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Keyword Search Shareable Encryption for Fast and Secure Data Replication

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Abstract—It has become a trend for clients to outsource their encrypted databases to remote servers and then leverage the Searchable Encryption technique to perform secure data retrieval. However, the method has yet to be considered a crucial need for replication on searchable encrypted data. It calls for challenging works on Dynamic Searchable Symmetric Encryption (DSSE) since clients must share the search capability of the encrypted data replicas and guarantee forward and backward privacy. We define a new notion called “Keyword Search Shareable Encryption” (KS²E) and the corresponding security model capturing forward and backward privacy. In our notion, data owners are allowed to share search indexes of the encrypted data with users. A search index will be updated with a new search key before sharing to guarantee the data privacy of the source database. The target database also inherits data search efficiency along with the shared data. We further construct an instance of KS²E called *Branch*, prove its security, and use real-world datasets to evaluate *Branch*. The evaluation results show that *Branch*’s performance is comparable to classical DSSE schemes on search efficiency and demonstrate the effectiveness of searching encrypted data replicas from multiple owners.

Index Terms—Searchable symmetric encryption, forward and backward privacy, encrypted data replication.

I. INTRODUCTION

IN 2012, Kamara et al. developed Dynamic Searchable Symmetric Encryption (DSSE) [1] to bring forward/backward privacy attention to newly updated retrieval entries on

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historical/future search results [2]. DSSE schemes usually employ search indexes to retrieve logical file addresses, and they should update search indexes along with the update of ciphertexts. Research works on DSSE consider data updating as adding or deleting outsourced encrypted data on a server via maintaining a solo search index table [3]. Besides, the high efficiency of DSSE also attracts researchers to extend the search function from a single client to multiple clients. This advantage avoids the fundamental efficiency bottleneck in public encryption schemes like PEKS [4].

Multi-Key Searchable Encryption (MKSE) [5] can be applied as a heuristic ciphertext-sharing approach, which enables search tokens to search shared ciphertexts via the adjust-key that can re-encrypt the search token from one search key to another search key. In our view, the search capability relies on the corresponding search key other than the client. Moreover, [6] additionally uses shared data information as the input of sharing process in the Multi-Key Searchable Encryption scheme. We call such key-sharing processes for searching shared ciphertexts “static” methods since this scheme pre-defines the usage of outsourced ciphertexts. Inspired by the DSSE research of adding and deleting data along with search indexes, we consider that sharing processes may need variability on data users and ciphertexts in “dynamic” ways. Hence, we seek efficient schemes that can search the shared data via search indexes without long-term maintenance of auxiliary keys, which is desirable in realistic scenarios:

A. Data Backup

Inter-cloud data replication [7], [8] is an essential operation for backup, even though the data are stored as ciphertexts. The data owner delivers replicas to the nearest backup database, and the replicas will be re-encrypted with the key of the backup database. The indexes associated with the data play a critical role in determining the performance of search queries [9]. In this example, sharing search indexes can help clients to achieve a natural efficiency in securely searching encrypted replicas authorized by the data owner.

B. Health Monitoring

Electronic health record (EHR) data [10] are usually encrypted to guarantee privacy. When patients are transferred among different health centers, their EHR should be replicated and shared with doctors automatically. Recall that the replication is encrypted with patients’ private keys. Without those keys, the doctors should be able to find a way to put the encrypted records into the local database but also to search them correctly and efficiently.

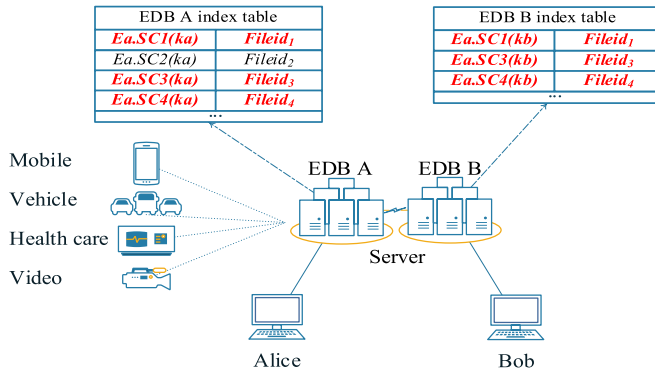


Fig. 1. The generic scenario of secure data replication. Alice and Bob outsource EDB A and EDB B to Server, respectively. The highlighted search indexes are shared from Alice to Bob. In the index table, $Ei.SCj(k_i)$ denotes the j th searchable ciphertext encrypted by k_i from EDB i , and $Fileid_j$ denotes the corresponding logical file address id_j .

Fig. 1 illustrates this scenario. Alice and Bob use separate encrypted databases and search indexes, but they need to exchange data. When the data is shared from Alice to Bob, Alice takes the owner role while Bob is the user. Firstly, the server sends the encrypted data from Alice's database to Bob as the replication. Secondly, Bob derives the search capability over the shared data. To this end, Bob should derive the search index for the replica.

1) *Motivation*: Most prior works *predefine* sharing keys to establish long-term sharing relationships. For example, MKSE [5], [6] delegates the search capability by adjust-keys. An adjust-key is predefined to re-encrypt the search tokens from the user to the owner on the server side. Multi-user Searchable Encryption (MUSE) [11], [12], [13] allows multiple users to search an owner's encrypted data securely. To do so, the owner must enroll the keys of the users in advance. We conclude the characteristic of MKSE and MUSE in Tab. I. Key-Aggregate Searchable Encryption (KASE) [14] uses aggregate-keys to share the search capability over encrypted ciphertexts. An aggregate-key enables re-encrypt search tokens to be generated from the keywords specified by the owner. Hybrid Searchable Encryption (HSE) [15] combines the cryptographic primitives of DSSE and ID-Coupling Key-Aggregate Encryption (ICKAE). This scheme prepares a set of initialized keys for owners who can use the keys to update their local database, in which the updated ciphertexts are related to the user's public key.

Considering scalability and search efficiency, Alice and Bob should dynamically and securely share search indexes with a *temporary* sharing key. We further observe the followings:

- For dynamic sharing, it is impractical to preset any sharing keys beforehand for each dynamic request of search indexes.
- For securely sharing, the sharing process should not cause serious leakages on the previously updated search index, which is not shared, or the later updated search index.
- For the efficiency of search and index sharing, the search complexity after sharing should rely on the number of keywords.

2) *Our Contributions*: Inspired by the primitives of DSSE, we propose a novel scheme called *Keyword Search Shareable Encryption* (KS²E) to achieve our goals. KS²E is applied to construct the outsourced encrypted database with shareable search indexes authorized by *sharing tokens*. The owner and

user temporarily negotiate the sharing tokens to “snapshot” search indexes. The “snapshot” is achieved by a specific structure among encrypted search indexes, and the internal state generates this structure. In this approach, the sharing token represents the search indexes to be shared in the current internal state, and Server can retrieve replicas by shared search indexes with a search token. The main contributions are summarized as follows:

- We define the KS²E scheme. KS²E has functions of searching and sharing indexes, and it differs from other models in the sharing process or the solo search index table. We analyze potential threats and security risks accordingly by redefined leakage functions and semantic security from the indistinguishability between real and ideal games, i.e., simulation-based security.

- To instantiate KS²E, we construct an instance called *Branch*. Branch uses cryptographic hash functions and secure Pseudo Random Functions (PRFs) to generate search indexes, sharing tokens, and search tokens. Branch generates a bi-directional structured search index to achieve sub-linear efficiency in the search processes. We prove that Branch has adaptive security under our security model.

- We code Branch in Python and simulate it in the inter-cloud data replication circumstances with the leverage of real-world datasets to perform efficiency analysis. We compare Branch with the heuristic KS²E instance based on a DSSE instance Diana [3], an MKSE instance constructed from Perfect Hash Table [6], and a MUSE instance Q- μ SE [11]. We thoroughly evaluate time overheads on actual sharing performance. Branch achieves sub-linear efficiency according to its sharing and search performances. As shown in Tab. I, the bi-directional index brings KS²E significant advantages in search time complexity as compared to MUSE and MKSE.

3) *Organization*: Section II introduces the related work on Searchable Symmetric Encryption (SSE) schemes. Section III introduces the KS²E model. Section IV proposes KS²E's instance called *Branch*. Section V presents our evaluation results. Section VI concludes this paper.

II. RELATED WORKS

The first SSE scheme [16] proposes a secure system that designs search tokens to retrieve ciphertexts. Inspired by that, researchers have delivered massive works on designing secure and fast SSE schemes in the single-user setting, which are applicable to many real-world applications [17], [18]. Other works investigate how to securely share the search capability in the context of SSE. None of the prior SSE schemes has considered the followings: (1) data owner and data users may probably have different setups, initializing respective private search keys; (2) encrypted database could have replications, and search is required over those replicas.

A. Dynamic Searchable Symmetric Encryption (DSSE)

DSSE [1], [19], [20], as compared to traditional SSE, provides the addition and deletion of search indexes via update queries. In 2017, Bost et al. [3], [21] proposed the forward and backward privacy notions for DSSE. Forward privacy forbids implying new *add* contents to current search results, while backward privacy is used to securely exclude previous

TABLE I

COMPARISON OF PREVIOUS REPRESENTATIVE WORKS AND OUR WORKS. a_s REPRESENTS ALL SHARING KEYS. a_w REPRESENTS ALL KEYWORD/IDENTIFIER PAIRS (w, id) ABOUT w . $|\text{DIFF}|$ REPRESENTS THE SIZE OF THE DIFFERENCE QUEUE. $n(I_w)$ AND $n(I_{id})$ REPRESENTS THE NUMBER OF KEYWORD w AND FILE id ITERATORS, RESPECTIVELY. F^Q AND B^Q REPRESENT THE MUSE'S FORWARD AND BACKWARD PRIVACY, RESPECTIVELY. F^B AND B^B REPRESENT THE KS²E'S FORWARD AND BACKWARD PRIVACY, RESPECTIVELY

Scheme	Unlimited users	Sharing Token	Computation			Communication			FP	BP
			Search	Share	Update	Search	Share	Update		
PKC18 (MKSE) [6]	✓	✗	$O(a_s)$	$O(1)$	-	$O(a_s)$	$O(1)$	-	-	-
Q- μ SE (MUSE) [11]	✗	✗	$O(a_w + \text{diff})$	$O(1)$	$O(1)$	$O(a_w + \text{diff})$	$O(1)$	$O(1)$	F^Q	B^Q
Branch (Ours)	✓	✓	$O(n(I_w))$	$O(n(I_{id}))$	$O(1)$	$O(n(I_w))$	$O(n(I_{id}))$	$O(1)$	F^B	B^B

del content from search [3], [22], [23]. The definition acts as an essential security requirement in practical applications, such as non-interactive communication [24], small client storage [25], and efficient search [26]. Recently, several works have extended the security definitions to particular contexts, such as Robustness [27], multi-user [11], and document collections [28]. In this work, we refer to the forward and backward privacy as basic security notions of KS²E, capturing the leakage from the addition and deletion of search indexes.

B. Multi-User Searchable Encryption (MUSE)

Curtmola et al. [12] first proposed a multi-user SSE (MUSE) scheme where an arbitrary group of users can submit search queries for the owner's database. Following that work, many researchers have designed various variants covering security [29], [30], efficiency [31], non-interactive [32], and sharing revocation [13]. Recently, Chamani et al. [11] developed a new concept called Multi-user Dynamic Searchable Symmetric Encryption (DMUSE). They considered the leakage of the sharing upon the forward and backward privacy in the multi-user setting and proposed an OMAP-base μ SE instance and a queue-base μ SE instance.

Assigning sharing keys to users beforehand (by data owners) easily limits the dynamic search delegation. For example, the number of users needs to be pre-fixed [11], [13]. Besides, existing schemes supporting search sharing may consume more search complexity while multiple scheme instances are needed. Users leverage the sharing keys of all owners to compute the search tokens and search for a keyword in each MUSE instance. In KS²E, we enable users to leverage private search keys to retrieve the shared data replication in their databases so that we do not have to limit the number of users in advance. We also ensure search efficiency regardless of the number of owners.

C. Multi-Key Searchable Encryption (MKSE)

Popa and Zeldovich [5], [33] introduced the definition of MKSE. It enables users to search keywords over data encrypted with different keys. The first MKSE scheme [5] is based on elliptic curves with bi-linear mappings, but it is vulnerable to Leakage-Abuse attacks [34]. Later, Hamlin et al. [6] revisited the concept and explicitly defined a setting for MKSE where users who store the encrypted documents on a remote server can selectively share documents with each other. They proposed the indistinguishability-based security notions and two concrete constructions from a perfect hash function and an indistinguishability obfuscation, respectively. Recently, some works [35], [36] proposed alternative formulations to verify users' privileges.

In MKSE (e.g., [6]), we observe that a sharing key must be established for each encrypted file so as to delegate a search successfully. The search token can not directly match the search index, as it first needs to be computed with the sharing key. In KS²E, the token is directly performed on search indexes, which results in much higher efficiency (see Section V).

III. KEYWORD SEARCH SHAREABLE ENCRYPTION

In this section, we introduce the definitions and the generic construction of KS²E for sharing search indexes during data replication. We also define the security model with forward and backward privacy.

A. Problem Statement

We consider two parties:

- **Clients:** They initialize the system by a setup with security parameters and outsource the encrypted databases to a Server. A client can have two roles separately - owner and user. As an owner (Owner), it can send queries to update search indexes, delegate the sharing token to a user (User), and issue search queries to Server. User can receive the sharing token from Owner and send sharing requests and search queries to Server.
- **Server:** Server (which could be one or more server devices) manages the encrypted databases outsourced by clients. It further handles the requests to setup and update the databases, converts search sharing, and launches secure search over encrypted data. Upon receiving an update query, Server adds or deletes a search index from the corresponding database. Server also participates in sharing the index from Owner to User. It can use a search token to match the indexes to return the corresponding search results.

B. Scheme Definitions

Definition 1 (The KS²E Scheme): A KS²E scheme consists of the following protocols:

- **Setup(λ):** This protocol is run between Owner and Server or User and Server for initialization by inputting the security parameter $\lambda \in \mathbb{N}$. When it is run between Owner and Server, it probabilistically outputs an internal state σ , master secret key $K_\sigma \in \mathcal{K}$ stored by Owner, and an encrypted database EDB_σ outsourced on Server. In another case, it probabilistically outputs an internal state σ' , master secret key $K_{\sigma'} \in \mathcal{K}$ stored by User, and an encrypted database $\text{EDB}_{\sigma'}$ outsourced on Server.
- **Update($K_\sigma, \sigma, (w, id, op)$; EDB_σ):** This protocol is run between Owner and Server to update the search index in the encrypted database. Owner inputs K_σ , internal state σ , and a tuple of the keyword w , file address id , operation

$op \in \{add, del\}$. Server inputs the encrypted database EDB_σ . At the end of this protocol, it adds (add) or deletes (del) the search index about the pair (w, id) in EDB_σ .

- **Search**($K_\sigma, \sigma, w; EDB_\sigma$): This protocol is run between Owner and Server or User and Server to search the index. When it is run between Owner and Server, Owner inputs K_σ, σ , and a specific keyword w . Server inputs the EDB_σ . At the end of this protocol, Owner gets the search result S or \perp (null output). In another case, User inputs $K_{\sigma'}, \sigma'$, and a specific keyword w . Server inputs the $EDB_{\sigma'}$. At the end of this protocol, User gets the search result S' or \perp .

- **Sharetoken**(K_σ, σ, id): This protocol is run between Owner and User to generate the sharing token P_{id} which can access a specific set of search indexes. It takes as input the K_σ , internal state σ , and the shared file id from Owner. Finally, it outputs P_{id} to User.

- **Share**($P_{id}, \sigma', K_{\sigma'}; EDB_{\sigma'}, EDB_\sigma$): This protocol is run between User and Server for sharing search indexes. User inputs sharing token P_{id} , internal state σ' , and the $K_{\sigma'}$. Server inputs the databases $EDB_{\sigma'}$ and EDB_σ . At the end of this protocol, $EDB_{\sigma'}$ stores the shared search indexes.

Correctness: Except with negligible probability, the KS^2E scheme is correct if the **Search** protocol returns correct results for the searched keyword and if the **Share** protocol shares correct search indexes for the accessed P_{id} . For formalism, we ignore the case where Owner attempts to add a file with an existing identifier or delete/edit with an identifier not present in EDB .

C. The Generic Construction of KS^2E Scheme

Fig. 2 illustrates our generic construction that includes four functions:

- **Setup:** Owner and User can separately run the protocol **Setup** with the security parameter λ . At the end, Owner stores a master secret key K_σ and the internal state σ locally, and outsources the encrypted database EDB_σ to Server. Similarly, User stores a master secret key $K_{\sigma'}$ and the internal state σ' locally, and sends the $EDB_{\sigma'}$ to Server.

- **Index Updating:** This function directly uses the protocol **Update** between Owner and Server to update search indexes. Let the file id be associated with a set of keywords $\{w_1, w_2, \dots, w_n\}$ and $C_{w,id}$ denote the encrypted search index. The two parties can run the protocol **Update** to update indexes $\{C_{w_i,id} | i \in [1, n]\}$ in EDB_σ .

- **Sharing:** Owner shares search indexes with User. Firstly, Owner runs the protocol **Sharetoken** to generate the sharing token P_{id} associated with the search indexes for User, where P_{id} can be securely received (via a key encapsulation [37]). Secondly, User and Server run the protocol **Share** to generate the search indexes $\{C'_{w_i,id} | i \in [1, n]\}$. At last, the User's search indexes are stored in $EDB_{\sigma'}$.

- **Search:** Owner and User can run the protocol **Search** to retrieve matched indexes, respectively. When Owner and Server run the protocol **Search**, Owner chooses a keyword $w_i \in \{w_1, w_2, \dots, w_n\}$ and sends the search token T_{w_i} to Server. Server makes result S from the matched indexes in $\{C_{w_i,id} | i \in [1, n]\}$ and sends it back to Owner. We note that User can perform a similar **Search** with Server.

We consider that all clients are provided search capability at the initialization; User is allowed to obtain the shared index

specified in the sharing stage by Owner. Unlike existing works, our construction does not require any sharing token in the search stage; instead, we reflect the sharing directly into the search index, leading to an efficient search process.

D. Security Model

According to the fact that the server is always considered as honest-but-curious, Server should know the query content and the outsourced encrypted databases as little as possible. More precisely, the adversary tracks the queries from Owner and User, views the full transcripts of queries on Server, knows who initiates these transcripts, and wants to learn anything beyond some explicit leakages.

We typically capture the security model from the indistinguishability between *Real and Ideal Game* formulation [3], [38]. In the real game, the adversary runs all operations originally and observes all transcripts from her view. In the ideal game, the adversary runs all operations constructed from the set of simulator \mathcal{S} with the leakage function \mathcal{L} and gets all simulated transcripts from her view. Finally, the adversary decides on which game is running and distinguishes the real and ideal game.

Definition 2 (Adaptive Security of KS^2E): Assume a KS^2E scheme is \mathcal{L} -adaptively secure iff for all sufficiently large security parameters $\lambda \in \mathbb{N}$ and PPT adversary \mathcal{A} , there is a set of efficient simulators \mathcal{S} with a set of leakage functions \mathcal{L} that has:

$$|\mathbb{P}[\text{Real}_{\mathcal{A}}^{\text{KS}^2\text{E}}(\lambda) = 1] - \mathbb{P}[\text{Ideal}_{\mathcal{A}, \mathcal{S}, \mathcal{L}}^{\text{KS}^2\text{E}}(\lambda) = 1]| = \text{negl}(\lambda)$$

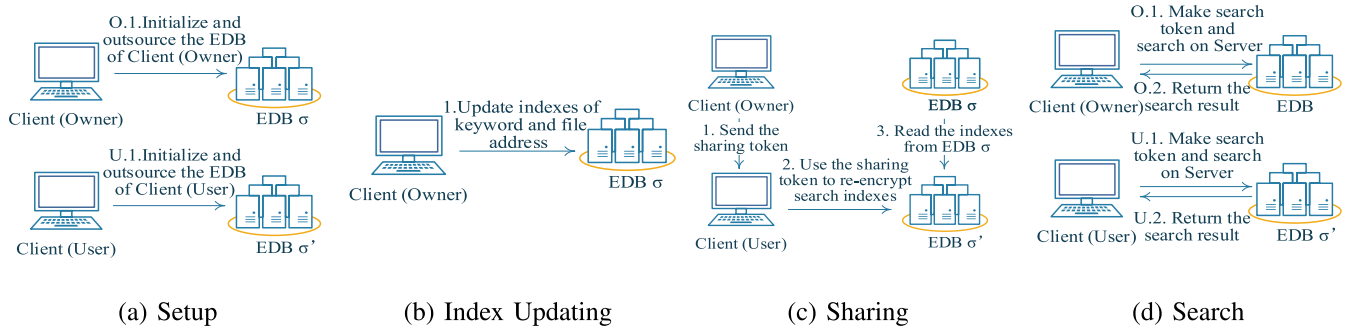
where the game $\text{Real}_{\mathcal{A}}^{\text{KS}^2\text{E}}(\lambda)$ and the game $\text{Ideal}_{\mathcal{A}, \mathcal{S}, \mathcal{L}}^{\text{KS}^2\text{E}}(\lambda)$ are defined as below:

- **Game $\text{Real}_{\mathcal{A}}^{\text{KS}^2\text{E}}(\lambda)$:** In this game, the adversary triggers Owner and User to run real protocols. Firstly, the adversary triggers the protocol **Setup** and gets two encrypted databases $EDB_\sigma, EDB_{\sigma'}$. Secondly, with the parameters of her choice, she adaptively triggers **Update**, **Search**, **Share**, **Sharetoken**, and then she gets the real transcript (q_1, q_2, \dots, q_n) from the view of Server. Finally, she outputs a bit b decided from the real transcript $(EDB_\sigma, EDB_{\sigma'}, q_1, \dots, q_n)$.

- **Game $\text{Ideal}_{\mathcal{A}, \mathcal{S}, \mathcal{L}}^{\text{KS}^2\text{E}}(\lambda)$:** In this game, all of the real protocols are replaced by the set of simulators \mathcal{S} . The adversary first triggers the simulator $\mathcal{S}_{\text{setup}}$ and gets two simulated databases $EDB_\sigma^S, EDB_{\sigma'}^S$. Secondly, with the parameters of her choice, she adaptively triggers $\mathcal{S}_{\text{update}}, \mathcal{S}_{\text{search}}, \mathcal{S}_{\text{share}}, \mathcal{S}_{\text{sharetoken}}$, and then she gets the simulated transcript $(q_1^S, q_2^S, \dots, q_n^S)$ from the view of Server. Finally, she outputs a bit b decided from the simulated transcript $(EDB_\sigma^S, EDB_{\sigma'}^S, q_1^S, q_2^S, \dots, q_n^S)$.

E. Leakage Function

In this section, we borrow and define some basic leakage functions. Without the construction from data-oblivious memory, *search pattern* leakage $sp(w)$ is a common leakage in most DSSE schemes [3]. Let u denote a timestamp. Let Q denote a set of all queries issued up to now. $sp(w) = \{u : (u, w) \in Q\}$ leaks which search queries are related to the same keyword w . We borrow this leakage function to capture the search pattern leakage in KS^2E .

Fig. 2. The generic KS²E construction.

Inspired by [21], we also use the update history $UpHist(w)$ in KS²E. $UpHist(w)$ outputs the list of all updates on keyword w . Moreover, the adversary also knows which database is updated. Let O denote the Update queries issued on the database from Owner. Let U denote the Update queries associated with the sharing requests from User. In our notion, the types of elements in the list $UpHist(w)$ include (O, u, w, id, op) and (U, u, w, id, op) .

We define the replication pattern $rp(id)$ of each replication in the sharing process. Inspired by search patterns [12], the adversary observes the memory accessed on the Owner's database during the sharing process. The adversary can find out which sharing tokens access the same memory address and speculate that these sharing tokens share the same file id .

$rp(id)$ reveals the repetition of which id generates a replicated index and is defined as $rp(id) = \{u : (u, id) \in Q\}$

F. Forward and Backward Privacy

Forward and backward privacy is important security definitions when the SSE scheme has addition and deletion [3]. Especially, Zhang et al. [39] prove that adaptive file injection attack is powerful on encrypted databases without forward privacy. Meanwhile, search queries leak the deleted entries after the search protocol without backward privacy. We analyze the leakage from each protocol to determine forward and backward privacy in KS²E.

1) *Forward Privacy*: As KS²E holds forward privacy, the adversary should not use the previous transcript to imply the newly added content before executing the protocol Search or Share. More precisely, the source of the disclosure is the protocol Update and Share. If Owner and User want to replicate the id , the sharing token needs to access previous indexes about the id . Besides, if Owner and User want to search the index, the protocol Search needs to access the matched index by the keyword w . Hence, the leakage from Update can not disclose the content about w and id . Besides, the Share protocol should not use previous sharing tokens to imply the content of newly added indexes.

To capture the forward privacy in KS²E, we introduce the functions $TimeST(id)$ and $UpSHist(id)$. $TimeST(id)$ captures the timestamps when Owner generates the sharing token about id . With the input id , $TimeST(id)$ returns the timestamp set of sharing tokens P_{id} .

$$TimeST(id) = \{u \mid (u, P_{id}) \in Q\}$$

$UpSHist(id)$ captures the leakage of replicating the encrypted search indexes from Server. With the input id ,

$UpSHist(id)$ returns the set of the timestamps from Update queries. The timestamps should not be larger than the maximum of $TimeST(id)$.

$$UpSHist(id) = \{u \mid (O, u, w, id, op) \in Q \text{ and } u \leq \max [TimeST(id)]\}$$

Combining our analysis of the protocols of Update and Share, we have the definitions of forward privacy in KS²E.

Definition 3 (Forward Privacy): An \mathcal{L} -adaptively-secure KS²E scheme has forward privacy iff its Update, Share leakage functions \mathcal{L}_{Up} , \mathcal{L}_{Shr} can be written as:

$$\begin{aligned} \mathcal{L}_{Up}(w, id, op) &= \mathcal{L}'(op) \\ \mathcal{L}_{Shr}(id) &= \mathcal{L}''(UpSHist(id)) \end{aligned}$$

where \mathcal{L}' , \mathcal{L}'' are state-less functions.

Recall that Bost et al. [3] introduced the forward and backward privacy for DSSE. As for the forward privacy, KS²E further considers the leakage in the sharing stage. This leakage reveals the timestamp of the Owner's update operations before the last sharing on a file id . The adversary can not use the previous timestamp information to infer the content of the new update operation, and thus we can achieve the forward privacy.

2) *Backward Privacy*: As KS²E holds backward privacy, the adversary should not use new transcripts to imply previous content after executing Search. In the protocol Share, the search index (in the Owner's database) is only re-encrypted and is further restored in the User's database. The Share does not leak deleted indexes about id . Besides, transcripts from the protocol Search should not reveal the deleted file address id . Considering the independence of search index tables on all databases, we require that Search can not disclose the content of id at most.

To capture backward privacy in KS²E, we introduce the functions $TimeDBS(w)$ and $DelHistS(w)$. Let u^{add} denote the timestamp of an addition Update query. Let u^{del} denote the timestamp of a deletion Update query. $TimeDBS(w)$ is the list of all id matching w , excluding the deleted ones, together with the timestamps of when they were inserted in all databases.

$$\begin{aligned} TimeDBS(w) &= \{(u, O, id) \mid (O, u, w, add, id) \in Q \\ &\text{and } \forall u', (O, u', w, del, id) \notin Q\} \\ &\cup \{(u, U, id) \mid (U, u, w, add, id) \in Q \\ &\text{and } \forall u', (U, u', w, del, id) \notin Q \\ &\text{and } u \leq \max [TimeST(id)]\} \end{aligned}$$

DelHistS(w) captures the leakage of the relation among addition and deletion about the same (w, id) in all databases. With the input w , DelHistS(w) returns the set of addition and deletion timestamp tuples. These timestamps reveal the correspondence between *add* and *del* queries about w in all databases.

$$\begin{aligned} & \text{DelHistS}(w) \\ &= \left\{ \left(u^{add}, u^{del} \right) \mid \exists id \text{ s.t. } \left(\mathbf{U}, u^{add}, w, add, id \right) \in \mathcal{Q} \right. \\ & \quad \text{and } \left(\mathbf{U}, u^{del}, w, del, id \right) \in \mathcal{Q} \\ & \quad \text{and } u^{add} \leq \max [\text{TimeST}(id)] \\ & \quad \left. \text{and } u^{del} \leq \max [\text{TimeST}(id)] \right\} \\ & \cup \left\{ \left(u^{add}, u^{del} \right) \mid \exists id \text{ s.t. } \left(\mathbf{O}, u^{add}, w, add, id \right) \in \mathcal{Q} \right. \\ & \quad \left. \text{and } \left(\mathbf{O}, u^{del}, w, del, id \right) \in \mathcal{Q} \right\} \end{aligned}$$

From these functions and our analysis, we define the backward privacy of the KS²E scheme.

Definition 4 (Backward Privacy): An \mathcal{L} -adaptively-secure KS²E scheme has backward privacy if its *Update*, *Search* leakage functions \mathcal{L}_{Upt} , \mathcal{L}_{Srch} can be written as:

$$\begin{aligned} \mathcal{L}_{Upt}(w, id, op) &= \mathcal{L}'(op) \\ \mathcal{L}_{Srch}(w) &= \mathcal{L}'''(\text{TimeDBS}(w), \text{DelHistS}(w)) \end{aligned}$$

where \mathcal{L}' , \mathcal{L}''' are state-less functions.

The backward privacy is inspired by the Type-III backward privacy [3]. The search leakage on the Owner's database is included in the original Type-III backward privacy. The search leakage on the User's database is also included in the Type-III backward privacy, although the User's search is limited to the shared search indexes. After combining the two parts, the leakage does not speculate the preservation of the Type-III backward privacy.

Similarly, DMUSE [11] gives the forward and backward privacy definitions for multi-user sharing. The leakages of DMUSE are captured when multiple users use the Owner's search indexes. DMUSE tries to incorporate the leakage under static MUSE [13] into considering the forward and backward privacy in its work. In our model, Users use their keys to search their databases' index. The privacy definitions of KS²E inherit from traditional DSSE but also minimize the leakage incurred by the sharing stage.

G. Heuristic KS²E From DSSE

Through careful investigations, we find a heuristic instance of KS²E by using multiple DSSE instances. Specifically, Owner can prepare two DSSE instances. One instance is to search the keywords and retrieve the file's *id*. To securely share the search index, the other instance is to search the shared file's *id*, retrieve all keywords, and securely deliver this retrieval to User. Finally, User updates the information transferred by Owner to his DSSE instance.

This heuristic instance suffers from some efficiency defects. It requires many search tokens and sharing tokens from different keys, increasing the time overheads of matching search indexes in the database. Moreover, Owner should be online to update the two DSSE instances at the same time, handle

TABLE II
NOTATIONS

SE	A symmetric encryption scheme
\mathcal{E}	The encryption algorithm of an SE
\mathcal{D}	The decryption algorithm of an SE
EDB _{σ}	The Owner's encrypted database
EDB _{σ'}	The User's encrypted database
lastID _{σ}	An Owner's map, which stores the latest <i>id</i> associated with w
lastW _{σ}	An Owner's map, which stores the latest w associated with <i>id</i>
lastID _{σ'}	A User's map, which stores the latest <i>id</i> associated with w
lastW _{σ'}	A User's map, which stores the latest w associated with <i>id</i>
$C_{w, id}$	A searchable ciphertext generated from (w, id)
L	An identifier of a search index in an encrypted database
I_w	A ciphertext that is used to iterate to the previous search index associated with w
J_w	A ciphertext that starts the iterate to the previous search index associated with w
J_{id}	A ciphertext that is used to iterate to the previous search index associated with <i>id</i>
I_{id}	A ciphertext that starts the iterate to the previous search index associated with <i>id</i>
R_w	A random number used in the iteration of w
R_{id}	A random number used in the iteration of <i>id</i>
C_w	A symmetric ciphertext generated from the <i>id</i> of (w, id)
C_{id}	A symmetric ciphertext generated from the w of (w, id)
S	A result set from Server
T_w	A search token generated from keyword w
P_{id}	A sharing token generated from <i>id</i>
D_{id}	A partial token from sharing token P_{id}

many sharing requests, and transfer sharing information. These cause considerable local computational resources to handle User's request. We say that the heuristic instance can maintain security as a DSSE, but it is not practical and efficient.

IV. INSTANCE CONSTRUCTION

This section introduces our approach to constructing the instance of KS²E. To construct our KS²E instance, we introduce our notation, preliminary, and bi-directional index. Following the KS²E generic construction, we show the detailed workflow of Branch. Finally, we analyze the correctness, security, and performance.

A. Notation and Preliminary

1) *Notation:* Let $\lambda \in \mathbb{N}$ be the security parameter. Symbol $r \xrightarrow{\$} \mathcal{R}$ means randomly choosing r from the space \mathcal{R} . Symbol $a||b$ means the concatenation between a and b . Symbol $\{0, 1\}^\lambda$ means a λ -bit length string. Symbol $\{0, 1\}^*$ means an arbitrary-bit length string. Symbol H means a cryptographic hash function. Symbol F means a secure Pseudo Random Function (PRF). The frequently used notations are given in Tab. II.

2) *Symmetric Encryption:* Given a security parameter $\lambda \in \mathbb{N}$, message space $\mathcal{M} = \{0, 1\}^*$, ciphertext space $\mathcal{C} = \{0, 1\}^*$, and the key space $\mathcal{K}_{SE} = \{0, 1\}^*$, a symmetric encryption scheme consists of two algorithm (\mathcal{E}, \mathcal{D}) and runs under the following syntax:

- $\mathcal{E}(k, m)$: Input a symmetric key $k \in \mathcal{K}_{SE}$ and a message $m \in \mathcal{M}$, output a ciphertext $c \in \mathcal{C}$.
- $\mathcal{D}(k, c)$: Input a symmetric key $k \in \mathcal{K}_{SE}$ and a ciphertext $c \in \mathcal{C}$, recover a message $m \in \mathcal{M}$.

For correctness, with each message $m \in \mathcal{M}$ and secret key $k \in \mathcal{K}_{SE}$, the equation $c \leftarrow \mathcal{E}(k, m)$ should always make sense, and $m \leftarrow \mathcal{D}(k, c)$ can always recover the message m from c using the secret key k . In the security definitions, a popular requirement of symmetric encryption is the semantic security under chosen plaintext attack (SS-CPA).

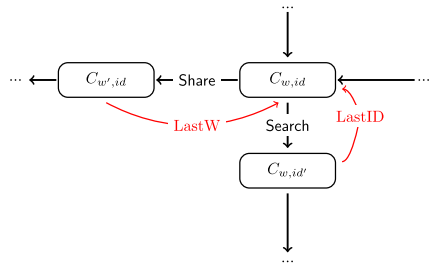


Fig. 3. Relation between tokens and encrypted search indexes.

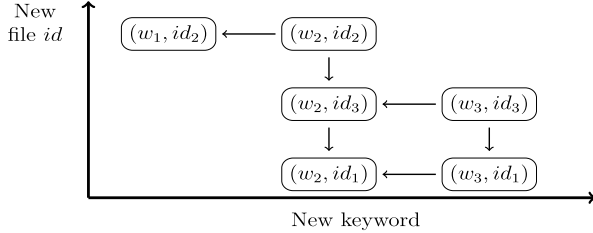


Fig. 4. Logical bi-directional index structure for the example sequence.

B. Bi-Directional Index

The bi-directional index is a structure constructed from encrypted search indexes. A popular method in constructing DSSE is to reveal the following index from the calculation between tokens and the current matched index. Following this idea, we construct iterations from the last matched encrypted search index using the tokens of **Share** or **Search**.

Fig. 3 shows the relation between tokens and encrypted search indexes. The index $C_{w,id}$ needs to generate from two types of internal state $LastID$, $LastW$. $LastID$ is a map storing the latest id updated with w . $LastW$ is a map storing the latest w updated with id . In this approach, the search token can match the current $C_{w,id}$ and utilize previous $LastID$ state information to find the previous index $C_{w,id'}$. Similarly, the sharing token can match the current $C_{w,id}$ and utilize previous $LastW$ state information to find the previous index $C_{w',id}$.

Fig. 4 shows a logical bi-directional index table after adding the example sequence: (w_2, id_1) , (w_1, id_2) , (w_2, id_3) , (w_3, id_1) , (w_2, id_2) , (w_3, id_3) . We can observe that each unique pair (w, id) pointing to the horizontal has the same id , and the pair pointing to the vertical has the same w .

Advantage: The bi-directional index avoids performance degradation. This structure enables the same private key to perform search and sharing operations. The generation of sharing and search tokens only needs the internal states about the chain tail's information. Compared with MUSE [11], [13] and MKSE [6], [36], we find that the bi-directional index enables Users to merge several search indexes collected from multiple owners into one index. Batching into one approach is naturally more practical and efficient when there are multiple owners that share files simultaneously.

C. The Workflow of Branch Instance

We use functions defined in the KS²E construction to perform search index sharing from Owner to User. More precisely, we take inputs in the example sequence of Fig. 4 as update queries of Owner and let Owner share search indexes

about id_1 with User. At the *Search*, we let User and Server run the protocol **Search** with the keyword w_2 and retrieve the search result.

Protocol 1 Branch: Setup

Setup(λ): // the example between Owner and Server, otherwise Gen. $K_{\sigma'}$, $lastID_{\sigma'}$, $lastW_{\sigma'}$, $EDB_{\sigma'}$

- 1: Initialize $H : \{0, 1\}^\lambda \times \{0, 1\}^\lambda \rightarrow \{0, 1\}^{2\lambda}$
- 2: Initialize $F : \{0, 1\}^\lambda \times \{0, 1\}^* \rightarrow \{0, 1\}^\lambda$
- 3: Initialize SE: $\{\mathcal{E}, \mathcal{D}\}$
- 4: Gen. $K_{\sigma} : K_{\sigma,1} \xleftarrow{\$} \{0, 1\}^\lambda, K_{\sigma,2} \xleftarrow{\$} \{0, 1\}^\lambda$
- 5: Gen. empty maps $lastID_{\sigma}, lastW_{\sigma}$ as the internal state σ
- 6: Gen. encrypted database $EDB_{\sigma} \leftarrow MAP$

1) *Setup:* Owner and User separately run the protocol **Setup**. Owner gets the master secret key K_{σ} and the maps $lastID_{\sigma}$, $lastW_{\sigma}$ as internal state σ . User gets the master secret key K'_{σ} and the maps $lastID_{\sigma'}$, $lastW_{\sigma'}$ as internal state σ' . Owner sends EDB_{σ} to Server, and User sends $EDB_{\sigma'}$ to Server. The $lastID$ maps the latest added w to id , and the $lastW$ maps the latest added id to w . These maps capture the endpoint in the bi-directional index structure.

Protocol 2 Branch: Update

Update($K_{\sigma}, \sigma, (w, id, add); EDB_{\sigma}$):

- Owner:**
- 1: $w' \leftarrow lastW_{\sigma}[id], id' \leftarrow lastID_{\sigma}[w]$
 - 2: $L \leftarrow F(K_{\sigma,1}, w||id), R_w \xleftarrow{\$} \{0, 1\}^\lambda, R_{id} \xleftarrow{\$} \{0, 1\}^\lambda$ //location and two random numbers for pointers
 - 3: $k_w \leftarrow F(K_{\sigma,2}, w), k_{id} \leftarrow F(K_{\sigma,2}, id)$
 - 4: $C_w \leftarrow \mathcal{E}(k_w, id), C_{id} \leftarrow \mathcal{E}(k_{id}, w)$
 - 5: if id' is \perp then
 - 6: $I_w \leftarrow H(F(K_{\sigma,2}, w||id), R_w) \oplus \perp||\perp$
 - 7: else
 - 8: $L' \leftarrow F(K_{\sigma,1}, w||id'), J'_w \leftarrow F(K_{\sigma,2}, w||id')$
 - 9: $I_w \leftarrow H(F(K_{\sigma,2}, w||id), R_w) \oplus (L'||J'_w)$
 - 10: end if
 - 11: if w' is \perp then
 - 12: $I_{id} \leftarrow H(F(K_{\sigma,2}, id||w), R_{id}) \oplus \perp||\perp$
 - 13: else
 - 14: $L' \leftarrow F(K_{\sigma,1}, w'||id), J'_{id} \leftarrow F(K_{\sigma,2}, id||w')$
 - 15: $I_{id} \leftarrow H(F(K_{\sigma,2}, id||w), R_{id}) \oplus (L'||J'_{id})$
 - 16: end if
 - 17: $C_{w,id} \leftarrow (L, I_w, R_w, C_w, I_{id}, R_{id}, C_{id})$
 - 18: Update $lastW_{\sigma}[id] \leftarrow w, lastID_{\sigma}[w] \leftarrow id$
 - 19: Send $C_{w,id}$
- Server:**
- 20: Parse $C_{w,id}$ as $(L, I_w, R_w, C_w, I_{id}, R_{id}, C_{id})$
 - 21: $EDB_{\sigma}[L] \leftarrow (I_w, R_w, C_w, I_{id}, R_{id}, C_{id})$

2) *Index Updating:* Owner and Server run the protocol **Update** with the example sequence, and Server stores these indexes in EDB_{σ} with a logical bi-directional index structure like the left part of Fig. 5. Owner uses his internal state $lastID_{\sigma}$, $lastW_{\sigma}$ to find the latest w' added on id and the latest id' added on w (step 1). He generates L as the identifier of the search index in EDB_{σ} and generates random R_w, R_{id} to make randomness to the previous index pointer (step 2). He generates symmetric encryption keys k_w, k_{id} and encrypts the id, w to the C_w, C_{id} as the retrieval of this search index (steps 3-4).

To construct the vertical index chain for search tokens, Owner checks the latest id' whether it is \perp or not. Then he creates a vertical chain head like the I_w in nodes L_{w_1, id_2} , L_{w_2, id_1} , L_{w_3, id_1} or makes the I_w point at the vertical chain tail by encrypting the latest $L'||J'_w$ (steps 5-10). To construct the horizontal index chain for sharing tokens, Owner checks whether the latest keyword w' is \perp or not. Then he creates a

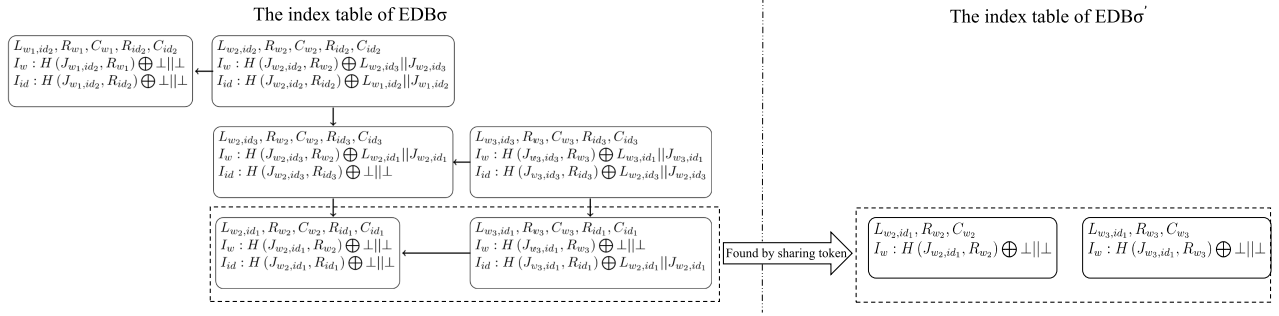


Fig. 5. The encrypted search indexes on Server.

horizontal chain head like the I_{id} in nodes L_{w_1, id_2} , L_{w_2, id_3} , L_{w_2, id_1} or makes the I_{id} point at the horizontal chain tail by encrypting the latest $L' || J'_{id}$ (steps 11-16). Finally, Owner sends this index $C_{w, id}$, and Server store it in EDB_σ .

Protocol 3 Branch: Sharetoken

Sharetoken(K_σ, σ, id) :

Owner:
 1: $w \leftarrow \text{lastW}_\sigma[id]$
 2: **if** w is \perp **then**
 3: return \perp
 4: **end if**
 5: $L \leftarrow F(K_{\sigma,1}, w || id)$, $J_{id} \leftarrow F(K_{\sigma,2}, id || w)$
 6: $k_{id} \leftarrow F(K_{\sigma,2}, id)$
 7: **Send** $P_{id} \leftarrow (L, J_{id}, id, k_{id})$ to User

Protocol 4 Branch: Share

Share($P_{id}, \sigma', K_{\sigma'}$; $EDB_{\sigma'}$, EDB_σ) :

User: // the first-round interaction
 1: Decapsulate and parse $P_{id} = (L, J_{id}, id, k_{id})$
 2: **Send** $D_{id} \leftarrow (L, J_{id})$
Server:
 3: $S \leftarrow \emptyset$, read the uploaded EDB_σ
 4: **while** L is not \perp **do**
 5: Choose (I_{id}, R_{id}, C_{id}) from $EDB_\sigma[L]$
 6: $(L' || J'_{id}) \leftarrow H(J_{id}, R_{id}) \oplus I_{id}$ // get the previous location that has the same id
 7: $S \leftarrow S \cup C_{id}$
 8: Update $L \leftarrow L'$, $J_{id} \leftarrow J'_{id}$
 9: **end while**
 10: **Return** S
User: // the second-round interaction
 11: $S' \leftarrow \emptyset$
 12: **for** each symmetric ciphertext s in S **do**
 13: $w \leftarrow \mathcal{D}(k_{id}, s), id' \leftarrow \text{lastID}_{\sigma'}[w]$
 14: $C_w \leftarrow \mathcal{E}(F(K_{\sigma',2}, w), id)$
 15: $L \leftarrow F(K_{\sigma',1}, w || id), R_w \xrightarrow{\$} \{0, 1\}^\lambda$
 16: **if** id' is \perp **then**
 17: $I_w \leftarrow H(F(K_{\sigma',2}, w || id), R_w) \oplus \perp || \perp$
 18: **else**
 19: $L' \leftarrow F(K_{\sigma',1}, w || id')$, $J'_w \leftarrow F(K_{\sigma',2}, w || id')$
 20: $I_w \leftarrow H(F(K_{\sigma',2}, w || id), R_w) \oplus (L' || J'_w)$
 21: **end if**
 22: Update $\text{lastID}_{\sigma'}[w] \leftarrow id$
 23: Update $S' \leftarrow S' \cup (L, I_w, R_w, C_w)$
 24: **end for**
 25: **Send** S'
Server:
 26: **for** each ciphertext s' in S' **do**
 27: parse s' as (L, I_w, R_w, C_w) , $EDB_{\sigma'}[L] \leftarrow (I_w, R_w, C_w)$
 28: **end for**

3) *Sharing*: First, Owner runs Sharetoken to generate the sharing token P_{id} and sends it to User. He checks the internal state LastW_σ to find the latest keyword w about id or returns when the keyword w is \perp (steps 1-4). He generates L and J_{id} , which can identify the horizontal chain tail about id , and he sends the sharing token P_{id} to User (steps 5-7).

After that, User and Server run Share. User sends the partial token of P_{id} to Server in the first round of interaction (step 2). Server iteratively matches search indexes in the EDB_σ of Owner by the identifier L (steps 4-9). Server executes XOR to restore the previous $L' || J'_{id}$ (step 6). After finding all encrypted keywords C_{id} on the file id , Server returns the encrypted keywords set S to User.

In the second round of interaction, User decrypts each $C_{id} \in S$ and gets the keyword w . Then he checks his internal state $\text{lastID}_{\sigma'}[w]$ to get the latest id' about this keyword (step 13). He generates symmetric ciphertext C_w for retrieving the id on Search (step 14). He generates L as the identifier of this search index (step 15). He checks id' whether it is \perp or not, and then he creates a vertical chain head I_w or makes the I_w point at the chain tail (steps 16-21). Finally, User generates the set of search indexes S' , and Server stores it in $EDB_{\sigma'}$. In our example sequence, the shared search indexes in $EDB_{\sigma'}$ are shown like the right part in Fig. 5.

Protocol 5 Branch: Search

Search($K_{\sigma'}, \sigma', w$; $EDB_{\sigma'}$) : // the search example between User and Server, otherwise input $K_\sigma, \sigma, EDB_\sigma$ for Owner

User:
 1: $id \leftarrow \text{lastID}_{\sigma'}[w]$
 2: **if** id is \perp **then**
 3: return \perp
 4: **end if**
 5: $L \leftarrow F(K_{\sigma',1}, w || id)$, $J_w \leftarrow F(K_{\sigma',2}, w || id)$
 6: **Send** $T_w = (L, J_w)$ to the Server
Server:
 7: Parse T_w as (L, J_w)
 8: $S \leftarrow \emptyset$
 9: **while** L is not \perp **do**
 10: Choose (I_w, R_w, C_w) from $EDB_{\sigma'}[L]$ // get the previous location that has the same w
 11: $(L' || J'_w) \leftarrow H(J_w, R_w) \oplus I_w$
 12: $S \leftarrow S \cup C_w$
 13: Update $L \leftarrow L'$, $J_w \leftarrow J'_w$
 14: **end while**
 15: **Return** S

4) *Search*: User and Server run the protocol Search to retrieve the file address by the keyword w_2 . User checks $\text{lastID}_{\sigma'}[w]$ to get the latest id then sends the search token T_w (step 5). Server uses this search token to restore the last step $L' || J'_w$ iteratively until the identifier L is \perp (steps 9-14). Finally, Server makes a set S of matched symmetric ciphertexts C_w from accessed indexes and returns this search result. To retrieve the actual file, User can extract the file id and send it to Server who is then able to retrieve the file.

D. Instance Analysis

1) *Correctness*: According to the correctness in Def. 1, the correctness of Branch can be reduced to the collision resistance of tokens in the protocol Sharetoken and Search [21]. To prove the correctness of Branch, we further reduce the probability of finding the tokens' collision to that of solving the Birthday problem [40]. Let μ denote the maximum number of inputs to the PRF F . The collision probability is $\mathbb{P}_{Coll} = 1 - e^{-\frac{\mu^2}{2\lambda+1}}$, which should be negligible in practice. For example, when using the SHA256 hash function in Branch and choosing $\lambda = 256$ and $\mu = 10^{20}$, the \mathbb{P}_{Coll} is $4.32 \cdot 10^{-38}$. Hence, the probability is small enough to achieve collision resistance. Accordingly, the correctness is proved. In Appendix B, we present the calculation of the collision probability in detail.

2) *Security*: Branch securely generates the search indexes, search tokens, and sharing tokens, guaranteed by the cryptographic hash function and the secure PRF. Based on the security parameters used in the protocol Setup, external adversaries are prevented from violently cracking ciphertexts. For forward and backward privacy, Owner always uses the master secret key K_σ and secure PRF to generate new tokens when adding new (w, id) . After receiving a new sharing request generated from the same id , Server only knows the new keyword count on specific search indexes. We use this to restrict the previous tokens from matching the newly added index, and thus we have the following theorem.

Theorem 1 (Adaptive Security of KS²E-Branch): Let PRF F with a specific key be modeled as the random oracle \mathcal{H}_F , and let cryptographic hash function H be modeled as the random oracle \mathcal{H} . We define $\mathcal{L}_B = (\mathcal{L}_B^{Up}, \mathcal{L}_B^{Shr}, \mathcal{L}_B^{Sch})$ as:

$$\begin{aligned} \mathcal{L}_B^{Up}(w, id, op) &= \emptyset \\ \mathcal{L}_B^{Shr}(id) &= rp(id), \text{UpSHist}(id) \\ \mathcal{L}_B^{Sch}(w) &= sp(w), \text{UpHist}(w) \end{aligned}$$

Branch is \mathcal{L}_B -adaptively-secure.

In our proof of instance Branch, we use a game hop method to replace the real output of the hash function and PRF step by step. The cryptographic hash function H is modeled as a random oracle \mathcal{H} . In the real game, the initial game is identical to the instance Branch. First, we replace F with the random oracle \mathcal{H}_F and replace Symmetric Encryption ciphertexts with randomly chosen strings. Next, we replace all strings generated from random oracles in the protocol Update with randomly chosen strings. In this step, we program the random oracles for the input to simulate the original hash functions in Search, Sharetoken, and Share. Finally, we make the game only use the pattern and the history to generate tokens. In this condition, the game only needs the output of leakage functions. In other words, we have a simulator.

In Appendix A, we give formal proof for our instance with an indistinguishable advantage.

3) *Complexity*: We analyze the complexity of our instance Branch. In the function Setup, Branch initializes the cryptographic hash function H , master secret key K_σ , internal state lastID_σ , lastW_σ , PRF F , and encrypted database EDB_σ in constant time. In the protocol Update, Branch generates the search index $C_{w,id}$ in constant time using the cryptographic

hash function H and PRF F . A one-way interaction exists between Owner and Server for the EDB_σ . In the protocol Search, the PRF F is used to make the search token T_w within constant time, and the search token has a constant length. During the one interaction, Server needs $O(n(I_w))$ time to find all indexes matching this search token, where $n(I_w)$ denotes the count of I_w in the current encrypted database. In the protocol Sharetoken, Owner uses the constant time to generate the constant length sharing token P_{id} . In the protocol Share, User has two round interactions with Server. In the first-round interaction, User needs constant time to compute D_{id} , and Server needs $O(n(I_{id}))$ time to retrieve ciphertexts where $n(I_{id})$ denotes the count of I_{id} in the current encrypted database. In the second-round interaction, User needs constant time to generate each index, and Server takes constant time to add each index in $\text{EDB}_{\sigma'}$.

4) *Deletion*: Although Branch can only store the *add* index, we can fix it using a ‘‘copy’’ of Branch instance [3]. We can use an instance to record the search index containing the *add* operation and another instance for the *del*. After Owner or User sends the same content search queries to the two instances, the actual result can be implied by comparing the results of the two instances locally.

After creating two instances, the server can know the type of instance [3], enabling it to know when the deletion occurred during the update stage. Therefore, this multi-instance approach can provide forward privacy with weak backward privacy. To overcome this leakage during the update stage, we can deploy branch instances separately on the non-collusive servers, preventing the server from knowing the type of instances.

It is challenging to construct efficient KS²E instances supporting both add and delete operations. Previous works usually added a file deletion content $id||del$ to the EDB in the update protocol [22], [26], which is indistinguishable from the file addition content. We tried adding a search index deletion content but faced a situation where we could not use the same private key to perform searching and sharing. Therefore, we implemented Branch with the bi-directional index to make the index efficiently searchable and shareable. Another possible solution is to use Puncturable Encryption [3], [23], [24], allowing the server to decrypt the non-deleted content. However, the puncture key should be renewed after each search and require rebuilding the bi-directional index structure.

V. EVALUATION OF INTER-CLOUD CONTEXT

In this section, we select the Inter-cloud data replication context to test the performance of Branch. We specify Server as Cloud A and B to hold EDB_σ and $\text{EDB}_{\sigma'}$. First, we describe several popular datasets for testing our instance's performance. Then, we show our test platform with cryptographic tools. Next, we introduce our evaluation setup for each aspect of KS²E. Finally, we observe and analyze our evaluation results.

A. Dataset

1) *Enron Email Dataset*:¹ We choose this popular dataset to create an EDB [39], [41]. Enron email dataset stores the data

¹Enron email dataset: online at <https://www.cs.cmu.edu/enron/>

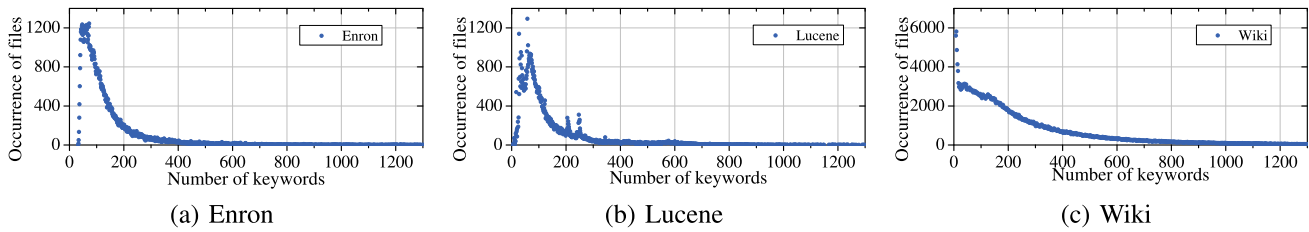


Fig. 6. The occurrence of files.

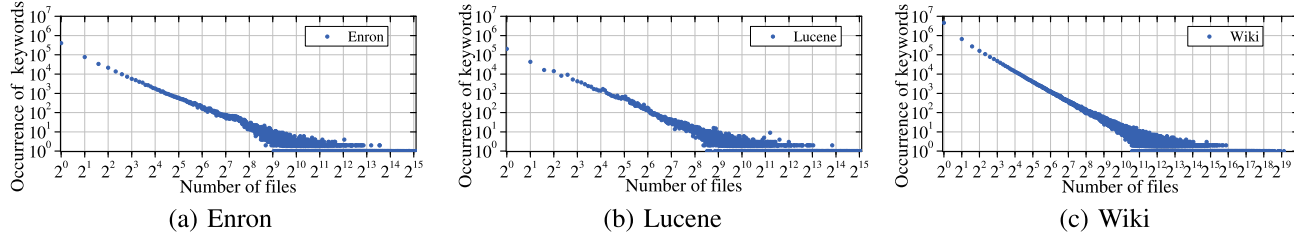


Fig. 7. The occurrence of keywords.

converted from Enron corporation emails into text files. Our test applies 128,103 documents in the ‘all-documents’ folder as the data source. We remove the HTML format code to eliminate the effects caused by stop-words. Then we use the Python NLTK package to remove stop-words like punctuations and other unnecessary texts in these text files. After that, we use Porterstemmer [42] to extract keywords from the text files. The size of extracted keyword space is 638,070, and the total number of (w, id) pairs is 18,309,166.

2) *Lucene*.² We choose the Lucene email dataset for testing on a similar scale dataset with another frequency distribution. Lucene email dataset is a java-user mailing list from the Lucene project. It stores the email sent by the developer of this project and is divided into each author. Our test samples this email data in its mailing list from Sept. 2001 to May 2020. After removing stop-words and extracting from Porterstemmer, we get the Lucene email dataset. It has 110,008 documents corresponding to a keyword space of 354,708 keywords, and the total number of (w, id) pairs is 14,935,640.

3) *Wikipediadump*.³ In order to test the performance of Branch in a larger scale data, we use the sample from Wikipediadump to create a large dataset. The Wikipediadump is an image from the content of Wikipedia. Our test downloaded the dump on Sept. 20, 2021, and sampled the first 2,000,000 dumped data without non-text files. Before extracting keywords, we use Wikiextractor [43] to remove the HTML format code and get the sliced Json structure data. Then we use the Python NLTK package [44] to ‘tokenize’ these data and remove stop-words. After this extraction, the size of Wikipediadump has 1,983,819 valid documents corresponding to a 6,441,361 keyword space, and the total number of (w, id) pairs is 272,897,980.

Fig. 6 shows the occurrence of files in each dataset. It represents the number of files that meet a certain number of keywords. As we can see, all distributions of datasets are similar to the log-normal distribution. Most files contain less than 300 keywords. When the number of keywords is less than 100, many files have few words in the Wikipediadump dataset.

²Lucene: online at <https://lists.apache.org/list.html?dev@lucene.apache.org>

³Wikipediadump: online at <https://dumps.wikimedia.org/>

TABLE III
TEST ENVIRONMENT

Platform	Intel® Xeon Gold 5120 CPU @ 2.20GHz, 128GB, Dell RERC H730 Adp SCSI Disk Device, Windows Server 2016 Standard
OpenSSL	v1.0.2g, 1 Mar 2016
MongoDB	v3.0.15
Python	v3.60
VMware	v15.5.6 build-16341506 with the instance of Ubuntu 16.04

Besides, we observe that the presence of words in common use affects the log-normal distribution of the Lucene dataset.

Fig. 7 shows the occurrence of keywords in each dataset. It represents the number of the same keywords in certain files. As we can see, all datasets follow the rule that the number of the same keywords decreases with the increment in the number of files.

B. Test Platform

Tab. III shows our test environment to simulate our test context. To build Owner, User, Cloud A, and Cloud B, we choose VMware Workstation and assign CPU cores/memory on each side. We allocate 16 cores and 64GB of memory for each Cloud. MongoDB [45] is a popular NoSQL database with high performance on query responses. We choose MongoDB to store the encrypted data, which fits the storage structure of the key-value pair.

When coding the instance Branch, we use SHA512 from OpenSSL [46] to construct the cryptographic hash function H , and we use the HMAC function from OpenSSL to construct the secure PRF F . The SHA256 parameter initializes the HMAC function. We use the Python object model to encapsulate this part of the OpenSSL code.

When coding the connection of databases, we use the MongoDB database in single-node mode to store search indexes, and we use the PYmongo package to connect the database on each side. In accessing the database, we use data slicing and thread parallelism to reduce the additional time overhead of accessing the database content. In order to make interactions between Cloud A and B, we use the Python socket package to transfer the encrypted data under ASCII encoding by LAN.

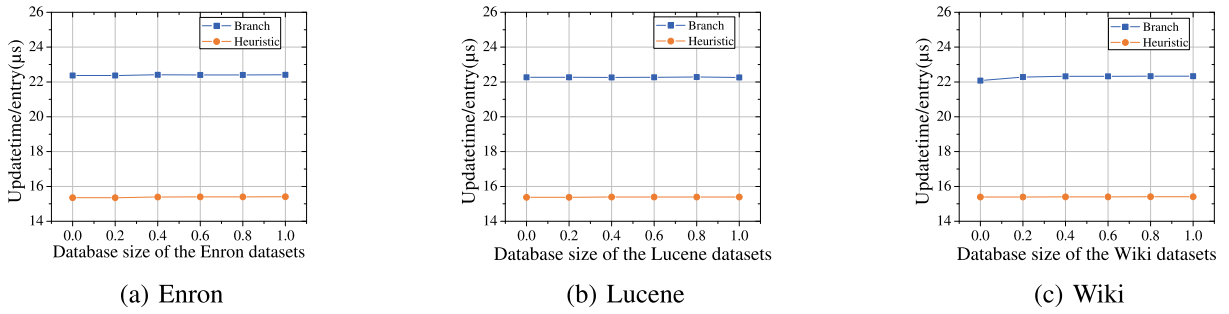


Fig. 8. Comparison of updating time overheads between Branch and the heuristic KS²E instance in LAN.

C. Evaluation Setup

Inspired by the previous evaluation setup in DSSE and the functionality of sharing search indexes, we confirm that the time expenses of operations represent the primary performance of each scheme. We record the expansion of plaintext and ciphertext databases and design experiments to test the time expenses of sharing and searching.

1) *Updating Time Overhead in Branch*: In the test of encryption time overhead, we use the protocol `Update` of Branch to encrypt entries of each dataset. In detail, we let Owner read and update all plaintexts and then write ciphertexts into Cloud A database in ciphertext collections. We record the entire updating time overhead three times and take the average overhead on each dataset. After that, we get the average updating time overhead from the percentage of encrypted data in the whole dataset.

2) *Sharing Time Overhead in Branch*: To test the sharing time overhead of Branch, we use the protocols `Sharetoken` and `Share` to share all indexes from Cloud A to B. Then we record the average sharing time overhead of User and Cloud B in the protocol `Share`, respectively. During this test, Owner generates the sharing tokens of each *id* and sends them to User. After receiving them, User executes the protocol `Share` with Cloud B. We sample the sharing time overhead of documents with different numbers of keywords by the number of matching keywords in the document.

3) *Search Time Overhead in Branch*: We test the search time overhead from User and Cloud B running the protocol `Search`. During this test, User sends search tokens to Cloud B. Cloud B queries the database using the search token and returns search results. We search the top 30 keywords ordered by the highest frequency on the three datasets and sample the average search time overhead on each keyword.

4) *Compare With the Heuristic KS²E Instance*: We construct a heuristic KS²E instance from the DSSE instance Diana. Recall that Diana is a forward private scheme which is applied to the forward and backward secure DSSE [3], [22], [23], [24]. It uses the Constrained Pseudo Random Function (CPRF) to guarantee forward privacy. We instantiate Diana based on the tree-based GGM PRF [47], [48]. Then we use our datasets to test the heuristic KS²E and compare it with Branch.

5) *Compare With MUSE and MKSE*: To evaluate the performance of searching for files shared by multiple Owners, we measure the time overheads of database generation, sharing, and keyword searching using MKSE and MUSE under LAN and WAN. We employ multiple Owners and

TABLE IV
COMPARISON OF STORAGE OVERHEAD BETWEEN
BRANCH AND THE HEURISTIC KS²E

Dataset	Size (GB)	Branch		Heuristic	
		State(MB)	EDB(GB)	State(MB)	EDB(GB)
Enron	0.72	21.46	8.47	10.66	1.97
Lucene	0.59	21.46	6.91	10.66	1.59
Wiki	10.81	320.33	126.27	166.64	29.14

outsource encrypted databases to Cloud A to adapt the setting. We compare the sub-linear search instance PKC18 [6], which uses the Perfect Hash Function, with MKSE. We choose the queue-based μ SE instance (Q- μ SE) in [11] for comparison. Both instances focus on efficient database generation, sharing, and search. The PKC18 instance uses the *DataKeyGen* algorithms for each Owner and the *QueryKeyGen* for one User. The queue-based μ SE instances are separately initialized for multiple Owners, and each Owner can *Enroll* the identity of one User. We uniformly sample 2,000 files from our Enron email dataset to generate an appropriate dataset. The MUSE and MKSE instances are coded in Python, and they use the same construction of the secure PRF.

D. Comparison With the Heuristic KS²E Instance

Fig. 8 shows the updating time overhead of Branch and the heuristic KS²E on each dataset. They roughly share a constant time overhead for updating a search index in a database, while they have updating overheads of about 22.5 μ s and 15.3 μ s per entry, respectively. Branch does spend more time in constructing the bi-directional index structure among searchable ciphertexts. In the updating stage, we see that Branch incurs more cost on calling the hash function than the heuristic KS²E.

Tab. IV shows the storage overhead of Branch and the heuristic KS²E. Branch consumes more memory to store the internal state for search and sharing than the heuristic KS²E. Besides, the EDB storage cost by Branch is about 4.3 times that of the heuristic KS²E. We note that Branch requires more storage cost to maintain the bi-directional index structure.

Fig. 9 shows the entire sharing time cost. The costs of Branch on Enron, Lucene, and Wikipedia are 614.87s, 563.81s, and 1.73×10^4 s, respectively. Branch's cost increases by 28.0 times between Enron and Wikipedia datasets and 30.7 times between Lucene and Wikipedia datasets. The overheads of the heuristic are at least nine *times* larger. We see that the bi-directional index accelerates the retrieval in the data-collection slice of MongoDB and speeds up the index sharing.

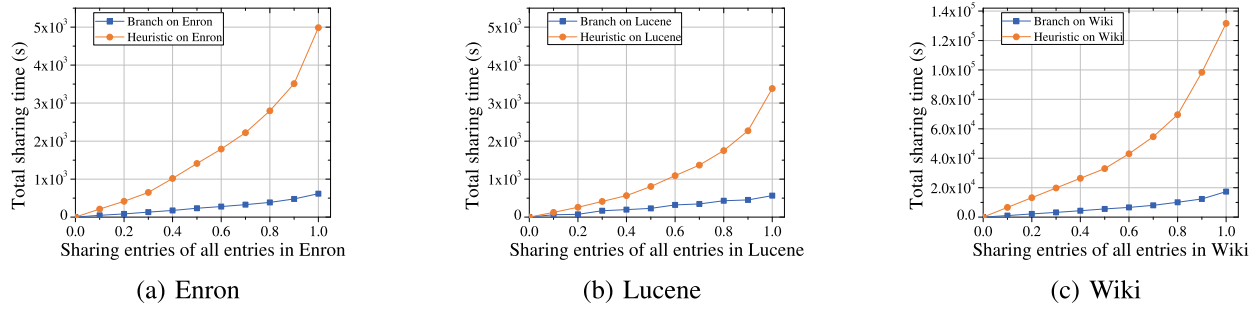


Fig. 9. Comparison of sharing time overheads between Branch and the heuristic KS^2E instance in LAN.

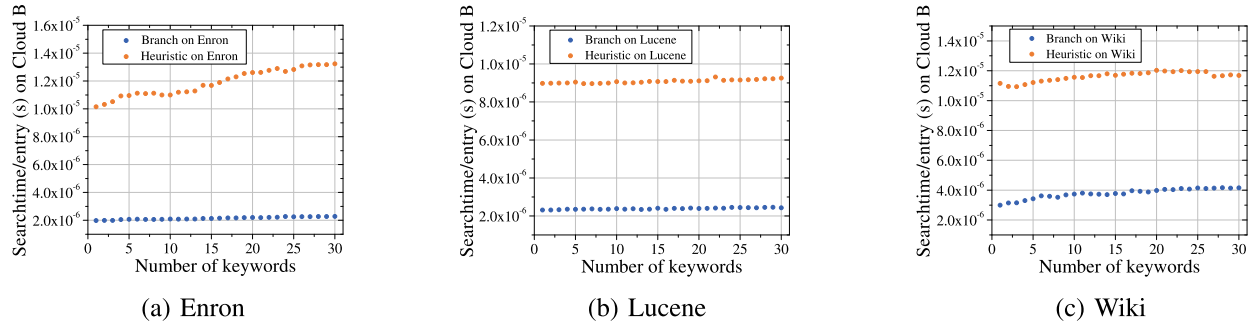


Fig. 10. Comparison of search performance between Branch and the heuristic KS^2E instance in LAN.

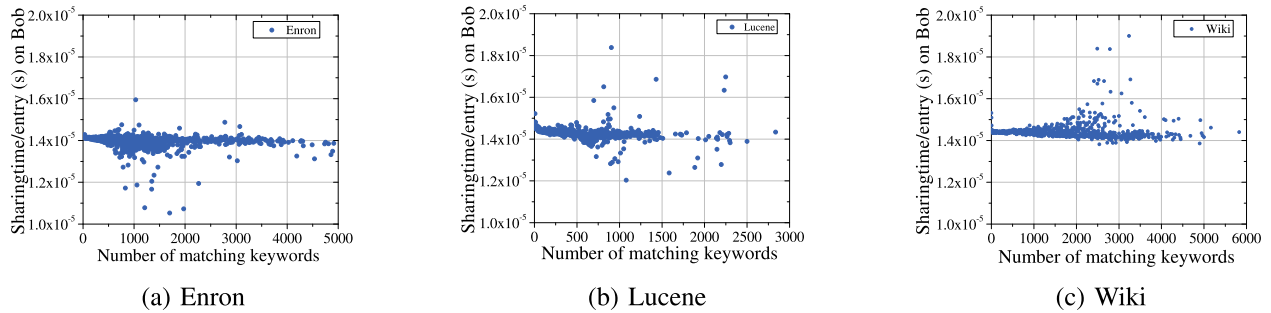


Fig. 11. Sharing performance on User in LAN.

Fig. 10 shows the search time overhead of Branch and the heuristic KS^2E on each dataset. From these search times of Branch, we observe that the search time overhead of each index is gradually increasing with the gradual reduction of keyword frequency. The frequency of the 30th keyword decreases by 0.37, 0.38, and 0.45 times that of the first keyword. At the same time, the keyword frequency significantly affects the search time on each dataset. The search time overheads of the 1st frequency keyword in Branch is separately 1.98×10^{-6} s, 2.31×10^{-6} s, 2.99×10^{-6} s, and that of the 30th frequency keyword is separately 2.27×10^{-6} s, 2.45×10^{-6} s, 4.15×10^{-6} s. The search performance of Branch is nearly 4.3 times faster than that of the heuristic KS^2E in our test.

E. The Sharing Performance of Branch

Fig. 11 shows the sharing time overhead of User on each dataset in detail. These graphs show the average cost of sharing an index according to the number of matches from one Share. From Fig. 11a, 11b, 11c, we observe that the sharing time presents a constant level. The average overhead of Fig. 11a, 11b, 11c, is separately 1.40×10^{-5} s, 1.43×10^{-5} s, 1.44×10^{-5} s, and the variance is separately 9.86×10^{-14} , 2.30×10^{-13} , 2.39×10^{-13} . Therefore, we confirm that the sharing

time overhead on User is linear with the matching number of keywords in one Share.

Fig. 12 shows the sharing time overhead of Cloud B on each dataset in detail. Similarly, these graphs show the average cost of sharing an index according to the number of matches from one Share. We observe that each entry's average sharing time overhead decreases significantly with the increment of matching keywords. Fig. 12a, 12b, 12c show that the sharing time overhead drops to 3.44×10^{-6} , 1.37×10^{-6} , and 3.05×10^{-6} , respectively. Besides, the time overhead tends to be nearly constant on all datasets when the number of matching keywords exceeds 1,000. Therefore, we confirm that the sharing time overhead on Cloud B is sub-linear with the matching number of keywords in one Share.

F. Comparison With the Instances of MUSE and MKSE

Fig. 13 shows the comparison results of KS^2E , MUSE, and MKSE instances in the setting of multiple Owners under LAN. We see that KS^2E is comparable to MKSE and MUSE on the time overheads of database generation and sharing. By Fig. 13a, we confirm that the database generation overheads are constant and they are not significantly affected by the number of Owners. In Fig. 13a, the overheads of Branch,

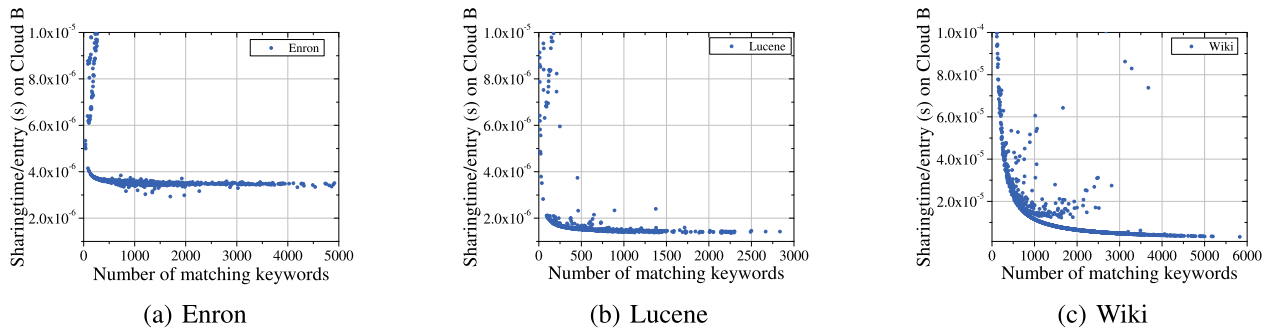
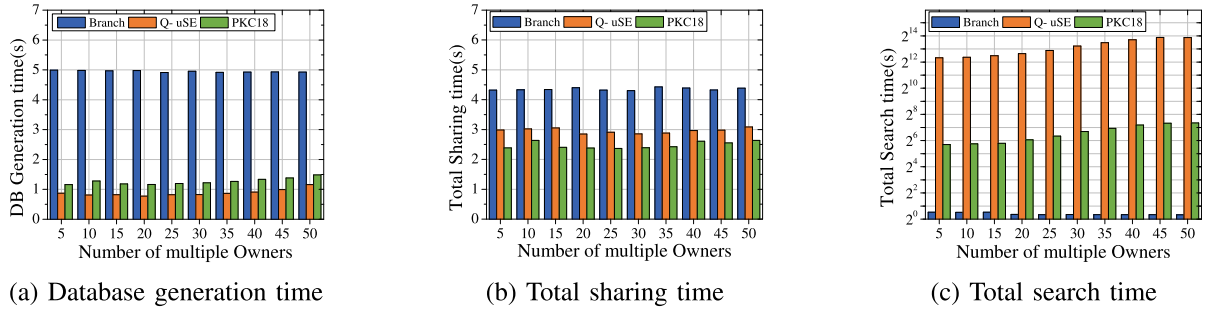
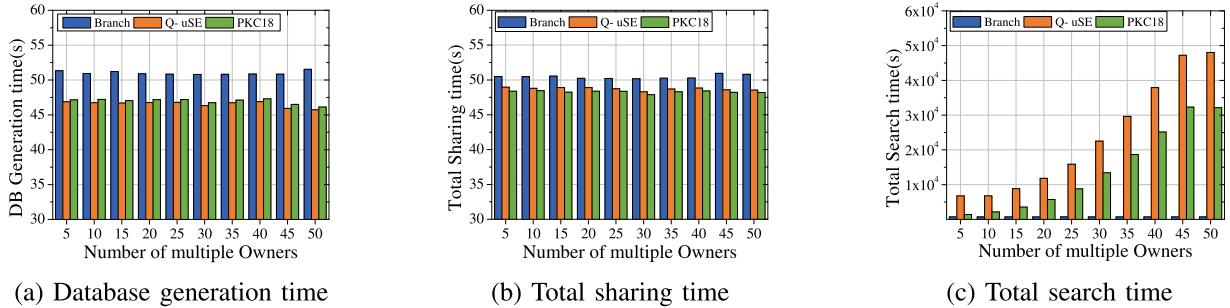


Fig. 12. Sharing performance on Cloud B in LAN.

Fig. 13. Comparison of performance between the instances of KS^2E , MUSE, and MKSE in LAN.Fig. 14. Comparison of performance between the instances of KS^2E , MUSE, and MKSE in WAN (average 23ms delay).

$Q-\mu SE$, and PKC18 generating the databases (with 2,000 files) are about 4.96s, 0.82s, and 1.14s, respectively.

From Fig. 13b, we see that the total overheads of KS^2E , MUSE, and MKSE instances are also constant which is not restricted by the number of Owners. In Fig. 13b, the sharing time overheads of Branch, $Q-\mu SE$, and PKC18 are about 4.33s, 2.95s, and 2.51s, respectively.

It should be noted that Fig. 13c shows our advantages over MUSE and MKSE in terms of search time overheads. Fig. 13c presents the total search overheads for searching all keywords from the database (with 2,000 files). The cost of Branch is much lower than that of $Q-\mu SE$ and PKC18. Fig. 13c illustrates that the increasing number of Owners does harm the performance of MKSE and MUSE. We say that the KS^2E instance benefits from the bi-directional index to search the shared files efficiently.

Fig. 14 shows the comparison results for the KS^2E , MUSE, and MKSE instances in the context of multiple Owners under WAN with an average delay of 23ms. The instances also incur constant costs here. But it is clear that the gaps among them in Fig. 14a and 14b are smaller than those in Fig. 13. In Fig. 14c, the overhead of Branch is nearly 730.46s, while the costs of $Q-\mu SE$ and PKC18 increase from 6,757.23s and 1,386.12s to

48,030.73s and 32,152.72s, respectively. We see that Branch still brings efficiency in the search.

VI. CONCLUSION

In this paper, we propose the concept of KS^2E . By analyzing previous Searchable Encryption with sharing problems, we conclude that the forward and backward privacy and bi-directional index structure are the keys to our challenge. We propose the forward and backward privacy definitions of KS^2E and construct the instance Branch. From our security analysis and evaluation, we confirm that Branch can efficiently search and share search indexes while providing forward and backward privacy.

Although Branch provides all functions under the defined security model, the deletion operation still requires multiple instances consuming extra local users' computational costs in the sharing stage. We will aim to construct non-interactive KS^2E constructions with more efficient deletion.

APPENDIX A

Theorem 1 (Adaptive Security of KS^2E -Branch): Let PRF F with a specific key be modeled as the random oracle \mathcal{H}_F ,

Game 3 G_3 Description

Setup (λ) :

- 1: Generate empty maps $\text{EDB}_\sigma, \text{EDB}_{\sigma'}$ as encrypted database
- 2: Generate two timestamps $u \leftarrow 0, u' \leftarrow 0$

Update ($\sigma, (w, id, add)$) ; EDB_σ :**Owner**:

- 1: $u \leftarrow u + 1$
- 2: $(L, I_w, R_w, C_w, I_{id}, R_{id}, C_{id}) \xleftarrow{\$} \{0, 1\}^\lambda \times \{0, 1\}^{2\lambda} \times \{0, 1\}^\lambda \times \mathcal{C}_w \times \{0, 1\}^{2\lambda} \times \{0, 1\}^\lambda \times \mathcal{C}_{id}$
- 3: $J_w \xleftarrow{\$} \{0, 1\}^\lambda, J_{id} \xleftarrow{\$} \{0, 1\}^\lambda$
- 4: If $\text{Kid}[id]$ is \perp , then $k_{id} \xleftarrow{\$} \{0, 1\}^\lambda$ and set $\text{Kid}[id] \leftarrow k_{id}$, else $k_{id} \leftarrow \text{Kid}[id]$
- 5: Set $\text{Dec}[k_{id}, C_{id}] \leftarrow w$
- 6: Program \mathcal{H}_F^1 s.t. $\mathcal{H}_F^1(w||id) = L$
- 7: Program \mathcal{H}_F^2 s.t. $\mathcal{H}_F^2(w||id) = J_w, \mathcal{H}_F^2(id||w) = J_{id}$
- 8: Insert $(u, J_{id}, C_{w,id})$ to $\text{UptIDHist}[id]$
- 9: $C_{w,id} \leftarrow (L, I_w, R_w, C_w, I_{id}, R_{id}, C_{id})$
- 10: **Send** $C_{w,id}$

The view of Server:

- 1: Parse $C_{w,id}$ as $(L, I_w, R_w, C_w, I_{id}, R_{id}, C_{id})$
- 2: $\text{EDB}_\sigma[L] \leftarrow (I_w, R_w, C_w, I_{id}, R_{id}, C_{id})$

Search (σ', w) :**User**:

- 1: Read $((u^1, J_w^1, C_{w,id}^1), \dots, (u^j, J_w^j, C_{w,id}^j))$ from $\text{UpWHist}'[w]$
- 2: **for** $i = j$ to 1 **do**
- 3: Parse $C_{w,id}^i$ as (L, I_w^i, R_w^i, C_w^i)
- 4: **if** $\mathcal{H}(J_w^i, R_w^i) \neq \perp \wedge \mathcal{H}(J_w^i, R_w^i) \neq I_w^i \oplus (L^{i-1}||J_w^{i-1})$ **then**
- 5: set Break
- 6: **else** Program \mathcal{H} s.t. $\mathcal{H}(J_w^i, R_w^i) = I_w^i \oplus (L^{i-1}||J_w^{i-1})$
- 7: **end if**
- 8: **end for**
- 9: Send search token $T_w \leftarrow (L^j, J_w^j)$ to Server

The view of Server:

- 1: $S \leftarrow \emptyset$
- 2: **while** L is not \perp **do**
- 3: Choose (I_w, R_w, C_w) from $\text{EDB}_{\sigma'}$ [L]
- 4: $(L' || J_w') \leftarrow \mathcal{H}(J_w, R_w) \oplus I_w$
- 5: $S \leftarrow S \cup C_w$
- 6: Update $L \leftarrow L', J_w \leftarrow J_w'$
- 7: **end while**
- 8: Return S

Sharetoken (σ, id) :**Owner**:

- 1: $((u^1, J_{id}^1, C_{w,id}^1), \dots, (u^j, J_{id}^j, C_{w,id}^j)) \leftarrow \text{UptIDHist}[id]$

- 2: Insert u to $\text{TimeST}[id]$

for $i = j$ to 1 **do**

- 4: Parse $C_{w,id}^i$ as $(L^i, I_w^i, R_w^i, C_w^i, I_{id}^i, R_{id}^i, C_{id}^i)$

- 5: **if** $\mathcal{H}(J_{id}^i, R_{id}^i) \neq \perp \wedge \mathcal{H}(J_{id}^i, R_{id}^i) \neq I_{id}^i \oplus (L^{i-1}||J_{id}^{i-1})$ **then**

- 6: set Break

- 7: **else** Program \mathcal{H} s.t. $\mathcal{H}(J_{id}^i, R_{id}^i) = I_{id}^i \oplus (L^{i-1}||J_{id}^{i-1})$

end if**end for**

- 10: Read $k_{id} \leftarrow \text{Kid}[id]$

- 11: **Return** $(L^j, J_{id}^j, id, k_{id})$

Share (P_{id}, σ ; $\text{EDB}_\sigma, \text{EDB}_{\sigma'}$) :**User**:

- 1: Parse $P_{id} = (L, J_{id}, id, k_{id})$
- 2: Send $D_{id} \leftarrow (L, J_{id})$

The view of Server:

- 1: $S \leftarrow \emptyset$
- 2: **while** L is not \perp **do**
- 3: Choose (I_{id}, R_{id}, C_{id}) from EDB_σ [L]
- 4: $(L' || J_{id}') \leftarrow \mathcal{H}(J_{id}, R_{id}) \oplus I_{id}$
- 5: $S \leftarrow S \cup C_{id}$
- 6: Update $L \leftarrow L', J_{id} \leftarrow J_{id}'$
- 7: **end while**
- 8: Return S

User:

- 1: $S' \leftarrow \emptyset$
- 2: **for** each symmetric ciphertext C_{id} in S **do**
- 3: Read $w \leftarrow \text{Dec}[k_{id}, C_{id}]$
- 4: $u' \leftarrow \max[\text{TimeST}[id]]$
- 5: $(L, I_w, R_w, C_w) \xleftarrow{\$} \{0, 1\}^\lambda \times \{0, 1\}^{2\lambda} \times \{0, 1\}^\lambda \times \mathcal{C}_w$
- 6: $J_w \xleftarrow{\$} \{0, 1\}^\lambda$
- 7: Program \mathcal{H}_F^1 s.t. $\mathcal{H}_F^1(w||id) = L$
- 8: Program \mathcal{H}_F^2 s.t. $\mathcal{H}_F^2(w||id) = J_w$
- 9: $C_{w,id} \leftarrow (L, I_w, R_w, C_w)$, and insert $C_{w,id}$ to S'
- 10: Insert $(u', J_w, C_{w,id})$ to $\text{UpWHist}'[w]$
- 11: **end for**
- 12: Send S'

The view of Server:

- 1: **for** each keyword ciphertext $C_{w,id}$ in S' **do**
 - 2: parse $C_{w,id}$ as (L, I_w, R_w, C_w)
 - 3: $\text{EDB}_{\sigma'}[L] \leftarrow (I_w, R_w, C_w)$
 - 4: **end for**
-

and let cryptographic hash function H be modeled as the random oracle \mathcal{H} . We define $\mathcal{L}_B = (\mathcal{L}_B^{\text{Upt}}, \mathcal{L}_B^{\text{Shr}}, \mathcal{L}_B^{\text{Srch}})$ as:

$$\begin{aligned} \mathcal{L}_B^{\text{Upt}}(w, id, op) &= \emptyset \\ \mathcal{L}_B^{\text{Shr}}(id) &= rp(id), \text{UpSHist}(id) \\ \mathcal{L}_B^{\text{Srch}}(w) &= sp(w), \text{UpHist}(w) \end{aligned}$$

Branch is \mathcal{L}_B -adaptively-secure.

Proof: We analyze the indistinguishability between the instance Branch and simulator $\mathcal{S}_{\text{Branch}}$, and we use the game hop method to analyze indistinguishability.

Game G_0 : This game is identical with the real scheme Branch so that:

$$\mathbb{P} \left[\text{Real}_{\mathcal{A}, \mathcal{S}, \mathcal{L}}^{\text{KS}^2\text{E-Branch}}(\lambda) = 1 \right] = \mathbb{P}[G_0 = 1]$$

Game G_1 : In this game, we replace the PRF F used in G_0 with a random oracle \mathcal{H}_F . The adversarial distinguishing advantage between G_0 and G_1 is exactly the distinguishability between PRF F and \mathcal{H}_F . The adversary constructs a reduction B_1 making at most call on PRF F with a specific key, then he can decide whether to call F or get a random string from \mathcal{H}_F . In this condition, we have:

$$\mathbb{P}[G_0 = 1] - \mathbb{P}[G_1 = 1] \leq \text{Adv}_{F, B_1}^{\text{prf}}(\lambda)$$

Game G_2 : In this game, we replace the symmetric ciphertexts C_w, C_{id} with a random string from the space $\mathcal{C}_w, \mathcal{C}_{id}$. We can see that the distinguishability between game G_1 and game G_2 is from the symmetric ciphertexts C_w, C_{id} . The adversary constructs a reduction B_2 making at most call on CPA secure symmetric encryption (SE-CPA), then he can decide whether to call random string or symmetric ciphertexts. In this condition, we have:

$$\mathbb{P}[G_1 = 1] - \mathbb{P}[G_2 = 1] \leq \text{Adv}_{\mathcal{E}, B_2}^{\text{SE-CPA}}(\lambda)$$

Game G_3 : In this game, we replace I_w, I_{id} with random string, then we use random oracle \mathcal{H} to replace the cryptographic hash function H , which is programmed to hold the correctness of search and sharing. In this game, table $\text{UptIDHist}[id]$ records the history of the id when updating the search index. Table $\text{UpWHist}'[w]$ records the history of the w when re-encrypting search indexes. From this game, we can see that the distinguishability between game G_2 and game G_3 is from the cryptographic hash function value I_w, I_{id} . To break the program on oracle \mathcal{H} , the adversary constructs a reduction B_3 from a distinguisher \mathcal{A} which inserts $N(w, id)$ pairs in encrypted database. B_3 firstly guesses which (w^*, id^*) pair will be set to **Break** for the first time among the N pairs.

For (w^*, id^*) , reduction B_3 will call its oracle $\mathcal{H}_F^{2^*}$ to evaluate the same J_w^* or J_{id}^* at q times and try to make a **Break** on \mathcal{H} . If we assume the random R_w, R_{id} are uniformly random, the probability of \mathcal{H} called on (J_w^*, R_w) and (J_{id}^*, R_{id}) is $2 \cdot q \cdot 2^{-\lambda}$ so that:

$$Pr[\text{set Break}] \leq Adv_{F, B_3}^{prf}(\lambda) + \frac{2q}{2^\lambda}$$

As guessing the pair (w^*, id^*) has N loss in the advantage of the reduction B_3 , we have:

$$\mathbb{P}[G_2 = 1] - \mathbb{P}[G_3 = 1] \leq N \cdot Adv_{F, B_3}^{prf}(\lambda) + \frac{2Nq}{2^\lambda}$$

The Simulator: We slightly replace the w, id used in G_3 with the *min* pattern $sp(w)$ and $rp(id)$ to hold **Search** and **Share**, and we can observe that $\text{UpIDHist}[id]$ leaks the $\text{UpSHist}(id)$ to recover the previous search index by D_{id} . Besides, the $\text{UpWHist}'[w]$ reveals the $\max[\text{TimeST}[id]]$ and historical entries left in the current database. When search query finds a record based on keyword w , the last sharing timestamp of id is also revealed. In this condition, we can note $\text{UpWHist}'[w]$ as $\text{UpHist}(w)$, and we have:

$$\mathbb{P}[G_3 = 1] - \mathbb{P}[\text{Ideal}_{\mathcal{A}, \mathcal{S}, \mathcal{L}}^{\text{KS}^2\text{E-Branch}}(\lambda) = 1] = 0$$

Conclusion: By combining all contributions from all games, there exists the adversary such that:

$$|\mathbb{P}[\text{Real}_{\mathcal{A}}^{\text{KS}^2\text{E-Branch}}(\lambda) = 1] - \mathbb{P}[\text{Ideal}_{\mathcal{A}, \mathcal{S}, \mathcal{L}}^{\text{KS}^2\text{E-Branch}}(\lambda) = 1]| \leq Adv_{F, B_1}^{prf}(\lambda) + Adv_{\mathcal{E}, B_2}^{\text{SE-CPA}}(\lambda) + N \cdot Adv_{F, B_3}^{prf}(\lambda) + \frac{2Nq}{2^\lambda}$$

□

APPENDIX B

Claim 1 (The correctness of KS²E-Branch): Based on the birthday problem, the Branch instance can make the collision resistance by making the $1 - e^{-\frac{\mu^2}{2\lambda+1}}$ negligible, where μ represents the maximum number of inputs to the PRF F .

Proof: Based on the birthday problem, we see that the probability of all μ inputs occur collision after hashing is:

$$\mathbb{P}_{\text{Coll}} = 1 - 1 \cdot \left(1 - \frac{1}{2^\lambda}\right) \dots \left(1 - \frac{\mu-1}{2^\lambda}\right)$$

Base on the Taylor expansion for e^x , we have the following equation when x is around 0:

$$e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \dots \approx 1 + x$$

Following this Taylor expansion, we can approximately obtain:

$$\mathbb{P}_{\text{Coll}} \approx 1 - e^{-\frac{1}{2^\lambda}} \cdot e^{-\frac{2}{2^\lambda}} \dots e^{-\frac{\mu-1}{2^\lambda}} = 1 - e^{-\frac{\mu(\mu-1)}{2 \cdot 2^\lambda}}$$

For simplicity, we can approximately obtain the collision probability when μ is small relative to 2^λ :

$$\mathbb{P}_{\text{Coll}} = 1 - e^{-\frac{\mu^2}{2\lambda+1}}$$

□

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