



Delft University of Technology

Foreword

Bisagni, Chiara

Publication date

2022

Document Version

Final published version

Published in

Springer Aerospace Technology

Citation (APA)

Bisagni, C. (2022). Foreword. *Springer Aerospace Technology*, vii.

Important note

To cite this publication, please use the final published version (if applicable). Please check the document version above.

Copyright

Other than for strictly personal use, it is not permitted to download, forward or distribute the text or part of it, without the consent of the author(s) and/or copyright holder(s), unless the work is under an open content license such as Creative Commons.

Takedown policy

Please contact us and provide details if you believe this document breaches copyrights. We will remove access to the work immediately and investigate your claim.

Green Open Access added to TU Delft Institutional Repository

'You share, we take care!' - Taverne project

<https://www.openaccess.nl/en/you-share-we-take-care>

Otherwise as indicated in the copyright section: the publisher is the copyright holder of this work and the author uses the Dutch legislation to make this work public.

Springer Aerospace Technology

Series Editors

Sergio De Rosa, DII, University of Naples Federico II, Napoli, Italy

Yao Zheng, School of Aeronautics and Astronautics, Zhejiang University,
Hangzhou, Zhejiang, China

The series explores the technology and the science related to the aircraft and spacecraft including concept, design, assembly, control and maintenance. The topics cover aircraft, missiles, space vehicles, aircraft engines and propulsion units. The volumes of the series present the fundamentals, the applications and the advances in all the fields related to aerospace engineering, including:

- structural analysis,
- aerodynamics,
- aeroelasticity,
- aeroacoustics,
- flight mechanics and dynamics,
- orbital maneuvers,
- avionics,
- systems design,
- materials technology,
- launch technology,
- payload and satellite technology,
- space industry, medicine and biology.

The series' scope includes monographs, professional books, advanced textbooks, as well as selected contributions from specialized conferences and workshops.

The volumes of the series are single-blind peer-reviewed.

To submit a proposal or request further information, please contact:
Mr. Pierpaolo Riva at pierpaolo.riva@springer.com (Europe and Americas)
Mr. Mengchu Huang at mengchu.huang@springer.com (China)

The series is indexed in Scopus and Compendex

More information about this series at <http://www.springer.com/series/8613>

Simon Appel · Jaap Wijker

Simulation of Thermoelastic Behaviour of Spacecraft Structures

Fundamentals and Recommendations

 Springer

Simon Appel 
ATG Europe (Netherlands)
Noordwijk, The Netherlands

Jaap Wijker
Velsbroek, The Netherlands

ISSN 1869-1730

ISSN 1869-1749 (electronic)

Springer Aerospace Technology

ISBN 978-3-030-78998-5

ISBN 978-3-030-78999-2 (eBook)

<https://doi.org/10.1007/978-3-030-78999-2>

© The Editor(s) (if applicable) and The Author(s), under exclusive license to Springer Nature Switzerland AG 2022

This work is subject to copyright. All rights are solely and exclusively licensed by the Publisher, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, reuse of illustrations, recitation, broadcasting, reproduction on microfilms or in any other physical way, and transmission or information storage and retrieval, electronic adaptation, computer software, or by similar or dissimilar methodology now known or hereafter developed.

The use of general descriptive names, registered names, trademarks, service marks, etc. in this publication does not imply, even in the absence of a specific statement, that such names are exempt from the relevant protective laws and regulations and therefore free for general use.

The publisher, the authors and the editors are safe to assume that the advice and information in this book are believed to be true and accurate at the date of publication. Neither the publisher nor the authors or the editors give a warranty, expressed or implied, with respect to the material contained herein or for any errors or omissions that may have been made. The publisher remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.

This Springer imprint is published by the registered company Springer Nature Switzerland AG
The registered company address is: Gewerbestrasse 11, 6330 Cham, Switzerland

*The authors dedicate this book to their
former boss ir. Marinus P. Nieuwenhuizen of
the structural department of Fokker Space
and Systems BV, The Netherlands.*

Foreword

Thousands of spacecraft have been launched into space since the launch of Sputnik in 1957. We have landed men on the moon, visited planets in our Solar System with robotic probes, stationed scientists on the International Space Station, established global telecommunication and navigation systems, and nowadays, we are setting new goals in exploration, and consequently, despite all the reached accomplished, we are facing new, as well as old, challenges. Among them, the design of spacecraft structures subjected not only to mechanical, vibrational and acoustic loading, but also to extreme thermal conditions, requires new analysis techniques, and the consideration of thermal and structural analyses at the same time.

This book covers the topics necessary to the understanding of the thermoelastic behaviour of spacecraft structures, describing the essential steps of the analysis and verification, without missing important aspects such as handling uncertainties in the thermoelastic analysis process. Each topic is presented in the book with the necessary background, the up-to-date perspective and the essential theory–practical connection, thanks to the extensive industrial experience of both authors.

This book will be of immense help to thermal and mechanical engineers who are looking for concrete answers of their problems and, at the same time, to graduate students who would like to acquire knowledge in the fascinating world of the space structures.

October 2020

Chiara Bisagni
Professor of Aerospace Structures
Delft University of Technology
Delft, The Netherlands

Preface

The authors, Jaap Wijker and Simon Appel, have been working together since October 1986. In that month, Simon Appel arrived as a trainee at the Space division of Fokker Aircraft in the group led by Jaap. In those years, the group was running a technology development activity for ESA-ESTEC of which part of it aimed at developing an interface between the most important lumped parameter thermal analyser of those days, SINDA (“Systems Improved Numerical Differencing Analyser”) and the finite element code used in the structures section at ESTEC, which was ASKA (“Automatic System for Kinematic Analysis”). In this activity, the foundation was established of the PAT method, the prescribed average temperature method, that you will find explained in this book and its implementation in the SINAS software: SINda—ASKa. The mentioned technology development was initiated by Steve Stavrinidis, and the follow-on developments were run under the supervision of Michel Klein.

Jaap with his team members could be called the founding fathers of the PAT method. In the years after, Simon matured the implementation of the method in the SINAS software and the interface with the commonly used finite element tool MSC Nastran. The tool has been used at ESTEC and at Fokker Space and Systems. These were the years that Jaap and Simon worked on several thermoelastic problems.

In 2001, Simon joined the company that is called today ATG Europe and has been supporting ESA-ESTEC since then. An important part of his time has been, and still is, dedicated to thermoelastic problems and further development of the SINAS software. Jaap kept working for Fokker Space as it was called in the meantime, but also started to lecture at the Delft Technical University. Inspired by his students, he got motivated to write several books on spacecraft structures and related structural dynamics. Despite his retirement in 2009, Jaap continued writing books and managed even to obtain his Ph.D. degree.

The never-ending drive of Jaap to share his knowledge and keep on studying made him to contact Simon to join him writing the book that is now in your hands on the subject of thermoelastic simulations. It is the subject with which the relationship between Jaap and Simon started in the 1980s.

Jaap and Simon enjoyed writing this book. They know there is much more to discuss, and it will never be finished, but at some point, the time is there to share the results with you as reader.

Whether you are a graduate student or an experienced senior thermal or mechanical engineer, Jaap and Simon hope you find some useful information in this book. They also invite you to send feedback and suggestions that they may consider for a potential next edition.

Velserbroek, The Netherlands
Voorhout, The Netherlands
May 2021

Jaap Wijker
Simon Appel

Acknowledgements

The authors thank all those supported them with advices and encouragements during preparation of this book.

A special thanks goes to:

- The management and colleagues of ATG Europe and the Mechanical Department of ESA-ESTEC for their moral support and advice
- The colleagues of Engineering Lab of ATG Europe, and specifically Alexander van Oostrum and Alberto Peman, for the inspiring discussions
- Dick Wijker for facilitating the application of scientific software for the preparation of the numerical examples for this book
- Daniele Stramaccioni for his thorough and critical review of the thermal aspects discussed in the book
- MSC Software and In Summa Innovation for their support on the structural aspects of the numerical examples prepared for this book
- ITP for providing an ESATAN-TMS licence for solving the thermal problems that are part of the examples

and many others who contributed with ideas without realising this.

Perspective

How did the Universe originate, what are its fundamental physical laws and what is it made of? How does the Solar System work? What are the conditions for planet formation and the emergence of life within and outside our Solar System? How can we monitor, understand and preserve the delicate ecosystem of our planet and how can we protect life on Earth? These are some of the fundamental questions we, as human beings, are trying to answer since hundreds of years, and these are the questions the European Space Agency (ESA) is addressing within its world leading space science programmes and Earth observation missions.

With our “time-machine” Planck and his astronomy observer companion Herschel, we changed our view of the Universe, by revealing the relic radiation left by the Big Bang to an unprecedented level of accuracy, measuring fluctuations in temperature of a few millionth of degree, hence tracing the birth of stars and the evolution of galaxies throughout time. Gaia, the billion-star surveyor, has been making precise measurements of the positions, motions and characteristics of stars in order to create a three-dimensional map of our Milky Way and explore the past and future evolution of the Galaxy. PLATO will soon hunt Sun–Earth analogue systems in relatively nearby stars, identifying and studying thousands of exoplanetary systems, with emphasis on discovering and characterising Earth-sized planets in the habitable zone of their parent star. Closer to us, the Copernicus Earth observation programme is taking the pulse of our planet and is providing decision-makers with indisputable data in order to understand global changes and intervene effectively to resolve them.

These are some of the most recent missions ESA has developed in an attempt to shed light in the understanding of the Universe and our home, the Earth. They have posed exceptional challenges to the thousands of European engineers and scientists working in industry, academia and in the agency for the development, the verification and the operation of these scientific jewels. Space is probably the most demanding and hostile operational environment in which human engineering products are required to function. Without any repair or maintenance option, space structures shall guarantee years of performances reaching incomparable levels of accuracy and stability of the telescopes and optical instruments on-board. And, this shall be ensured despite the very demanding mechanical loading during launch and separation and the prohibitive temperature environment and fluctuations encountered in-service.

Although more and more the design of our space and Earth exploring satellites is driven by thermomechanical performances, hardly any textbooks or university classes are devoted to this fundamental discipline. Many good references exist addressing spacecraft thermal analysis and control, and also, a vast amount of literature can be found on the different satellite structural engineering subjects. However, these sources only cover the relevant topics within the context of the respective disciplines. In some cases, the interaction with the other subject is briefly discussed, but mainly from the perspective of their own domains. Moreover, the thermal and structural disciplines for the development of spacecraft structures and payloads have been traditionally supported by two distinct entities in most space engineering organisations, and in addition, both disciplines are using different analysis methodologies and associated numerical tools.

Thermomechanical and thermoelastics analyses aim to predict the deformations and the stresses affecting a structure or a component due to temperature fields and variations. In order to have a complete understanding of the problem, the two above-mentioned disciplines need to be addressed in a synergistic and cross-sectorial manner. A structural finite element model for thermoelastic predictions of a structure under a given thermal environment cannot be precisely established without a detailed knowledge of the temperature fields described by the thermal model. Conversely, a thermal engineer needs to have an adequate understanding of what are the temperature results and the resolution required by the structural model for reliable analysis.

The present book aims at capitalising the vast experience of the authors in the field of thermoelastic predictions applied to spacecraft structures and provides a coherent approach to solve practical and real-life problems. While analysing the different modelling and verification objectives of the thermal and structural domains, the authors address the current state-of-the-art approaches and limitations for both analyses and provide a suitable and verified method for addressing both disciplines in a synergistic fashion. This also includes specific numerical tools for transferring results from the structural to the thermal numerical environment and vice versa.

The book is most welcome in the space community and will provide a unique guidance to senior and the younger generation of engineers involved in the structural and thermal analyses of sophisticated spacecraft structures and instruments. It can constitute a sound basis for the building of a dedicated European Cooperation for Space Standardisation (ECSS) standard related to thermoelastic analysis and verification, ultimately leading to more performing space missions, improving our understanding of the Universe and contributing to a better life on our planet.

May 2021

Tommaso Ghidini
Head of the Structures, Mechanisms
and Materials Division
European Space Agency
The Hague, The Netherlands

Contents

1	Thermoelastic Verification	1
1.1	The Thermoelastic Problem	1
1.2	Structure of This Book	2
2	Occurrence of Thermoelastic Phenomenon in Spacecraft	5
2.1	Introduction	5
2.2	Hubble Space Telescope	5
2.3	Korean Observation Satellite	7
2.4	Gaia	7
2.5	Surface Water and Ocean Topography (SWOT)	9
2.6	PLATO	11
3	Physics of Thermoelastics	15
3.1	Introduction	15
3.2	Coefficient of Thermal Expansion	16
3.3	Young's or Elasticity Modulus	20
3.4	Constitutive Laws of Linear Thermoelasticity	26
3.4.1	General 3-D Constitutive Laws of Linear Thermoelasticity	26
3.4.2	1-D Stress–Strain Relation	27
3.4.3	Plane Stress State	29
3.4.4	Plane Strain State	31
3.5	Summary Governing Equilibrium and Constitutive Equations	37
3.5.1	Equilibrium	37
3.5.2	Strain–Displacement Relations	37
3.5.3	Constitutive Law	38
	Problems	39
4	Modelling for Thermoelastic	45
4.1	Introduction	45
4.2	What Is a Thermal Gradient?	46
4.3	What to Model?	53

- 4.4 Structural and Thermal Modelling for Thermoelastic: An Integrated Process 53
- 4.5 Integrated Model Convergence Checks 54
- 4.6 Modelling Features 64
 - 4.6.1 Features and How These Are Commonly Modelled 64
 - 4.6.2 Assessment of a Box on a Plate 65
 - 4.6.3 Simplifying Feature Modelling: Preserve the Physics 71
- 4.7 Need for Automation of the Analysis Chain 73
- 4.8 Summary and Recommendations 75
 - 4.8.1 Which Deformations Cause Degradation of Performance of Instruments? 76
 - 4.8.2 Which Mechanisms Can Make the Degradation of Performance of Instruments Happen? 76
 - 4.8.3 What Is Needed to Simulate the Thermoelastic Mechanisms? 77
 - 4.8.4 Mesh Resolution and Level of Detail 82
 - 4.8.5 Temperature Mapping 84
 - 4.8.6 Selection of Worst Cases 84
 - 4.8.7 Uncertainties 85
 - 4.8.8 Concluding Recommendations 85
- Problems 85
- 5 Thermal Modelling for Thermoelastic Analysis 87**
 - 5.1 Introduction 87
 - 5.2 Space Thermal Environment 88
 - 5.2.1 On Ground Phase 88
 - 5.2.2 Launch and Ascent Phase 89
 - 5.2.3 Orbital Phase 90
 - 5.2.4 Direct Solar Flux 91
 - 5.2.5 Planet Reflected Solar Flux (Albedo) 91
 - 5.2.6 Planet Flux, Infrared Radiation 91
 - 5.2.7 Internal Dissipation 92
 - 5.3 Heat Transfer Mechanisms 92
 - 5.3.1 Conduction 92
 - 5.3.2 Contact Conductance 93
 - 5.3.3 Convection 94
 - 5.3.4 Thermal Radiation Heat Transfer 94
 - 5.4 Spacecraft Thermal Modelling with the Lumped Parameter Method 96
 - 5.4.1 Thermal Network Modelling with the Lumped Parameter Method 97
 - 5.4.2 Thermal Node in a Thermal Lumped Parameter Model 98
 - 5.4.3 Geometric Mathematical Model 99

- 5.4.4 Thermal Mathematical Model 99
- 5.5 Thermal Transient Analysis 100
 - 5.5.1 Transient Phenomena in Space Thermal Analysis 100
 - 5.5.2 Solution Approach for Thermal Transient Problems 101
- 5.6 Thermoelastic Analysis for Transient Problems 102
- 5.7 Thermal Analysis for Thermoelastic Versus Thermal Control 103
 - 5.7.1 Objectives of Thermal Analysis for Thermal Control 103
 - 5.7.2 Objectives of Thermal Analysis for Thermoelastic 104
 - 5.7.3 Selection of Worst Case Temperature Fields 105
 - 5.7.4 Thermal Mesh Convergence for Thermoelastic 106
 - 5.7.5 Level of Detail in Models for Thermoelastic 107
 - 5.7.6 Thermal Analysis Uncertainties for Thermoelastic 107
 - 5.7.7 Concluding Thermal Analysis for Thermal Control Versus Thermoelastic 109
- Problems 109
- 6 Structural Modelling for Thermoelastic Analysis 111**
 - 6.1 Introduction 111
 - 6.2 The Finite Element Method for Thermoelastic Simulations 113
 - 6.3 Characteristics of Finite Elements for Thermoelastic Analysis 113
 - 6.4 Elastic Finite Elements 114
 - 6.4.1 0-D, Scalar Element 114
 - 6.4.2 1-D, Rod Element 119
 - 6.4.3 1-D, Bar and Beam Element 121
 - 6.4.4 2-D, Membrane Element 122
 - 6.4.5 2-D, Plate, Shell, Sandwich Element 122
 - 6.4.6 3-D, Volume (Solid) Element 124
 - 6.5 Constraint Equations and Rigid Elements 126
 - 6.5.1 Principle of Constraint Equations 126
 - 6.5.2 The Interpolation Element 127
 - 6.5.3 The Rigid Body Element 128
 - 6.6 Boundary Conditions 132
 - 6.6.1 Iso-static Supports 132
 - 6.6.2 Statically Indeterminate Supports 134
 - 6.6.3 Intertia Relief Method 136
 - 6.7 Refurbishing a Dynamic Finite Element Model for Thermoelastic 139
 - 6.7.1 Introduction 139
 - 6.7.2 Required Mesh Resolution for Dynamic and Thermoelastic Models 140

- 6.7.3 Finite Element Models for High-Frequency Response Analysis 150
- 6.7.4 Simulation of Joints 151
- 6.7.5 Check on Adequacy of Rigid Body Elements for Thermoelastic 156
- 6.8 Finite Element Model Health Checks Thermoelastic FE Models 158
 - 6.8.1 Introduction 158
 - 6.8.2 Strain Energy as Rigid Body 158
 - 6.8.3 Free Iso-thermal Expansion 159
- Problems 160
- 7 Transfer of Thermal Analysis Results to the Structural Model 165**
 - 7.1 The Interface Problem 165
 - 7.2 Thermal Lumped Parameter Node Versus Finite Element Node 166
 - 7.3 Building Correspondence Between Models 166
 - 7.4 Temperature Mapping Methods 168
 - 7.4.1 Geometric Temperature Interpolation Method 168
 - 7.4.2 Centre-Point Prescribed Temperature Method 170
 - 7.4.3 Patch-Wise Temperature Application Method 171
 - 7.4.4 Prescribed Average Temperature Method 172
 - 7.5 Comparing Mapping Methods on a 1-D Problem 173
 - 7.5.1 One-Dimensional Model Description 173
 - 7.5.2 Temperature Mapping Results 174
 - 7.5.3 Thermoelastic Responses 175
 - 7.5.4 Conclusion of One-Dimensional Problem 179
 - 7.6 Benchmarking of Temperature Mapping Methods on a Two-Dimensional Problem 179
 - 7.6.1 Geometry, Mesh and Boundary Conditions 179
 - 7.6.2 Temperature Field to Be Mapped 180
 - 7.6.3 Reference Temperature, Displacement and Stress 180
 - 7.7 Comparing Performances of Mapping Methods 182
 - 7.7.1 Performance Criteria for the Mapping Methods 182
 - 7.7.2 Qualitative Comparison of the Mapped Temperature Fields 184
 - 7.7.3 Average Temperature Comparison 185
 - 7.7.4 Displacement Comparison 186
 - 7.7.5 Stress Comparison 187
 - 7.7.6 Concluding the 2-D Benchmark Model 189
 - 7.8 Summary Temperature Mapping/Interpolation Methods 190
 - Problems 193

8	Prescribed Average Temperature Method	195
8.1	Introduction	195
8.2	Relating Thermal Nodes and FEM Nodes	195
8.3	Creation of Consistent Values of A-Matrix Coefficients with a Finite Element Code	199
8.4	Coupling TMM to the FE Model	203
8.5	Evaluating PAT Method Results	209
8.6	Mathematical Models Checks for PAT Method	211
8.6.1	Introduction	211
8.6.2	Conduction FE Model Health Check	211
8.6.3	Checking A-Matrix Input to the PAT Method	213
8.7	Effect of Incomplete Correspondence	217
	Problems	221
9	Generation of Linear Conductors for Lumped Parameter Thermal Models	225
9.1	Need for Automated Conductor Generation	225
9.2	Calculation of a Single Linear Conductor with a Conduction FE Model	228
9.2.1	Calculation of a Conductor Through Reduction of the Conduction Matrix	228
9.2.2	Conductor Calculation Through Steady-State Thermal Analysis	235
9.2.3	Far Field Method for Generation of 1-D Linear Conductors	237
9.3	PAT-Based Methods for Generating TMM Conductors	242
9.3.1	Extracting Conductors from Lagrange Multipliers Λ	242
9.3.2	Reduction of FE Model Conduction Matrix	243
9.3.3	Consideration for the Use of the PAT-Based Conductors	252
	Problems	255
10	Estimating Uncertainties in the Thermoelastic Analysis Process	261
10.1	Uncertainties in the Thermoelastic Analysis Process	261
10.1.1	Uncertainties from the Thermal Analysis	262
10.1.2	Uncertainties from the Temperature Mapping Process	262
10.1.3	Uncertainties from the Thermoelastic Structural Response Analysis	263
10.1.4	Uncertainties from the Instrument Performance Impact Analysis	264
10.2	Use of Factors of Safety for Covering the Uncertainties	264
10.3	Uncertainty Assessment of Thermoelastic Analysis Using Probabilistic Analysis	266

10.4	Monte Carlo Simulation Method	268
10.5	Modified MCS, Latin Hypercube Sampling Method	269
10.6	The Rosenblueth $2k + 1$ Point Estimates Probability Moment Method	273
10.7	Sensitivity Analysis	293
	Problems	300
Appendix A: Detailed Description of “Box on Plate” Experiment		309
Appendix B: One-Dimensional (1-D) Conduction Finite Element		345
Appendix C: One-Dimensional (1-D) Thermoelastic Finite Element		359
Appendix D: Theory of Introduction Multipoint Constraint Equations in Thermoelastic Problems		377
Appendix E: Solutions		383
References		389
Index		395

Acronyms and Abbreviations

Abbreviations

ASM	Aerospace Specification Metals
AU	Astronomical unit
CNES	Centre National d'Etudes Spatiales
CPPT	Centre-Point Prescribed Temperature method
CSA	Canadian Space Agency
CTE	Coefficient of thermal expansion
DOF	Degree of freedom
ECSS	European Cooperation for Space Standardisation
ESA	European Space Agency
ESTEC	European Space Technology Center
FE	Finite element
FEA	Finite element analysis method
FEM	Finite element model
FoS	Factor of safety
GMM	Geometric mathematical model
HST	Hubble Space Telescope
IDW	Inverse distance weighting
IR	Infrared
JAXA	Japan Aerospace Exploration Agency
KARI	The Korean Aerospace Research Institute
KSASS	Korean Society Aeronautical Space Science
LEOP	Launch early orbit phase
LHS	Latin hypercube sampling
LoS	Line of sight
LPM	Lumped parameter method
MCRT	Monte Carlo ray tracing
MCS	Monte Carlo sampling
MoS	Margin of safety
MPC	Multipoint constraint

NAFEMS	National Agency for Finite Element Methods and Standards
NASA	National Aeronautical and Space Administration
PAT	Prescribed average temperature
PCB	Printed circuit board
PCL	MSC.Patran Command Language
PEM	Point estimates moments
RBE	Rigid body element
RF	Radio frequency
SM	Service mission
SRC	Standardised regression coefficient
STOP	Structural, thermal and optical performance
SWOT	Surface Water Ocean Topography
TMM	Thermal mathematical model
TN	Thermal node
TRP	Temperature reference point

Symbols

A	Area, cross-section (m^2)
a	Radius (m), constant, coefficient, $[A]$ -matrix term
$[B]$	Interpolation matrix
b	Radius (m), width (m)
C	Heat capacitance
$[C]$	Conduction matrix
CTE	Coefficient of thermal expansion ($n/m/^\circ C$, $m/m/K$)
c_p	Specific heat ($J/kg/^\circ C$, $J/kg/K$)
$^\circ C$	Centigrade (degree Celsius)
E	Energy (W)
$[E]$	Unitary diagonal matrix
$[D]$	Elasticity tensor
E	Young's modulus (Pa)
F	Force (N), view factor
(F_T)	Equivalent thermal load vector
$F(x)$	Function of x
$F(x, y)$	Function of x, y
\mathcal{F}	Hottel's total view
f	Frequency (Hz)
G	Shear modulus (Pa), conductance ($W/^\circ C,K$)
G_{sc}	Solar constant (W/m^2)
G_{ss}	Transformation matrix
GL	Conductor, thermal conductance coefficient ($W/^\circ C,K$)
GR	Radiative conductor
h	Height, thickness (m), convective heat transfer coefficient ($W/m/^\circ CK$)

H	Panel height
I	Second moment of area (m^4), integral
J	Joules, Jacobian, thermal functional
K	Kelvin
K	Stiffness matrix
K_c	Conduction matrix
k	Conductivity coefficient ($W/m/K$, $W/m/^\circ C$)
\mathcal{N}	Normal distribution
l, L	Length (m)
L_{ref}	Reference length (m)
\mathcal{LN}	Log normal distribution
m	Discrete mass (kg)
M_T	Equivalent thermal moment (NM)
M	Mass matrix
M_{eff}	Modal effective mass (kg)
N	Number of samples
P	Pressure (Pa)
P_T	Thermal force (N)
q	heat transfer rate
Q_s	Solar constant (W)
Q	Heat flux (W/m^2)
(R_Q)	Heat flow vector
r	Radius (m)
R	Distance (m), resistance ($^\circ C/W$, K/W), residual, Rayleigh quotient
T	Temperature ($^\circ C$, K)
T_{ref}	Reference temperature ($^\circ C$, K)
t	Time (s), thickness (m)
r	Radius (m)
u	Displacement m, stochastic variable
\mathcal{U}	Uniform distribution
U	Strain energy
U_m	Matrix of coefficients (MPC equations)
v	Displacement (m)
V	Volume (m^3), coefficient of variation
W	Watts
w	Deflection (m)
x	Coordinate, variable
y	Coordinate, variable
Y	Stochastic variable

Greek Symbols

α Absorption coefficient (-), absorptivity

α	Coefficient of thermal expansion (m/m/K, m/m/°C)
β	(Thermal) Expandability (m/m/°C), thermal stress modulus (Pa), constant
β_x	Sensitivity index
ε	(Thermal) Emittance (-), emissivity, (engineering) strain (m/m)
δ_{ij}	Kronecker delta
δ	Displacement (m), differential operator, virtual (displacement)
Δ	Difference, evaluated (temperature), prescribed displacement
ζ	Isoparametric coordinate, dummy variable
μ	(Ensemble) average value
η	Bond thickness (m), isoparametric coordinate, dummy variable
λ	Lamé modulus (Pa), constant
Λ	Lagrange multiplier
ν	Poisson's ratio (-)
Π	Potential energy
σ_{ii}	Component stress tensor (Pa)
σ	Boltzmann constant (W/m ² /K ⁴), standard deviation, constant
θ	Angle
τ	Shear stress (Pa)
ϕ_R	Rigid body mode
Φ	Probability function
Ψ	Trial function, (nodal) shape function
ω^2	Eigenvalue