - t_m)}{6}$, $\hat{b}_{m,k+1} = \frac{(t_m - t_{u_k})^3}{6\Delta t_{u_k}} - \frac{\Delta t_{u_k}(t_m - t_{u_k})}{6}$ ($m \in [0, u_{k+1}]$, $k = 1$; $m \in [u_k, u_{k+1}]$, $k = 2, ..., N_u - 2$; $m \in [u_k, N]$, $k = N_u - 1$) and others equaling 0. C is a $N_u \times N_u$ matrix with entries $c_{1,1} = c_{N_u,N_u} = 1$, $c_{k,k-1} = \frac{\Delta t_{u_{k-1}}}{6}$, $c_{k,k} = \frac{\Delta t_{u_{k-1}} + \Delta t_{u_k}}{3}$, $c_{k,k+1} = \frac{\Delta t_{u_k}}{6}$ and others equaling 0. D is a $N_u \times N_u$ matrix with entries $d_{1,1} = d_{N_u,N_u} = 0$, $d_{k,k-1} = \frac{1}{\Delta t_{u_{k-1}}}$, $d_{k,k} = -(\frac{1}{\Delta t_{u_{k-1}}} + \frac{1}{\Delta t_{u_k}})$, $d_{k,k+1} = \frac{1}{\Delta t_{u_k}}$ and others 0.

$$\widehat{A_1} = \begin{bmatrix} \ddots & \vdots & \vdots & \iddots \\ \cdots & \frac{t_{u_{k+1}} - t_m}{\Delta t_{u_k}} & \frac{t_m - t_{u_k}}{\Delta t_{u_k}} & \cdots \\ \cdots & \frac{t_{u_{k+1}} - t_{m+1}}{\Delta t_{u_k}} & \frac{t_{m+1} - t_{u_k}}{\Delta t_{u_k}} & \cdots \\ \iddots & \vdots & \vdots & \ddots \end{bmatrix} \tag{4}$$

Matrix C is strictly diagonally dominant and is thereby invertible [32]–[34]. Let $\vec{e}_n$ represent an all ones vector with length n, and $\vec{\mathbf{0}}_n$ represent a zero vector with length n. Since the summation of each row from matrix D equals 0, $\vec{e}_{N_u}$ is the eigenvector of D corresponding to the eigenvalue 0. Considering the $N \times N_u$ matrix $\widehat{A_2} = \widehat{B} C^{-1} D$, we can obtain that

$$\widehat{A_2} \vec{e}^\dagger_{N_u} = \widehat{B} C^{-1} D \vec{e}^\dagger_{N_u} = \vec{\mathbf{0}}^\dagger_N \tag{5}$$

$$S^\dagger = (\widehat{A_1} + \widehat{A_2}) Y_u^\dagger \tag{6}$$

According to Eq. (5), the summation of each row from matrix $\widehat{A_2}$ equals 0. The next step is upsampling the maxima Y_u to the signal Y, where vector $Y = \{y_m \mid m = 1, 2, ..., N\}$. Correspondingly, the matrices $\widehat{A_1}$ and $\widehat{A_2}$ are zero-padded with $u_{k+1} - u_k - 1$ columns between the column k and $k+1$, $k = 1, 2, ..., N_u - 1$ to achieve the $N \times N$ matrices A_1 and A_2. Eq. 7 shows the zero-padding of $\widehat{A_1}$.

$$A_1 = \begin{bmatrix} \ddots & \vdots & 0 & \cdots & 0 & \vdots & \iddots \\ \cdots & \frac{t_{u_{k+1}} - t_m}{\Delta t_{u_k}} & 0 & \cdots & 0 & \frac{t_m - t_{u_k}}{\Delta t_{u_k}} & \cdots \\ \cdots & \frac{t_{u_{k+1}} - t_{m+1}}{\Delta t_{u_k}} & 0 & \cdots & 0 & \frac{t_{m+1} - t_{u_k}}{\Delta t_{u_k}} & \cdots \\ \iddots & \vdots & 0 & \cdots & 0 & \vdots & \ddots \end{bmatrix} \tag{7}$$

We obtain the upper envelope interpolation matrix $A_u = A_1 + A_2$ and the upper envelope S as

$$S^\dagger = A_u Y^\dagger \tag{8}$$

Therefore, (1) according to the zero-padding process, only the columns u_k of A_1 and A_2 contain non-zero value and so for the matrix A_u. (2) Since the summation of each row of A_1 equals 1 and that of A_2 equals 0, the summation of each row of A equals 1. (3) Since the entries of $\widehat{A_1}$ and $\widehat{A_2}$ are only dependent on t_m and t_{u_k} and the zero-padding process is only dependent on u_k, A_u is only dependent on t_m and u_k.

Similarly, we can achieve the lower envelope matrix A_v that satisfies: (1) only the columns v_l contain non-zero value; (2) the summation of each row equals 1; and (3) all entries are only dependent on t_m and v_l. Therefore, the EMF matrix $Q = (A_u + A_v)/2$ meets the first three properties of our definition.

Since $N - N_u - N_v$ columns of Q are all zero, the matrix rank of Q is at largest $N_u + N_v$. The eigenspace for eigenvalue 0 contains at least $N - N_u - N_v$ linear-independent eigenvectors, which are $\vec{X}_i = (x_{i,1}, x_{i,2}, ..., x_{i,j}, ..., x_{i,N})$ with $x_{i,i} = 1$ and $x_{i,j} = 0$ ($j \neq i$), $i \neq u_k, v_l$. We consider the vectors $\vec{P}_1 = (p_{1,1}, ..., p_{1,j}, ..., p_{1,N})$ with $p_{1,j} = 1$ ($j = u_1, u_2, ..., u_{N_u}$) and others equaling 0, and $\vec{P}_2 = (p_{2,1}, ..., p_{2,j}, ..., p_{2,N})$ with $p_{2,j} = 1$ ($j = v_1, v_2, ..., v_{N_v}$) and others equaling 0. Since the summation of each row of A_u or A_v equals 1, we can obtain

$$A_u \vec{P}_2^\dagger = A_v \vec{P}_1^\dagger = A_u \vec{P}_1^\dagger = A_v \vec{P}_2^\dagger = \vec{e}_N^\dagger \tag{9}$$

Therefore, Q has another eigenvector $\vec{X} = \vec{P}_1 - \vec{P}_2$ corresponding to eigenvalue 0. The geometric multiplicity of eigenvalue 0 is at least $N - N_u - N_v + 1$.

The last property is to determine the range of all eigenvalues. We consider the $(N_u + N_v)^2$ matrix $\widehat{Q}$ that only contains the rows and columns $u_k \cup v_l$ of Q. According to property **1.1**, except eigenvectors $\vec{X}_i$ of eigenvalue 0, the eigenvalue λ of Q is also that of $\widehat{Q}$ and the corresponding eigenvetor of Q is decimated at $u_k \cup v_l$ to that of $\widehat{Q}$. First, $\widehat{Q}$ has eigenvector $\vec{e}_{N_u+N_v}$ corresponding to eigenvalue 1. For any other eigenvector $\vec{P} = \{p_i \mid i = 1, 2, ..., N_u + N_v\}$ of $\widehat{Q}$

($\widehat{Q}\vec{P}^{\dagger} = \lambda\vec{P}^{\dagger}$), it should have the first local maximum p_{s_1} and minimum p_{s_2}. Since $\widehat{Q}$ calculates the mean of interpolations $\hat{f}(p)$ for $\{p_{2i-1} \mid i = 1, 2, ...\}$ and $\hat{g}(p)$ for $\{p_{2_i} \mid i = 1, 2, ...\}$, we have the following relationships

$$\begin{aligned} &p_{s_1} - \widehat{Q}\vec{P}^{\dagger} \mid_{s_1} = (1-\lambda)p_{s_1} > 0 \\ &p_{s_2} - \widehat{Q}\vec{P}^{\dagger} \mid_{s_2} = (1-\lambda)p_{s_2} < 0 \\ &p_{s_1} > p_{s_2} \end{aligned} \tag{10}$$

Therefore, we can obtain $\lambda < 1$. Considering the entry p of $\vec{P}$ with maximum absolute value and $p \in \{p_{2i-1} \mid i = 1, 2, ...\}$ without loss of generality, we can obtain

$$\mid \hat{g}(p) \mid \leq \mid p_{2i} \mid_{max} \leq \mid p \mid \tag{11}$$

Thus, we can derive that

$$\lambda p^2 = (\frac{\hat{g}(p) + p}{2})p \geq 0 \tag{12}$$

Therefore, we can obtain $\lambda \geq 0$. In addition, we can use mathematical induction to prove that λ of $\widehat{Q}$ are all real: (1) 1st-order $\widehat{Q}$ satisfies obviously; (2) when kth-order $\widehat{Q}$ satisfies, its generalized eigenvector can be extended Δt signal interval to a new extrema with the original cubic splines. In this case, the extended vector is the generalized eigenvector of the $k+1$th-order $\widehat{Q}$. Therefore, $k+1$th-order $\widehat{Q}$ has at least k generalized eigenvectors, as well as generalized real eigenvalues. Since imaginary eigenvalues should appear in pair, the last generalized eigenvalue is also real. Considering all the aforementioned, $0 \leq \lambda \leq 1$ for matrix Q.

B. Sifting process

Huang et al. [1] developed the sifting process to extract an IMF. We assume that the extrema locations u_k, v_l are invariant when extracting one IMF. This is an approximation because the extrema locations only moves a little surrounding the initial ones during sifting process. Since Q is dependent on t_m, u_k and v_l and independent on any y_m, Q becomes invariant when extracting one IMF. Therefore, the sifting process can be expressed as

$$\vec{\xi}^{\dagger} = \lim_{\beta \to +\infty} (I_N - Q)^{\beta} Y^{\dagger} = GY^{\dagger} \tag{13}$$

where, $\vec{\xi}$ is an IMF, Y is the input signal, and I_N is the N^2 identity matrix. $G = \lim_{\beta\to+\infty}(I_N - Q)^{\beta}$ is the sifting matrix and the convergence of G should be determined.

Considering the unit linear-independent generalized eigenvectors [36] $\vec{\varrho}_i$ of Q and their corresponding eigenvalues λ_i ($i = 1, 2, ..., N$), the eigenvalues of $R = I_N - Q$ are $\mu_i = 1 - \lambda_i$. Thus we have

$$(R - \mu_i I_N)^{\tau} \vec{\varrho}_i^{\dagger} = \vec{\mathbf{0}}_N^{\dagger}, \quad \exists\, \tau \in \mathbb{N}^{+} \;\&\; \tau < N \tag{14}$$

$$\begin{aligned} R^{\beta} - \mu_i^{\beta} I_N &= (R - \mu_i I_N)(\sum_{j=0}^{\beta-1} \mu_i^j R^{\beta-1-j}) \\ &= (\sum_{j=0}^{\beta-1} \mu_i^j R^{\beta-1-j})(R - \mu_i I_N) \end{aligned} \tag{15}$$

Therefore, we can obtain

$$\begin{aligned} &(R^{\beta} - \mu_i^{\beta} I_N)^{\tau} \vec{\varrho}_i^{\dagger} \\ &= (\sum_{j=0}^{\beta-1} \mu_i^j R^{\beta-1-j})^{\tau} (R - \mu_i I_N)^{\tau} \vec{\varrho}_i^{\dagger} = \vec{\mathbf{0}}_N^{\dagger} \end{aligned} \tag{16}$$

Therefore, $\vec{\varrho}_i$ are the linear-independent generalized eigenvectors for eigenvalues μ_i^{β} of R^{β}. Since $\mu_i \in [0, 1]$, $G = \lim_{\beta\to+\infty} R^{\beta}$ only has eigenvalues 0 and 1. Matrix G is similar to a Jordan normal form [37] with all diagonal entries of 0 and 1, and thus G is convergent. We also have

$$G^2 = \lim_{\beta\to+\infty} R^{2\beta} = G \tag{17}$$

Therefore, G is an idempotent matrix [38] and all the generalized eigenvectors $\vec{\varrho}_i$ are indeed eigenvectors.

C. Intrinsic mode functions

Eq. (13) is used to calculate an IMF. Considering the input signal Y represented by the unit linear-independent eigenvectors $\vec{\varrho}_i$, we can obtain

$$\vec{\xi}^{\dagger} = GY^{\dagger} = G\sum_{i=0}^{N} \alpha_i \vec{\varrho}_i^{\dagger} = \sum_{\mu_i = 1} \alpha_i \vec{\varrho}_i^{\dagger} \tag{18}$$

where α_i are the projection scalar of Y on $\vec{\varrho}_i$. Thus an IMF is indeed the projection of input signal Y on the generalized eigenspace for eigenvalue 0 of EMF Q, which corresponds to a high-pass filtering process.

The $\mathbb{C}^N$ Euclidean space W is the direct sum of two invariant eigenspaces W_0 and W_1 corresponding to the eigenvalues 0 and 1 of G, and $W_0 \perp W_1$. Since $\xi \in W_1$ and $Y - \xi \in W_0$, we have $\xi \perp (Y - \xi)$. Therefore, the sifting process to extract one IMF is an orthogonal decomposition.

Considering two IMFs $\xi_1^{\dagger} = G_1 Y^{\dagger}$ and $\xi_2^{\dagger} = G_2(Y - \xi_1)^{\dagger}$, for any Y, $\xi_1 \perp \xi_2$ is equivalent to $(I_N - G_1^{\dagger})G_2^{\dagger}G_1$ is an anti-symmetric matrix with all 0 diagonals. However, different Q mostly do not satisfy this condition. The classic EMD using cubic spline interpolation does not provide orthogonality between IMFs.

III. Numerical experiments

A. Filter bandwidth

First, we decompose Gaussian white noise by classic EMD procedures and by projection on the generalized eigenspace for eigenvalue 0 of Q. The sifting process iterates 100 times for the classic EMD procedures. The sampling frequency is 10 kHz and the length is 1 s for the Gaussian white noise. Fig. 1 illustrates the fast Fourier spectra of IMFs to demonstrate the filter bandwidths. Both FFT spectra present similar bandwidths for the corresponding IMFs. EMD works as overlapped bandpass filters on the time series [12]. The corresponding low-frequency IMFs with two decomposition procedures present different energy, which may arise from our assumption of invariant extrema locations and the finite iteration times of classic EMD.

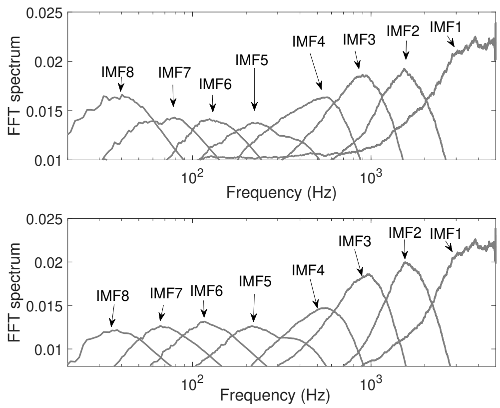

Fig. 1. Top panel: FFT spectra of IMFs by projection on the generalized eigenspace for eigenvalue 0 of Q. Bottom panel: FFT spectra of IMFs by the classic EMD.

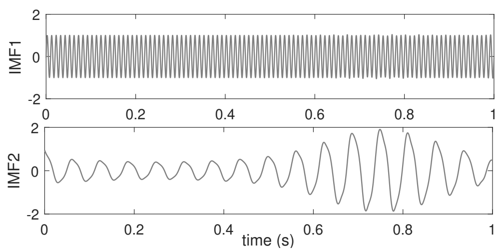

Fig. 2. IMF1 and IMF2 achieved by projection on the generalized eigenspace for eigenvalue 0 of Q.

B. Decomposition results

Second, a numerically generated signal Y consisting of a time-varying and a stationary components (Eq. (19)) is decomposed using the classic EMD and our framework. The sampling frequency is 10 kHz and the duration is 1 s.

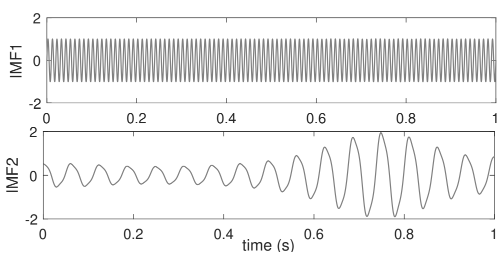

Fig. 3. IMF1 and IMF2 achieved by the classic EMD.

$$
\begin{aligned}
Y_1 &= \frac{cos(32\pi t + cos(64\pi t))}{1.5 + sin(2\pi t)} \\
Y_2 &= sin(200\pi t) \\
Y &= Y_1 + Y_2
\end{aligned}
\tag{19}
$$

Fig. 2 illustrates the IMFs achieved by projection on the generalized eigenspace for eigenvalue 0 of Q. IMF1 and IMF2 agree well with Y_1 and Y_2, respectively. This indicates the decomposition accuracy of our EMD framework. The classic EMD also achieves similar results, as shown in Fig. 3.

Third, we decompose a seismic signal taken from [39], as shown in Fig. 4. Fig. 5 presents the Hilbert spectra by our

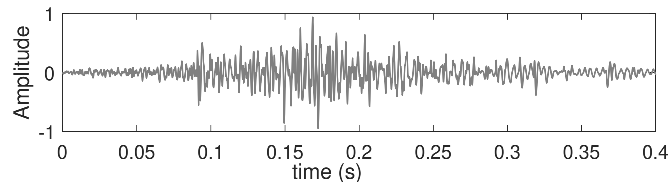

Fig. 4. The seismic signal taken from [39].

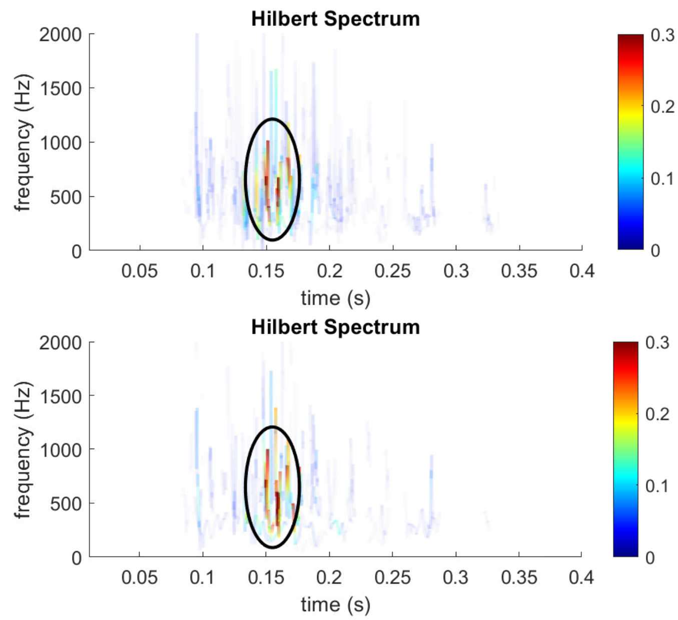

Fig. 5. Top panel: Hilbert spectrum of IMFs by projection on the generalized eigenspace for eigenvalue 0 of Q. Bottom panel: Hilbert spectrum of IMFs by the classic EMD.

EMD framework and the classic EMD. The spectra demonstrate similar distribution, e.g., large energy concentration at three close locations marked with the circles. The little decomposition difference may arise from our assumption of invariant extrema locations and the finite iteration times of classic EMD.

IV. Conclusion

We propose a theoretical framework for EMD in this letter. The cubic spline interpolation works as an EMF with the filter matrix satisfying five properties. The sifting process matrix is convergent to an idempotent matrix only with eigenvalues 0 and 1. An IMF is the projection of the input signal on the generalized eigenspace of EMF matrix Q, which corresponds to a high-pass filtering process. Numerical experiments demonstrate that our framework achieves similar results to the classic EMD, although difference may result from the assumption of invariant extrema locations and the finite iteration times of classic EMD.

However, different IMFs are not orthogonal. Future work will concern the construction of the EMF matrix for IMF orthogonality. In addition, experiments should consider decomposing complicated signals and compare more EMD algorithms in the future work.

REFERENCES

[1] Huang, N., Shen, Z., Long, S., Wu, M., Shih, H., Zheng, Q., Yen, N., Tung, C. and Liu, H., "The empirical mode decomposition and the Hilbert spectrum for nonlinear and non-stationary time series analysis," *Proc. Math. Phys. Eng. Sci.*, vol. 454, no. 1971, pp. 903–995, Mar. 1998.

[2] Z. Wu and N. E. Huang, "Ensemble Empirical Mode Decomposition: A noise-assisted data analysis method," Adv. Adapt. Data Anal., vol. 01, no. 01, pp. 1–41, 2009.

[3] S. Meignen and V. Perrier, "A new formulation for empirical mode decomposition based on constrained optimization," IEEE Signal Process. Lett., vol. 14, no. 12, pp. 932–935, 2007.

[4] M. E. Torres, M. A. Colominas, G. Schlotthauer, and P. Flandrin, "A complete ensemble empirical mode decomposition with adaptive noise," in 2011 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP), 2011.

[5] H. Li, Y. Hu, F. Li, and G. Meng, "Succinct and fast empirical mode decomposition," Mech. Syst. Signal Process., vol. 85, pp. 879–895, 2017.

[6] J. Han and M. van der Baan, "Empirical mode decomposition for seismic time-frequency analysis," Geophysics, vol. 78, no. 2, pp. O9–O19, 2013.

[7] R. J. Martis et al., "Application of empirical mode decomposition (emd) for automated detection of epilepsy using EEG signals," Int. J. Neural Syst., vol. 22, no. 6, p. 1250027, 2012.

[8] L. Wang and Y. Shao, "Fault feature extraction of rotating machinery using a reweighted complete ensemble empirical mode decomposition with adaptive noise and demodulation analysis," Mech. Syst. Signal Process., vol. 138, no. 106545, p. 106545, 2020.

[9] T. Oberlin, S. Meignen, V. Perrier, "An alternative formulation for the empirical mode decomposition," IEEE Trans. Signal Process., vol. 60, no. 5, p. 2236-2246, 2012.

[10] N. Pustelnik, P. Borgnat, P. Flandrin, "Empirical mode decomposition revisited by multicomponent non-smooth convex optimization," Signal Processing, vol. 102, p. 313-331, 2014.

[11] M.A. Colominas, G. Schlotthauer, M.E. Torres, "An unconstrained optimization approach to empirical mode decomposition," IEEE Trans. Signal Process., vol. 40, p. 164-175, 2015.

[12] P. Flandrin, G. Rilling, and P. Goncalves, "Empirical mode decomposition as a filter bank," IEEE Signal Process. Lett., vol. 11, no. 2, pp. 112–114, 2004.

[13] Z. Wu and N. E. Huang, "A study of the characteristics of white noise using the empirical mode decomposition method," Proc. Math. Phys. Eng. Sci., vol. 460, no. 2046, pp. 1597–1611, 2004.

[14] G. Rilling, P. Flandrin, and P. Goncalves, "Empirical Mode Decomposition, fractional Gaussian noise and Hurst exponent estimation," in Proceedings. (ICASSP '05). IEEE International Conference on Acoustics, Speech, and Signal Processing, 2005, 2006.

[15] P. Flandrin and P. Gonçalvès, "Empirical mode decompositions as data-driven wavelet-like expansions," Int. J. Wavelets Multiresolut. Inf. Process., vol. 02, no. 04, pp. 477–496, 2004.

[16] Z. Wu and N. E. Huang, "On the filtering properties of the empirical mode decomposition," Adv. Adapt. Data Anal., vol. 02, no. 04, pp. 397–414, 2010.

[17] Y. Yang, C. Miao, and J. Deng, "An analytical expression for empirical mode decomposition based on B-spline interpolation," Circuits Syst. Signal Process., vol. 32, no. 6, pp. 2899–2914, 2013.

[18] Y. Yang, "Empirical mode decomposition as a time-varying multirate signal processing system," Mech. Syst. Signal Process., vol. 76–77, pp. 759–770, 2016.

[19] H. Li, Z. Li, and W. Mo, "A time varying filter approach for empirical mode decomposition," Signal Processing, vol. 138, pp. 146–158, 2017.

[20] E. Delechelle, J. Lemoine, and O. Niang, "Empirical mode decomposition: an analytical approach for sifting process," IEEE Signal Process. Lett., vol. 12, no. 11, pp. 764–767, 2005.

[21] El Hadji S Diop, R. Alexandre, and A. O. Boudraa, "Analysis of intrinsic mode functions: A PDE approach," IEEE Signal Process. Lett., vol. 17, no. 4, pp. 398–401, 2010.

[22] E. H. S. Diop, R. Alexandre, and V. Perrier, "A PDE based and interpolation-free framework for modeling the sifting process in a continuous domain," Adv. Comput. Math., vol. 38, no. 4, pp. 801–835, 2013.

[23] L. Lin, Y. Wang, and H. Zhou, "Iterative filtering as an alternative algorithm for empirical mode decomposition," Adv. Adapt. Data Anal., vol. 01, no. 04, pp. 543–560, 2009.

[24] Y. Kopsinis and S. McLaughlin, "Investigation and performance enhancement of the empirical mode decomposition method based on a heuristic search optimization approach," IEEE Trans. Signal Process., vol. 56, no. 1, pp. 1–13, 2008.

[25] S. D. Hawley, L.E. Atlas, and H. J. Chizeck, "Some properties of an empirical mode type signal decomposition algorithm," IEEE Signal Process. Lett., vol. 17, no. 1, pp. 24–27, 2010.

[26] G. Wang, "The most general time-varying filter bank and time-varying lapped transforms," IEEE Trans. Signal Process., vol. 54, no. 10, pp. 3775–3789, 2006.

[27] I. Sodagar, K. Nayebi, and T. P. Barnwell, "Time-varying filter banks and wavelets," IEEE Trans. Signal Process., vol. 42, no. 11, pp. 2983–2996, 1994.

[28] M. Vetterli and G. Strang, "Time-varying filter banks and multiwavelets," in Proceedings of IEEE 6th Digital Signal Processing Workshop, 2002.

[29] C. A. Hall, "Natural cubic and bicubic spline interpolation," SIAM J. Numer. Anal., vol. 10, no. 6, pp. 1055–1060, 1973.

[30] S. A. Dyer and J. S. Dyer, "Cubic-spline interpolation. 1," IEEE Instrum. Meas. Mag., vol. 4, no. 1, pp. 44–46, 2001.

[31] T. Lyche, "Discrete cubic spline interpolation," BIT, vol. 16, no. 3, pp. 281–290, 1976.

[32] O. Taussky, "A recurring theorem on determinants," Am. Math. Mon., vol. 56, no. 10, p. 672, 1949.

[33] F. O. Farid, "Criteria for invertibility of diagonally dominant matrices," Linear Algebra Appl., vol. 215, pp. 63–93, 1995.

[34] M. Neumann, "A note on generalizations of strict diagonal dominance for real matrices," Linear Algebra Appl., vol. 26, pp. 3–14, 1979.

[35] G. H. Golub and C. F. Van Loan, Matrix Computations, 3rd ed. Baltimore, MD: Johns Hopkins University Press, 1996.

[36] H. Anton, Elementary Linear Algebra, 5th ed. Nashville, TN: John Wiley & Sons, 1987.

[37] C. G. Cullen, Matrices and Linear Transformations: Second Edition. Dover Publications, 2012.

[38] K. Wainwright and A. C. Chiang, Fundamental methods of mathematical economics, 4th ed. Maidenhead, England: McGraw Hill Higher Education, 2005.

[39] WU, "seismic data." figshare, 08-Jul-2019 [Online]. Available: https://doi.org/10.6084/m9.figshare.8797502.v1