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Optimizing public transport transfers by integrating timetable coordination and vehicle scheduling

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ABSTRACT

Transfer optimization in public transport (PT) networks can be achieved through coordinated timetabling and vehicle scheduling. Traditionally, the coordinated timetabling problem is solved first before proceeding to the vehicle scheduling problem. The integration of these two problems can help further reduce the total operation cost and improve the level of service, especially when timetables of different PT lines are well-coordinated at transfer stations. This work addresses the integrated PT timetable coordination and vehicle scheduling problem while ensuring that each PT line is dispatched with an even headway. We first separately formulate two integer linear programming models for the timetable coordination and vehicle scheduling problems. Next, the two models are integrated into a bi-objective integer linear programming model for the integrated timetable coordination and vehicle scheduling problem. For small size PT networks, the model can be solved by using an ε-constraint method, together with off-the-shelf optimization solvers. For large-size problems, two constraint-reduction procedures are developed to reduce the number of redundant constraints so as to reduce the computation complexity and improve the solution process. Finally, the models and solution method are applied to a numerical example and a real-world bus rapid transit (BRT) network in Chengdu, China. Computation results show that the solution generated by the sequential optimization approach is usually dominated by the Pareto-optimal solutions generated by the integrated optimization approach. Our findings suggest that it is not a wise decision to use the solution generated by the sequential optimization approach or the solution with the minimum fleet size generated by the integrated optimization approach. For practical implementation, it is recommended to choose the solution that has a fleet size of one more vehicle than the minimum fleet size.

1. Introduction

1.1. Background and motivation

Transfers in public transport (PT) networks are needed to create more efficient service network by allowing for more flexible route planning and reducing the total operation cost (Vuchic, 2005; Gkiotsalitis, 2022a, 2022b). A survey of the PT systems in Melbourne, Australia shows that bus ridership has a 48% transfer rate, i.e. the share of passengers that transfer at least once along their journey, amounting to almost half of all trips made (Currie & Loader, 2010). Another recent survey of bus transport in Beijing, China shows that transfer time – the time associated with walking and waiting at an interchange location - and access time together account on average for 35% of total journey travel time (The State Council of the People's Republic of China, 2016). Transfers are often cited as a key reason for PT being less attractive than private cars (Ceder, 2016; Chowdhury & Ceder, 2016). Missed transfer connection and long transfer waiting time will significantly reduce the attractiveness of PT services, which subsequently frustrates existing PT users and deters potential new users (Susilo & Cats, 2014). Consequently, reducing transfer waiting time can improve the ride experience for passengers, and thus lead to an increase in PT ridership and competiveness with private cars (Chowdhury et al., 2015; Kuo et al., 2023). Hence, one important consideration of PT policy makers, service planners and operators is how to optimize transfer coordination so as to achieve well-connected and 'seamless' transfers.

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The goal of providing well-coordinated transfers can be achieved through different levels of PT operations planning activities (Liu et al., 2021). At the strategic level, PT planners can design a network with a minimal number of transfers, and optimize the layout of transfer stations (Zhao & Ubaka, 2004; Guihaire & Hao, 2008; Yu et al., 2012). At the tactical level, coordinated timetables and optimized frequencies/headways can be developed to reduce passenger transfer waiting times (Domschke, 1989; Daganzo, 1990; Ceder et al., 2001; Ibarra-Rojas & Rios-Solis, 2012; Aksu & Akyol, 2014; Liu & Ceder, 2017a; Wang et al., 2020).

At the operational level, the vehicle scheduling process can be optimized to coordinate the arrival and departure times of PT vehicles at transfer stations so as to facilitate transfers (Salzborn, 1980; Désilets & Rousseau, 1992; Xiao et al., 2016). At the control level, various control strategies, such as vehicle holding, stop-skipping, speed control, short-turning, and boarding limits, can be employed to increase the actual occurrence of coordinated transfers (Lee & Schonfeld, 1994; Hall et al., 2001; Dessouky et al., 2003; Hadas & Ceder, 2010; Nesheli et al., 2015; Daganzo & Anderson, 2016; Gavriilidou & Cats, 2019; Gkiotsalitis et al., 2023).

It is preferable for all these different levels of PT operations planning activities to be conducted simultaneously in order to exploit the system's capability to the greatest extent and further maximize the coordination of transfers with minimal operational costs (Desaulniers & Hickman, 2007; Ibarra-Rojas et al., 2015; Ceder, 2016). First, the integrated optimization of different PT operations planning activities has been demonstrated to be methodologically feasible with great potential of further optimizing the performance of PT systems in a series of studies (e.g., Guihaire & Hao, 2010; Ibarra-Rojas et al., 2014; Liu & Ceder, 2017a; Fonseca et al., 2018; Wu et al., 2022; Xu et al., 2023). Second, commercial computer-aided transit scheduling software packages, such as SYNCRO (Désilets & Rousseau, 1992), HASTUS (Fleurent et al., 2004) and more recently Optibus, conduct integrated operations planning, thereby demonstrating that the integrated optimization approach is technically feasible. Third, recent reforms and changes of the operations planning activities in the PT systems in Beijing and Chengdu showcase that an integrated approach can significantly reduce the total operation cost while improving the level of service, demonstrating that the integrated approach is practically feasible (Liu & Ceder, 2017a; Liu & Ceder, 2017b). Therefore, nowadays, the integrated optimization of timetable coordination and vehicle scheduling has become methodologically, technically, and practically feasible.

This study focuses on integrating two fundamental and essential PT operations planning activities, namely timetable coordination and vehicle scheduling to further optimize transfers in PT networks with minimum operational costs. We consider a multiple-depot transfer-based PT network in which each line has an even headway. Both sequential and integrated optimization models are developed. An ε-constraint method, together with two constraint-reduction procedures, is employed to solve the integrated optimization model. The performances of the optimization models and solution method are demonstrated with both a numerical example and a large real-world bus rapid transit (BRT) network in Chengdu, China. It is anticipated that the models and solution method can serve as useful optimization framework and tools in supporting PT operators to conduct integrated optimization of timetable coordination and vehicle scheduling so as to further explore the trade-off between operational cost and level-of-service.

1.2. Literature review

1.2.1. Public transport timetable coordination

PT timetable coordination is the problem of determining the arrival and departure times of PT vehicles at stops/stations, especially at transfer stations, to facilitate passenger transfers (Bookbinder & Désilets, 1992; Ceder, 2016). Previous studies have developed various solution approaches to address this problem. A recent systematic review by Liu et al. (2021) classified the solution approaches into four categories, namely heuristic rule-based approach, analytical modelling approach, mathematical programming (MP) approach, and simulation-based approach. Among them, the MP approach is the most commonly adopted one, accounting for more than 68% of the studies. One advantage of the MP approach is that it usually uses discrete parameters and decision variables, and can generate more realistic results that can be directly applied into practice.

Almost all the MP models are in the form of an integer programming model using vehicle departure times from terminal stations, i.e. offset times, as the main decision variables (Liu et al., 2021). Some MP models further considered other decision variables, such as line headway/frequency (Shrivastava et al., 2002; Wu et al., 2019; Estrada et al., 2021), inter-station vehicle running times (Wong et al., 2008; Kwan & Chang, 2008; Wu et al., 2015), and dwell times (Wong et al., 2008; Kwan & Chang, 2008; Shang et al., 2018; Tian & Niu, 2019). There are also some studies which included an extra stopping time at a transfer station, i.e. slack time, to increase the coordination of transfers (Shafahi & Khani, 2010; Wu et al., 2015; Dou et al., 2015; Wu et al., 2019). However, the inclusion of slack times may increase the in-vehicle time of passengers. Thus, a trade-off between the transfer waiting time reduction and in-vehicle passenger travel time increase should be made.

PT timetable coordination MP models mostly differ in their optimization objectives. Several different objectives are considered in previous studies. The typical objective is to minimize the total transfer waiting time (Rapp & Gehner, 1976; Domschke, 1989; Cevallos & Zhao, 2006; Wong et al., 2008; Shafahi & Khani, 2010; Parbo et al., 2014; Gkiotsalitis & Maslekar, 2018; Abdolmaleki et al., 2020). Another commonly used objective is to maximize the number of successful transfers (Ceder et al., 2001; Liu et al., 2007; Ibarra-Rojas & Rios-Solis, 2012; Ibarra-Rojas et al., 2016; Cao et al., 2019; Guo et al., 2020). A successful transfer is usually defined by whether the arrival times of vehicles from two different lines at a transfer station either coincide or are within a time window (Ceder et al., 2001; Eranki, 2004). By doing so, passengers can transfer from one line to another with the minimum transfer waiting time. Some studies incorporated the number of transfer passengers into the optimization objective, i.e., to maximize the number of transfer passengers that are benefited from the coordinated transfers (Ibarra-Rojas et al., 2014; Fouilhoux et al., 2016; Wu et al., 2016). However, information concerning the number of transferring passengers is not always available (Ma et al., 2013; Yap et al., 2019). In addition, a few studies further considered some other performance metrics, such as reducing headway deviation and bus bunching (Ibarra-Rojas & Rios-Solis, 2012; Wu et al., 2016; Gkiotsalitis et al., 2019). Different objective formulations will lead to different results. Thus, as suggested by Ansarilari, Nesheli, Bodur, & Shalaby, 2023, PT agencies need to compare and assess the importance of different objective formulations and choose the most appropriate one when optimizing transfers. To consider the interests of different stakeholders, a multi-objective optimization approach may be adopted rather than a single-objective optimization approach.

As for solution methods to PT timetable coordination MP models, Liu et al. (2021) classified them into three different groups, i.e., exact solution method, heuristic and meta-heuristic methods. Interested readers may be referred to Liu et al. (2021) for details. Recent studies on PT timetable coordination are focused on integration with other operations planning activities (Ibarra-Rojas et al., 2014; Liu & Ceder, 2017a; Fonseca et al., 2018), incorporating passenger demand assignment (Parbo et al., 2014; Chu et al., 2019; Wu et al., 2019; Wang et al., 2022), coordinating multimodal PT transfers (Chowdhury & Chien, 2002; Ceder, 2021; Huang et al., 2021; Kang et al., 2021; Gkiotsalitis, 2022a, 2022b), and considering the stochastic or time-dependent features of PT systems (Gkiotsalitis et al., 2019; Yin et al., 2021; Lee et al., 2022).

1.2.2. Public transport vehicle scheduling

PT vehicle scheduling is the problem of optimally scheduling a set of vehicles to conduct all the service trips in a given timetable with the objective of minimizing the total operation cost. It is usually conducted after a PT timetable is created, i.e., in a sequential manner. There are mainly two tasks involved in the PT vehicle scheduling process: (i) determining the minimum number of vehicles, i.e., minimum fleet size, required for performing the scheduled service trips; (ii) constructing detailed vehicle trip chains, which are assigned to vehicles (Ceder, 2016; Gkiotsalitis, 2020). The PT vehicle scheduling problem, in essence, is a combinatorial optimization problem. For the case of multiple-depot PT vehicle scheduling problem, it is known to be a NP-hard problem (Bodin et al., 1983; Carraresi & Gallo, 1984; Bunte & Kliewer, 2009; Ceder, 2016). Various modelling and solution approaches have been developed during the last decades.

Recent comprehensive reviews and comparisons of PT vehicle scheduling models and solution methods can be found in Daduna and Paixão (1995), Haghani et al. (2003), Desaulniers and Hickman (2007), Bunte and Kliewer (2009), and Ceder (2016). Daduna and Paixão (1995) reviewed mathematical programming models for three PT vehicle scheduling problems, namely a basic vehicle scheduling problem, the vehicle scheduling problem with fixed number of vehicles, and the multiple-depot vehicle scheduling problem. They further reviewed some extensions of these models, and the practical experiences of using computer-aided scheduling systems. Haghani et al. (2003) made a comparative analysis of three bus scheduling models, including two single-depot and one multiple-depot vehicle scheduling models. The performances of these models are evaluated using data collected from the Mass Transit Administration (MTA) in the city of Baltimore, Maryland. Computation results show that both single-depot and multiple-depot vehicle scheduling models can reduce the total operation cost, fleet size, and deadhead and layover times. The multiple-depot vehicle scheduling model generally has a better performance than the single-depot vehicle scheduling models. In addition, they found that the vehicle deadheading speed has a significant impact on the final results. Desaulniers and Hickman (2007) also investigated a multiple-depot vehicle scheduling model, which was originally proposed by Ribeiro and Soumis (1994). Three solution methods, namely a column generation method, a branch-and-bound, and a time–space network modelling method, were described, which were demonstrated can solve real-world large-scale PT vehicle scheduling instances. Recently, Bunte and Kliewer (2009) conducted a comprehensive review of modelling approaches to both single-depot and multiple-depot PT vehicle scheduling problems. For the case of single-depot PT vehicle scheduling problems, four MP models are reviewed, namely a minimal decomposition model, an assignment model, a transportation model, and a network flow model. For the case of multiple-depot PT vehicle scheduling problems, they classified the related models into four categories, namely single-commodity models, multi-commodity models, time–space network models, and set-partitioning models. They further compared the lower bound qualities of these different multiple-depot vehicle scheduling models and discussed some practical extensions.

In addition to the MP models, there is a graphical scheduling model, named deficit function (DF) model, which was also used for PT vehicle scheduling (Ceder & Stern, 1981; Ceder, 2016). The visual nature of the DF model can allow PT schedulers to further improve the computer-generated solutions by interjecting their own practical considerations (Liu & Ceder, 2017b). The DF model is equivalent to a maximum network flow-based MP model. Both models can yield the same minimum fleet size (Liu & Ceder, 2021).

As for solution methods, some exact solution methods (e.g., branch-and-bound), which can provide exact optimal solutions in a short computation time, were developed to solve small and medium-size problems. While for large-scale problems, heuristic or meta-heuristic algorithms (e.g., genetic algorithm, simulated annealing, ant colony optimization, particle swarm optimization) are adopted to generate nearly optimal solutions within an acceptable computation time.

Table 1
Comparisons of previous and our studies on integrating PT timetable coordination and vehicle scheduling.

Authors (year)	PT system					Optimization model			Solution method
	Depot	Headway	Deadhead	Transferring passengers	Network size	Optimization objective	Decision variable	Model characteristic	
Rapp and Gehner (1976)	Multiple	Even	No	Yes	23 lines	Min transfer delay and fleet size	Offset time	N.A.	Interactive graphic optimization approach
Guihaire and Hao (2010)	Multiple	Uneven	Yes	No	50 lines	Min fleet size and length of deadheads, Max transfer opportunities and headway evenness	Offset time and vehicle assignment	IP model	Iterated local search
Petersen et al. (2013)	Multiple	Even	Yes	Yes	Up to 8 lines	Min waiting time, total cost	Offset time	IP model	Large neighborhood search meta-heuristic
Ibarra-Rojas et al. (2014)	Single	Uneven	No	Yes	Up to 50 lines, 5 transfer nodes	Max number of transfer passengers, Min vehicle operating cost	Departure times of vehicles	Bi-objective IP model	ε-constraint method, CPLEX
Liu et al. (2017)	Multiple	Even	Yes	No	2 lines, 1 transfer node	Max number of simultaneous arrivals of vehicles, Min fleet size	Offset time	Bi-objective IP model	Heuristic algorithm
Fonseca et al. (2018)	Multiple	Uneven	Yes	Yes	Up to 8 lines	Min transfer and operational costs	Departure times, Dwell time	MIP model	Matheuristic
Ataeian et al. (2021)	Multiple	Even	No	No	8 lines	Max number of synchronized arrivals, Min fleet size	Offset time, headway, number of vehicle departures	IP model	NSGA II, GAMS
This study	Multiple	Even	Yes	No	18 lines, 9 transfer nodes	Max number of coordinated transfer connections, Min fleet size	Offset time	Bi-objective IP model	ε-constraint method, Gurobi, constraint-reduction procedures

Note: IP = integer programming, MIP = mixed integer programming, NSGA II = non-dominated sorting genetic algorithm II, GAMS = general algebraic modeling system, N.A. = not applicable.

1.2.3. Integrated public transport timetable coordination and vehicle scheduling

The potential advantage of integrating PT timetable coordination and vehicle scheduling stems from allowing PT decision makers and schedulers to make a trade-off between the operational cost and level-of-service of a PT system (Rapp & Gehner, 1976; Ibarra-Rojas et al., 2014; Liu & Ceder, 2017a). They can choose the most appropriate timetable and vehicle scheduling scheme, based on their practical considerations, from a set of Pareto-optimal solutions.

Previous solution approaches to the integrated PT timetable coordination and vehicle scheduling problem can be classified into two categories, namely the sequential interactive optimization approach and the integrated optimization approach. The sequential interactive optimization approach solves the integrated PT timetable coordination and vehicle scheduling problem in an interactive iteration manner. That is, at each iteration, one sub-problem, either timetable coordination or vehicle scheduling, is solved first. Then, the results are used as input for solving the other sub-problem. The two sub-problems are interactively solved in each iteration. The iteration process stops when a set of Pareto-optimal solutions are found. In contrast, the integrated optimization approach solves the two sub-problems simultaneously. That is, both a timetable and a vehicle schedule (or minimum fleet size) can be generated by solving the integrated optimization model in a single step. One earliest study using the sequential interactive optimization approach was conducted by Rapp and Gehner (1976) who adopted a computer-aided, coordinated four-stage interactive graphic system for PT timetable coordination and fleet size determination. The optimization objectives are to reduce passenger transfer delay and the required minimum fleet size. Another recent study using this solution approach by Liu and Ceder (2017a) adopted the DF-based graphical interactive approach to optimize the fleet size as well as the number of simultaneous arrivals of vehicles at a transfer node. One limitation of the sequential interactive optimization approach is that it is a heuristic solution method and cannot guarantee generating all the Pareto-optimal solutions.

In recent years, there is an increasing number of studies using the integrated optimization approach to solve the integrated PT timetable coordination and vehicle scheduling problem (e.g., Guihaire & Hao, 2010; Petersen et al., 2013; Ibarra-Rojas et al., 2014; Fonseca et al., 2018; Ataeian et al., 2021). Guihaire and Hao (2010) employed an iterated local search heuristic method to optimize line offset times and the assignment of vehicles. Four cost components, namely the number and quality of transfer possibilities, headway evenness, fleet size, and length of deadheading (DH) trips, are considered, which are combined as a weighted cost objective function. Based on a time–space network modelling approach, Petersen et al. (2013) developed an integration optimization model, which was solved by using a large neighborhood search meta-heuristic algorithm. Fonseca et al. (2018) further considered using bus dwell times as decision variables, and also employed the time–space network modelling approach to build the integrated optimization model. Based on the structure of the model, a meta-heuristic solution method was developed to solve the integrated optimization model. Except for using a time–space network, there is one study (Ibarra-Rojas et al., 2014) using a vehicle trip-connection based vehicle scheduling model to develop the integrated optimization model. Ibarra-Rojas et al. (2014) proposed to use an ε-constraint method, together with the CPLEX optimization solver, to solve the integrated model. Ataeian et al. (2021) included the fleet size as the second objective in the timetable coordination optimization model. The fleet size was not determined by using vehicle scheduling models, but by using a well-known single-line fleet size determination equation that is based on the line cycle time and headway.

Past studies on integrating PT timetable coordination and vehicle scheduling are summarized in Table 1. These studies are compared in terms of PT system characteristics, optimization model features, and solution methods. It can be observed that most studies considered multiple-depot PT systems with either even or uneven headways. Vehicle DH trips are usually allowed in multiple-depot PT systems. The current largest PT network considered is a network of 50 lines and 5 transfer nodes. As for model features, all the models use offset times as the decision variables, which take the form of an IP model. Two categories of objectives are usually considered in the integrated optimization model: (i) reducing the transfer delay/waiting time or maximizing the number of coordinated transfer connections; (ii) reducing the total operation cost, usually measured by the required minimum fleet size. Most studies employ heuristic or meta-heuristic methods. Only the studies of Ibarra-Rojas et al. (2014) and Ataeian et al. (2021) employed an ε-constraint method and commercial optimization solvers, e.g., CPLEX or GAMS, to generate all the Pareto-optimal solutions.

1.3. Research gaps, contributions and organization

The above literature review clearly indicates that better-integrated optimization models and more efficient solution methods are needed in order to solve the integrated PT timetable coordination and vehicle scheduling for larger size and more complex networks considering different PT systems characteristics. For model development, since both the vehicle trip-connection-based vehicle scheduling model and timetable coordination model use vehicle departure times as decision variables, it is sensible to combine them into an integrated optimization model. Thus, in this study we adopt the vehicle trip-connection-based vehicle scheduling model in developing the integrated optimization model. Compared to the trip-connection-based vehicle scheduling model proposed in Ibarra-Rojas et al. (2014), our model is capable of considering multiple-depot vehicle scheduling and allowing for vehicle DH operations. In addition, our model considers each line with an even headway, which is different from the uneven headway case considered in Ibarra-Rojas et al. (2014).

As for solution method, it is better to use exact solution methods, rather than heuristic or meta-heuristic methods, to generate all Pareto-optimal solutions so as to facilitate their assessment by PT schedulers. Following the solution method developed in Ibarra-Rojas et al. (2014), we also adopted an ε-constraint method for multi-objective optimization, together with a mixed-integer linear programming solver, to solve the integrated model. More importantly, we further develop two constraint-reduction procedures to reduce the number of variables and constraints of the optimization models before solving them. By doing so, we can considerably reduce the computation complexity and improve solution efficiency.

The contributions of this paper are threefold. First, it offers a new integrated optimization model that combines a trip-connection-based multiple-depot vehicle scheduling model and a timetable coordination model. It considers vehicle DH operation. We ensure that each line is operated with an even headway. To the best of our knowledge, this is the first time that such an integrated optimization model is established. Second, two useful constraint-reduction procedures are developed to reduce the computation complexity of the model by reducing the number of variables and constraints. This makes it possible to solve large-size problems within an acceptable computation time. Third, a numerical example is used as an expository device to illustrate the solution method developed, followed by a real-world case study of the BRT network in Chengdu, China. The results demonstrate that the proposed optimization models and solution methods are very effective in solving the integrated PT timetable coordination and vehicle scheduling problem.

This work comprises seven sections including this introductory section. Section 2 provides a formal description of the integrated PT timetable coordination and vehicle scheduling problem, together with an illustrative example. Section 3 presents two optimization models for the PT timetable coordination problem and vehicle scheduling problem, respectively. Then, the two models are combined into an integrated optimization model. The ε-constraint method, together with the two

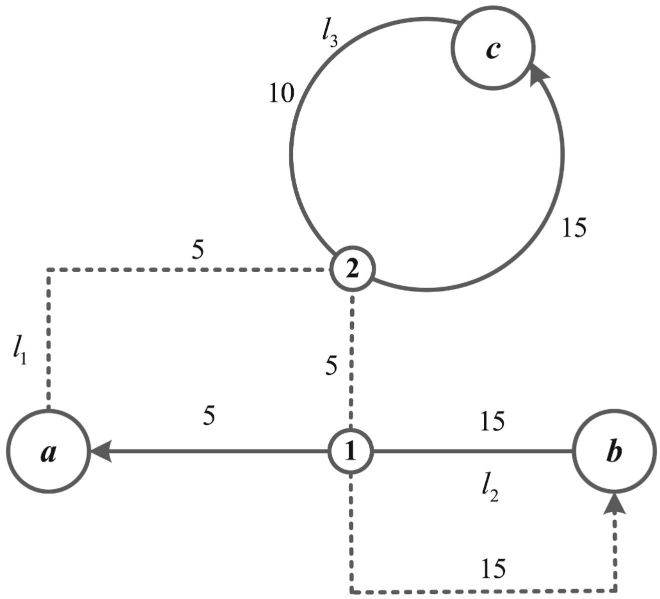

Fig. 1. An illustrative public transport network example.

Table 2
Timetable of the PT network example given in Fig. 1.

Trip ID	Departure terminal	Departure time	Arrival terminal	Arrival time
1	*a*	7:10	*b*	7:35
2	*a*	7:20	*b*	7:45
3	*a*	7:30	*b*	7:55
4	*a*	7:40	*b*	8:05
5	*a*	7:50	*b*	8:15
6	*a*	8:00	*b*	8:25
7	*b*	7:00	*a*	7:20
8	*b*	7:20	*a*	7:40
9	*b*	7:40	*a*	8:00
10	*c*	7:05	*c*	7:30
11	*c*	7:15	*c*	7:40
12	*c*	7:25	*c*	7:50
13	*c*	7:35	*c*	8:00
14	*c*	7:45	*c*	8:10
15	*c*	7:55	*c*	8:20

constraint-reduction procedures, is described in Section 4. Section 5 presents a numerical example to illustrate the model and solution method. A case study of the Chengdu BRT network is detailed in Section 6. Finally, Section 7 concludes our work and discusses limitations, as well as possible directions for future research.

2. Problem description

Consider a PT network with a set of PT lines $K = \{1, 2, 3\cdots k\}$, a set of terminal stations D and a set of transfer stops/stations N. For a given PT line, the average vehicle running times between terminal stations and transfer stops/stations, and the average vehicle running times between transfer stops/stations are given. The terminal layover times are also given. For a given planning period T, the PT timetable coordination problem aims to design a timetable, including vehicle departure and arrival times at terminal and transfer stations, to maximize the total number of coordinated transfer connections.

A coordinated transfer connection is defined as a situation where passengers can successfully make a transfer between two different PT lines at a transfer stop/station. The coordinated transfer connection is related to the arrival times of vehicles from two different lines and the dwell times at the transfer stop/station. A successful transfer connection

Table 3
Required vehicles and the related trip chains for the original timetable of the PT network example given in Fig. 1.

Vehicle ID	Served trip chains
1	1-9-6
2	3
3	5
4	7-2
5	8-4
6	10-13
7	11-14
8	12-15

Table 4
Modified timetable of the PT network example.

Trip ID	Departure terminal	Departure time	Arrival terminal	Arrival time
1	*a*	7:05	*b*	7:30
2	*a*	7:15	*b*	7:40
3	*a*	7:25	*b*	7:50
4	*a*	7:35	*b*	8:00
5	*a*	7:45	*b*	8:10
6	*a*	7:55	*b*	8:20
7	*b*	7:00	*a*	7:20
8	*b*	7:20	*a*	7:40
9	*b*	7:40	*a*	8:00
10	*c*	7:05	*c*	7:30
11	*c*	7:15	*c*	7:40
12	*c*	7:25	*c*	7:50
13	*c*	7:35	*c*	8:00
14	*c*	7:45	*c*	8:10
15	*c*	7:55	*c*	8:20

may help reduce passengers' transfer waiting time and improve the level of service. On the other hand, PT agencies usually aim to create an optimal vehicle schedule with the objective of minimizing the total operation cost, which is usually measured by the required minimal fleet size. Thus, the integrated PT timetable coordination and vehicle scheduling problem has two optimization objectives. The first objective is to maximize the number of successfully coordinated transfer connections. The second objective is to minimize the required fleet size.

2.1. Illustrative example

A PT network example is used to illustrate the integrated PT timetable coordination and vehicle scheduling problem at hand. The PT network example, as shown in Fig. 1, includes three PT lines (l_1, l_2, and l_3), three terminal stations (a, b, and c), and two transfer stations (Stations 1 and 2). The numbers appearing next to the line segment in Fig. 1 denote the average vehicle running times, in minutes. Let us consider a planning period T = [7:00, 8:00]. A timetable was created for the PT network example, as shown in Table 2. If we assume that vehicle dwell times at the two transfer stations are 1 min each, then we can obtain a maximum number of coordinated transfer connections of six. That is, there are six successful transfer connections at transfer Station 2 between lines l_1 and l_3. However, transfer connections at transfer Station 1 between lines l_1 and l_2 are not successful. To perform the fifteen scheduled vehicle trips, the required minimum fleet size amounts to eight vehicles. The associated vehicle trip chains are shown in Table 3.

Consider a modified timetable for the PT network example, as shown in Table 4. The original timetable of Table 2 is modified by shifting the vehicle departure times of line l_1 by five minutes earlier. Then, the total number of successfully coordinated transfer connections becomes nine. That is, there are three successful transfer connections at transfer Station 1, and six successful transfer connections at transfer Station 2. The required minimum fleet size, together with the related vehicle trip

Table 5
Required vehicles and the related trip chains for the modified timetable of the PT network example.

Vehicle ID	Served trip chains
1	1–9
2	2
3	4
4	6
5	7–3
6	8–5
7	10–13
8	11–14
9	12–15

Table 6
Notations.

Sets	
K	Set of PT lines
X	Set of the first vehicle trip departure times, i.e., offset times, of all lines
I_k, I_l	Set of vehicle departure trips from terminal stations of lines k and l
N_{kl}	Set of transfer stops/stations common to lines k and l
Indexes	
k, l	Indexes of PT lines,$k, l \in K$
i, j	Indexes of vehicle departure trips from terminal stations
n	Index of transfer stops/stations
Parameters	
h_k, h_l	Headways of line k and l
t_{kin}	Vehicle running time from the terminal station to the transfer station n of the i-th trip of line k
t_{ljn}	Vehicle running time from the terminal station to the transfer station n of the j-th trip of line l
t_{ki}	Vehicle running time of the i-th trip of line k
t_{ij}	Vehicle deadheading time from the ending terminal of the i-th trip to the beginning terminal of the j-th trip
$[w_n^-, w_n^+]$	Transfer waiting time window at transfer station n
T	Planning period, in minutes
M_1, M_2	Large positive constants
Auxiliary variables	
y_{kiljn}	is 1 if trip i of line k coordinates with trip j of line l at transfer stop n within the time window $[w_n, W_n]$; otherwise is set to 0.
z_{kilj}	is 1 if trip j of line l is conducted by the same vehicle after trip i of line k; otherwise is set to 0.
x_{ki}	Vehicle departure time of trip i ($i \geq 2$) of line k
x_{lj}	Vehicle departure time of trip j ($j \geq 2$) of line l
Decision variables	
x_{k1}, x_{l1}	departure times of the first trip (offset times) of lines k and $l, k, l \in K$

chains, is shown in Table 5. We can see that compared to the original timetable, the modified timetable results in more successful transfer connections. However, it leads to an increase of the required minimum fleet size. Thus, there is a trade-off between the two optimization objectives.

3. Model formulations

3.1. Notations, assumptions and key definitions

For the sake of simplicity and presentation, the following notations listed in Table 6 are used in formulating the optimization models.

To facilitate the presentation of the essential ideas, without loss of generality, the following basic assumptions are made.

A1. It is assumed that each PT line considered has an even headway within the planning period. This is a common situation in practice following the widely used passenger average waiting time formula, i.e., $E(w) = \frac{E(h)}{2}\left(1+\frac{\mathrm{Var}(h)}{E^2(h)}\right)$. The use of even-headway has the advantage of approximating the minimal average waiting time of $\frac{E(h)}{2}$ for randomly arriving passengers at the initial boarding stops. Previous studies, such as Daganzo (1990), Aksu and Akyol (2014), and Ting and Schonfeld (2005), show that the use of even headway can achieve better timetable coordination, especially using integer-ratio headways.

A2. Vehicle travel times are assumed to be fixed, i.e., not time-variant, during the planning period. Since we are solving a tactical-level timetable coordinating design problem, not an operational-level control problem, this assumption is reasonable. It has been commonly used in previous PT timetable design problems, such as Ceder et al. (2001), de Palma and Lindsey (2001), Ibarra-Rojas and Rios-Solis (2012), and Wu et al. (2016).

A3. Since we consider even headways, fixed vehicle travel and dwell times, it is assumed that bus bunching will not occur.

A4. It is assumed that the planning period is discrete in minutes; that is, vehicle departure times are discrete integer variables in minutes. This is common practice in daily PT planning and operations.

A5. The capacity of transfer stations, i.e., number of berths, is assumed to be sufficient to accommodate the number of coordinated arrival vehicles.

A6. The vehicle fleet is homogeneous in operating costs. In addition, it is assumed that the vehicle capacity is enough to accommodate passenger demand; that is, no passengers will be left behind. This can be realized when setting the line service frequency/headway (Niu & Zhou, 2013; Ceder, 2016; Daganzo & Ouyang, 2019; Wang et al., 2020).

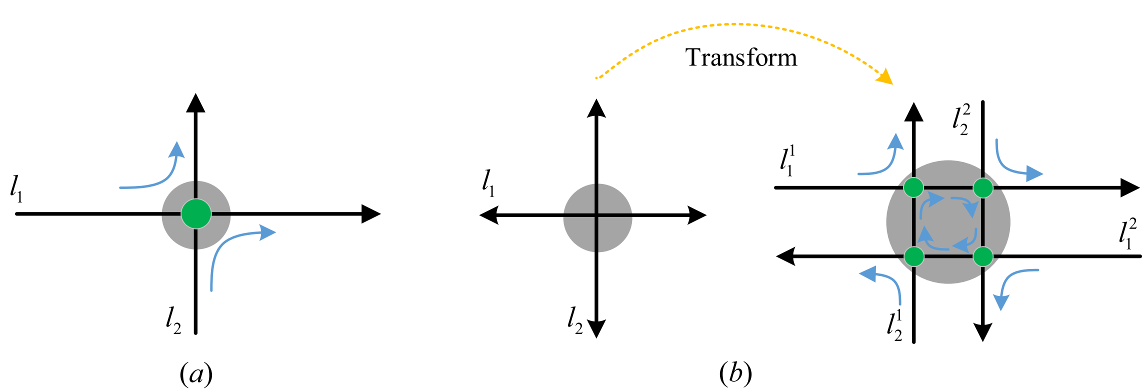

Fig. 2. Illustration of the concepts of transfer node, transfer movement, and transfer connection for: (a) two unidirectional lines; (b) two bidirectional lines.

A7. Vehicle deadheading (DH) operation is allowed if the travel times of DH trips comply with DH travel time constraints. In practice, DH trips are employed to reduce the required fleet size and reduce operations cost.
A8. PT drivers fully comply with the planned vehicle trips and timetable, i.e. they do not deviate from the planned service routes.

To facilitate the understanding of the optimization models, some key definitions related to PT transfers are hereby provided. These concepts are illustrated using two examples shown in Fig. 2. Fig. 2(a) and 2(b) illustrate the transfers between two unidirectional (l_1 and l_2) and bidirectional lines (l_1^1, l_1^2 and l_2^1, l_2^2), respectively. A bidirectional line is transformed into two unidirectional lines, as shown in Fig. 2(b).

- ***Transfer node***: A PT transfer node is the intersection point area where two unidirectional or bidirectional PT lines intersected with each other. The grey areas in Fig. 2(a) and 2(b) represent the transfer nodes.
- ***Transfer movement***: A PT transfer movement is defined as a transfer between two direction-specific lines at a transfer node (Rapp & Gehner, 1976). It describes the possible transfer movement of passengers between two different direction-specific lines. For example, the blue arrows in Fig. 2(a) and 2(b) represent the possible transfer movements between different lines.
- ***Transfer connection***: A PT transfer connection is defined as a transfer node between two unidirectional lines where possible transfer movements can be made. It can be seen from Fig. 2(a) and 2 (b) that for a given transfer connection, there are two possible transfer movements between two different lines.

In this study, the PT timetable coordination model aims to maximize the number of successfully coordinated transfer connections in a given PT network. For transfer nodes with more than two directional lines, the number of possible transfer connections can be calculated in the same way as the case of two directional lines.

3.2. Sequential optimization approach

The sequential optimization approach solves the PT timetable coordination and vehicle scheduling problems in two stages. In the first stage, the timetable coordination optimization problem is solved to generate a maximal coordinated timetable. Using the maximal coordinated timetable as an input, the second stage solves the vehicle scheduling problem to minimize the vehicle operation cost. Mathematical formulations for each of the two optimization problems are provided below.

3.2.1. First-stage PT timetable coordination optimization model

The first-stage PT timetable coordination optimization model aims to maximize the number of successfully coordinated transfer connections, which is measured by a binary coordination variable y_{kiljn}. That is, if the arrival time of the i-th trip of line k at transfer station n minus the arrival time of the j-th trip of line l at transfer station n is within a predefined transfer waiting time window $[w_n^-, w_n^+]$, then the binary coordination variable is activated to be one, otherwise it is zero. The first-stage PT timetable coordination optimization model is formulated as follows.

$$\max F_{TT}(X) = \sum_{k\in K}\sum_{l\in K, l\neq k}\sum_{n\in N_{kl}}\sum_{i\in I_k}\sum_{j\in I_l} y_{kiljn} \tag{1}$$

s.t.

$$(x_{ki}+t_{kin})-(x_{lj}+t_{ljn}) \geq w_n^- - M_1(1-y_{kiljn}),\ \forall k\in K, i\in I_k, l\in K, j\in I_l, n\in N_{kl} \tag{2}$$

$$(x_{ki}+t_{kin})-(x_{lj}+t_{ljn}) \leq w_n^+ + M_1(1-y_{kiljn}),\ \forall k\in K, i\in I_k, l\in K, j\in I_l, n\in N_{kl} \tag{3}$$

$$x_{ki}-x_{k,i-1} = h_k,\ \forall k\in K, i\in I_k \tag{4}$$

$$x_{lj}-x_{l,j-1} = h_l,\ \forall l\in K, j\in I_l \tag{5}$$

$$T-h_k \leq x_{k|I_k|} \leq T,\ \forall k\in K \tag{6}$$

$$T-h_l \leq x_{l|I_l|} \leq T,\ \forall l\in K \tag{7}$$

$$x_{k1} \in \{0,1,2,\cdots,h_k\},\ \forall k\in K \tag{8}$$

$$x_{l1} \in \{0,1,2,\cdots,h_l\},\ \forall l\in K \tag{9}$$

$$y_{kiljn} \in \{0,1\},\ \forall k\in K, i\in I_k, l\in K, j\in I_l, n\in N_{kl} \tag{10}$$

where Eq. (1) is the objective function that maximizes the total number of successfully coordinated transfer connections in a PT network. Constraints (2) and (3) are the transfer coordination constraints ensuring that the binary coordination variable takes the value one when there is a successfully coordinated transfer connection; otherwise, the variable is set to zero. Constraints (4) and (5) are the headway constraints that ensure that each line has an even headway. Constraints (6) and (7) ensure that the last vehicle departure time of each line is within the scheduling horizon T and not earlier than the time that is one headway less than T. Constraints (8) and (9) specify the possible values of the decision variables. Constraint (10) specifies the possible values of the binary coordination variables y_{kiljn}.

This model is an extension of the timetable coordination model described in Ceder et al. (2001) by including the time window consideration in the transfer coordination constraints and limiting line headway as even headway. Ibarra-Rojas and Rios-Solis (2012) have proved that in the case of uneven headway the problem is NP-hard.

3.2.2. Second-stage PT vehicle scheduling optimization model

A trip-connection-based multiple-depot vehicle scheduling model was developed to conduct vehicle scheduling. It uses vehicle terminal departure times as decision variables. A binary auxiliary variable z_{kilj} is introduced to measure whether two vehicle trips i and j can be chained or not. Trips are connected into vehicle chains, which are assigned to vehicles. The objective of vehicle scheduling is to minimize the total operation cost, which is usually measured by the required minimum fleet size. To further reduce the fleet size, vehicle DH trips between different terminals are considered in the model. The formulated multiple-depot PT vehicle scheduling model is described as follows.

$$\max C_{VS}(X) = \sum_{k\in K}\sum_{l\in K}\sum_{i\in I_k}\sum_{j\in I_l} z_{kilj} \tag{11}$$

s.t.

$$x_{lj} - (x_{ki}+t_{ki}+t_{ij}) \geq -M_2(1-z_{kilj}),\ \forall k\in K, i\in I_k, l\in K, j\in I_l \tag{12}$$

$$\sum_{l\in K}\sum_{j\in I_l} z_{kilj} \leq 1,\ \forall k\in K, i\in I_k \tag{13}$$

$$\sum_{k\in K}\sum_{i\in I_k} z_{kilj} \leq 1,\ \forall l\in K, j\in I_l \tag{14}$$

$$x_{ki}-x_{k(i-1)} = h_k,\ \forall k\in K, i\in I_k \tag{15}$$

$$x_{lj}-x_{l(j-1)} = h_l,\ \forall l\in K, j\in I_l \tag{16}$$

$$T-h_k \leq x_{k|I_k|} \leq T,\ \forall k\in K \tag{17}$$

$$T-h_l \leq x_{l|I_l|} \leq T,\ \forall l\in K \tag{18}$$

$$x_{k1} \in \{0,1,2,\cdots,h_k\},\ \forall k \in K \tag{19}$$

$$x_{l1} \in \{0,1,2,\cdots,h_l\},\ \forall l \in K \tag{20}$$

$$z_{kilj} \in \{0,1\},\ \forall k \in K, i \in I_k, l \in K, j \in I_l \tag{21}$$

where Eq. (11) is the objective function that maximizes the number of vehicle trip connections, which is equivalent to minimizing the required fleet size. The equivalence is shown in the below Theorem 1. Constraint (12) forces the binary auxiliary variable z_{kilj} to be 0, if trip j cannot be conducted after trip i with the same vehicle. Constraints (13) and (14) ensure that each trip can be connected with no more than one successor and predecessor trips, respectively. Constraints (15) and (16) are the headway constraints ensuring that each line has an even headway. Constraints (17) and (18) ensure that the last vehicle departure time of each line is within the scheduling horizon T and not earlier than the time that is one headway less than T. Constraints (19) and (20) are the decision variable constraints. Constraint (10) specifies the possible values of the binary auxiliary variable z_{kilj}.

Theorem 1. (*(The minimum fleet size theorem)*) To perform a given PT timetable with $|I|$ scheduled vehicle trips, the minimum number of vehicles required $\min F_{VS}(X)$, i.e., minimum fleet size, can be calculated by

$$\min F_{VS}(X) = |I| - \max C_{VS}(X) \tag{22}$$

where $\max C_{VS}(X)$ is the optimal solution of the model described by Eqs. (11)–(21), which indicates the maximum number of vehicle trip connections. A formal proof of this theorem can be found in Levin (1971) and Ceder (2016).

3.3. Integrated optimization approach

Both the timetable coordination model and vehicle scheduling model use the same decision variables. They also have the same headway and last trip departure time constraints. That is, constraints (4)–(9) are the same as constraints (15)–(19). Combing the two models and merging the same constraints, we obtain the below integrated bi-objective optimization (IBO) model.

[IBO model:]

Objective functions:	$[\max F_{TT}(X), \min F_{VS}(X)]$	(23)
s.t.		
TT constraints:	Eqs. (2)-(3) and (10)	(24)
TT + VS constraints:	Eqs. (4)-(9)	(25)
VS constraints:	Eqs. (12)-(14) and (21)	(26)

The IBO model is a bi-objective integer linear programing model. One objective is the objective of the timetable coordination model, and the other is the objective of the vehicle scheduling model. There are three groups of constraints. The first group is only related to the timetable coordination model (TT constraints), the second group is only related to the vehicle scheduling model (VS constraints), and the third group is the common constraints used by both the timetable coordination and vehicle scheduling models (TT + VS constraints). Note that the optimization model considers one-shot planning. That is, the number of vehicle trips, together with their departure and arrival terminals, are given. The optimization model aims to optimize vehicle trip departure times.

4. Solution approach

The bi-objective optimization problem is a special case of multi-objective optimization problems (Ehrgott, 2005). Various solution methods for multi-objective optimization problems have been developed, such as the weighted sum method, interactive method, lexicographic method, ε-constraint method, and evolutionary algorithms (e.g., NSGA-II). Among these methods, the ε-constraint method can generate all Pareto-optimal solutions and result in an exact Pareto front. Thus, the ε-constraint method is employed to solve the IBO model.

4.1. ε-constraint method

To use the ε-constraint method, the IBO model needs to be transformed into an integrated single-objective optimization (ISO) model. That is, we need to put one objective with one of its feasible values as a new constraint. In our case, since PT schedulers are more interested in knowing the number of coordinated transfer connections that can be increased (decreased) by increasing (decreasing) one more vehicle, we thus put the objective of the vehicle scheduling model as the new constraint, and transform the IBO model into the following ISO model.

[ISO model:]

Objective functions:	$[\max F_{TT}(X)]$	(27)
s.t.		
TT constraints:	Eqs. (2)-(3) and (10)	(28)
TT + VS constraints:	Eqs. (4)-(9)	(29)
VS constraints:	Eqs. (12)-(14) and (21)	(30)
	$\max C_{VS}(X) = \varepsilon$	(31)

where the initial value of ε is set to the maximum number of vehicle trip connections $\max C^*_{VS}(X)$, which is obtained by independently solving the second-stage PT vehicle scheduling optimization model. After performing this transformation, the resulting ISO model is an integer linear program that can be solved using branch-and-cut and the simplex method. This approach is used in off-the-shelf optimization solvers, such as Gurobi, CPLEX and GAMS. By using the branch-and-cut method, we can find one Pareto-optimal solution. To get another Pareto-optimal solution, the value of ε is reduced by one. Then, we solve the ISO model again with the updated value of ε. By doing so, we get a new Pareto-optimal solution. This iteration process will not terminate until $\max F_{TT}(X) = F^*_{TT}(X)$ or $\varepsilon = 0$, where $F^*_{TT}(X)$ is the maximum number of coordinated transfer connections that are obtained by independently solving the first-stage timetable coordination optimization model. The overall solution procedure is outlined in Algorithm 1.

Algorithm 1
The ε-constraint method-based solution procedure.

Input:	PT network, line headway, vehicle running times, deadheading times
Output:	Set of Pareto-optimal solutions S
Step 1:	Initialize $S=\varnothing$;
Step 2:	Solve the first-stage timetable coordination model (Eqs. (1)–(10)) independently and get the maximum number of coordinated transfer connections $F^*_{TT}(X)$;
Step 3:	Solve the second-stage vehicle scheduling model (Eqs. (11)–(21)) independently and get the maximum number of vehicle trip connections $C^*_{VS}(X)$;
Step 4:	Let $\varepsilon = C^*_{VS}(X)$;
Step 5:	Calculate the value of $\min F_{VS}(X)$ using Eq. (22).
Step 6:	Solve the ISO model to get the optimal number of coordinated transfer connections $F_{TT}(X)$;
Step 7:	Generate one Pareto-optimal solution $(F_{TT}(X), F_{VS}(X))$, and update $S := S \cup (F_{TT}(X), F_{VS}(X))$;
Step 8:	If $F_{TT}(X) = F^*_{TT}(X)$ or $\varepsilon = 0$, then stop and output the Pareto-optimal solution set S; otherwise set $\varepsilon := \varepsilon - 1$, $\max C_{VS}(X) = \varepsilon$, and go to Step 5.

4.2. Constraint-reduction procedures

Compared to previous studies considering PT systems with uneven headways (e.g., Guihaire & Hao, 2010; Ibarra-Rojas et al., 2014; Fonseca et al., 2018), the use of even headway can significantly reduce the number of decision variables. That is, we only need to determine the

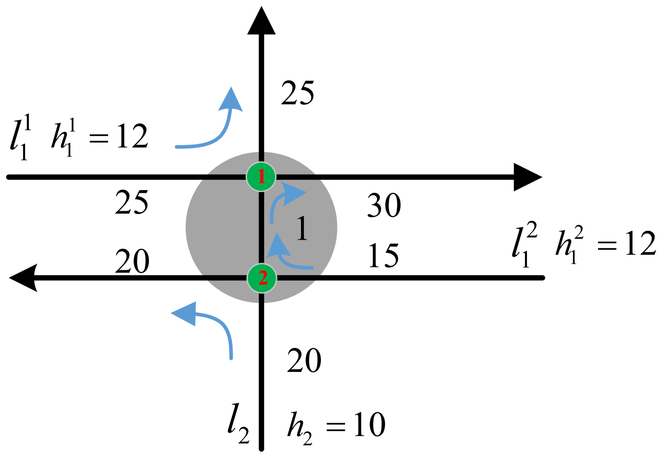

Fig. 3. An illustrative example with one bidirectional line, one unidirectional line, and two transfer connections.

Table 7
List of arrival time windows for the second transfer connection related to lines l_2 and l_1^2.

PT lines	Arrival time windows for the second transfer connection					
	1st arrival	2nd arrival	3rd arrival	4th arrival	5th arrival	…
l_2	[8:20, 8:30]	[8:30, 8:40]	[8:40, 8:50]	[8:50, 9:00]	[9:00, 9:10]	…
l_1^2	[8:15, 8:27]	[8:27, 8:39]	[8:39, 8:51]	[8:51, 9:03]	[9:03, 9:15]	…

vehicle departure time from the terminal station for the first trip of each line, i.e., offset time. Then, departure times of other trips can be obtained with the offset time and headway. Notwithstanding, there are many constraints for both the timetable coordination and vehicle scheduling models, especially for medium and large size networks. This may make the models insolvable within a reasonable computation time. Similar to the network reduction procedure used in the time–space network-based vehicle scheduling models (Kliewer et al., 2006; Li & Balakrishnan, 2016; Niu et al., 2018), two constraint-reduction procedures are developed to reduce the number of constraints.

The first procedure is used to reduce the constraints related to the binary coordination variable y_{kiljn}. The rationale behind this procedure is that firstly, for two given lines, if there are no transfer connections between these two lines, the related variable y_{kiljn} will be zero, and all the related constraints, i.e., constraints (2), (3) and (10), will be removed from the optimization model. Let us illustrate this using the simple network shown in Fig. 3 as an example. Since there are no transfer connections between line l_1^1 and line l_1^2, the related variables y_{kiljn} and constraints will be removed.

Secondly, for PT lines with mutual transfer connections, we check the arrival time windows of vehicle trips of each line for the mutual transfer connection, based on the terminal departure time, running time, and headway. If the difference between the earliest arrival time of trip i from one line and the latest arrival time of trip j from the other line is bigger than the predefined transfer window threshold w_n^+, then the transfer connection between trip i and trip j is infeasible. This means that the binary coordination variable y_{kiljn} is zero, and the related constraints can be removed from the optimization model. We also use the example network shown in Fig. 3 to explain the underlying principle. Consider that the planning period starts at 8:00 am, which is the earliest vehicle departure time from the terminal station for the three unidirectional lines. The headways for lines l_2, l_1^1, and l_1^2 are 10 min, 12 min, and 12 min, respectively. The numbers appearing next to the route segment in Fig. 3 denote vehicle running times, including the dwell times at non-transfer stations, in minutes. The transfer window threshold w_n^+ is set to be 1 min. For the second transfer connection, which is related to lines l_2 and l_1^2, the arrival time windows for the first five arrivals of the two lines are calculated and listed in Table 7. From it we can see that for the first arrival trip of line l_2, it is possible to have a successful transfer connection with the first and second trips of line l_1^2. While for the third and the rest of trips of line l_1^2, it is impossible to have a successful transfer connection, because the differences between the earliest arrival times of these trips (8:39, 8:51, 9:03) and the latest arrival time of the first arrival trip of line l_2 (8:30) are larger than w_n^+. Thus, the variables y_{kiljn} for all these transfer connections are zero, and all the related constraints can be removed from the optimization model. By doing so, we can significantly reduce the number of such redundant variables and constraints. This procedure is described in Algorithm 2.

Algorithm 2
The constraint-reduction procedure based on the binary coordination variable y_{kiljn}.

Step 1:	For $\forall$ pair of PT lines $k, l \in K$, if there is no mutual transfer connection between the two lines, set all y_{kiljn}=0, and remove all the related constraints, i.e., constraints (2), (3) and (10); otherwise, go to Step 2.
Step 2:	**For** $\forall(k,i,l,j,n), k,l \in K, i \in I_k, j \in I_l, n \in N_{kl}$ **do**
Step 3:	Based on the offset times of lines k and l, i.e., $x_{k1} \in [0,1,2,\cdots h_k]$ and $x_{l1} \in [0,1,2,\cdots h_l]$, and the line headways h_k and h_l, calculate the departure time window for the i-th trip of line k, $[(i-1)h_k, ih_k]$, and the departure time window for the j-th trip of line l, $[(j-1)h_l, jh_l]$.
Step 4:	Based on the route segment running times, calculate the arrival time windows for the transfer connection n, for trips i and j, which are $[(i-1)h_k + t_{kin}, ih_k + t_{kin}]$, and $[(j-1)h_l + t_{ljn}, jh_l + t_{ljn}]$, respectively.
Step 5:	If $\left|((ij-1)h_l + t_{ljn}) - (ih_k + t_{kin})\right| \rangle w_n^+$ or $\left|((i-1)h_k + t_{kin}) - (jh_l + t_{ljn})\right| \rangle w_n^+$, then set $y_{kiljn} = 0$, and remove all the related constraints.
Step 6:	**End for**

The second procedure is used to reduce the constraints related to the binary auxiliary variable z_{kilj}. The rationale behind this procedure is that if the earliest arriving time of trip i is later than the latest departure time of trip j, then trip i cannot be connected with trip j. This means that trips i and j cannot be conducted by the same vehicle. Thus, the binary auxiliary variable z_{kilj} will be zero, and all the related constraints, i.e., constraint (12), can be removed. By doing so, we can remove a large number of constraints before solving the optimization model. This procedure is described in detail in Algorithm 3.

Algorithm 3
The constraint-reduction procedure based on the binary auxiliary variable z_{kilj}.

Step 1:	**For** $\forall(k,i,l,j,n), k,l \in K, i \in I_k, j \in I_l$ **do**
Step 2:	Based on the offset times of lines k and l, i.e., $x_{k1} \in [0,1,2,\cdots h_k]$ and $x_{l1} \in [0,1,2,\cdots h_l]$, and the line headways h_k and h_l, calculate the departure time window for the i-th trip of line k, $[(i-1)h_k, ih_k]$, and the departure time window for the j-th trip of line l, $[(j-1)h_l, jh_l]$.
Step 3:	Calculate the latest departure time of the j-th trip of line l, which is jh_l.
Step 4:	Calculate the earliest arrival time of i-th trip of line k, which is $((i-1)h_k + t_{ki} + t_{ij})$.
Step 5:	If $((i-1)h_k + t_{ki} + t_{ij}) - jh_l > 0$, then set $z_{kilj} = 0$ and remove all the related constraints.
Step 6:	**End for**

5. Numerical example

A toy network is presented in this section to comprehend the optimization model and solution method. Computational results are presented in detail.

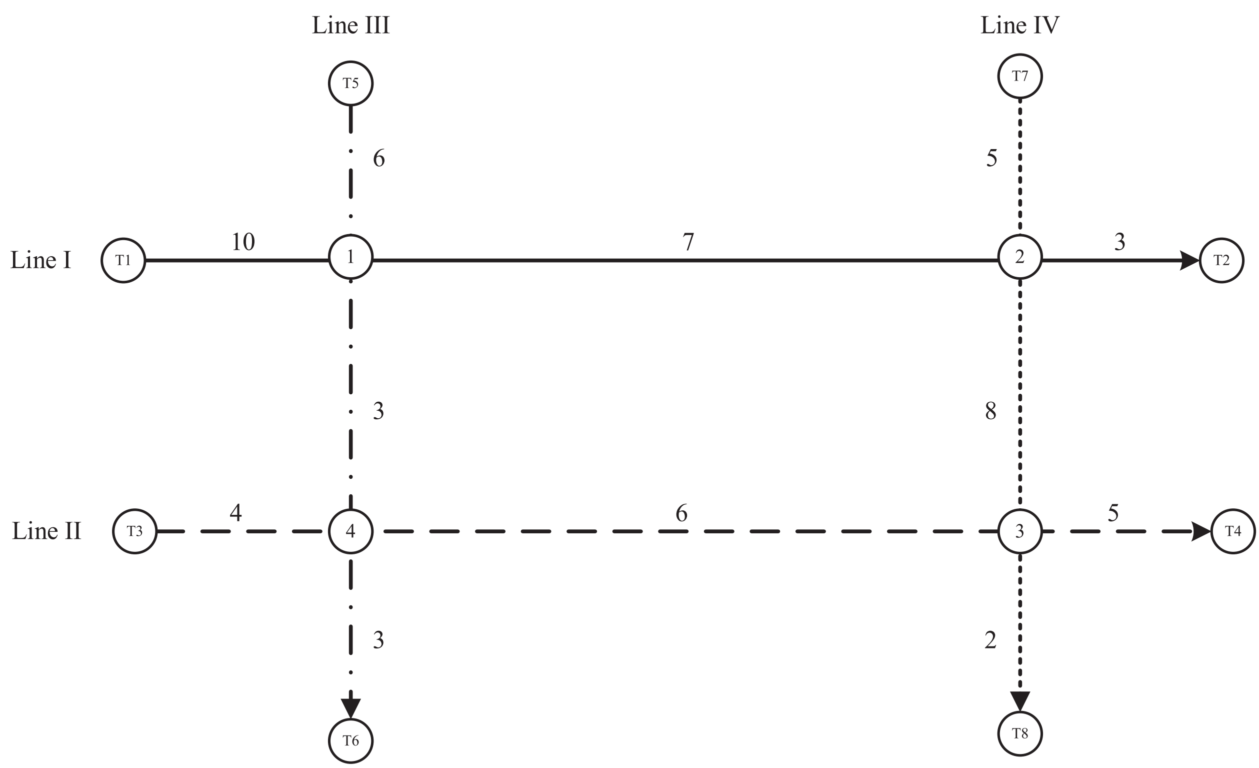

Fig. 4. A toy network adapted from Ceder et al. (2001).

Table 8
Vehicle DH times (minutes) between terminal stations.

	T1	T2	T3	T4	T5	T6	T7	T8
T1	0	10	3	15	12	13	20	18
T2	10	0	20	5	12	12	5	11
T3	3	20	0	10	10	5	12	10
T4	15	5	10	0	12	9	10	3
T5	12	12	10	12	0	8	5	16
T6	13	12	5	9	8	0	15	4
T7	20	5	12	10	5	15	0	8
T8	18	11	10	3	16	4	8	0

5.1. Toy network

A toy network adapted from Ceder et al. (2001) is shown in Fig. 4. It has four unidirectional PT lines (lines I, II, III, and IV) with four transfer nodes (nodes 1, 2, 3, and 4), and eight terminal stations (stations T1, T2, T3, T4, T5, T6, T7, and T8). The numbers appearing next to the route segment in Fig. 4 denote the vehicle running times, in minutes. Vehicle DH running times between terminal stations are listed in Table 8, which is a symmetric matrix, i.e. the DH times between two terminal stations are equal for both directions. The headways of lines I, II, III, are set to 10 min. While for line IV, it is 20 min. The planning period is set as [8:00, 8:30], which means vehicles can only depart from the terminal stations within this time window. Following Ceder et al. (2001), the transfer waiting time window $[w_n^-, w_n^+]$ is set to be [0, 0], which means a successful transfer connection is the simultaneous arrival of vehicles from two different lines at a transfer node.

5.2. Numerical results

With this input data, we first solve the timetable coordination and vehicle scheduling models in a sequential way. Both optimization models are solved with Gurobi, together with Python. Gurobi offers a Python interface that enables us to work with individual variables and constraints. By solving the first stage timetable coordination optimization model, we obtain the maximum number of successfully coordinated transfer connections, which is $F_{TT}^{*}(X) = \max F_{TT}(X) = 8$. The resulting timetable (vehicle departure times from the terminal stations) is presented in Table 9. Table 10 lists the coordinated transfer connections and the related trip departure times.

Using the timetable generated at the first stage, we solve the second-stage vehicle scheduling model. The solution of the model gives us the maximum number of trip connections, which is $\max C_{VS}(X) = 2$. Because the total number of vehicle trips is 11, according to Theorem 1 the

Table 9
Vehicle departure times from the terminal stations of the four lines (sequential approach).

Departure number Line	1	2	3
I	8:00	8:10	8:20
II	8:09	8:19	8:29
IIII	8:04	8:14	8:24
IV	8:06	8:26	–

Table 10
List of coordinated transfer connections and the related line departure times (sequential approach).

Coordinated transfer connection	Transfer node	Coordinated time	Departure time			
			I	II	III	IV
1	1	8:10	8:00		8:04	
2	1	8:20	8:10		8:14	
3	1	8:30	8:20		8:24	
4	4	8:13		8:09	8:04	
5	4	8:23		8:19	8:14	
6	4	8:33		8:29	8:24	
7	3	8:19		8:09		8:06
8	3	8:39		8:29		8:26

Table 11
Vehicle departure times from the terminal stations of the four lines (1st iteration).

Departure number Line	1	2	3
I	8:00	8:10	8:20
II	8:10	8:20	8:30
IIII	8:04	8:14	8:24
IV	8:05	8:25	–

Table 12
List of coordinated transfer connections and the related line departure times (1st iteration).

Coordinated transfer connection	Transfer node	Coordinated time	Departure time			
			I	II	III	IV
1	1	8:10	8:00		8:04	
2	1	8:20	8:10		8:14	
3	1	8:30	8:20		8:24	

Table 13
Vehicle departure times from the terminal stations of the four lines (2nd iteration).

Line	Departure number		
	1	2	3
I	8:01	8:11	8:21
II	8:10	8:20	8:30
IIII	8:05	8:15	8:25
IV	8:07	8:27	–

Table 14
List of coordinated transfer connections and the related line departure times (2nd iteration).

Coordinated transfer connections	Transfer node	Coordinated time	Departure time			
			I	II	III	IV
1	1	8:11	8:01		8:05	
2	1	8:21	8:11		8:15	
3	1	8:31	8:21		8:25	
4	4	8:14		8:10	8:05	
5	4	8:24		8:20	8:15	
6	4	8:34		8:30	8:25	
7	3	8:20		8:10		8:07
8	3	8:40		8:30		8:27

minimum fleet size is $\min F_{VS}(X) = 11–2 = 9$ vehicles. Thus, with the sequential solution approach, we obtain the solution of $F_{TT}(X) = 8$ and $F_{VS}(X) = 9$ for the toy network.

To solve the integrated optimization model, we first independently solve the second stage vehicle scheduling model and get the optimal value of the objective, which is $C^{*}_{VS}(X) = 3$. Thus, we initialize $\varepsilon = C^{*}_{VS}(X)$=3, which means that the minimum fleet size is $\min F_{VS}(X) = 11–3 = 8$. By solving the resulted ISO model, we obtain the result of $\max F_{TT}(X) = 3$. Then, we get the first Pareto-optimal solution, which is $(F_{TT}(X), F_{VS}(X))$=(3, 8). The resulting timetables and list of coordinated transfer connections are shown in Table 11 and Table 12, respectively.

In the second iteration, we reduce ε by one, i.e., setting $\varepsilon = 2$, which means that the minimum fleet size is$\min F_{VS}(X) = 11–2 = 9$. We then solve the resulting ISO model, which yields $\max F_{TT}(X) = 8$. Then, we get the second Pareto-optimal solution, which is $(F_{TT}(X), F_{VS}(X)) = (8, 9)$. The resulting timetables and list of coordinated transfer connections are shown in Table 13 and Table 14, respectively.

After the second iteration, we observe that the value of $F_{TT}(X)$ has

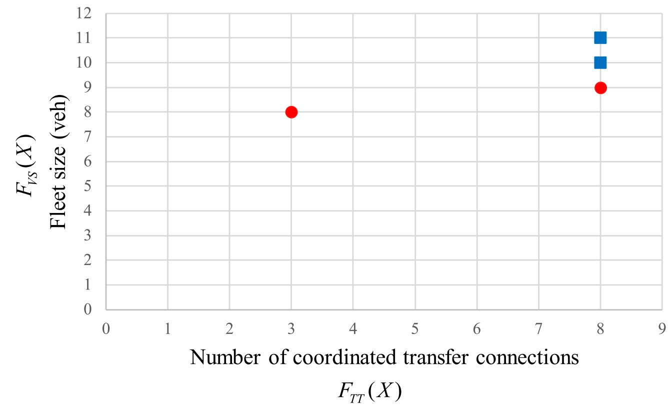

Fig. 5. Trade-off between the number of coordinated transfer connections and the required minimal fleet size.

reached its optimal value. According to Algorithm 1, the iteration will stop and we get two Pareto-optimal solutions. However, to further illustrate the solution methodology, we continue reducing ε by one in each iteration. After two additional iterations, ε reaches zero and the iteration stops. We obtain two additional solutions: $(F_{TT}(X), F_{VS}(X))$=(8, 10), and $(F_{TT}(X), F_{VS}(X))$=(8, 11).

These two solutions, together with the two Pareto-optimal solutions, are displayed in a two-dimensional (2D) graph in Fig. 5. The two red circle dots indicate the two objective function values of the two Pareto-optimal solutions, and the other blue square dots indicate the other two solutions which are not Pareto-optimal. It is interesting to see that by increasing one more vehicle from the minimum fleet size, i.e., 8 vehicles, the number of successfully coordinated transfer connections is increased from 3 to 8 (a 166.67% increase). The PT decision makers and schedulers thus can make a trade-off between the number of coordinated transfer connections and the required fleet size, and choose a proper Pareto-optimal solution for practical implementation.

It should be noted that since the timetable coordination model, vehicle scheduling model, and the transformed ISO model all can be solved in less than one second with Gurobi for this toy network, the two constraints reduction procedures are thus not used. Their applications are demonstrated for the larger size case study network described in the next section.

6. Application

The optimization models and solution methods described in the previous sections are next applied to solve a real-world larger size BRT network. This section presents the case study results, together with some sensitivity analyses, to better understand the performances of the optimization models and solution method.

6.1. Real-world BRT network in Chengdu

The BRT network in Chengdu, China is selected as the case study network. Chengdu, a city with a population of 21.2 million by the end of 2021, is a large city located in the southwest of China. Bus, BRT, and metro, are the main public transport modes. Fig. 6 shows the current BRT network in Chengdu.

The first BRT line in Chengdu opened for operation on June 11, 2013. As of May 2022, there are 19 bidirectional BRT lines, including 13 normal lines, 5 high-frequency lines, and 1 tourist line, with a total length of 347 km. The BRT system uses dedicated lanes that reduces the interactions of other road users. Thus, the vehicle running time is more stable and reliable compared to that of the bus system. This makes our assumption on fixed vehicle running times reasonable.

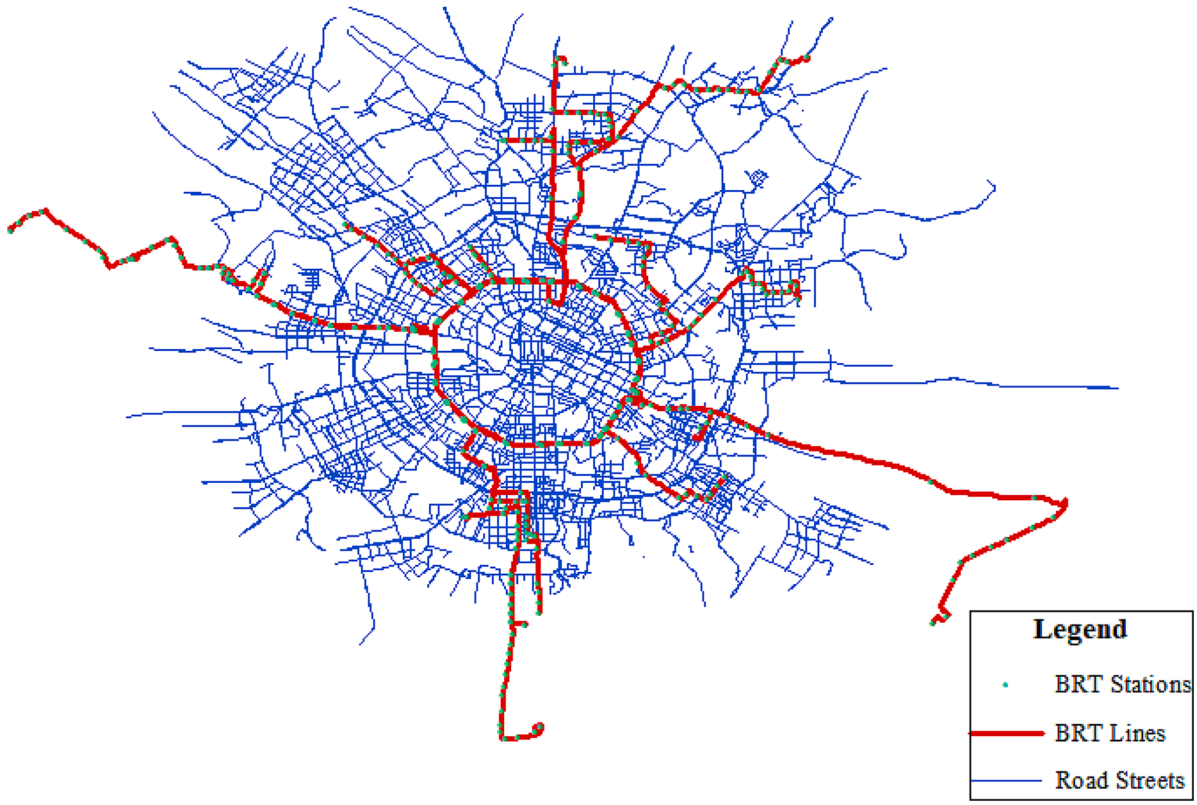

Fig. 6. The BRT network in Chengdu, China.

Table 15
BRT Line headways (for both directions) used in the case study.

BRT Line ID	K1	K3	K5	K6	K11	K12	K13	K17	K19
Headway (min)	8	12	15	10	8	10	8	15	10

6.2. Data collection

Since this study aims to coordinate transfer connections so as to reduce passenger transfer waiting time, we only consider the 13 normal BRT lines. The high-frequency and tourist lines are not considered, because for high-frequency lines the benefit of timetable coordination is expected to be negligible, in terms of transfer waiting time reduction (Bookbinder & Désilets, 1992; Vuchic, 2005). Network preprocessing was first conducted to remove the lines that were not connected with the BRT network and combine some lines that were mostly overlapping with each other. After doing the network preprocessing, 9 normal bidirectional BRT lines with 9 transfer nodes (36 transfer connections) are selected for the case study.

The BRT network and stations are visualized on the existing road streets, as shown in Fig. 6, using ArcMap 10.8. The line headways, vehicle running times, and DH running times, are collected from multi-source data, including the published timetable, a smartphone transit App, named Chelaile, and the AutoNavi (Gaode) map App. The data is processed by using Python and stored in Microsoft Excel Spreadsheet. Table 15 lists the headways considered for the 9 BRT lines. It should be noted that all the BRT lines are bidirectional and the headways for both directions are identical. The vehicle DH running times between terminal stations are presented in Table A of Appendix A. The DH running times between two terminal stations for both directions are the same, i.e. the vehicle DH running time matrix is a symmetric matrix. The terminal layover times and non-transfer station dwell times are included in the vehicle running times.

Table 16
Results of constraints reduction.

Planning period	Number of constraints related to variables y_{kiljn}			Number of constraints related to variables z_{kilj}		
180 min	Before	After	Reduction	Before	After	Reduction
	13,064	1185	90.93%	101,124	12,162	87.97%

6.3. Results

PT systems usually demonstrate a multi-period operation characteristic (Chang & Schonfeld, 1991; Bie et al., 2015; Ceder, 2016; Ibarra-Rojas et al., 2016). Vehicle running times and passenger demand vary across different periods. Therefore, in the case study, we first considered a late evening planning period, i.e., 20:00–23:00 (180 min), because BRT line headways are usually longer in the late evening planning period compared to those in the daytime periods, especially the morning and evening peak hour periods. As mentioned, timetable coordination is more beneficial for PT systems with long headways. A total number of $|I| = 318$ vehicle trips is considered in the planning period. The transfer waiting time window $[w_n^-, w_n^+]$ is set to [0, 0].

All the optimization modes and solution algorithms were implemented in Python 3.10 and solved by Gurobi 9.5.1 on a personal computer (PC) with an Intel(R) Core(TM) i9-10850K CPU @3.60 GHz, 16 GB RAM, and a 64-bit Windows 10 operating system. Table 16 shows the results of constraints reduction. We observe that after implementing the first constraint-reduction procedure (Algorithm 2), the number of constraints related to variables y_{kiljn} is reduced by 90.93%, and after implementing the second constraint-reduction procedure (Algorithm 3), the number of constraints related to variables z_{kilj} is reduced by 87.97%. In total, 100,841 constraints can be removed from the original optimization model for the case study problem, which significantly improved the solution process. Without doing these constraints reductions, we

Table 17
Offset times of both directions of each line for the case study (sequential approach).

Direction 1	K1	K3	K5	K6	K11	K12	K13	K17	K19
	20:05	20:00	20:13	20:01	20:06	20:10	20:08	20:09	20:09
Direction 2	K1′	K3′	K5′	K6′	K11′	K12′	K13′	K17′	K19′
	20:06	20:01	20:15	20:06	20:08	20:02	20:05	20:12	20:06

Table 18
List of solutions of the integrated and sequential optimization approaches.

Solution approach	Solutions	Fleet size (veh)	Number of coordinated transfer connections	CPU time (s)
Integrated optimization approach	Solution 0	148	195	2504.44
	Solution 1	149	228	1090.96
	Solution 2	150	238	512.15
	Solution 3	151	240	305.22
Sequential optimization approach	Solution 4	152	240	195.44

cannot solve the integrated optimization model within an acceptable computation time, e.g., 24 h. After implementing the two constraint-reduction procedures, the first stage timetable coordination optimization model is solved in just 1.11 s. The solution gives the maximum number of coordinated transfer connections $F_{TT}(X) = 240$. The resulted offset times, for both directions of the 9 BRT lines, is shown in Table 17. Using the timetable generated at the first stage as input, the second stage vehicle scheduling optimization model is solved in only 0.06 s, which results in a minimum fleet of $F_{VS}(X) = 152$ vehicles. Thus, we have the solution of $(F_{TT}(X), F_{VS}(X))$=(240, 152) for the sequential optimization models.

To solve the integrated optimization model, we first independently solve the second-stage vehicle scheduling model. The solution gives a result of maximum $C^*_{VS}(X) = 170$ trip connections, which indicates that a minimum fleet of $|I|$-$C^*_{VS}(X) = 148$ vehicles is required. Thus, according to Algorithm 1, the initial value of ε is set to 170. Then, we use the ε-constraint method to solve the integrated optimization model. After four iterations, the objective function $F_{TT}(X)$ of the integrated optimization model is equal to the optimal solution $F^*_{TT}(X) = 240$, which is obtained by the sequential optimization model. Thus, according to Algorithm 1, the iteration process terminates. Consequently, we obtain four Pareto-optimal solutions (Solutions 0, 1, 2, and 3). The related two objective function values and required CPU times for these four Pareto-optimal solutions, together with the solution (Solution 4) obtained by the sequential optimization models, are summarized in Table 18. It shows that indeed the integrated optimization model takes considerably more computation time compared to the sequential optimization models. However, the integrated optimization model can generate a full set of Pareto-optimal solutions.

Note: K1 and K1′ indicate the two different directions of the same line. The same to other K*i* and K*i*’ ($i = 3, 5, 6, 11, 12, 13, 17, 19$).

Compared to previous integrated optimization using exact solution methods (e.g., Ibarra-Rojas et al., 2014; Ataeian et al., 2021), our solution method is much faster. In the worst case (Solution 0), it takes 2504.44 s to solve the integrated optimization model, which is much faster than the 73093.8 s reported by Ibarra-Rojas et al. (2014). In addition, Ibarra-Rojas et al. (2014) considered a network with one or five transfer nodes, while our models were applied to a network with nine transfer nodes, which is more complex. The computation time reduction can be mainly attributed to the two constraint-reduction procedures as well as to the even headway consideration. The computation time for the integrated optimization model stays even in the worst case under one hour, which means that our models and solution method can reasonably be applied in practice.

The resulting offset times of each BRT line, for both directions, of the four Pareto-optimal solutions (Solutions 0, 1, 2, and 3) are listed in Tables 19, 20, 21, 22, respectively.

The four Pareto-optimal solutions, together with the solution obtained using the sequential solution approach, are further displayed in a 2D figure, in Fig. 7. It can be seen that by using one more vehicle from a fleet size of 148, 149, and 150, another 33, 10, and 2 coordinated transfer connections can be achieved, respectively. The BRT operator can make a trade-off between the number of coordinated transfer connections and the required fleet size.

6.4. Sensitivity analysis

To further understand the performance of the optimization models and solution method, sensitivity analyses for two key parameters, i.e., transfer waiting time window and planning period, are conducted.

6.4.1. Transfer waiting time window

The solutions obtained from the sequential and integrated optimization approaches when specifying three alternative transfer waiting time windows, i.e., $[w^-_n, w^+_n]$=[−1,+1], $[w^-_n, w^+_n]$=[−2,+2], and $[w^-_n, w^+_n]$ = [−3,+3], are shown in Fig. 8(a), 8(b), and 8(c), respectively. The red dots in the figures indicate the Pareto-optimal solutions, while the blue dots indicate non-Pareto-optimal solutions.

It is clear that for a wider transfer waiting time window, there are more coordinated transfer connections. For a wider transfer waiting time window, the chance of a transfer connection being coordinated is higher than that for a smaller transfer waiting time window. For a given transfer waiting time window, the solution obtained by the sequential optimization approach is always dominated by the solutions generated

Table 19
Offset times of both directions of each line (Solution 0 under the integrated approach).

Direction 1	K1	K3	K5	K6	K11	K12	K13	K17	K19
	20:04	20:10	20:09	20:00	20:05	20:01	20:06	20:00	20:00
Direction 2	K1′	K3′	K5′	K6′	K11′	K12′	K13′	K17′	K19′
	20:08	20:04	20:08	20:07	20:07	20:03	20:05	20:15	20:05

Table 20
Offset times of both directions of each line (Solution 1 under the integrated approach).

Direction 1	K1 20:04	K3 20:02	K5 20:09	K6 20:00	K11 20:05	K12 20:09	K13 20:08	K17 20:11	K19 20:08
Direction 2	K1′ 20:08	K3′ 20:08	K5′ 20:09	K6′ 20:05	K11′ 20:07	K12′ 20:01	K13′ 20:05	K17′ 20:11	K19′ 20:05

Table 21
Offset times of both directions of each line (Solution 2 under the integrated approach).

Direction 1	K1 20:05	K3 20:08	K5 20:13	K6 20:01	K11 20:06	K12 20:10	K13 20:08	K17 20:05	K19 20:09
Direction 2	K1′ 20:06	K3′ 20:01	K5′ 20:15	K6′ 20:06	K11′ 20:08	K12′ 20:02	K13′ 20:05	K17′ 20:05	K19′ 20:06

Table 22
Offset times of both directions of each line (Solution 3 under the integrated approach).

Direction 1	K1 20:04	K3 20:07	K5 20:14	K6 20:00	K11 20:05	K12 20:09	K13 20:07	K17 20:09	K19 20:08
Direction 2	K1′ 20:05	K3′ 20:12	K5′ 20:14	K6′ 20:05	K11′ 20:07	K12′ 20:01	K13′ 20:04	K17′ 20:11	K19′ 20:05

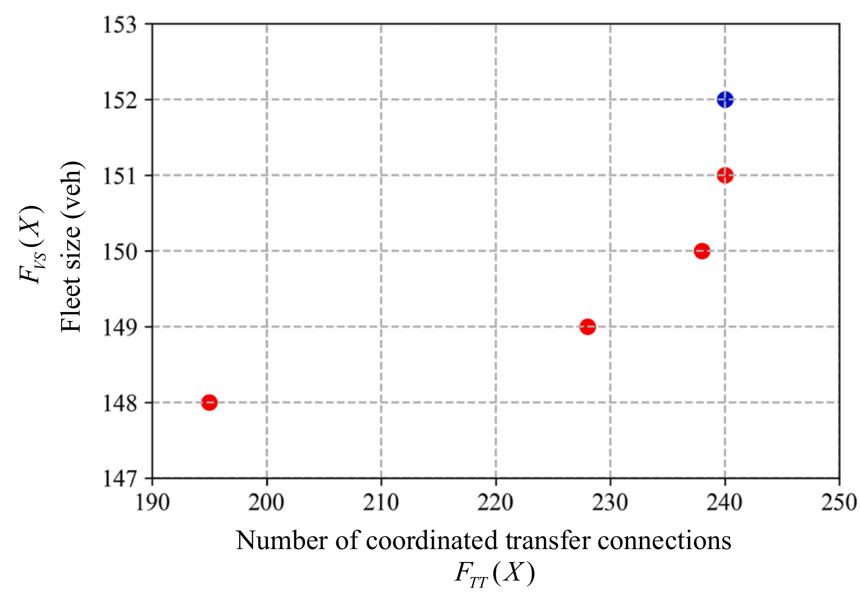

Fig. 7. Trade-off between the number of coordinated transfer connections and the required fleet size for the case study network.

by the integrated optimization approach. This further demonstrated the benefits of using the integrated optimization approach. One useful finding is that an increase of one more vehicle (fleet size) may not bring increased number of coordinated transfer connections. This is demonstrated by the related red dots positioned directly under each of the blue dots, which is dominated by the red dot solutions. This finding can help PT operators to deploy the minimum number of vehicles (operation cost) to keep the desired number of successfully coordinated transfer connections (level of service).

6.4.2. Planning period

Except for the benchmarking planning period $T = 180$ min, three additional planning periods, i.e., T=150 min, 210 min, and 240 min, are further considered in this sensitivity analysis. The impacts of different planning periods on the number of constraints related to variables y_{kiljn} and z_{kilj} is first analyzed. The analysis results are shown in Table 23. It shows that with an increase of the length of the planning period, the number of constraints related to variables y_{kiljn} and z_{kilj} both increase. The number of constraints related to variables z_{kilj} is far more than the one related to variables y_{kiljn}. After implementing the first constraint reduction procedure, more than 89.46% of the constraints related to

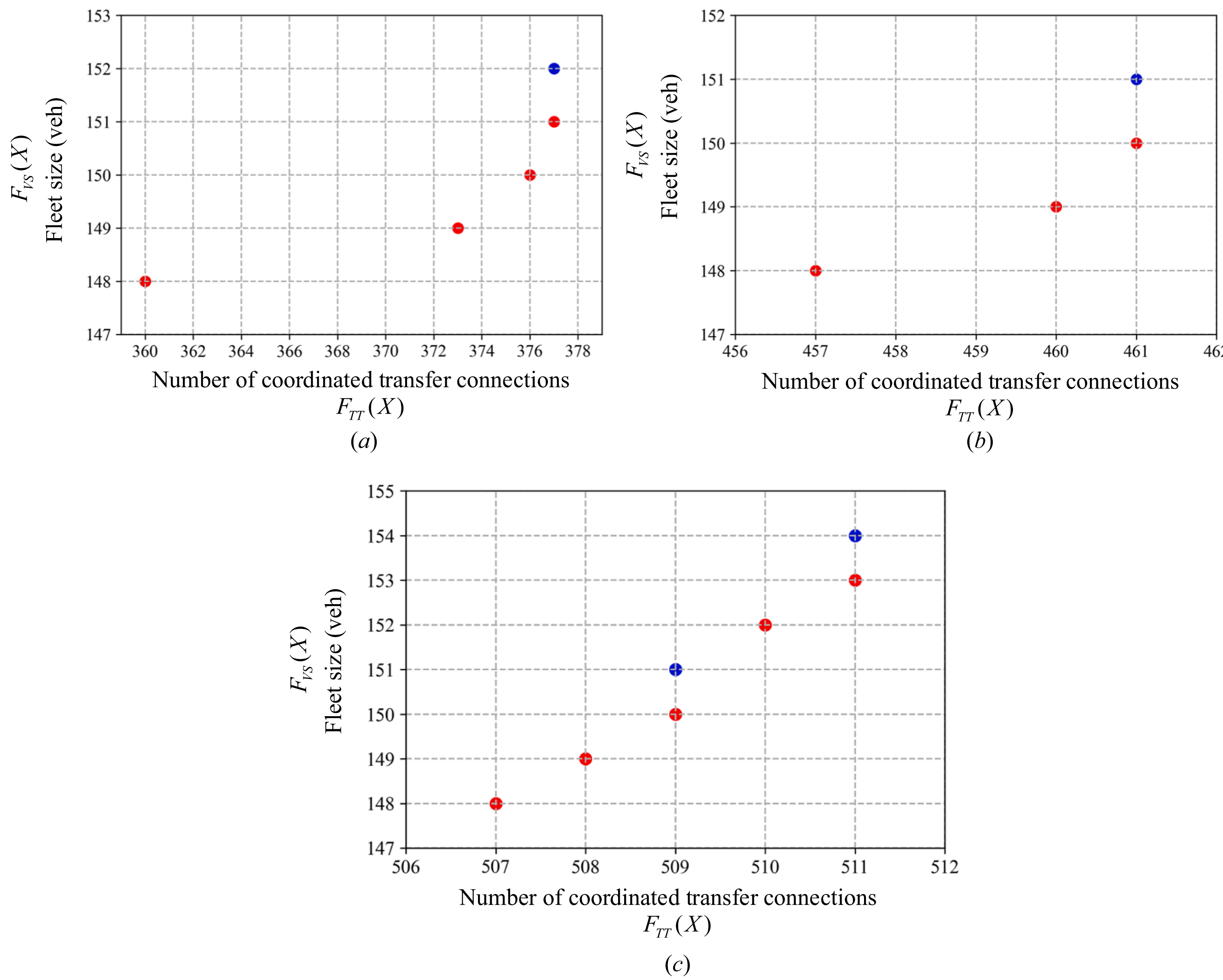

Fig. 8. Solutions under different transfer waiting time windows: $(a)\left[w_n^-, w_n^+\right] = [-1,+1]$, $(b)\left[w_n^-, w_n^+\right] = [-2,+2]$, and $(c)\left[w_n^-, w_n^+\right] = [-3,+3]$.

Table 23
Results of constraints reduction under different planning periods.

Planning period T	Number of constraints related to variables y_{kiljn}			Number of constraints related to variables z_{kilj}		
	Before	After	Reduction	Before	After	Reduction
150 min	8856	933	89.46%	68,644	5096	92.58%
180 min	13,064	1185	90.93%	101,124	12,162	87.97%
210 min	17,984	1433	92.03%	138,384	21,957	84.13%
240 min	23,808	1686	92.92%	183,184	35,189	80.79%

variables y_{kiljn} can be reduced, and after implementing the second constraint reduction procedure, more than 80.79% of the constraints related to variables z_{kilj} can be reduced. With the increase of the planning period length, the reduction percentage related to variables y_{kiljn} increases, while in the case of variables z_{kilj}, it decreases. The reason is that the prolonging of the planning period length will increase the number of total vehicle trips $|I|$. For a given trip i, under the same headway and transfer waiting time window, the number of possible coordinated trips will not increase. It means that for this trip i the number of impossible coordinated trips will increase, which contributes to the reduction percentage increase for constraints related to variables y_{kiljn}. For the case of constraints related to variables z_{kilj}, the increased number of total vehicle trips $|I|$ will increase the number of feasible trip connections, which contributes to the reduction percentage decrease.

All in all, with the increase of the planning period, after implementing the two constraint reduction procedures, the total number of constraints related to variables y_{kiljn} and z_{kilj} will still increase. The number of constraints related to variables y_{kiljn} will slightly increase, while the number of constraints related to variables z_{kilj} will significantly increase. Thus, it is recommended to use short planning period rather than long ones (e.g., no more than 240 min).

The solutions yielded by the sequential and integrated optimization approaches under the four different planning periods, i.e., T=150 min, 180 min, 210 min, and 240 min, are shown in Fig. 9(a), 9(b), 9(c), and 9(d), respectively. The red dots in the figure indicate the Pareto-optimal solutions, while the blue dots indicate non-Pareto-optimal solutions. It can be seen that under each planning period, three or four Pareto-optimal solutions can be obtained. In each case, all the solutions obtained by the integrated optimization approach dominate the solution obtained by the sequential optimization approach. One useful finding is that the increased number of coordinated transfer connections by using one more vehicle for the case of the minimum fleet size, which is 149 vehicles in Fig. 9(a) and 148 vehicles in Fig. 9(b), 9(c), and 9(d), is the most significant compared to the cases of other fleet sizes. It implies that it may not be a wise decision to use the minimum fleet size obtained by

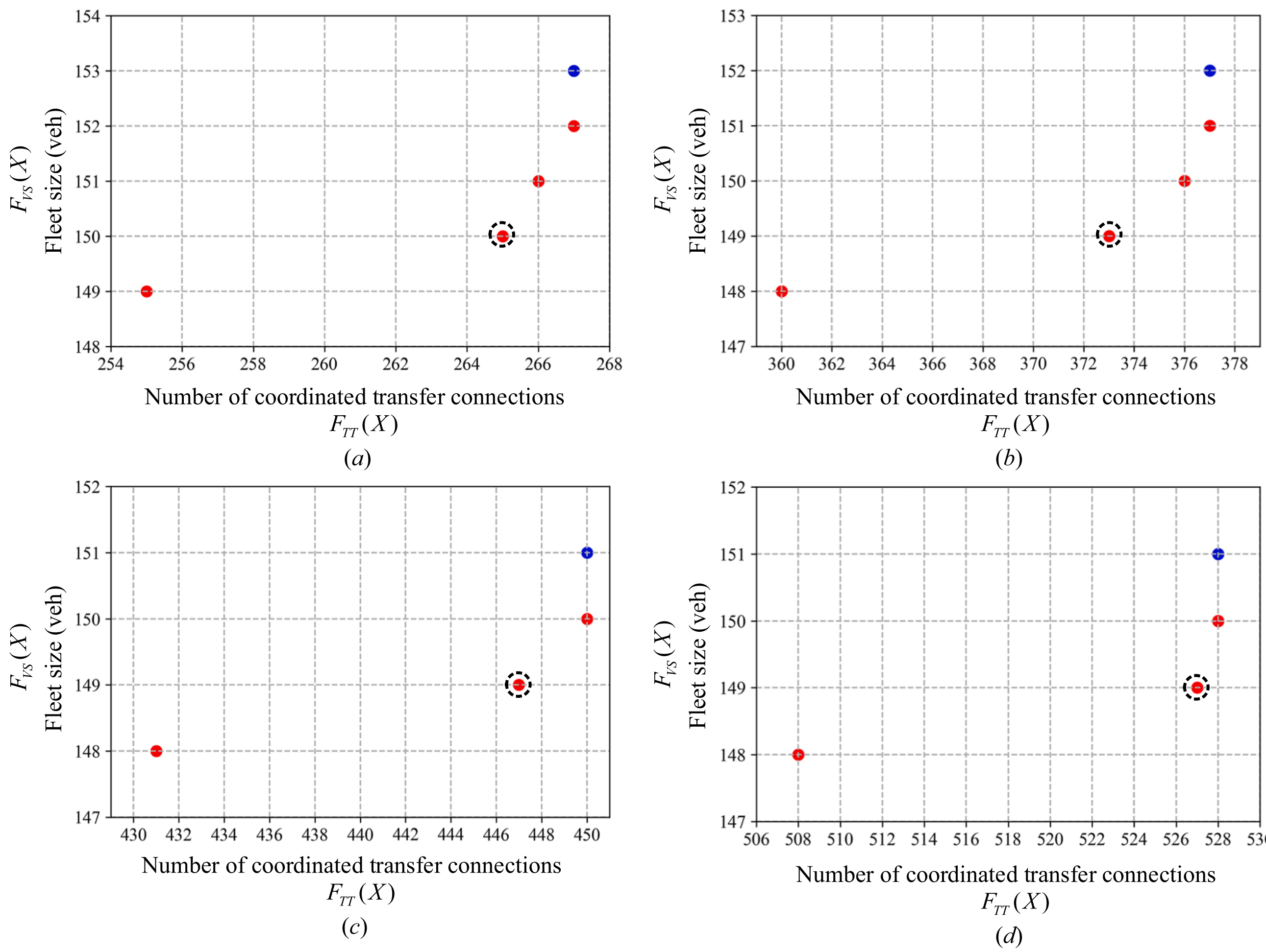

Fig. 9. Solutions under different planning periods: $(a)T = 150$ min, $(b)T = 180$ min, $(c)T = 210$ min, and $(d)T = 240$ min.

the second stage vehicle scheduling optimization model. Instead, it might be preferable to use a relatively larger fleet size so as to increase the number of coordinated transfer connections. After the insertion of one additional vehicle, the benefit (increased number of coordinated transfer connections) is marginal when using more vehicles.

7. Conclusions

This study provides a new and novel bi-objective optimization model for public transport (PT) transfer optimization by integrating timetable coordination and vehicle scheduling. The model is solved using an ε-constraint method to generate a full set of Pareto-optimal solutions. Two constraint-reduction procedures were developed to reduce the number of constraints and make the integrated optimization model solvable within an acceptable computation time. A toy network and a real-world BRT network in Chengdu, China were used to demonstrate the performance of the optimization models and solution method. The key findings of this study are summarized as follows:

(1) The solution generated by the sequential optimization approach is usually dominated by the solutions generated by the integrated optimization approach. The integrated optimization approach can generate a full set of Pareto-optimal solutions, which allows PT operators to make a trade-off between the required fleet size (operation cost) and the maximum number of successfully coordinated transfer connections (level of service). However, the integrated optimization approach requires a longer computation time than that of the sequential optimization approach.
(2) The even-headway consideration and two constraint-reduction procedures are very effective in reducing the number of constraints, which makes the integrated optimization model solvable within an acceptable computation time and thus applicable in practice.
(3) The transfer waiting time window has a significant impact on the two objectives of the integrated optimization model, i.e., maximum number of coordinated transfer connections and the minimum required fleet size. A wider transfer waiting time window will lead to increased number of coordinated transfer connections while not increasing the fleet size.
(4) The planning period has significant impacts on the performances of the model and solution method. Increasing the planning period will increase the reduction percentage for constraints related to variables y_{kiljn}, and decrease the reduction percentage for constraints related to variables z_{kilj}. In total, the number of constraints related to variables y_{kiljn} and z_{kilj} will increase. Thus, it is recommended not to use a long planning period (e.g., a period in excess of four hours) in practice.
(5) It is preferable not to use the solution generated by the sequential optimization models or the solution with the minimum fleet size generated by the integrated optimization model. For practical implementation, it is better to select the solution that has a fleet size of one more vehicle than the minimum fleet size. By selecting

this solution, a large marginal increase in the number of coordinated transfer connections can be obtained. However, a further increase of one more vehicle will not necessarily bring a further increase in the number of coordinated transfer connections.

These findings offer some valuable managerial insights for making informed decisions in PT transfer optimization within an integrated timetable coordination and vehicle scheduling optimization framework. However, there are some limitations that deserve further explorations in future studies:

(1) This study only considered two objectives, i.e., number of coordinated transfer connections and fleet size. Other objectives could be further incorporated. For example, the number of transfer passenger flow at each transfer node can be included in the timetable coordination optimization objective to allow minimizing a weighted total passenger transfer waiting time objective. However, it is not trivial to obtain the accurate number of transfer passengers for each transfer node. In addition, other objective functions, such as minimizing the longest transfer waiting time and number of failed transfer connections (Wang et al., 2020), could be considered. As for the vehicle scheduling optimization objective, except for minimizing the fleet size, other objectives, such as minimizing the total DH trip cost, improving vehicle headway regularity, and reducing vehicle crowdedness, might be considered.
(2) The ε-constraint method might become limiting when solving the integrated optimization model with larger-scale size problems. More efficient solution approaches, such as the use of heuristic or meta-heuristic methods can be further explored. Comparisons of solution quality and efficiency between the heuristic/meta-heuristic methods and the ε-constraint method can be further studied to understand the performance of each solution method.
(3) PT systems are known to exhibit stochasticity and uncertainty, such as variations in vehicle running times, passenger demand, and driver driving behavior. Future studies can relax some of the assumptions considered in this study. For example, by relaxing the assumption on drivers' compliance on the planned vehicle trip and timetable, one can allow for more flexible vehicle routing and scheduling to further improve the optimization results. This might be realized in the context of connected and autonomous vehicles.
(4) For some PT systems, such as rail, a time or distance gap between two arriving trains at a transfer station could be considered in the optimization model, e.g., defined in the transfer waiting time window, to ensure safe operations.
(5) This study considered the transfer optimization within a single PT mode, i.e., BRT system. It will be an interesting research topic to further study the transfer optimization between different passenger transport modes, such as the transfers between bus and rail, bus and ferry, bus and plane, and rail and plane (Jin et al., 2014; Yan et al., 2020; Chen et al., 2022). Transfer coordination between different PT models will contribute to improving the integration of different modes and achieve seamless door-to-door travel.

CRediT authorship contribution statement

Tao Liu: Conceptualization, Methodology, Formal analysis, Investigation, Writing – original draft, Writing – review & editing, Project administration. **Wen Ji:** Data curation, Investigation, Writing – original draft. **Konstantinos Gkiotsalitis:** Conceptualization, Methodology, Writing – review & editing. **Oded Cats:** Conceptualization, Methodology, Writing – review & editing.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

The authors do not have permission to share data.

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Appendix A. Vehicle deadheading running time matrix

Table A
Vehicle deadheading running times (in minutes) between terminal stations in the studied BRT network.

	T1	T1′	T3	T3′	T5	T5′	T6	T6′	T11	T11′	T12	T12′	T13	T13′	T17	T17′	T19	T19′
T1	0	0	30	13	0	17	39	19	30	0	22	18	22	10	21	10	22	15
T1′		0	30	13	0	17	39	19	30	0	22	18	22	10	21	10	22	15
T3			0	25	30	35	39	27	43	30	28	36	37	24	21	26	27	26
T3′				0	13	26	40	12	32	13	16	24	24	12	17	14	20	11
T5					0	17	39	19	30	0	22	18	22	10	21	10	22	15
T5′						0	43	30	37	17	35	26	30	20	26	20	31	25
T6							0	31	34	39	30	28	28	40	38	40	21	38
T6′								0	26	19	8	18	20	20	23	21	11	18
T11									0	30	29	18	11	32	42	31	24	36
T11′										0	22	18	22	10	21	10	22	15
T12											0	17	19	20	24	21	7	16
T12′												0	10	21	33	21	17	26
T13													0	26	37	25	18	30
T13′														0	16	7	22	9
T17															0	19	28	15
T17′																0	23	13
T19																	0	19
T19′																		0

Note: T1 and T1′ indicate the departure terminals for the two different directions of the same line. The same to other T*i* and T*i*' ($i = 3, 5, 6, 11, 12, 13, 17, 19$).

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