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Abstract

Embodied learning technologies have shown efficacy in laboratories with ideal supportive conditions, but their effectiveness in classroom with "real-world" constraints is yet understudied. Inspired by the innovation implementation framework, we compare the classroom-situated engagements of two student pairs and their teachers with the action-based embodied design for proportions with earlier laboratory and classroom study findings and conjecture on influential factors. Much of these classroom students' sensorimotor learning resembled laboratory findings, but they had more opportunities to be overtly engaged with their hands and self-directed in including artifacts, likely influenced by (unintended) technological changes and setting-specific environmental affordances. Their teachers' engagements resembled laboratory findings to some extent, but showed less perceptiveness to students' qualitative multimodal expressions and more directedness in introducing new quantitative forms of engagements, likely influenced by setting-specific fragmented access and novelty of

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the embodied pedagogy. We discuss the importance of focusing on teachers and conducting semi-natural efficacy research.

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Keywords

embodied cognition – mathematics education – classroom implementation – proportions – educational technology

1 Introduction

For decades, mathematics education stakeholders such as teachers, researchers, and policymakers have been developing a variety of theories, teacher practices, policies, and programs with the goal of bringing about change in classrooms, organizations, and educational systems. Until the 1960s and 1970s, these innovations were viewed as replicable technologies (Century & Cassata, 2016) that could bring about large-scale improvements because implementers would simply copy these programs as the originators had done (Rogers, 2003). Evaluations of practice, on the other hand, revealed a very different picture, with implementers as creative adapters of a new idea (Rogers, 2003) and instances of implementation critically influenced by local contexts and conditions (e.g., Fullan & Pomfret, 1977). Components of the program may be left out or adapted due to administrative structures, insufficient training, scheduling or spatial constraints, or personal preferences (e.g., Greenberg et al., 2005). Replication is not always possible or desirable, and improving the quality and effectiveness of services thus necessitates looking beyond the innovations themselves and taking into account the diverse and dynamic interactions with individuals, organizations, and environments (Domitrovich et al., 2008; Century et al., 2012), the domain of the newly emerging field of implementation research (Jankvist et al., 2021).

The current work is part of implementation studies that focus on integrating new research-based learning activities, notably digital technologies, into established classroom practices. Classrooms are booming, buzzy and exciting, fascinating places, anchored in bigger systems of schools and traditions (e.g., Brown, 1992), and changing their teacher and procedural-oriented methods is a difficult and complex process with variable results. According to Uwurukundo et al. (2020), while GeoGebra programs are typically beneficial, their effectiveness is contingent on how they are integrated into the teaching and learning process. In general, as digital programs strive to scale larger, their effect sizes appear to shrink (Drijvers, 2019). Other creative tools, such as interactive whiteboards, have been demonstrated to reinforce rather than replace traditional teaching methods (Rudd, 2007). These variable outcomes are not, of course, unique to digital technologies. Gravemeijer et al. (2016) demonstrated how teacher adaptations made during the introduction of Realistic Mathematics Education textbooks can result in overarching goals of coherent conceptual understanding being replaced with solutions to local problems. As Fixsen et al. (2005) elegantly summarized, "the challenges and complexities of implementation far outweigh the efforts of developing the practices and programs themselves" (Fixsen et al., 2005: p. vi).

The present work is part of a sub-category of digital technologies in the early stages of implementation: embodied learning technologies. These emerging activity genres view the body's interaction with the physical, material, and cultural environment as essential to mathematical cognition (e.g., Radford, 2014) and capitalize on this by designing interaction with motion-responsive devices such as touchscreens and Kinect sensors (see, e.g., Abrahamson et al., 2020). Numerous studies have shown the efficacy (or internal validity) of these programs in relatively controlled laboratory settings with one-to-one expert tutoring, but considerably fewer studies provide insights about their effectiveness (or external or ecological validity) in natural classroom settings operating under "real-world" constraints. Whereas the inability to produce the desired outcome under the most favorable conditions requires consideration of theory, the inability to reproduce desired beneficial effects in actual use also requires consideration of implementation (O'Donnell, 2008).

Along with wider systemic questions about ideology, theory, and professional development, the laboratory to classroom transition will unavoidably bring about variance in how the programs are engaged due to (1) methods to "up-scaling" designs to serve not one or two but thirty students, (2) students of varied abilities interacting under less continuous supervision by teachers who are less familiar with (or convinced of) the embodied pedagogy (Abrahamson et al., 2021), and (3) a variety of local contexts and conditions inherent in any setting change and thus far not captured by laboratory studies (Cai et al., 2020). Anderson and Wall (2016), for example, showed how students become distracted as technologies are occupied by others and the need of instructor scaffolding in consolidating multimodal discoveries. With considerable support from researchers, Ferrari and Ferrara (2018) showed how a sample student pair interacting in front of the classroom can ground rich discussion by observers' post-interaction. Georgiou and Ioannou (2021) showed good results using a learning-station model, but also limitations in terms of time on tasks, fighting over turn-taking, and classroom noise. The question is thus not *if* changes occur, but the extent and nature of the changes and adaptations, the factors influencing the quality of program implementation and the effects these have on accomplishing the desired learning outcomes.

Based on their review on embodied learning technologies, Georgiou and Ioannou (2019) recommend that "For this field to grow and become a more mainstream one, future studies should be more oriented toward (…) the integration and evaluation of technology-enhanced embodied learning environments in authentic school settings, considering the school curricula, both content-wise and time-wise" (Georgiou & Ioannou, 2019: p. 169). The current paper addresses these aspects for the action-based embodied design for proportions, which was developed based on extensive prior (laboratory) research and is part of a larger activity genre that covers a variety of mathematical topics (Abrahamson et al., 2014; Alberto et al., 2021). We present two pairs of students and their teachers situated in two Dutch classrooms and compare their engagements with the tool with findings from similar laboratory (e.g., Duijzer et al., 2017) and classroom (e.g., Negrete, 2013) investigations. We explore two questions inspired by the innovation implementation framework (Century et al., 2012; Century & Cassata, 2016). First, which action-based core components revealed in laboratory investigations are enacted by students and teachers in these classrooms, and to what extent? Second, what factors, in terms of the individual, the environment, the technology, and the support strategies, could potentially influence enactment differences across settings? More broadly, we hope to offer some practical insights into the potential and challenges of incorporating embodied learning technologies into established cultural practices, as well as to highlight the potential of cross-setting fertilization in insights in innovations and their implementation (Cai et al., 2020).

2 Implementation Research

Implementation research is an interesting field that has only recently gained prominence in its own right with appropriate organized outlets across several domains, including health science and promotion (Greenberg et al., 2005; Eccles & Mittman, 2006), educational sciences (Fixsen et al., 2005; Century & Cassata, 2016), and, most recently, mathematics education (Jankvist et al., 2017, 2021). Implementation research has been defined in various ways within and across domains, including "the scientific study of methods to promote the systematic uptake of research findings and other evidence-based practices into routine practice" (Eccles & Mittman, 2006: p. 1), "a specified set of activities designed to put into practice an activity or program of known dimensions" (Fixsen et al., 2005: p. 5), or "a change-oriented process of adapting and enacting a particular resource (…) that occurs in partnership of (…) a community of the resource proponents (CRP) and a community of the resource adapters (CRA)" (Aguilar et al., 2019: p. 3774). The definition proposed by Century and Cassata (2016), "the systematic inquiry regarding innovation enacted in controlled settings or in ordinary practice, and the relationship between innovations, influential factors and outcomes" (Century & Cassata, 2016: p. 170), was the best match for the current work, which is still in the early stages of implementation. Their innovation implementation framework (Century et al., 2012; Century & Cassata, 2014) provides a useful conceptual framework for evaluating innovation implementation and identifying hindering or supporting factors (see application for K-12 STEM curricula in Gale et al., 2020). In Section 2.1, this framework is used to frame conceptualizations on innovations (the what), as well as two commonly acknowledged measures. The enactment of innovation (the how) is discussed in Section 2.2, and the process of implementation (the why) — in Section 2.3.

2.1 *Innovations*

The object of most implementation research can be summarized under the notion of *innovations*, which might be ideas, practices, programs, policies or technologies. Innovations are generally perceived to be novel (Dearing et al., 2012) and to aim for changes in behaviors or practices of individual end-users (Century & Cassata, 2016) that are important to some stakeholders (Aguilar et al., 2019). Innovations are "the it" or "the what" of the desired change (Century & Cassata, 2016). The review by Koichu et al. (2021) demonstrates the diversity of innovation associated with implementation-related research in mathematics, which differs in shape (material or interaction-oriented), pre-definition of behavior, and intended system levels (classrooms, schools, districts and/or states). Within the implementation framework, innovations are viewed as complex, containing (a varied number of) core components (Century & Cassata, 2016): active ingredients that are theoretically or empirically linked to the desired outcomes (see also Domitrovich et al., 2008; Fixsen et al., 2005). Structural components include the organizational design and support elements; interactional components include intended behaviors, interactions, and practices of teachers, students, and other user groups (Century et al., 2012).

In terms of innovation development, the current definition of implementation research in mathematics education states innovators have ultimate agency over the resource at the outset (when innovation are shapes), whereas implementers construct agency and make adaptations only during implementation (Aguilar et al., 2019). The majority of innovations found in the implementation literature is initiated by researchers, with very few documentations on initiatives by teachers (in collaboration with researchers) solving problems in their classrooms (Koichu et al., 2021). Cai and Hwang (2021), among others, have questioned the separation of responsibilities and engagements with innovations and its implementation research, pointing to a major danger of learning goals not being accomplished. They argue that, to create a productive learning environment, implementation should be seen as an inextricable part of innovation development, which could be done by having researchers and practitioners collaborate to continually improve an artifact (such as a lesson plan) (Cai & Hwang, 2021). These ideas are in line with broader trends moving away from the traditional conception of innovations that are "predesigned by centralized change-agents" and toward co-production by research and practice stakeholders (Dearing et al., 2012, p. 56).

2.2 *Innovation Enactment*

The first part of the innovation implementation framework (Century et al., 2012; Century & Cassata, 2014) is about *innovation enactment* and assesses the extent to which intended components of the innovation model are actually enacted or in use at a particular location and moment in time. This analysis examines *how* innovations are enacted or used. Initially, these authors intended to develop instruments for measuring implementation fidelity, comparing actual enactment to an "ideal" (O'Donnell, 2008). Fidelity is important in effectiveness studies because faithful replication is likely to elicit similar positive results in natural settings (Lendrum & Humphrey, 2012; Century & Cassata, 2014). However, there is a tension in the field between fidelity and adaptations, as it is doubtful that programs can be implemented exactly as intended due to inherent differences in settings and users (O'Donnell, 2008; Lendrum & Humphrey, 2012). A lack of fidelity was described by Lendrum and Humphrey (2012) as discarding program components, whereas adaptations are considered either additions of new components or modifications to existing ones. Changes may be advantageous, providing a better match in current situations and so increasing sustainability (Lendrum & Humphrey, 2012). However, a greater number of changes or extreme mutations may cause important components to change in such a way that the program's integrity and effectiveness are jeopardized (O'Donnell, 2008; Lendrum & Humphrey, 2012).

The innovation enactment analysis (Century & Cassata, 2014) evaluates the *use* or status of component enactment at a given time, which can be summarized under an "implementation profile" (Century & Cassata, 2014: p. 97) with sufficient data. In similar vein, Buxton et al. (2015: p. 499) reframed fidelity of implementation as "multiplicities of enactment", in which students and teachers are actors taking ownership of practices that may gradually lead to transformation based on a variety of personal and contextual factors. The innovation enactment analysis, which uses a component approach to innovations, is concerned with determining which portions of the program (core components) are or are not enacted, the extent and character of these enactments, and how each component's enactment relates to intended results (Century et al., 2012). In this regard, tracking variations in innovative enactments, as well as the kind and range of helpful, acceptable, and undesirable adaptations, is critical (Century & Cassata, 2014).

2.3 *Implementation Process*

The second part of the innovation implementation framework examines the *implementation process* and assesses "what factors contribute and/or inhibit innovation implementation at a given point in time" (Century et al., 2012; Century & Cassata, 2014). It evaluates the contextual factors and conditions that may influence why end-users engage with the innovation in the manner that they do. The importance of context has been well established across and within domains, such as Domitrovich et al. (2008: p. 8), stating that "Implementation of evidence-based practices in schools does not occur in a vacuum; it is influenced by a broad array of school, district, state, and federal policies and practices". Cai et al. (2020) are also concerned with studying implementation in multiple contexts to understand how innovations interact with local conditions to continuously improve innovation. They define scaling up as learning from replications with variations, which is encapsulated in the concept of "replimentation".

Factors that influence implementation quality or innovation implementation have been grouped in a variety of ways, including macro, school, and individual levels, all of which are interrelated and influence implementation quality and learning outcomes (Domitrovich et al., 2008). The influential factors in the implementation process analysis were divided into five spheres (Century & Cassata, 2016): (1) individual user characteristics (e.g., understanding, experience, and motivation, but also openness to new experiences and perspectives on teaching), (2) organizational and environment characteristics (e.g., class size, physical space, and scheduling, but also administration, decision-making, and collective attitudes), (3) attributes of the innovation (perceived or actual), (4) implementation support strategies or implementation drivers, and (5) time.

3 Action-Based Embodied Design

The main aim of this paper is to apply the innovation implementation framework to compare the engagements of students and teachers with the *Action-Based Embodied Design for Proportions* situated in two Dutch elementary school classrooms, with engagements from similar laboratory (e.g., Duijzer et al., 2017) and classroom (e.g., Negrete, 2013) studies. We are interested in assessing whether core components for action-based learning appeared in these classrooms, to what extent adaptations are made, and the factors — in terms of the individual, the environment, the technology, and the support strategies — that might affect cross-setting differences. While the latter parameters were not explicitly measured in the current investigation, they were used to construct conjectures regarding cross-setting differences. In the following sections, we describe the core components and learning and teaching outcomes from laboratory studies (Section 3.1) and transitions and outcomes from prior classroom studies (Section 3.2).

3.1 *Previous Research in Laboratories*

3.1.1 Ecological Dynamics

The central premise of all action-based designs is that all learning, whether juggling, playing the piano, or proportional reasoning, is "learning to move in new ways" (Abrahamson & Sánchez-García, 2016). The genre adopted the ecological dynamics framework (Araújo et al., 2006; Abrahamson & Sánchez-García, 2016) developed by sport scientists to investigate how individuals develop athletic skills and combines dynamical systems theory (Smith & Thelen, 1996) and ecological psychology (Gibson, 1979). The process towards skilled behavior (coordination) of any kind is viewed as a self-organizing and non-linear phenomenon (Smith & Thelen, 1996) that emerges from the continuous dynamic relationship between the learner, the task, and the possibilities for action (affordances) offered in a specific environment (Gibson, 1979). Rather than the mechanical or direct instruction approach commonly used in mathematics education, practitioners following a constraint-led approach manipulate constraints to facilitate and direct the evolving organism — environment relationship (Araújo et al., 2006).

3.1.2 Motor-Control Problems

Figure 1 depicts an action-based approach to proportional reasoning for the 1:2 proportion (e.g., Duijzer et al., 2017). Instead of presenting students with the standard symbolic notation of proportions, such as 1:2 = 2:4 = 3:?, students

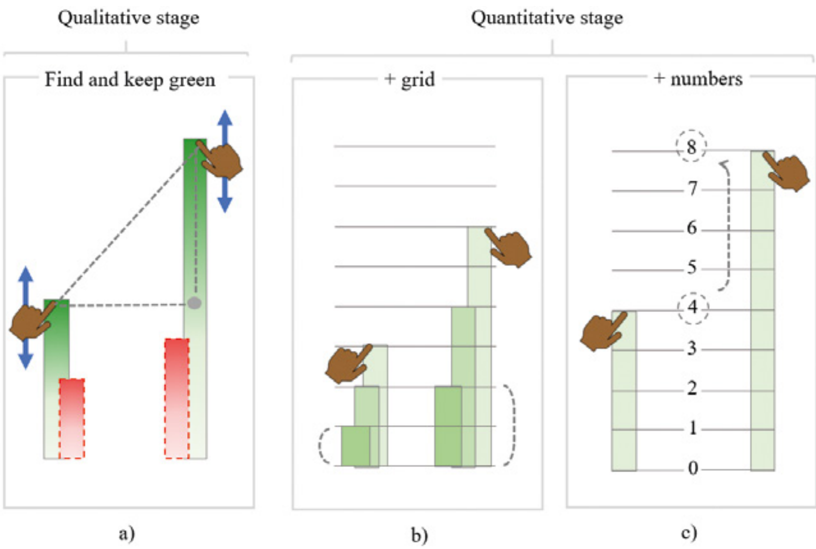

FIGURE 1 Learning sequence in the action-based embodied design for proportions exemplified for the 1:2 challenge. (a) Enactment with green and red (stripes) feedback eliciting a triangular shaped attentional anchor; (b) discretized enactment in the presence of an overlaid grid; (c) arithmetic recruitment with the supplemented numbers
ADAPTED FROM ABRAHAMSON ET AL. (2014)

solve a motor control problem (Bernstein, 1996) on tablets or Kinect devices, in which they are tasked with moving the two bars so that they turn green (Figure 1a). Green feedback occurs only when the right bar is twice as high or moves twice as fast as the left bar; otherwise, the feedback is red. Students are given ample opportunities to achieve coordination (to act) during this qualitative stage, and tutors then prompt them to articulate their strategy (to reflect). The acting and reflecting stages are repeated in the quantitative stage, but the constraints are altered as the instructor overlays measuring instruments onto the interaction space, such as a grid (Figure 1b), which is later supplemented with numbers (Figure 1c). The parallel bars condition is used in this study (e.g., Duijzer et al., 2017), but there are several other options, including the original condition of moving cursors (e.g., Abrahamson et al., 2014), real-life icons like balloons (e.g., Rosen et al., 2016), and orthogonal directions (e.g., Abrahamson et al., 2016). Extensive analyses of multimodal data (actions, perceptions, and reflections) from students' learning paths in laboratories reveal idiosyncratic, nonlinear, and condition-dependent but consistent patterns of student behavior.

3.1.3 Learning Trajectories

Students typically begin by exploring the environment without a clear strategy (alternating the bars or moving them at the same height), haphazardly finding green, only to realize that the relationship between the elements is important (Abrahamson et al., 2014; Duijzer et al., 2017). An additive enactment strategy (Abrahamson et al., 2014; Duijzer et al., 2017) — keeping the bars at a constant distance from each other — is frequently brought to the fore, coinciding with well-reported additive numerical strategies like thinking that 1:2 = 2:3 = 3:4 (e.g., Lamon, 2007), but resulting in red feedback and signaling the need for reconsideration. With enough time, all students can achieve a functional level of performance. Eye tracking studies consistently show that the emergence of so-called attentional anchors, self-imposed perceptual constraints that provide a "steering wheel," drives performance improvement (e.g., Hutto et al., 2015). Figure 1a depicts the typical triadic structure for the bars task, which includes a point halfway up the right bar (Duijzer et al., 2017); for attentional anchors across the proportion variations and other topics, see Alberto et al. (2021).

Students describe their methods of gaining control through these attentional anchors in interactions with the tutor. Students' initial descriptions may be out of sync with their actions, such as "you stay the same distance apart" (Abrahamson et al., 2011: p. 68) or "the bars turn green when they move at the same pace" (Abrahamson et al., 2016: p. 230), and, predictably, competing strategies can be expressed within pair work (Abrahamson et al., 2011). Tutors are critical in both challenging students and progressively transforming students' informal descriptions into mathematical discourse through multimodal re-voicing, which includes techniques such as repeating, omitting, elaborating, or modifying verbalizations and gestures (Abrahamson et al., 2012; Flood, 2018; Flood et al., 2020). All students eventually reach an "aha moment" while interacting with the tool and/or the tutor and state an effective rule that represents their actions, such as "The smallest is half of the other" (Abrahamson et al., 2016: p. 230) or "it has to get, like, farther away, the higher up we are" (Abrahamson et al., 2011: p. 68).

Consistent with the ecological dynamics theory, introducing quantitative artifacts into the interaction-space introduces a new constraint that reconfigures students' actions, resulting in the discovery of new affordances (Abrahamson & Sánchez-García, 2016). When the grid is present (Figure 1b), students' motor strategy changes from continuous and simultaneous to discrete and sequential along the grid lines, optimizing control (Abrahamson et al., 2011). As a result, their descriptions change, such as "If this one moves up two, this one moves up one" (Duijzer et al., 2017, p. 13), which is an a-per-b strategy similar to cultural forms. Within pair work, the grid can act as a deciding tool between opposing solutions (Abrahamson et al., 2011). Supplementing numbers (Figure 1c) allows for the creation of ordered pairings as well as arithmetic techniques (Abrahamson et al., 2016). Timing, familiarity, and compatibility (Abrahamson et al., 2011) as well as tutor responsiveness (Abrahamson et al., 2012) all play a role in the adoption of these artifacts.

3.2 *Previous Studies in Classrooms*

While the majority of research on action-based designs has been done in laboratories, two initiatives have used distinct techniques to bring them into classrooms. Petrick (2012) compared the learning outcomes of students who took turns enacting the cursor Wii task remotes (e.g., Abrahamson et al., 2014) in front of the class to students who watched a video of a typical learning progression. In both situations, the entire class was invited to make hand gestures while sharing thoughts with a partner every few minutes, and the measurement tools were added at around two-thirds of the time. A detailed analysis of students' multimodal engagements was lacking, but experimental results revealed that, while the 1:2 relationship emerged in both conditions, observing students used significantly fewer words overall, with less mathematical detail and specification of the proportional relationship. This finding was also found when the two levels of engagement in classrooms for the topic of angles were compared (King & Petrick Smith, 2018), suggesting the importance of having students enact mathematical concepts.

The current study closely resembles the second implementation project (Lee, 2013; Negrete, 2013), in which students worked in small groups enacting the bars task on tablets (e.g., Duijzer et al., 2017). The pilot with a small break-out group revealed that students' everyday tablet fluency could derail the intended activity sequence, resulting in the learning goals not being fully met (Negrete, 2013). A case study group missed an opportunity to define the qualitative non-numerical description since they only looked at the qualitative features for a minute or so before pressing the accessible buttons, swiftly settling on the grid plus numbers mode, where they remained. These findings were used to modify the instruction (keeping green in as many ways as possible) for the scaled-up classroom study (Lee, 2013; Negrete, 2013). Outcomes demonstrated teachers negotiating different solution strategies in accordance with action-based components (Lee, 2013), as well as new engagements, such as students taking the initiative in mathematizing display features by using artifacts available in the classroom, such as requesting rulers (Negrete, 2013). The results of this classroom study were used to position our findings alongside those of the laboratory, but the results were not included in the study's design.

4 Methods

4.1 *The Innovation*

The central action-based technology in the lesson was the bar version for proportions operationalized on tablets, which was evaluated in prior laboratory (Abrahamson et al., 2016; Duijzer et al., 2017) and classroom (Lee, 2013; Negrete, 2013) studies. Three proportion tasks were included in the digital tool: 1:2, 1:4 and 1:3. Students worked in pairs on the tablet, taking turns and completing the objectives. A worksheet was created to guide students through a series of exercises with the overarching goal of "cracking the code," and all three challenges were set up in the same way. The students began in a continuous interaction space with only the bars visible (Figure 1a). They were given the instructions to (1) take turns to find and count as many green locations as possible, (2) take turns to find the smallest green positions and keep them green while moving the bars up, and (3) reflect and construct a rule together. Students then continued in the quantitative mode, overlaying 20 lines and numbers onto the bars (see Figure 1c; the lines-only mode, shown in Figure 1b, was omitted for time reasons). Students were instructed to (4) collaborate while remaining green and moving the bars up and (5) solve a mathematical problem (e.g., a ratio table). A teacher's manual was created to walk teachers through the lesson in depth. This included components on (1) preparation, (2) task instructions, (3) guidance during collaborative work (varying from encouraging participants to look for additional green to asking for clarity, proposing counterexamples, and appropriating artifacts), and (4) plenary discussion. The first and second authors met with the teachers prior to the study to discuss the action-based lesson.

4.2 *The Context*

The implementation study was carried out in two elementary school classrooms in the Netherlands. The lesson was delivered to all students in their classrooms during one of their regular mathematics teaching hours with their regular teacher. Permission from the school head and written consent from the caregivers of all students were given. The first study was conducted in a Grade 6 classroom with 20 students from a traditional curriculum school. In the classroom, students' desks were arranged two-by-two with predetermined placements. The 6th grade teacher cooperated with the research institute and was interested in the project. The second study was carried out in a mixed Grade 3 and 4 classroom of 12 students from a progressive didactics school. The 4th grade teacher was an enthusiastic adopter. During both lessons, the first and second authors were present as assistant teachers, with some and no prior experience guiding action-based design, respectively.

4.3 *Cross-Setting Analysis*

Cameras were employed in both classrooms to capture interactions between student pairs, with teachers, and general classroom activity. The innovation enactment analysis included a thorough examination of students' and teachers' multimodal interactions, with a particular focus on the extent and nature with which the core components — as described in Section 3 and based on condensed laboratory findings — were enacted in these classrooms. We were particularly interested in the similarities and differences between (1) the various motor patterns based on tablet actions (explorative, default) and the presence of imaginary objects mediating coordination (speech, gestures, materials), (2) tutor guidance and tactics (idiosyncratic, constraint-led, re-voicing of words and gestures), and (3) the adoption of quantitative artifacts (changes in strategies and descriptions). In the implementation process analysis, deviations from the "laboratory model" were analyzed, with an emphasis on influencing factors within the spheres of the individuals, the environment, the innovation, and the teacher support strategies.

We chose two student pairs to describe here, one from each classroom, and various vignettes from their learning paths in the 1:2 and 1:4 challenges. These were chosen because they show how action-based designs may be used across various classes, schools, and grades, as well as the parallels and distinctions that can be found when using action-based designs in laboratories or classrooms. We are careful in making broad generalizations from only two pairs and a 60-minute intervention.

5 Results from Case Observations

The current section describes four vignettes that contain several scenes from the learning trajectories of two student pairs and their teachers during the 1:2 and 1:4 challenge. These classroom examples' engagements are compared to engagements found in similar laboratory (Abrahamson et al., 2016; Duijzer et al., 2017) and classroom (Lee, 2013; Negrete, 2013) research, as well as to each other. The first two vignettes are from a sixth-grade student pair, Eline and Stan (all names are pseudonyms), guided by their teacher. These vignettes focus on similarities in enacting and guiding coordination patterns towards effective rule statement with attentional anchors (Section 5.1) and extra opportunities in being self-directed in included artifacts such as rulers with teacher responsiveness (Section 5.2). The third and fourth vignettes are from a fourth-grade student pair, Iris and Frida, who are guided by their teacher and a researcher. These vignettes address a different trajectory and guidance towards effective rule statement but with similar coordinative solutions and attentional anchors

(Section 5.3) and extra opportunities in covert unit measurement though with less responsiveness and more directedness of classroom tutors (Section 5.4). Each vignette is followed by a discussion of the extent and nature of adaptations, as well as potential influencing factors.

5.1 *Similarities in the 1:2 Challenge*

5.1.1 Students' Initial Strategies

In the 1:2 challenge, 6th grade students Eline and Stan explored the interaction space by doing things like alternating the bars, which led to the student pair haphazardly finding green. When given the objective of keeping green, the students attempted a fixed interval, but in the end were able to keep at least partially green by, for example, inching their way upwards. The students explained their approach to achieving green in various ways, at first to each other (verbalizations are put in bold):

> Eline: **That one** [left bar] **should always stay lower than the other one** [right bar].
>
> Stan: **This one** [left bar] **you should just put half-way and the other one** [right bar] **totally at the top.**

Eline, like many laboratory students, was concerned by the uneven height of the bars (e.g., Abrahamson et al., 2014). She did not specify the distance between the bars; thus it is unclear whether she expected the spacing to remain constant or increase as the bars become bigger. Stan described a positional strategy that includes the phrase "half-way," which most likely refers to the screen dimension rather than the other bar. Similar expressions have been found in the proportion task's balloon (Rosen et al., 2016) and orthogonal (Abrahamson et al., 2016) conditions, which treat the axes individually rather than as related.

5.1.2 Teacher's Interactivity

Tutors encourage students to specify their solutions in laboratory studies. When the 6th grade teacher inquired about the students' early strategies, the following multimodal conversation ensued (actions on bars and gestures are enclosed in parentheses):

> Eline: **They can't be equal.**
>
> Stan: **One a bit slower than the other.**
>
> Teacher: (Moves the bars unequally but in red). **Yes, but here it is also not equal and yet it is red.** (Puts the bars back in a green position). **What do you see?**

Eline: (Figure 2a. Gestures a diagonal line connecting the two tops of the bar). **You have to do them like that.**
Stan: (silence). **One a bit slower.**
Teacher: (moves one bar a bit slower but with red).
Stan: **You are going too fast, it has to stay exactly in the middle.** (Figure 2a: Gestures a horizontal line from the top of the left bar to the right bar landing it halfway up along the larger bar).
Eline: **O, that one halfway, we got it!**
Stan: **Yes I got it too, […], yes, it should be half**

While Eline continued to use her unequal-distance technique, Stan devised a new one that was also seen in laboratories (e.g., Abrahamson et al., 2014), taking the varied speeds of the bars into account. The teacher interacted with both students' responses by offering counterexamples that stayed true to the students' ideas but resulted in red feedback. For distance, this approach was described in the instructor's guidebook and then it was extended to speed by this teacher. Using gestures, both students became more explicit about their plan (see Figure 2a). Eline used a diagonal gesture to emphasize the distance between the bars. Stan explained his approach for varying speeds by

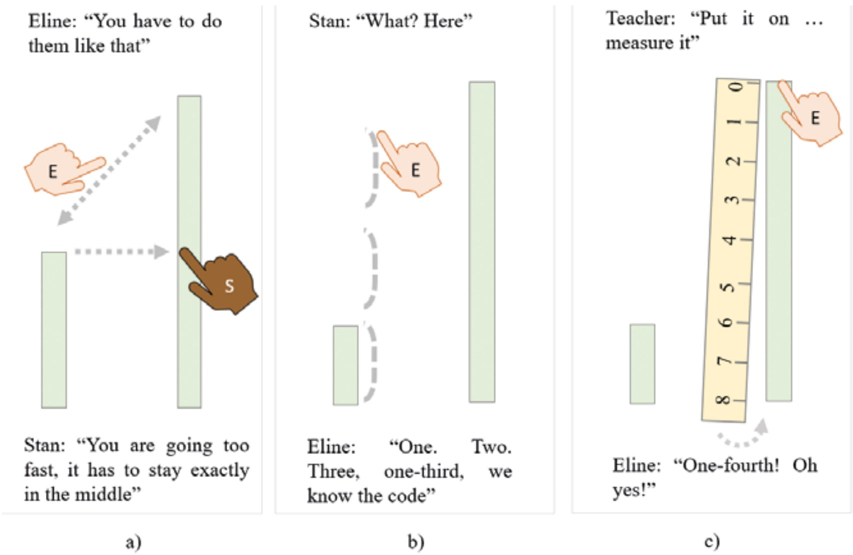

FIGURE 2 Scenes from the learning trajectory of 6th grade pair Stan (S) and Eline (E). (a) Students' multimodal expressions in the 1:2 challenge; (b) Eline's unit iteration above the left bar; (c) student–teacher interaction on ruler strategies in the 1:4 challenge

emphasizing the importance of being "in the middle" (which now refers to the other bar), which he underlined with a horizontal gesture. The gestures have a clear correspondence with the sites of attentional anchors established in laboratory experiments (pointing in Abrahamson et al., 2016; Figure 1a). The pair came to the conclusion that the left bar was half the size of the right, and their excitement was clearly visible as they solved the code of the 1:2 challenge.

5.1.3 Reflection

In terms of developmental patterns toward coordination (explorative, default and correct action strategies) and various descriptions attentive to distance and speed properties, this 6th grade student pair engaged with the innovation quite similarly to students performing in comparable laboratory research (Abrahamson et al., 2014; Duijzer et al., 2017). In interactions with the teacher, there were also significant signs of the mediation role of attentional anchors in successful rule-statement articulated in speech and supported by gestures. The 6th grade teacher's role in guiding these children' sensorimotor coordination was also similar to that of a tutor in a lab environment (e.g., Flood, 2018). The teacher was directly involved with students' unique and informal tactics, which helped the students improve and link the numerous solutions, in accordance with the action-based pedagogy (Abrahamson et al., 2014). The nature of teacher enactment has received little attention thus far, and we will see different adoption processes in the following sections.

5.2 *Students Including Rulers*

5.2.1 Students' Ruler Strategies

At the start of the 1:4 challenge, we met the 6th grade student pair again (we omitted the 1:2 quantitative stage). After completing the 1:2 challenge, the students expected similar rule types in the 1:4 challenge. Stan and Eline only interacted with the bars for a brief moment — finding green more easily, but with fewer examples — and Eline stated almost immediately that she knew the code, which she explained to Stan as follows:

Stan: (finds a green position). **What? Here**. (shows the green position to Eline).

Eline: Figure 2b. **Yes**. (points to the left bar). **One**. (slides her finger up). **Two**. (slides her finger up until it is as high as the right bar). **Three, one-third, we know the code**.

Eline iterated (left bar) units above the left bar with her index finger (Figure 2b): she was "guesstimating" as she measured without paying attention to accurate and fixed units, resulting in a three-times answer for the 1:4 challenge.

While working on the 1:4 challenge, the student pair noticed that several of their classmates were using rulers, and both rushed to grab a ruler from their drawers, recognizing its affordances. The sixth-graders used their rulers to measure the heights of the bars multiple times, but the results, like 1.5 and 6 cm, were insufficient to produce a four-times answer. Eline devised a recurring ruler strategy: finding a new green instance, measuring the bars' respective lengths with the ruler, and calculating their multiplication factor on paper. She was observed using skewed ruler positioning (Figure 2c) and imprecise ruler reading, as well as having problems with arithmetic calculations that involved decimal numbers. Furthermore, the required error margin for green feedback for coordination was unsuitable for precise measurement (e.g., green at 2 and 8.5 cm). In the end, Eline's ruler strategy did not (yet) yield enough information to crack the 1:4 code.

5.2.2 Teacher's Ruler Strategy

Though using the ruler was not explicitly stated in the teachers' manual, the teacher gladly accepted it. Eline presented her problem using the findings of her ruler measurements of 2 and 10.5 cm. The teacher responded by assisting her in measuring precisely by straightening the ruler (see Figure 2c), which might be considered a sort of "shaping" of artifact use that maximizes students' experiences in solving the problem at hand (Abrahamson & Sánchez-García, 2016). The right bar appeared to be "something over 10 [cm]" while the left bar was "even 2.5 [cm]." While these measurements could yield a 4-times arithmetic solution, the teacher suggested using a different ruler strategy, beginning with putting the bars at exactly 2 cm:

Teacher:	Figure 2c. **Put it on** [2 cm]**... measure it**
Eline:	(puts the ruler next to the left bar and tries to make the bar 2 cm).
Teacher:	**Accurately**. (helps Eline to make the bar precisely 2 cm by holding the ruler). **Ok, now that one**. (points to the left bar). **How much should that one be now?**
Eline:	(moves the right bar until they turn dark green and tries to measure it).
Teacher:	**Completely green. It can be even more green. This is still a bit brown**.
Eline:	(moves the right bar down a bit until it becomes bright green).
Teacher:	**Yes stop**.
Eline:	(looks at the ruler). **8.5**
Teacher:	**Approximately**.
Eline:	**8. No, 9**.

Teacher: **Put it on 9.**
Eline (uses the ruler to put the right bar on 9; The bars turn dark green).
Teacher: **Hey, it is turning brown again. And on 8?**
Eline: Figure 2c. (uses the ruler to put the right bar on 8; The bars turn bright green again). **One fourth! Oh yes! That took us really long.**

The teacher created a novel type of engagement in which the ruler is used in the same way as the gridlines are: the flexible bars are matched to a whole number line on the ruler (e.g., 2 cm). To convey this new technique to the students, the teacher used rather directive language, especially compared with his more open-ended questioning before the introduction of quantitative artifacts. A design constraint in play here might be that the action-based software was not optimized for precise ruler strategies with a green instance with the bars at 2 and 8.5 cm. Teacher guidance (along with knowledge of the rule) was required to encourage Eline to try both 9 (red feedback) and 8 (green feedback), bringing Eline to the correct code of one-fourth.

5.2.3 Reflection

Overall, the vignette depicts the student pair initiating the quantitative stage by incorporating rulers upon their interaction space — a fruitful idea that spread through the entire 6th grade classroom. While the use of rulers has been reported in a previous classroom study (Negrete, 2013), it has not been reported in laboratory studies in which tutors overlay digital measurement tools in the interaction field in response to students' expressed strategies (see Alberto et al. (2021). While both physical and "digital" rulers serve the same purpose, self-directed quantification may be beneficial to the evolution of action-based design (Cai et al., 2020) by potentially alleviating timing issues in the introducing of artifacts and allowing students to extend and improve their concrete measurement skills in the context of proportions. We hypothesized that the disparities in engagement in classroom and laboratory studies were largely due to technological and environmental factors. Unlike laboratory studies, both classroom studies (unintentionally) used technologies in which the bars remained rather than disappeared when the hands were released from the screen, relieving students' hands from gripping them and allowing operations on or beside them. This is not an intrinsic factor in setting, and comparable affordances would most likely be obtained in laboratories using remaining-bars tasks. However, incorporating remaining-bars tasks into classroom environments ensured that students had access to the measurement tools they found useful at any one time. This material richness is an inherent feature of mathematics natural settings that can be replicated in laboratory settings by providing materials.

Though the benefits of students' introduction of the ruler were not defined in the teachers' manual, the 6th grade teacher noted them. Teachers are usually familiar with rulers and the symbols they represent. In terms of how the ruler was used, it appears that more directive wording was used to introduce the new ruler strategy for students to adopt, rather than collaborative establishment and improvement on students' idiosyncratic (ruler) solutions as the teacher did with the pair earlier in line with the guidance component of action-based pedagogy (Flood et al., 2020). Paying attention to precision in measuring and arithmetic over several green occasions could have enhanced the pupils' distinctive ruler tactics. We hypothesized that this directedness stemmed from a lack of experience with the constraint-led approach contrasted to the direct instruction commonly used in classroom pedagogies, which could be particularly noticeable in the face of quantitative measures. Section 5.4 will demonstrate a similar level of teacher directedness.

5.3 *Differences in the 1:2 Challenge*

Fourth grade students Iris and Frida also solved the 1:2 and 1:4 challenge, although in a different way than 6th grade students Eline and Stan. Changes in technology and the environment continue to play a role, but with new affordances and responses from teachers.

5.3.1 Students' Initial Strategies

Iris and Frida both found and tallied several green positions in the 1:2 challenge. Both were able to move the bars while keeping them, at least partially, green in the keep-it-green task. Iris reflected to Frida on her solution strategy:

> Iris: **You have to go a little faster with that one** [right bar] **than with that one** [left bar].

Iris was paying attention to the speed of the bars (as did 6th grader Stan), which is a common expression among students in laboratories (Abrahamson et al., 2014). She correctly identified that one must move faster than the other, but she did not specify how much faster. At this time, Frida did not offer a solution.

5.3.2 Others Telling Rules

When the 4th grade teacher inquired about their code, they had the following brief conversation:

Iris: **The right bar has to be a little higher than the left** [bar].
Teacher: **Yes, that one has to be higher, but like this** (moves the left bar down). **one is also higher, so why is it not green? Think about it.**

Iris presented a novel approach, focusing on bars' varying height (as did 6th grader Eline) (Abrahamson et al., 2014). While the 4th grade teacher, like the 6th grade teacher, presented a counterexample, she did not engage in nearly as much contact with the students to let them better their strategies and express them in a multimodal manner. Furthermore, the student pair was not given enough time to investigate, since a whole class discussion began shortly after, during which the following scene involving Tom, another classmate, occurred:

Teacher: **Who found the rule?**
Tom: **The right bar should always be taller than the left bar, and the left bar should always be half as tall as the right bar.**
Teacher: **Do you want to test whether that is correct? Give it a try.**
Iris: Figure 3a. (grabs a die and places it at the top of the left bar, then moves it in a straight horizontal line towards the right bar, landing it halfway up along the larger bar).

Classmate Tom initially described his solution similar to what Iris told the teacher (one higher than the other), but then added a specification at which the

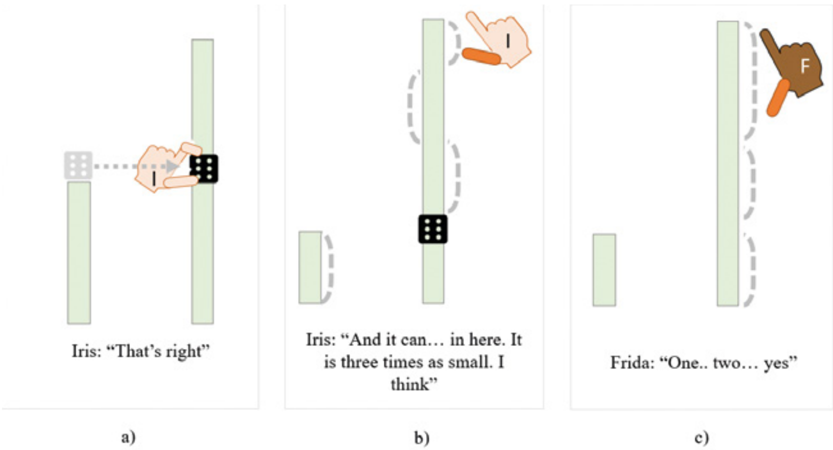

FIGURE 3 Scenes from the learning trajectory of 4th grade pair Iris (I) and Frida (F). (a) Iris's use of the die to validate Tom's solution for the 1:2 challenge; (b) Iris's 1-per-3 above hand measurement strategy; (c) Frida's hand measuring tailored to Iris's three-time solution

student pair had not yet arrived (one half the other). Prompted by the teacher to validate this solution, Iris employed the die that was introduced spontaneously by the teacher, who adapted the protocol to more clearly structure whose turn it was to interact with the tablet. Iris' horizontal-die-gesture — visible to Frida but inaccessible to other students or the teacher — resonated with 6th grade student Stan's horizontal-finger-gesture, and, as previously discussed, prior laboratory findings of gaze and pointing centering around halfway along the right bar (Figure 1a; Duijzer et al., 2017). The use of a die could be described as an intermediary object to "language" and to actualize attentional anchors in more tangible forms (Radford, 2014).

5.3.3 Reflection

In terms of establishing coordination, attending to speed and distance, and a clear role of attentional anchors, the 4th grade students exhibited similarities to the 6th grade and laboratory students. While attentional anchors typically emerge prior to and as a basis for rule statements in laboratory studies (Abrahamson et al., 2016), they now emerge to confirm another student's rule for green (see, Abrahamson and Sánchez-García (2016) for attentional anchors being suggested by instructors, a constraint of type augmented information). In the overall lesson plan described in the teacher support strategies, the timing of the classroom discussion was based not on individual students' insights but on a time schedule, which may have been preliminary in the eyes of these fourth grade students. Students being at different stages of the task is an inherent factor in classrooms. Prior to the general classroom discussion, it was unclear to what extent the other classroom students discovered and verbalized an effective rule. Overall, the fourth-grade teacher appeared less interactive than the sixth-grade teacher and laboratory tutors, but she did provide students with feedback to reconsider their solution.

5.4 *Students Including Their Hands*

5.4.1 Students' Unit Strategies

In the 1:4 challenge, the student pair, like the 6th grade pair, only briefly manipulate the bars, and Iris whispered almost immediately that she knew the code. The students engaged in the following multimodal conversation:

Iris: Figure 3b. **Uh, this is the line** (traces her finger horizontally from the top of the left bar to the right bar, placing the die a quarter up along the right bar). **And it can** (matches her thumb and index finger to match the left bar) **in here** (moves the pinch above the die, matches her thumb and index finger of the other hand with the left bar, and moves it above her other hand, then moves her right hand above the left while decreasing the space to match the remainder of the bar). **It is 3 times as small**. (removes the die). **I think.**

Frida: Figure 3c. (Matches her thumb and index finger to match the length of the left bar on the right bar). **One**. (moves the "empty space" upward on the right bar). **Two**. (moves the "empty space upward while increasing her hand span to match the top of the right bar). **Yes**.

Both students, like 6th grader Eline (see Figure 2b), applied unit measurement procedures, resulting in a three-times solution to the 1:4 challenge. Iris created a left-bar unit with her index and thumb and iterated it above the die she placed at a quarter of the right bar (Figure 3b), a strategy similar to the delta strategy in which the left bar is compared to the interval or difference between the elements (Abrahamson et al., 2014). It is important to note that, while the 1-per-3-above description is non-normative, transitioning from part-part to part-whole perceptual orientations is not wrong and, in fact, pedagogically desirable (e.g., Spinillo & Bryant, 1991). However, because she measured from the top of the die rather than the middle, she had to reduce the last unit to match the top of the right bar. While Frida had been largely unresponsive to Iris's actions and verbalizations thus far, she now appeared to have adopted Iris' strategy to iterate (left bar) units, but customized it by measuring the entire right bar. She, on the other hand, tailored her enactment to Iris's three-times statement (a constraint), thus she had to expand the spacing between her fingers to cover the remainder of the right bar (Figure 3c).

In a previous classroom study, a student demonstrated a related strategy by solving a 1:3 challenge by matching the length of the left bar with his pen and iterating this unit along the right bar (Negrete, 2013). These phenomena are not limited to classrooms; they also occur in laboratories, albeit to a lesser extent. One laboratory study (Flood et al., 2020) showed an experienced tutor (not students) manually iterating units in a 2:3 challenge alongside a student's midair gesture. Another student performing the balloon variant of the proportion task was shown to use this icon as a unit to measure the distance between the elements (Palatnik & Abrahamson, 2018), while another performing the cursors condition used the Dell logo on the screen encasement to project hand locations (see, e.g., Abrahamson et al., 2011). While these occurrences are not unique to classrooms and might occur everywhere, laboratory investigations were probably less likely to make these actions and objects available due to technological (disappearing bar) and environmental (materially poorer) factors.

5.4.2 Misalignment with Teachers

While 6th graders Eline and Stan introduced quantitative artifacts themselves, for 4th graders Iris and Frida the lines and numbers were added to the 1:4 challenge based on classroom instructions and the time schedule in the teacher's manual. The pair did not appear to have appropriated these quantitative artifacts for their usefulness, as has also been found in laboratory studies (e.g., Abrahamson et al., 2011). For example, the pair found green instances in which neither of the bars were positioned upon the lines. Furthermore, they explained explicitly to the researcher that they did not need these artifacts because the code was already found to be "three times as small." Unaware of the pair's history of behaviors that led to their three-times-as-large solution, the researcher directed the pair's attention to the measurement tool as a means to (dis)prove their code. After assisting the students in placing the bars exactly on lines 2 and 8, the following discussion ensued:

Researcher:	**Very good, now it is completely green. What would that mean?**
Frida:	(uses her one-handed measurement strategy like in Figure 3c). **Three times as small?**
Researcher:	**Does it fit in there three times? Look!?**
Frida:	(again starts with the measuring gesture, but then stops and grabs the die which she moves in a straight line from the top of the left bar to the right bar.)
Iris:	**Five times as small.**
Researcher:	**Does it fit in five times? How big is this one?** (points to the right bar).
Frida:	(uses her one-handed measurement strategy like in Figure 3c). **One, two, three.**
Researcher:	**How big is this one?** (points to the right bar and then to the number 8).
Frida:	**Eight.**
Researcher:	**And how big is this one?** (points to the left bar).
Frida:	**Two. Huh?!**
Iris:	(points to the right bar and makes a movement that indicates she is counting). **Two and a half.**
Researcher:	**Two and a half? How many times does 2 fit in 8?**

Frida: (uses her one-handed measurement strategy with better precision). **One, two … four.**
Iris: **Four**
Researcher: **Yes, and would that always work?**
Frida: **Yes, yes, no, no yes.**
Researcher: **Give it a try.** (walks away).
Iris: (moves right bar to 12 and searches with the left at first at 6 and then at 3 turning the bars green). **Yes, but then they do not work.** (uses her one-handed measurement strategy like in Figure 3c). **One, two, huh?**

The researcher guided the students in perceiving, using lines and numbers, that 8 is four times larger than 2, not three times. The students did not readily adopt this numeric strategy, instead expressing their previous unit strategies: Iris was pointing to the right bar as if counting units, while Frida frequently displayed her imprecise one-handed measuring strategy (Figure 3c), even using the die (like Iris did, shown in Figure 3b). The researcher missed these actions, instead reacting with astonishment to their incorrect numerical responses. To reinforce the artifact strategy, the researcher took a more direct instruction approach, pointing to the numbers vividly. This is similar to the 6th grade teacher's more directive wording in interaction with students' ruler strategies (see Section 5.2.2) and departs from the component of strengthening students' strategies (Abrahamson et al., 2012; Flood et al., 2020). While both students expressed a four-times solution, Iris most likely did so to answer the researcher's arithmetic question, whereas Frida did so by improving her measurement method with equal-sized units. The researcher, however, overlooked the improvement and thus it was not consolidated. As a result, the pair may have struggled to apply the four-times rule to a 3:12 situation, in which inaccurate hand measuring resurfaced (Figure 3c), with neither student using the arithmetic approach modeled by the researcher.

The researcher's unawareness of the students' unique methods and re-direction towards numerical strategies is also reflected in the pair's contact with their teacher. The students were still grappling with the 3:12 situation when the teacher asked about their code, and the following exchange ensued:

Iris: **No, we have … it** [presumably the space above the die] **is not three times as small.**
Frida: (uses her one-handed measuring strategy with precision). **One, two, three, it is four times!**

Teacher: **It is four times. You can see it, look. If you put it on 1** (puts the left bar on 1 unit), **where should the right one be?**
Frida: (drags the right bar down from 12, she quickly stops her movement around 5 then moves the bar further down to 4)
Teacher: **Not 5. Then it stands on 4. So how many times as big is it then?**
Frida: **Four** [mumbles].
Teacher: **Four times. So if you put left on 2, what fits with that? Iris?**
Frida: (puts the left bar on 2 and moves the right bar up and down)
Iris: **I did not know that the two** (points to the left bar), **like, counted here** (points to the bottom of the right bar).
Teacher: **Okay, yes, I get it. So if you have this one** [left bar] **on 2, where should the right one be? 1 fits with 4, what would fit with 2 then?**
Frida: (finds the bars turn green around 8 and leaves the bar just below 8)
Teacher: **It fits with 8. So 1 fits with 4 and 2 fits with 8. So how many times as big is it then?**
Iris: **Four**
Teacher: **Four times as big. Which one fits with three then?** (points to left bar). **Try to think first. What fits with 3.**
Frida: **Oh, I already know it** (puts the right bar on 12, then puts the left bar on 3).

While the teacher repeated Frida's numerical expression, she did not enquire about Frida's multimodal solution, and thus was unaware of the improvement in hand measurement accuracy and the resulting Eureka moment that the right bar is actually four times as large as the left. Instead, the teacher, as we have seen before, introduced a new strategy, this time a step-by-step strategy for generalizing the 4-times solution through several ratio pairs (1 and 4, 2 and 8, and 3 and 12). While watching Frida and the teacher interact, Iris had an epiphany when she pointed to the lower part of the right bar and realized she had forgotten to include the lowest unit when expressing her code. Iris referred to the lower unit as "the two," effectively transposing the unit to the item (Hutto et al., 2015). The teacher was unaware of the understanding and continued with the sequenced semi-direct instruction to have the pair anticipate what would fit with 3. Frida appeared to have accepted the proposed arithmetic technique (grounded in her idiosyncratic hand-measuring solution), as she did not search for green but instead placed the left bar on 12 right away.

5.4.3 Reflection

Overall, the tutors helping the 4th grade pair through the 1:4 problem appeared to be mostly reacting to (in)correct numerical conclusions, while being less perceptive and hence unresponsive to the students' valuable material and physical engagements that generated these solutions. Similarly, classroom tutors offered new types of enactments, which were designed to help students appropriate the digitally available artifacts while focusing on arithmetic skills. Teaching with embodied technologies implies specific embodied-behavior oriented practices (Flood et al., 2020), with responsiveness to students' full multimodal behavior and collaborative multimodal establishment and enhancement of students' strategies (Flood, 2018; Flood et al., 2020). Despite the classroom tutors' redirections, the student pair mostly stuck to their established approach, and in the process, both students improved their enacted and described solutions without being directly addressed. It is entirely possible, however, that other students' strategies went unnoticed and uncorrected, leading to less-than-ideal solutions at the end of the embodied lesson; thus, teachers' levels of perceptiveness, responsiveness, and directedness can have a real impact on student outcomes in classrooms.

We hypothesized that several factors were influencing these engagements. To begin with, teachers in classrooms can only view parts of students' learning paths as they move from one group to the next, whereas tutors in laboratories have constant access to an individual student's choices, tactics, and faulty reasoning. The classroom tutors did not enquire about the students' solutions which, along with the directedness, is most likely owing to a lack of student-centered approach training and limitations in teacher support strategies. Furthermore, both classroom tutor interventions occurred during the quantitative stage, with the goal of students appropriating the measuring instruments. Teachers may be more educated about the use of rulers and numeric expressions, whereas more "informal" expressions in the use of their hands or mundane materials (dice, pencils) may not be clearly credited as useful in the context of proportions. Both are vital in action-based pedagogy, but "Novices in any field are not able to make the distinctions of experts; they cannot see in the same detail, and therefore do not have the same nuanced repertoire of possible actions available," as Brown and Coles (2011: p. 862) put it. These will almost certainly necessitate more generalized training, including the use of meta-questions and tactics aimed at clarifying students' solutions, how they arrived at those solutions, and how they might be shared with others (Brown & Coles, 2011).

6 Discussion

This paper continues a series of studies aimed at understanding what, how, and why embodied learning technologies, which have so far been developed and tested primarily in laboratories, work in regular classroom settings. We compared the engagements of two student pairs and their teachers in classrooms with the action-based embodied design for proportions with similar laboratory (e.g., Duijzer et al., 2017) and classroom (e.g., Negrete, 2013) studies using the innovation implementation framework (Century et al., 2012; Century & Cassata, 2014). By implementing across settings, we obtain a more complete and realistic picture of potential engagements with the innovation and the factors that influence them.

Overall, both classroom pairs solved motor control problems in similar ways as students in laboratories in terms of sensorimotor coordination, with attentional anchors and the reflection upon them playing a central role (although ordering differed). Similar to previous classroom pairs (Negrete, 2013), one pair used overt hand measuring with a die in the qualitative stage, and one pair was self-directed to include rulers in the quantitative stage. While these affordances were present in laboratory studies, they were thought to be highlighted in these classroom studies due to non-setting technology differences (remaining vs. vanishing bars) and setting-specific environmental differences (material rich instead of poor). The three classroom tutors guided the students in both similar and dissimilar ways to laboratory tutors. In the qualitative stage, both teachers presented counterexamples, but the degree of involvement varied. In the quantitative stage, one teacher was responsive to the ruler, while two classroom tutors seemed unperceptive and unresponsive to students' manually expressed strategies, and all tended to "impose" new quantitative enactment strategies. These were thought to be caused by setting-specific limits in access to students' trajectories (fragmented vs. continuous access) and experience with the action-based pedagogy (embodied novices vs. experts). Future classroom investigations will demonstrate how distinctive these student pairs and teachers interactions are. Students and teachers in classrooms have a wide range of features and experiences, and they work under a variety of environmental, organizational (and sometimes technological) constraints and affordances.

Future research on embodied learning technologies may focus further on how to adapt delineated guidance practices for action-based (Abrahamson et al., 2012; Flood, 2018; Flood et al., 2020) and other embodied pedagogies

(Abrahamson et al., 2021) to classroom settings. Because research until now has mainly concentrated on one-on-one classes with a small group of qualified tutors, it is less obvious how to help actual teachers embrace these novel foreign strategies and serve as an individual tutor to every student. Although instructional strategies can be organized in teaching guides, learning to teach in a new way and changing classroom cultures may necessitate considering professional development. Maaß and Artigue (2013), while already at the stage of scaling up Inquiry-Based Learning techniques in routine teaching, underline the significance of paying attention to assumptions about the nature of mathematics learning and instruction. Embodied classrooms, as envisioned by Abrahamson et al. (2021: p. 157), legitimize a subjective mathematics experience that "may be personal, multimodal, pre-semiotic, and nuanced, yet is real, undeniable, and critical". This means that action and multimodal activity are valued as components of mathematics learning and teaching long before formal discretization and measurement tools become available. In practice, long-term and intense interventions that combine developing expertise within and outside the classroom are likely to help instructors (Maaß & Artigue, 2013). Using an enactivist approach to developing expertise, Brown and Coles (2011) proposed three principles for teacher collaborative learning, starting with shared experiences (e.g., reading the same text or watching the same video), focusing discussions on communal activities, and working through experiences to identify key and common learning points.

The relative scarcity of knowledge about teachers and classroom settings within the action-based embodied design genre may be a characteristic of a more general pattern, which is that innovation development and implementation research are not always intricately connected (Cai & Hwang, 2021). Although with new ideas, theories, and tools, efficacy trials within controlled settings are more than reasonable, including an early orientation toward traditional and realistic classroom constraints and affordances may have some advantages as well. This could mean to begin to innovate with more regard for the intended sample, instructional approach, existing cultures and settings. Such research does not exclude the laboratory setting as examples can include documenting teachers who are new to embodiment during their guidance of individual students or evaluating methods of having one tutor work with multiple student pairs. While maintaining considerable control (and access to multimodal monitoring technologies), the designs and conclusions of this branch of innovation development are expected to be more easily adaptable to classrooms in terms of quality. In addition to professional development and realistic constraints, thinking ahead about implementation of innovations also concerns other aspects of the educational system such as learning goals, assessment, and material resources (see, for example, van den Akker's (2007) curricular spider web). These and other considerations will become increasingly important as new technologies and programs with the potential to support learning will undoubtedly emerge in following decades.

Declaration of Conflicting Interests

The authors declare no conflicting interests.

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