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DOI
10.3850/978-981-18-8071-1_P171-cd

Publication date
2023

Document Version
Final published version

Published in
Proceedings of the The 33rd European Safety and Reliability Conference (ESREL 2023)

Citation (APA)
Torres Alves, G. A., Morales Napoles, O., & Mares-Nasarre, P. (2023). Reliability Analysis for Overturning and Sliding of Lacustrine Dikes: The Nezahualcoyotl's Dike Case. In M. P. Brito, T. Aven, P. Baraldi, M. Čepin, & E. Zio (Eds.), *Proceedings of the The 33rd European Safety and Reliability Conference (ESREL 2023)* (pp. 1376-1383). Article P171 Research Publishing. https://doi.org/10.3850/978-981-18-8071-1_P171-cd

Proceedings of the 33rd European Safety and Reliability Conference (ESREL 2023)
Edited by Mário P. Brito, Terje Aven, Piero Baraldi, Marko Čepin and Enrico Zio

Published by Research Publishing, Singapore.
doi: 10.3850/978-981-18-8071-1_P171-cd

Reliability analysis for overturning and sliding of lacustrine dikes: The Nezahualcoyotl's dike case

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Before the year 1519, the Valley of Mexico was a closed basin and at the bottom of the valley, an extensive system of shallow lakes was formed. Within this lacustrine system, the capital of the Aztec empire, Tenochtitlan, was built. The Aztecs were known for their impressive constructions and complex hydraulic structures, of which the most impressive structure was the Nezahualcoyotl dike. This structure was constructed across Lake Texcoco. Its principal function was to protect the city of Tenochtitlan from high water levels at the lake. However, there is not enough information about the reliability of this dike. Mainly due to two reasons, today there are no remains left of the dike and most of the lacustrine system is drained. In this paper, we present a method to study the reliability of the Nezahualcoyotl dike under two failure modes, overturning and sliding. This is done by following up on the work presented by Torres-Alves & Morales-Nápoles (2020) where they developed a hydrological characterization of the lacustrine system and studied the dike under one failure mode, overflow. The proposed analysis aims to provide a more realistic assessment of the reliability of the dike as a flood defense mechanism.

Keywords: Nezahualcoyotl, Texcoco, Dike, reliability, overturning, sliding.

1. Introduction

Mexico-Tenochtitlan was the capital city-state of the Aztec Empire, one of the most powerful and sophisticated civilizations in ancient Mesoamerica. The city was founded approximately 200 years before the arrival of the Spaniards on an island in the middle of Lake Texcoco (present-day Mexico City) (Smith, 2013). This lake was part of a system of lakes in the Valley of Mexico (Fig. 1).

Tenochtitlan was the cultural capital of the empire and it had important social, technological, and scientific developments. At the time of the arrival of the Spaniards in 1519, Tenochtitlan was one of the most populous cities in the world, the largest to ever flourish in the pre-Hispanic New World, and significantly wealthier and more grandiose than any community the Spanish soldiers have ever beheld in their home country (Smith, 2013). The city was a marvel of engineering and urban planning, with aqueducts and walkways-dikes that connected the city to the mainland. It had a complex system of hydraulic structures and floating gardens (chinampas) that allowed it to support a population of over 200.000 people (Smith, 2013).

Tenochtitlan was prone to flooding during the wet season. The intricate network of hydraulic structures served a variety of functions, including flood control, lake level regulation, catchment drainage control, irrigation, navigation, transportation, water supply, etc. The Nezahualcoyotl dike, however, was the most remarkable hydraulic structure at the time. This ancient structure, constructed around 1450, split Lake Texcoco in half from north to south (forming Lake Mexico on its west side). Additionally, the dike prevented saline water from Lake Texcoco from entering

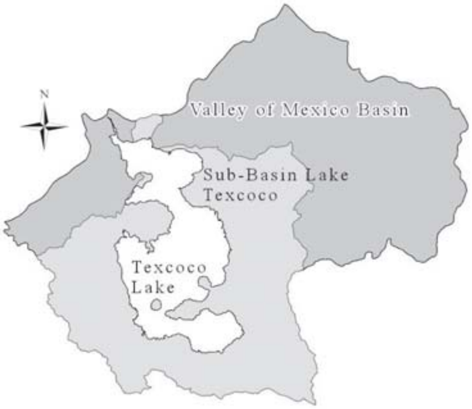

Fig. 1. Area of study.

Lake Mexico, which was where Tenochtitlan was located.

Scholars from several disciplines (art, religion, anthropology, among others) have been captivated by the history of Tenochtitlan. From a risk and reliability perspective, there is a lack of knowledge regarding the past of this city and the Valley of Mexico. Current attempts to investigate Tenochtitlan and its hydraulic features from an engineering standpoint include the research presented by Torres-Alves and Morales-Nápoles (2020), Pouliasis et al. (2021), and Mendoza-Lugo et al. (2023). These studies look into the probability that the Nezahualcoyotl dike would fail due to overflow (P_O). For example, Torres-Alves and Morales-Nápoles (2020) proposed a copula-based model to recreate the hydrological conditions at the basin in 1519 using current data along with a simplified water balance equation to estimate the water level at Lake Texcoco and then calculate the probability of overflow. Following up on that work, Pouliasis et al. (2021) developed a C-vine copula-based model to improve the simulation of the hydrological variables during the wet season. Finally, Mendoza-Lugo et al. (2023) included the hydrological conditions in the Valley during the dry season and provided an updated water balance equation that takes surface runoff and subsurface seepage losses into account. Using the data set generated by the model developed by Mendoza-Lugo et al. (2023) as input, we propose an extended and simplified methodology to test the Nezahualcoyotl dike on two other failure modes, overturning and sliding.

This paper is organized as follows, Section 2 starts by describing the methodology to estimate the probability of failure of the Nezahualcoyotl dike for overturning (P_{OV}) and sliding (P_S). Next, the case study and data of interest are presented in Section 3. Finally, the results, conclusions, and recommendations for future work are presented in Section 4 and Section 5, respectively.

2. Modeling approach

This section describes the approach used to estimate the probability of failure of the Netzahualcoyotl dike due to overturning and sliding. This approach is divided into four sections: i) characterization of the hydrology of the region, ii) probabilistic modeling of such hydrology, iii) characterization of the geometry and materials, and iv) definition of the limit state functions to estimate the probability of failure for overturning and sliding.

Data sets of precipitation, evaporation, and water levels at Lake Texcoco result from the statistical modeling presented in Torres-Alves and Morales-Nápoles (2020) and Mendoza-Lugo et al. (2023). Although this process is not the primary goal of this paper, it is briefly described in the following paragraphs. In order to define the building materials of the dike, historical sources were consulted. Finally, the probability that the structure may overturn (P_{OV}) or slide (P_S) is computed.

2.1. *Hydrology of the region*

To describe the hydrology in the region, Torres-Alves and Morales-Nápoles (2020) developed a model that characterizes the precipitation, and evaporation in the Valley of Mexico and the water levels at Lake Texcoco. This approach consists of three parts. First, one-dimensional probability distributions are fitted to the hydrological variables. Then, a discrete Markov chain model (MC) is used to generate a synthetic series of wet and dry days (days with and without precipitation respectively). Next, bi-variate copula models are used to simulate the amount of precipitation and evaporation.

This resulted in a synthetic time series of evaporation and precipitation. The time series was used as input into a water balance equation to estimate the water levels at Lake Texcoco. This process was performed only during the wet season.

Later, Mendoza-Lugo et al. (2023) extended this model by i) including the simulation of the hydrological variables during the dry season, and ii) updating the water balance equation by adding more water inflow and outflow sources. In this paper, the resulting synthetic time series of water levels from Mendoza-Lugo et al. (2023) are used as input to estimate the dike's failure probability for overturning and sliding. As mentioned, a brief description of those simulations is presented in the following paragraphs.

2.2. *Probabilistic preliminaries*

2.2.1. *Markov Chains*

Markov chains (MC) are models that describe a sequence of events or states where the probability of transitioning from one state to another depends only on the current state and is independent of past states. In other words, this is a "memoryless" process.

An MC is defined by a set of states (S) and a transition matrix (**P**). This matrix defines the probabilities of moving from one state to another. In Gabriel and Neumann (1962), Markov Chains are used in the context of characterizing the occurrence ($S = 1$) or absence ($S = 0$) of precipitation. Let X describe a random variable (precipitation), the respective transition probability matrix is presented in Eq. 1.

$$\mathbf{P} = \begin{bmatrix} P(X_{t+1}=0|X_t=0) & P(X_{t+1}=0|X_t=1) \\ P(X_{t+1}=1|X_t=0) & P(X_{t+1}=1|X_t=1) \end{bmatrix} \tag{1}$$

2.2.2. *Copulas*

A bi-variate copula is a two-variable joint distribution with uniform margins [0,1]. Any multivariate joint distribution can be expressed in terms of the univariate marginal distribution functions and a copula that describes the dependence between random variables (Sklar, 1959). For a complete overview of copulas, the reader is referred to Joe (1997). In the case of two variables:

$$H\left(X_{t+1}, X_t\right) = C_{\theta_X}\{F(x_{t+1})|F(x_t)\} \tag{2}$$

where $C_{\theta_X}(F(x_{t+1})|F(x_t))$ is the conditional copula. The parameter θ_X models the autocorrelation of order 1 for the time series of precipitation. This model is used to estimate the precipitation and evaporation values on a given day.

2.2.3. *Water Levels*

Based on a water balance equation first given in Torres-Alves and Morales-Nápoles (2020) and later modified and expanded by Mendoza-Lugo et al. (2023), the synthetic time series of water levels in Lake Texcoco is estimated. Water inflow sources are precipitation (X), streamflow into the water body (Q), surface runoff (Q_r), and subsurface runoff (Q_s). Evaporation (Y), streamflow discharge from the water body (Q_o), and subsurface permeability losses (Q_d) are sources of outflow. The time rate of change of the volume in a water body (dV/dt) is described as

$$dV/dt = X + Q + Q_r + Q_s - Y - Q_o - Q_d \tag{3}$$

Volume-Elevation and Volume-Area curves were used for the calculation of the water elevation values (E_t) at Lake Texcoco. These values were computed at the time step t from their respective dV/dt value. Torres-Alves and Morales-Nápoles (2020) and Mendoza-Lugo et al. (2023) provide a detailed description of the assumptions made regarding the inflows and outflows of the water balance equation.

2.3. *Characterization of materials*

Nowadays, there are no remains of the Nezahualcoyotl dike and the lacustrine system. As a result, it is difficult to describe the Nezahualcoyotl dike's geometry and building materials. To learn about this structure, a combination of historical sources and current data was used. Torres-Alves and Morales-Nápoles (2020) compiled and contrasted numerous others sources, including Palerm (1973); Lombardo de Ruiz (1973); Legorreta

(2006); Hernández Cruz (2013) to determine the dimensions of the Nezahualcoyotl dike (Section 3.1).

In sources gathered by Lombardo de Ruiz (1973), the Nezahualcoyotl dike is described as a "fence made up of wood and stones". Other sources in Lombardo de Ruiz (1973) state that the dike was a masonry wall, with stone and mud foundations within a stockade of thick wooden piles (Alvarez, 1919). For this study, we characterize the construction materials as wood, stone, and mud. Details about these materials are presented in Section 3.2

2.4. *Limit state functions*

To evaluate the probability of failure of the Nezahualcoyotl dike under sliding and overturning, their respective limit state functions (Z) are to be defined. A limit state function indicates the state of success or failure of a system. Failure is defined as when the structure or part of it fails to meet one of its criteria requirements.

For sliding (S) and overturning (OV), their respective limit state functions (Z_S and Z_{OV}) are evaluated in terms of the vertical and horizontal forces (F_V, F_H), and the stabilizing and destabilizing moments (M_{ST}, M_{DEST}) in the structure and the surrounding bodies of water (Eq. 4 and 5).

$$Z_S = \frac{\sum F_V}{\sum F_H} \tag{4}$$

$$Z_{OV} = \frac{\sum M_{ST}}{\sum M_{DEST}} \tag{5}$$

Consider the dike sitting on top of a rough surface, as shown in Figure 2. The water on both sides of the dike will exert horizontal forces on the dike. When the forces from the surrounding water ($\sum F_H$) exceeds the maximum force of static friction (F_f), the dike will slide across the surface (sliding). The normal and gravitational force (F_g and F_N) exert a moment (M_{EST}), this moment is counteracted by the moment exerted by the horizontal forces and the force of static friction (M_{DEST}). If the stabilizing moment is not able to counteract the destabilizing moment, the dike will overturn. Therefore, failure due to sliding and overturning is defined by $P_S = P(Z_S < 1)$ and $P_{OV} = P(Z_{OV} < 1)$.

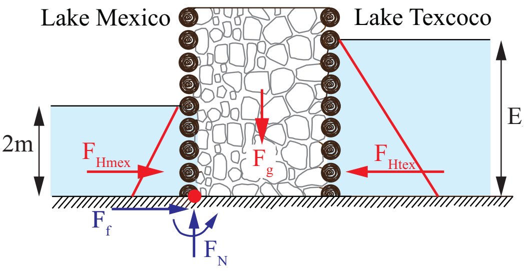

Fig. 2. Forces acting on the Nezahualcoyotl dike.

In summary, the probability that the dike would fail for sliding and overturning is evaluated as follows:

1. A synthetic time series of water levels in Lake Texcoco (E_t) is used to characterize Lake Texcoco during dry and wet seasons.
2. The vertical and horizontal forces as well as the stabilizing and destabilizing moments are computed for all the water level values (E_t) at Lake Texcoco.
3. The limit state functions are evaluated.
4. Repeat steps 2 and 3 for the 19 different dike cases (Table 1) and compute the probability of failure under both failure modes, overturning and sliding.

3. Case study

3.1. *The Nezahualcoyotl dike*

The Valley of Mexico basin was an endorheic (closed) basin at the time the Spaniards arrived in Tenochtitlan. Lakes, lagoons, and swamps covered around 1000 km^2 of the total surface of the Valley of Mexico. Five lakes stand out: Chalco, Xochimilco, Texcoco (saline water), Xaltocan, Zumpango, and Mexico (Fig. 3).

Moctezuma (the second Aztec emperor and fifth king of Tenochtitlan) and Nezahualcoyotl (the ruler of Texcoco) collaborated on the construction of the Nezahualcoyotl dike in 1450 (Palerm, 1973). As previously mentioned, the dike divided Lake Texcoco in half from north to south

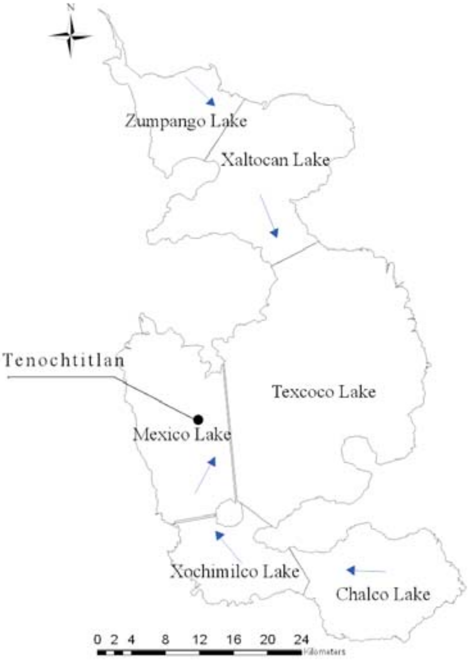

Fig. 3. Lacustrine System.

(Fig. 3), creating Lake Mexico on the west (freshwater) and Lake Texcoco on the east (saltwater). The main purpose of the Nezahualcoyotl dike was to prevent the inflow of salt water from Lake Texcoco and avoid floods in Lake Mexico, where Tenochtitlan was located. Within Lake Mexico, the water level was more or less constant at 2m approximately (Torres-Alves and Morales-Nápoles, 2020), as shown in Fig. 2. Depending on the source, the dimensions of the Nezahualcoyotl dike vary. Torres-Alves and Morales-Nápoles (2020) collected the dimensions provided by several sources (Section 2.3) to propose a final design. Therefore, it is assumed that the dike was 16 km long, 8 m high, and 3.5 m broad. Lombardo de Ruiz (1973) suggested the presence of gates in the dike that, in the event of high water levels, would let water from Lake Mexico flow into Lake Texcoco. However, the existence of these gates is neglected in this paper. Moreover, the dike was constructed on a base of pillars, in this study, the dike is assumed to be simply supported on the soil.

3.2. *Data*

In this paper, synthetic time series of daily water levels (E_t) at Lake Texcoco that spans 5,000 years (Section 2) are used. This data set is the result of the statistical model presented by Torres-Alves and Morales-Nápoles (2020) and Mendoza-Lugo et al. (2023) based on Markov Chains, bi-variate copulas, and water balance equations, as presented in Section 2.2.

To investigate the failure mechanisms associated with sliding and overturning of the Nezahualcoyotl dike, it is imperative to determine the properties of the materials used in its construction. As discussed in Section 2.3, available sources suggest that the dike was constructed from wood, stone, and mud obtained from the bottom of the lacustrine system. Due to the sinking of the soil in the lacustrine area, it has been reported that the Aztecs utilized a volcanic rock known as *tezontle* as a building material (Aguilar-Moreno, 2007). This rock possessed desirable attributes such as a high strength-to-weight ratio, ease of workability, and appealing color and texture. Its density is reported to range from 2.37 to 2.83 g/cm3. In this paper, we assume a density of 2.6 g/cm3.

Pine and oak trees were utilized to make the timber for the dike, with the former being particularly prevalent in the Valley of Mexico. Therefore, *Pinus teocote*, a kind of pine that is typically prevalent in the area, was chosen for this study. The density of this wood is reported to be 0.52 g/m3 (Ordóñez Díaz et al., 2015). Finally, the mud used to build the dike was extracted from the soil at the bottom of the lake (Palerm, 1973), which has a low bulk density of 0.28 MG m-3 (López-Ávila et al., 2004). This low density may be attributed to factors such as the composition of the lacustrine sediments or the presence of amorphous siliceous materials within the soil. The coefficient of friction of the soil is reported to be 43°(Díaz-Rodríguez et al., 2006).

The characteristics of the soil and wood presented herein, are used to compute the weight of the structure and the forces and moments to estimate the probability of failure of the dike due to sliding and overturning.

4. Results

Once the construction materials of the Nezahualcoyotl dike have been identified and characterized, the structure is tested for sliding and overturning. There are no known remnants of the dike, hence it is uncertain what precise proportion of the basic materials was employed in its construction. Thus, we propose 19 cases (Table 1), each of which displays a different ratio of rock and soil that leads to different estimates of the structure's weight. The ratio of wood is assumed as a fixed value of 15% of the structure's weight, serving as pillars to support the body of the dike. The 19 cases were then tested for sliding and overturning under the effect of the time series of water levels at Lake Texcoco.

Table 1. Cases determined by the ratio of rock and soil in the main body of the Nezahualcoyotl's dike.

Case	Rock	Soil	Weight $[kg]$
1	0	0.85	8848
2	0.02	0.83	10147
3	0.04	0.81	11446
4	0.06	0.79	12746
5	0.08	0.77	14045
6	0.1	0.75	15344
7	0.12	0.73	16643
8	0.14	0.71	17942
9	0.16	0.69	19242
10	0.18	0.67	20541
11	0.3	0.55	28336
12	0.32	0.53	29635
13	0.34	0.51	30934
14	0.36	0.49	32234
15	0.38	0.47	33533
16	0.4	0.45	34832
17	0.42	0.43	36131
18	0.44	0.41	37430
19	0.46	0.39	38730

According to the limit state functions presented in section 2.4, the dike fails due to sliding if the horizontal forces are larger than the vertical forces (Eq. 4). If the destabilizing moments cannot counteract the stabilizing moments, the dike fails due to overturning (Eq. 5). This was done for the 5,000 years of daily water level simulations at Lake Texcoco and for the 19 dike cases presented in Table 1. Moreover, it is assumed that the water level at Lake Mexico remains constant at 2m (2232 m.a.s.l). Justification for this assumption is presented in Torres-Alves and Morales-Nápoles (2020). The probability of failure for both sliding and overturning is computed by dividing the number of times the dike fails by the total number of days in the time series. The probability of failure due to sliding (P_S) and overturning (P_{OV}) together with the corresponding return periods (RP_S and RP_{OV}) are presented in Tables 2 and 3, respectively.

Table 2. Probability of failure due to sliding

Case	P_S	RP_S
1	0.377	2.64
2	0.069	14.37
3	0.003	285.82
4	0.0001	9759.35

Table 3. Probability of failure due to overturning

Case	P_{OV}	RP_{OV}
1-9	0.999	1.00
10	0.499	2.00
11	0.0590	16.92
12	0.0350	28.55
13	0.0202	49.48
14	0.0109	91.05
15	0.005	182.00
16	0.0028	351.63
17	0.0015	653.88
18	0.0007	1319.59
19	0.0003	2727.95

The results shown in Tables 2 and 3 indicate that failure probabilities are lower when the main body of the dike has a higher percentage of rock. This occurs for both sliding and overturning. For the cases where the percentage of rock in the main body of the dike is larger than 6%, very small probabilities of failure due to sliding are obtained. For this reason, only the first four cases are presented in Table 2. The probability of failure of the dike due to overturning remains relatively high for cases 1 to 9, where the proportion of rock varies between 0 and 16%. For the same dike composition, the probability of failure due to sliding is lower than to overturning, which declines quicker. Overall, a higher rock-to-soil ratio results in a heavier structure, thus both the probability of failure due to sliding and overturning are smaller resulting in a more reliable structure.

According to historical sources (SACMEX, 2012), the Nezahualcoyotl dike did not fail during its lifetime (70 years approximately). Therefore, to estimate a dike composition that could resemble the original structure, a dike case with a similar return period could be considered. Table 1 shows that Case 14, a dike with a rock ratio of 36% has a probability of failure due to overturning equal to 0.01 (Table 3), with a return period (RP_{OV}) of approximately 90 years. In other words, on average a single failure is due to occur every 90 years. This value does not refute the historical sources. However, these results are based on several assumptions about the dimensions of the dike and the construction materials' intrinsic characteristics and ratio.

5. Conclusions

This study presents an analysis of the reliability of the Nezahualcoyotl dike in the face of two failure mechanisms, sliding and overturning. Although this structure has been thoroughly examined from anthropological or historical perspectives, information about its engineering aspects is still scarce.

In this paper, materials used in constructing the Nezahualcoyotl dike are characterized, and synthetic time series of water levels during the dry and wet seasons at Lake Texcoco is used to model the structure loadings (methodology in Torres-Alves and Morales-Nápoles (2020) and expanded by Mendoza-Lugo et al. (2023)).

Given the uncertainty surrounding the ratio of the construction materials utilized in the Nezahualcoyotl dike, the structure was tested for multiple combinations (cases) of these materials. Overall, larger rock percentages are needed to ensure the safety of the dike under overturning than for sliding. The probability of failure of the dike due to sliding becomes very small for rock ratios larger than 6% (case 4). The probability of failure due to overturning is 0.01 (for case 14) when the rock percentage is 36%. This probability corresponds to a return period of approximately 90 years and is consistent with the historic accounts that the dike did not fail during its 70 years of lifetime. Future research should include the variation of water level in Lake Mexico due to irrigation and water consumption and incorporate a detailed design of the dike structure based on similar ancient structures in the region.

Acknowledgement

This research was supported by the Submerged Floating Tunnel (SFT) Team. This research project is commissioned by the Chinese engineering and construction company China Communications Construction Co., Ltd. (CCCC) and is jointly carried out by 8 institutions of universities, scientific research institutes, engineering consulting firms, and design and construction companies.

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