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Scenario-Based MPC for Real-Time Passenger-Centric Timetable Scheduling of Urban Rail Transit Networks

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Abstract: Effective timetable scheduling strategies are essential for passenger satisfaction in urban rail transit networks. Most existing passenger-centric timetable scheduling approaches generate a timetable according to deterministic passenger origin-destination (OD) demands. As passenger OD demands in urban rail transit networks generally show a high level of uncertainty, an effective timetable scheduling approach should take the uncertain passenger flows into account to generate a reliable timetable. In this paper, a scenario-based model predictive control (SMPC) approach is presented to handle uncertain passenger flows based on a passenger absorption model, where uncertainties are captured by several representative scenarios according to historical data. In each SMPC step, the optimization problem for generating the timetable can be reformulated as a mixed-integer linear programming (MILP) problem, which can be efficiently solved using current MILP solvers. A probabilistic performance level can be then determined based on the performance of SMPC under the representative scenarios. Numerical experiments based on the Beijing subway network are conducted to evaluate the efficacy of the proposed approach.

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Keywords: Urban rail transit network, Passenger-centric timetable scheduling, Uncertain passenger flows, Model predictive control, Scenario approach

1. INTRODUCTION

Urban rail transit has experienced significant development in many cities owing to its safety, high transport capacity, and eco-friendly characteristics. A reliable urban rail transit system is important for the competitiveness of the regional economy. In recent years, passenger demands for urban rail transit systems in many cities have been growing rapidly. Passenger-centric timetable scheduling explicitly includes passenger satisfaction in the timetable scheduling problem, and therefore it aims at providing high-quality service for passengers. Passenger demands are typically time-varying, and different passengers may have different origins and destinations in an urban rail transit network, which significantly increases the complexity of including passenger demands in timetable scheduling. Moreover, passenger demands generally show a high level of uncertainty, and a reliable timetable should be able to incorporate the demand uncertainty into the design. Real-time timetable scheduling has become increasingly challenging in recent years due to the rapidly growing passenger demands, expanding network scale, and the requirement for real-time application.

Real-time timetable scheduling with passenger origin-destination (OD) demands is regarded as an effective approach of enhancing passenger satisfaction. Cordone and Redaelli (2011) considered the interaction between the timetable and passenger demands through an event-activity network, and a heuristic algorithm and a branch-and-bound algorithm were applied to solve the resulting timetable scheduling problem. Based on time-varying passenger OD demands, Wang et al. (2015) presented an event-driven model for an urban rail transit network with the objective of minimizing the total train energy consumption and the total passenger travel time. Robenek et al. (2018) applied a logit model to reflect elastic passenger demands, and the ticket pricing problem was integrated into the passenger-centric timetable scheduling problem. Yin et al. (2021) described feasible travel paths of passengers in a metro network through a graph; then, a decomposition-based adaptive large-neighborhood search method was presented to minimize the crowdedness at the busiest station. The above studies imply that there is a trend to include more detailed passenger demands and to develop more efficient approaches for real-time passenger-centric timetable scheduling.

The timetable scheduling problem can be formulated as a constrained control problem. Model predictive control (MPC) is a widely acknowledged methodology for addressing multi-variable constrained control problems (Mayne et al., 2000). MPC has also been implemented in railway traffic management problems. Li et al. (2017) integrated the departure time and the occupancy of trains in a state-space model, and proposed an MPC approach to decrease the timetable deviation and the headway of a metro line. Liu et al. (2023a) proposed a simplified model to describe time-varying passenger OD demands in urban rail transit networks and developed an MILP-based MPC approach

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to optimize the timetable in real time. Cavone et al. (2022) formulated the MPC optimization problem as an MILP problem under a bi-level structure for timetable (re)scheduling in case of disruptions and disturbances. Liu et al. (2022) developed a passenger absorption model for real-time timetable scheduling with time-varying passenger OD demands, where the train departure frequency can be determined in real time under an MPC framework. However, the above studies investigate passenger demands under deterministic cases, which ignores uncertainties in the network, e.g., uncertain passenger demands and uncertain disturbances, leaving an open gap for improving the performance of timetable scheduling approaches.

Timetable scheduling under uncertainties has received much attention in recent years. The scenario approach (Calafiore and Campi, 2006; Campi et al., 2018) is a general data-driven decision-making methodology that explicitly takes uncertainties into account. The scenario approach generally describes uncertainties by a set of representative scenarios, and the solution is thus obtained by considering these representative scenarios. By using different scenarios to capture the uncertain train operation time in a metro network, Yang et al. (2016) proposed a two-stage stochastic integer programming model to reduce transfer activities and the expected travel time of passengers. Gong et al. (2021) formulated a mixed-integer nonlinear optimization problem to minimize the operational costs of a metro line where passenger distribution is represented through several scenarios. However, the above studies only consider the application of the scenario approach in timetable scheduling problems, and the theoretical performance analysis of the developed approach is still required to provide a performance indication for the scenario-based timetable scheduling approach.

In this paper, we investigate the real-time timetable scheduling problem considering uncertain time-varying passenger demands. A scenario-based MPC approach is adapted to the passenger-centric timetable scheduling problem, where uncertainties are represented by several representative scenarios based on historical data. The resulting MPC optimization problem can be transformed into an MILP problem, which can be efficiently solved using current MILP solvers. Moreover, probabilistic performance guarantees of the scenario-based MPC approach are derived based on the performance of the controller under the representative scenarios.

The remainder of the paper is organized as follows. In Section 2, the passenger absorption model used in this paper is introduced. In Section 3, a scenario-based MPC approach is developed, and the probabilistic performance guarantee of the scenario-based MPC approach is provided. Section 4 presents a case study. Section 5 summarized the paper and provides suggestions for future research.

2. MATHEMATICAL MODEL

The passenger absorption model is a macroscopic timetable scheduling model that can explicitly include time-varying passenger OD demands in urban rail transit networks (Liu et al., 2022). In the passenger absorption model, absorption refers to passengers boarding trains in each period, and a period is a certain time window where the passenger demands are constant. The model is summarized in this section, and interested readers are referred to Liu et al. (2022, 2023c) for a detailed description of the model.

The number of passengers $n_{p,d}(k)$ at platform p with destination d at the beginning of each period is:

$$\begin{aligned} n_{p,d}(k+1) = n_{p,d}(k) + \rho_{p,d}(k)T \\ + n_{p,d}^{\text{arr,tran}}(k) - n_{p,d}^{\text{absorb}}(k), \end{aligned} \tag{1}$$

where $\rho_{p,d}(k)$ refers to the passenger flow rate at platform p with destination d during period k, T is the length of a period, $n_{p,d}^{\text{arr,tran}}(k)$ denotes the number of transfer passengers reaching platform p with destination d during period k, and $n_{p,d}^{\text{absorb}}(k)$ represents the number of passengers at platform p with destination d boarding trains during period k.

Then, $\rho_{p,d}(k)$, $n_{p,d}^{\text{arr,tran}}(k)$, and $n_{p,d}^{\text{absorb}}(k)$ are computed by

$$\rho_{p,d}(k) = \lambda_{o,p,d}(k)\rho_{o,d}^{\text{station}}(k), \forall p \in P_o, \tag{2}$$

$$\begin{aligned} n_{p,d}^{\text{arr,tran}}(k) = \sum_{p' \in \text{cnp}(p)} \Big(\frac{T - \theta_{p',p}^{\text{transf}}}{T} n_{p',p,d}^{\text{transf}}(k) \\ + \frac{\theta_{p',p}^{\text{transf}}}{T} n_{p',p,d}^{\text{transf}}(k-1) \Big), \end{aligned} \tag{3}$$

$$n_{p,d}^{\text{absorb}}(k) = \alpha_{p,d}(k) n_p^{\text{absorb}}(k), \tag{4}$$

where $\rho_{o,d}^{\text{station}}(k)$ represents the arrival rate for passengers at period k with o and d as origin and destination, respectively; $\lambda_{o,p,d}(k)$ denotes the fraction of passengers at station o that choose platform p to reach destination d; P_o is the set of platforms at station o; $\text{cnp}(p)$ denotes the set of platforms at the same station as platform p, and $\theta_{q,p}^{\text{transf}}$ represents the mean transfer time from platform q to platform p; $\alpha_{p,d}(k)$ denotes the fraction of passengers absorbed by trains at platform p with destination d during period k, and $\alpha_{p,d}(k)$ can be estimated by using the historical data.

In (4), the total number of passengers $n_p^{\text{absorb}}(k)$ absorbed by trains at platform p during period k is computed by

$$n_p^{\text{absorb}}(k) = \min\left(C_p(k),\ n_p^{\text{want}}(k)\right), \tag{5}$$

$$C_p(k) = f_p(k) \cdot C_{\max} - \sum_{d \in D} \Big(n_{p,d}^{\text{board}}(k) - n_{p,d}^{\text{alight}}(k) \Big), \tag{6}$$

$$n_p^{\text{want}}(k) = n_p(k) + \rho_p(k)T + g_p(k), \tag{7}$$

with

$$\begin{aligned} n_p(k) = \sum_{d \in D} n_{p,d}(k), \quad \rho_p(k) = \sum_{d \in D} \rho_{p,d}(k), \\ g_p(k) = \sum_{d \in D} n_{p,d}^{\text{arr,tran}}(k), \end{aligned} \tag{8}$$

where $C_p(k)$ denotes the total remaining capacity of trains that visit platform p during period k; $n_p^{\text{want}}(k)$ represents the total number of passengers that want to depart from platform p during period k; $f_p(k)$ is the number of trains that visit platform p during period k, which is the decision variable of the absorption model; $C_{\max}$ is the maximum capacity of a train; D denotes the set of stations in the network; $n_{p,d}^{\text{board}}(k)$ and $n_{p,d}^{\text{alight}}(k)$ correspond to the number of passengers on board of trains and the number of passengers alighting from trains at platform p with destination d during period k, respectively.

We use γ_p to represent the mean time spent for a train from the first platform of a line to the platform p, and we define

$$\beta_p = \text{floor}\left\{\frac{\gamma_p}{T}\right\}, \quad \phi_p = \text{rem}\left\{\gamma_p, T\right\}, \tag{9}$$

with floor$\left\{\frac{\gamma_p}{T}\right\}$ being the greatest integer less than or equal to $\frac{\gamma_p}{T}$ and rem$\{\gamma_p, T\}$ the remainder obtained from dividing γ_p by T. Then, γ_p can be represented by

$$\gamma_p = \beta_p T + \phi_p, \quad 0 \leq \phi_p < T. \tag{10}$$

Then, the variable $f_p(k)$ in (6) is calculated by

$$f_p\left(k\right) = \frac{T-\phi_p}{T} f_{\text{fir}(p)}(k-\beta_p) + \frac{\phi_p}{T} f_{\text{fir}(p)}(k-\beta_p-1)\,, \tag{11}$$

where fir$\,(p)$ denotes the first platform of the line associated with platform p.

To ensure the safe operation of the trains in the urban rail transit network, $f_p\left(k\right)$ should be constrained by

$$f_p(k)\left(\tau_p^{\min} + h_p^{\min}\right) \leq T, \tag{12}$$

where $\tau_p^{\min}$ and $h_p^{\min}$ respectively denote the minimum dwell time and the minimum headway of platform p.

The number of passengers $n_{p,d}^{\text{depart}}\left(k\right)$ departing from platform p with destination d is computed by

$$n_{p,d}^{\text{depart}}\left(k\right) = n_{p,d}^{\text{board}}\left(k\right) - n_{p,d}^{\text{alight}}\left(k\right) + n_{p,d}^{\text{absorb}}\left(k\right), \tag{13}$$

Here, we have

$$n_{p,d}^{\text{board}}(k) = \frac{T - r_{\text{p}^{\text{pla}}(p)}}{T} n_{\text{p}^{\text{pla}}(p),d}^{\text{depart}}(k) + \frac{r_{\text{p}^{\text{pla}}(p)}}{T} n_{\text{p}^{\text{pla}}(p),d}^{\text{depart}}(k-1)\,, \tag{14}$$

$$n_{p,d}^{\text{alight}}\left(k\right) = \begin{cases} \sum\limits_{p' \in \text{cnp}(p)} n_{p,p',d}^{\text{transf}}\left(k\right), & \text{if } d \in D/\{\text{sta}(p)\}, \\ n_{p,d}^{\text{board}}\left(k\right), & \text{if } d = \text{sta}(p), \end{cases} \tag{15}$$

$$n_{p,p',d}^{\text{transf}}(k) = \chi_{p,p',d}\, n_{p,d}^{\text{board}}(k), \quad \forall p' \in \text{cnp}(p), \tag{16}$$

where $r_{\text{p}^{\text{pla}}(p)}$ denotes the average time of trains from the preceding platform $\text{p}^{\text{pla}}(p)$ to platform p; $n_{p,p',d}^{\text{transf}}\left(k\right)$ is the number of passengers transferring from platform p to platform p' with destination d during period k, and sta(p) denotes the station corresponding to platform p; $\chi_{p,p',d}$ represents the transfer rate for passengers from platform p to $p' \in \text{cnp}(p)$ with destination d.

3. SCENARIO-BASED MPC FOR REAL-TIME TIMETABLE SCHEDULING

We first introduce the basic MPC approach for real-time timetable scheduling. Then, a scenario-based MPC approach is introduced to deal with uncertain passenger flows. Finally, the performance analysis of the developed scenario-based MPC approach is given.

3.1 Problem Formulation in an MPC Set-Up

Based on the model introduced in Section 2, the total number of passengers $d_p(k)$ departing from platform p during period k is

$$d_p(k) = \sum_{d \in D} n_{p,d}^{\text{depart}}\left(k\right). \tag{17}$$

We consider two terms in the objective function representing the passengers' satisfaction and trains' energy consumption. The total travel time of passengers is used as the passenger-centric objective, as it significantly influences passenger satisfaction. The total travel time of passengers is estimated by

$$J^{\text{time}}(k) = \sum_{p \in P} \left(n_p(k)T + d_p(k)r_p + g_p(k)\theta_p^{\text{transf}}\right), \tag{18}$$

where $n_p(k)T$ represents the total waiting time of passengers who cannot depart from platform p during period k, $d_p(k)r_p$ denotes the total running time for passengers departing from platform p during period k to arrive at the next platform, and $g_p(k)\theta_p^{\text{transf}}$ denotes the total transfer time for passengers at platform p during period k.

The energy consumption for trains departing from the platform during period k can be represented by

$$J^{\text{cost}}(k) = \sum_{p \in P} f_p(k) E_p, \tag{19}$$

where E_p represents the average energy consumption for a train running from platform p to its succeeding platform.

Thus, for real-time timetable scheduling at control time step k_0, we have the MPC optimization problem $\mathbf{P}_{k_0}^{\text{MPC}}$:

$$\begin{aligned} &\min_{\boldsymbol{v}(k_0)} J(k_0) = \sum_{k=k_0}^{k_0+N-1} \left(J^{\text{time}}(k) + \mu J^{\text{cost}}(k)\right) + L_N(k_0), \\ &\text{subject to} \quad (1)-(8),\ (11)-(17), \end{aligned} \tag{20}$$

where N represents the prediction horizon, $\boldsymbol{v}(k_0)$ is a vector collecting all variables of the time window from time step k_0, μ is a weight balancing the objectives, and $L_N(k_0)$ is a penalty term for the passengers that can not board trains at the end of the prediction window. In this paper we set $L_N(k_0) = \sum\limits_{p \in P} n_p(k_0+N)T$, and for more details of designing $L_N(k_0)$, we refer to Liu et al. (2023b).

The MPC formulation $\mathbf{P}_{k_0}^{\text{MPC}}$ is a nonlinear nonconvex optimization problem, and the transformation methods of Williams (2013) can be applied to generate a fully equivalent mixed-integer linear programming problem:

$$\min_{\boldsymbol{v}(k_0)} J(k_0) = \sum_{k=k_0}^{k_0+N-1} \left(J^{\text{time}}(k) + \mu J^{\text{cost}}(k)\right) + L_N(k_0) \tag{21}$$

subject to

$$\boldsymbol{x}(k+1) = A_k \boldsymbol{x}(k) + B_{1,k}\boldsymbol{f}(k) + B_{2,k}\boldsymbol{\delta}(k) + B_{3,k}\boldsymbol{z}(k), \tag{22}$$

$$E_{2,k}\boldsymbol{\delta}(k) + E_{3,k}\boldsymbol{z}(k) \leq E_{1,k}\boldsymbol{f}(k) + E_{4,k}\boldsymbol{x}(k) + E_{5,k}, \tag{23}$$

$$k = k_0, \cdots, k_0+N-1,$$

where $\boldsymbol{x}(k)$ denotes the state variables, i.e., the variables associated with the number of passengers in period k; $\boldsymbol{\delta}(k)$ and $\boldsymbol{z}(k)$ respectively denote the vector of auxiliary binary variables and auxiliary continuous variables in period k; Eq. (22) describes the linear and mixed-integer linear formulations of the model explained in Section 2. Constraint (23) concatenates all the model constraints and operational constraints in a matrix form.

3.2 Scenario-Based Model Predictive Control

In practice, passenger demands typically exhibit highly uncertain characteristics. We, therefore, adopted a scenario-based approach (Karg et al., 2021) to handle uncertain passenger demands and to provide reliable service for passengers. To handle computational issues arising from the large number of scenarios, uncertainties are normally captured by several representative scenarios in the scenario-based approach:

$$\{s_1, s_2, \ldots, s_{N_{\text{total}}}\}, \tag{24}$$

where N_{total} denotes the total number of representative scenarios.

The scenario-based model predictive control (SMPC) approach develops the following optimization problem $\mathbf{P}_{k_0}^{\text{SMPC}}$ at each step:

$$\min_{\boldsymbol{v}(k_0)} \sum_{i=1}^{N_{\text{total}}} \mathbb{P}\{s_i\}\Bigg(\sum_{k=k_0}^{k_0+N-1}\left(J_i^{\text{time}}(k)+\mu J_i^{\text{cost}}(k)\right)+L_{N,i}(k_0)\Bigg) \tag{25}$$

subject to

$$\begin{aligned} \boldsymbol{x}_i(k+1) = A_{i,k}\boldsymbol{x}_i(k) + B_{1,i,k}\boldsymbol{f}(k) \\ + B_{2,i,k}\boldsymbol{\delta}_i(k) + B_{3,i,k}\boldsymbol{z}_i(k), \end{aligned} \tag{26}$$

$$\begin{aligned} E_{2,i,k}\boldsymbol{\delta}_i(k) + E_{3,i,k}\mathbf{z}_i(k) \leq \\ E_{1,i,k}\boldsymbol{f}(k) + E_{4,i,k}\boldsymbol{x}_i(k) + E_{5,i,k}, \end{aligned} \tag{27}$$

$$k = k_0, \cdots, k_0+N-1,$$
$$i = 1, \cdots, N_{\text{total}},$$

where $\mathbb{P}\{s_i\}$ represents the probability of scenario s_i; $J_i^{\text{time}}(k)$, $J_i^{\text{cost}}(k)$, $L_{N,i}(k_0)$, $\boldsymbol{x}_i(k)$, $\boldsymbol{\delta}_i(k)$, and $\boldsymbol{z}_i(k)$ denote the value of variables $J^{\text{time}}(k)$, $J^{\text{cost}}(k)$, $L_N(k_0)$, $\boldsymbol{x}(k)$, $\boldsymbol{\delta}(k)$, and $\boldsymbol{z}(k)$ under scenario s_i, respectively; $A_{i,k}$, $B_{1,i,k}$, $B_{2,i,k}$, $B_{3,i,k}$, $E_{1,i,k}$, $E_{2,i,k}$, $E_{3,i,k}$, $E_{4,i,k}$, and $E_{5,i,k}$ represent the corresponding matrices under scenario s_i, respectively.

Eq. (26) represents the linear and mixed-integer linear formulations of the model provided in (1)-(8), (11), (13)-(17) for the urban rail transit network under scenario s_i; (27) represents the corresponding constraints in the absorption model under scenario s_i, e.g., the safety constraint (12) for train operation. By solving problem $\mathbf{P}_{k_0}^{\text{SMPC}}$, we minimize the expected value of the objective function (25) while including the corresponding constraint satisfaction in (27). Problem $\mathbf{P}_{k_0}^{\text{SMPC}}$ is also an MILP problem and can be efficiently solved with current MILP solvers.

Solving problem $\mathbf{P}_{k_0}^{\text{SMPC}}$ yields a control sequence for period k_0 to k_0+N-1. In the MPC framework, only the control actions at period k_0 are applied to the practical urban rail transit network. In the next step, the optimization is done again by including the newly collected state information while the prediction time interval is shifted by one step.

3.3 Performance of the Developed Approach

Performance of the scenario-based approach depends to a large extent on the total number of representative scenarios used in the approach. Thanks to the theoretical results in Tempo et al. (1997); Karg et al. (2021), a probabilistic performance guarantee for the SMPC approach can be derived based on the ordered performances for the scenarios used in the SMPC approach.

Let's denote the performance of the resulting scenario-based model predictive control approach under scenario s_i by J_i^{SMPC}. Then, the performances under all the selected scenarios can be reordered as

$$J^{\text{SMPC}}(1) \leq J^{\text{SMPC}}(2) \leq \cdots \leq J^{\text{SMPC}}(N_{\text{total}}), \tag{28}$$

where $J^{\text{SMPC}}(o)$ represents the o-th smallest performance value among J_i^{SMPC}, $i \in \{1, \ldots N_{\text{total}}\}$; so o is the order index.

For the SMPC approach, we define $\mathbb{L}(v)$ as the probability of the objective function value smaller than v, $v \in \mathbb{R}$, that is

$$\mathbb{L}(v) := \mathbb{P}\{J < v\}. \tag{29}$$

Theorem 1. If the representative scenarios we use in the SMPC approach are independent and identically distributed (i.i.d.), we have

$$\mathbb{P}\left\{\mathbb{L}\left(J^{\text{SMPC}}(o)\right) \leq \varepsilon\right\} \geq 1-\rho, \tag{30}$$

for $o = 1, 2, \ldots, N_{\text{total}}$, provided that

$$\sum_{\zeta=0}^{o-1}\binom{N_{\text{total}}}{\zeta}\varepsilon^{\zeta}(1-\varepsilon)^{N_{\text{total}}-\zeta} \leq \rho. \tag{31}$$

In addition, (31) is satisfied (Karg et al., 2021) if

$$N_{\text{total}} \geq \frac{1}{\varepsilon}\left(o-1+\ln\frac{1}{\rho}+\sqrt{2\,(o-1)\ln\frac{1}{\rho}}\right). \tag{32}$$

For the proof of *Theorem 1*, we refer to Property 3 of Alamo et al. (2018).

The computational burden of problem $\mathbf{P}_{k_0}^{\text{SMPC}}$ increases rapidly as the number of scenarios grows, while less scenarios may negatively influence the performance of the scenario-based approach. The performance level of SMPC with different numbers of scenarios can be derived according to *Theorem 1*, which provides an indication for the performance of the developed SMPC approach.

4. CASE STUDY

Simulations are conducted in this section to demonstrate the efficacy of the scenario-based MPC approach for the timetable scheduling problem on two lines of the Beijing subway network.

The network we use is shown in Fig. 1, and it includes two bidirectional lines. The network consists of 19 stations with 40 platforms. The simulations are based on the passenger OD demands of the Beijing subway network, which can be generated according to the entering and existing flow data. We use the data related to Line 9 and Line 14 from 7:00AM for the simulation. The passenger flow data are recorded every half hour; thus, we set $T = 1800$ s. Table 1 presents the primary parameters for the simulation. In the simulation, the MILP problems are solved by the `gurobi` solver with MATLAB (R2019b).

The uncertain passenger demands are generated by Poisson distribution, where the passenger demands in the real-life passenger OD data are used as the expected value.

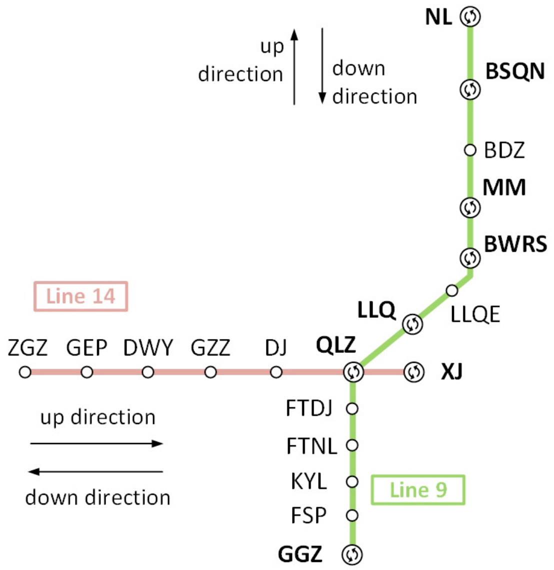

Fig. 1. Real-life network for simulation.

Table 1. Parameters for the simulation

Parameters	Line 9	Line 14
Minimum dwell time	30 s	30 s
Regular dwell time	60 s	60 s
Minimum headway	120 s	120 s
Regular headway	180 s	270 s
Mean transfer time	60 s	60 s
Train capacity	2400 persons	2400 persons
Period time	1800 s	1800 s

In this paper, we generate 5 scenarios with Poisson distribution as the representative scenarios for the SMPC. After getting the number of trains departing from each platform during a period by using the method in Section 3, we use the approach of Liu et al. (2023c) to calculate the specific departure and arrival times of each train, where a lower-level controller is applied for detailed train schedules considering departure/arrival constraints, running time constraints, and headway constraints. We use the passenger absorption model as the prediction model, while a more accurate timetable developed by Wang et al. (2015) is employed as the simulation model.

We first perform simulations under one uncertain scenario, and the performance of the basic timetable is also calculated for comparison, where the basic timetable is obtained with the regular headway and dwell time mentioned in Table 1. The MPC approach under the deterministic case uses the expected value of passenger demands to calculate the timetable, which is called nominal model predictive control (NMPC) in this paper. The prediction horizon for both NMPC and SMPC is set to $N = 3$.

Table 2. Comparison of different approaches under uncertain passenger demands

	Objective function value	CPU time (s)
Basic timetable	$2.3220 \cdot 10^4$	-
NMPC	$2.0891 \cdot 10^4$	9.5
SMPC	$2.0545 \cdot 10^4$	81.9

Table 2 and Fig. 2 display the simulation results. The results demonstrate that the performance of both NMPC and SMPC is improved compared with that of the basic timetable, with enhancements of 10.03% and 11.53%, respectively. Both NMPC and SMPC satisfy the real-time application requirement for the given case study. SMPC has a larger computational burden than NMPC, and the required CPU time increases from 9.5 s for NMPC to 89.1 s for SMPC. A suitable choice is required in real-life applications, i.e., when the CPU power is sufficient, SMPC is a better choice to get a higher quality solution; otherwise, NMPC can be used to calculate a timetable in a shorter time with acceptable performance.

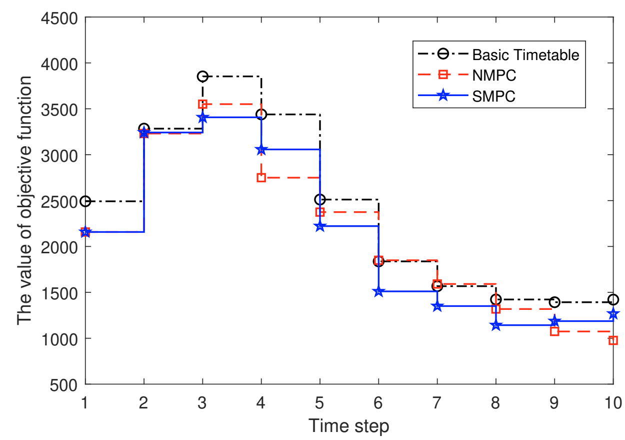

Fig. 2. Objective function value at each step.

The basic timetable and the timetable generated by SMPC for Line 9 (up direction) from 7:30AM and 8:00AM are given in Fig. 3 and Fig. 4, respectively. As the selected time window corresponds to the morning peak hour, compared with the basic timetable, SMPC schedules more trains to address the large passenger demands, which indicates that SMPC can optimize train schedules according to real-time passenger flow data.

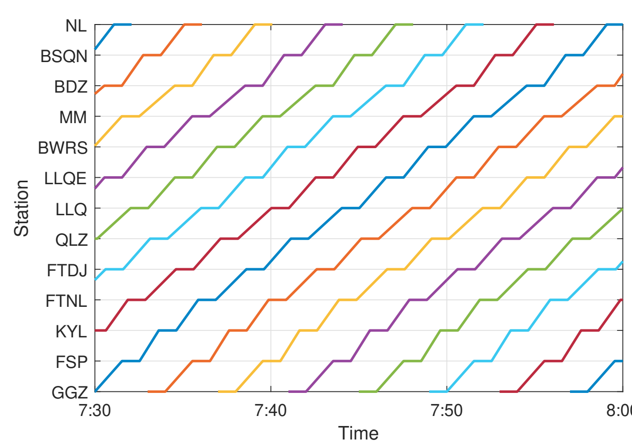

Fig. 3. Basic timetable of Line 9 (up direction).

Table 3. Comparison of the objective function values for different approaches

	Average	Standard deviation
Basic timetable	$2.3155 \cdot 10^4$	59.4700
NMPC	$2.1028 \cdot 10^4$	229.1080
SMPC	$2.0705 \cdot 10^4$	286.8022

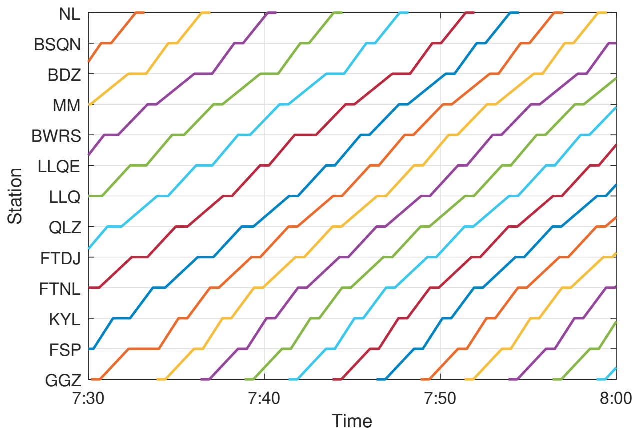

Fig. 4. Timetable of Line 9 (up direction) from SMPC.

To further illustrate the effectiveness of SMPC, simulations are carried out under 10 different scenarios, and the average value and the standard deviation for the basic timetable, the timetable obtained by NMPC, and the timetable obtained by SMPC are calculated, respectively. Table 3 shows that SMPC has the lowest average objective function values, while the standard deviation of SMPC is a bit larger than that of NMPC. Compared with the average objective function value of the basic timetable, NMPC and SMPC have the improvement of 9.19% and 10.58%, respectively. The simulation results imply that SMPC can be a suitable choice to handle uncertain passenger flows with an acceptable increase of the computational burden.

5. CONCLUSIONS

In this paper, we have investigated the real-time timetable scheduling problem of urban rail transit networks considering uncertain passenger flows. A scenario-based MPC approach has been adopted to handle uncertain passenger origin-destination demands for the timetable scheduling problem based on a passenger absorption model. For the scenario-based MPC approach, the uncertainties are captured by several scenarios based on historical data, and the probabilistic performance level can be derived based on the performance of the controller under the representative scenarios, which provides an indication for the performance of the scenario-based MPC approach. Simulation results indicate that, compared with the nominal MPC, the scenario-based MPC yields a better performance with an acceptable increase in computation time.

For future work, we will investigate how to further reduce the solution time of the scenario-based MPC approach by adopting a distributed control framework. The trade-off between reducing the prediction horizon and improving the solution quality will also be one future research topic.

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