Evaluation of Koopman-Based Fiber Parameter Estimation

Aamir, S.; Wahls, S.

DOI
10.1049/icp.2023.2548

Publication date
2023

Document Version
Accepted author manuscript

Published in
Proceedings of the 49th European Conference on Optical Communication (ECOC 2023)

Evaluation of Koopman-Based Fiber Parameter Estimation

Shahzeb Aamir[(1)], Sander Wahls[(1)]

[(1)]Delft Center for Systems and Control, Delft University of Technology, Netherlands, s.aamir@tudelft.nl

Abstract *Recently, a novel parameter identification method for partial differential equations based on the Koopman operator framework has been proposed. We evaluate its suitability for the identification of single span optical fiber links of various lengths in simulations.*

Introduction

The efficient transmission of data in optical fiber links requires knowledge of fiber parameters such as the loss, dispersion and Kerr nonlinearity coefficients. Fiber manufacturers provide these parameters on the data sheets for their fibers, which have been measured under lab conditions. However, fiber parameters can change during operation due to aging, bending, temperature changes, etc. The growing mismatch between the assumed and true fiber parameters can have a negative impact on the transmission quality. We therefore consider the problem of re-estimating the fiber parameters of an operational link, where we assume that input-output data from the link is available (i.e. transmitted and corresponding received signals), but that the inputs cannot be chosen freely. This rules out methods that require special hardware or specific probing signals[1].

One possible approach would be to use conventional partial differential equation (PDE) parameter estimation methods, e.g., based on sparsity[2],[3]. The disadvantage of these methods is that derivatives need to be estimated in both space and time. For fibers, where only inputs and outputs are measured, estimating the spatial derivatives is not possible for longer span lengths. Recently, an novel identification method for the dispersion and Kerr coefficients based on nonlinear Fourier transforms (NFT)[4],[5] was proposed. The advantage of this method is that no spatial derivatives have to be found. However, like all NFT methods, this method relies on a path-average approximation, which introduces additional errors. Furthermore, the loss coefficient has to be known a priori. A conceptually related approach is to exploit conserved quantities[4]. Yet another approach to identify the fiber parameters is to propagate the input data numerically, e.g. using a Fourier split-step method, for different candidate fiber parameters e.g. chosen from grids, and to keep the parameters for which the numerical output matches true output best[5]. This approach is simple and reliable, but the computational complexity suffers from the curse of dimensionality as the number of parameters increases. It is therefore ideally used to identify the Kerr coefficient when the other coefficients are already known. For such scenarios, there are furthermore adaptive digital backpropagation methods for estimating the Kerr coefficient[6],[7].

In this paper, we are however interested in methods that estimate all fiber parameters jointly. We therefore investigate the performance of the recently proposed PDE parameter estimation technique[8] (based on[9]) that exploits the Koopman operator framework. Koopman operators, originally introduced in[10], have recently obtained much attention as a tool to linearize nonlinear systems *globally*. This is in contrast to conventional *local* linearization. The Koopman operator thereby provides new observables under which the evolution of the system becomes linear. See, e.g.,[11],[12] for introductions to the topic. The method in[8] seems appealing for fiber identification because it is simple and does not require spatial derivatives, although it is known that the span length still cannot be arbitrary large. However, it is not clear from the current theory which span lengths would still be acceptable.

Our goal is thus to evaluate the suitability of the Koopman-based identification method (KIM) for optical fiber transmission scenarios. We consider the identification of a single span with an EDFA amplifier in this paper, and investigate the impact of the span length and transmit power.

Koopman-Based Identification Method (KIM)

In this section, we describe the KIM for PDE coefficients by Mauroy et. al[11], tailored to a nonlinear Schrödinger equation (NLSE) of the form

$$u_z = \sum_{k=1}^{3} c_k W_k(u) = c_1 \cdot u + c_2 \cdot u_{\tau\tau} + c_3 \cdot u|u|^2, \quad (1)$$

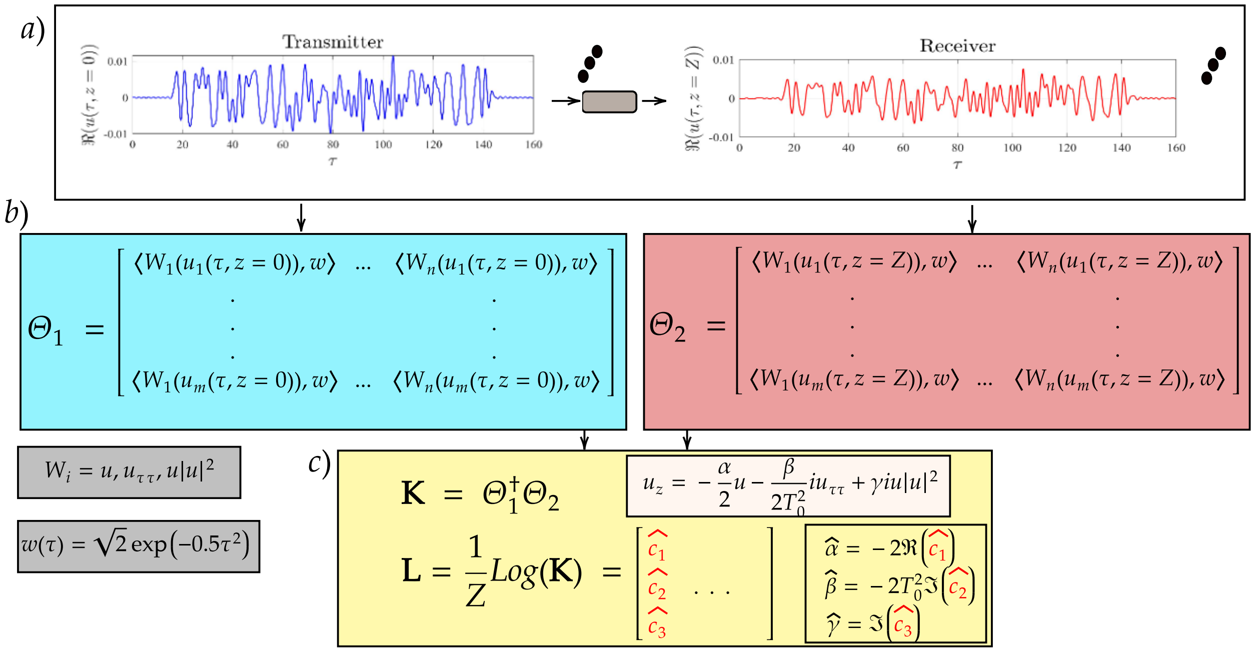

Fig. 1: Illustration of the Koopman-based Fiber Parameter Identification Method. The hats ($\hat{}$) indicate estimates.

which models the propagation of light in optical fibers[13]. Here $u = u(z, \tau)$, where z and τ denote the position and normalized retarded time, respectively. The subscripts $_z$ and $_\tau$ indicate partial derivatives. The KIM identifies the coefficients c_k from m data-pairs $\{u_j(\tau, z = 0), u_j(\tau, z = Z)\}_{j=1}^m$ of fiber inputs and corresponding outputs, where Z is the fiber span length. The coefficients c_k can be complex. They are related to the usual loss, dispersion and Kerr parameters by

$$c_1 = -\frac{\alpha}{2}, \quad c_2 = -i\frac{\beta}{2T_0^2}, \quad c_3 = i\gamma, \tag{2}$$

where $i = \sqrt{-1}$ and T_0 is a constant that arises since we normalized the time variable, $\tau = t/T_0$.

The theory behind the KIM[8] is quite complicated, but the method itself only has a few simple steps, which are also illustrated in Fig. 1:

1. Collect m input-output data pairs $\{u_j(\tau, z = 0), u_j(\tau, z = Z)\}_{j=1}^m$ as shown in Fig. 1a
2. Build data matrices Θ_1 and Θ_2 as shown in Fig. 1b, where w is a weighting function, the W_k are the library terms from the NLSE (1), and $\langle \cdot, \cdot \rangle$ is the usual inner product
3. Construct the matrix $\mathbf{K} = \Theta_1^\dagger \Theta_2$, where $\Theta_1^\dagger$ is the Moore-Penrose pseudoinverse (Fig. 1c)
4. Compute the matrix $\mathbf{L} = \frac{1}{Z}\,\mathrm{Log}(\mathbf{K})$, where Log is the matrix logarithm. The first column of $\mathbf{L}$ approximates the c_k (Fig. 1c).

Note that the KIM does not require the computation of spatial derivatives. Proposition 1 in [8] shows that the KIM recovers the true coefficients in the limit $Z \to 0$ in the noise-free case, given a sufficiently rich data set. It is pointed out in [8] that accurate estimates can also be obtained if Z is not small. However, the method will eventually break down when Z is too large. We are therefore interested in if it is suitable for fiber identification.

Simulation Setup

We identify the parameters α, β and γ for the NLSE (1)–(2) for a fiber-optic link with one span followed by an erbium doped fiber amplifier (EDFA) with a 6 dB noise figure. We vary the span length and transmit powers. For each length, the ground truth parameters are $\alpha = 0.0002$ dB/m, $\beta = -5 \cdot 10^{-27}$ s^2/m and $\gamma = 0.0012$ (Wm)$^{-1}$. The normalization constant is $T_0 = 2.5 \cdot 10^{-11}$ s, which ensures that c_2 has a magnitude of the same order as c_1 and c_3. The input-output data was obtained using the current development version of NFDMLab[14] (commit 361e23c). The fiber inputs were generated using Nyquist pulse-shaping with raised cosine pulses (roll-off 0.5) and a symbol spacing of $2.5 \cdot 10^{-11}$ s. Fiber inputs consisted of 128 16-QAM and 32 guard symbols, and were ideally low-pass filtered at 40 GHz bandwidth before transmission. The same filter was applied at the receiver. One input-output pair is shown in Fig. 1a. We remove the amplifier gain from the fiber output before the identification. The candidate term $W_2 = u_{\tau\tau}$ is approximated by a finite difference derivative. We choose a Gaussian weighting function, $w(\tau) = \sqrt{2}\exp(-0.5\tau^2)$, when computing the inner products in Fig. 1b. We used

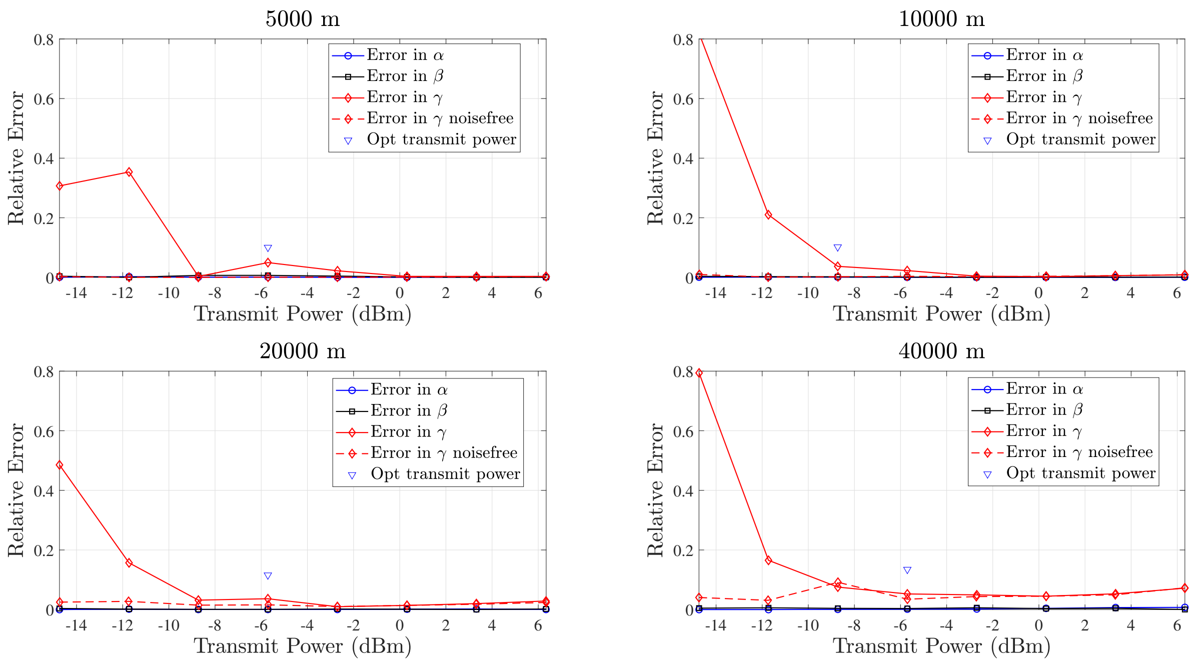

Fig. 2: Relative errors in the fiber parameters of transmit power for different span lengths.

$m = 5500$ input-output data pairs per identification, expect when the noise in the EDFAs was turned off. Then, $m = 1000$ pairs were used.

Results

The results of the simulations are shown in Fig. 2, where the dependence of the relative errors in the parameter estimates (e.g., $|\alpha - \hat{\alpha}|/|\alpha|$ for α) are shown as a function of the transmit power for the link lengths of 5, 10, 20 and 40 km. The plots also provide the performance of the KIM for the estimation of γ in the absence of noise (dashed lines). The optimal transmit powers, at which the error vector magnitude at the receiver (with an additional dispersion compensation block) are minimal, are marked with triangles. At 5, 20 and 40km, the optimal transmit power is ≈ -6dBm. At 10km there seems to be an outlier with an optimal transmit power of ≈ -9dBm, but the performance there is very close to that at ≈ -6dBm.

We observe that the identification of the loss parameter α and dispersion parameter β_2 succeeded for all considered span lengths and transmit powers, with relative errors very close to zero. The identification of the Kerr parameter γ is more difficult. For all span lengths, we observe that the transmit power must be sufficiently high for a successful identification of γ. (The zero error achieved at ≈ -9dBm and 5km appears to be an outlier that occurred because we used different random data for each identification.) By comparison with the noise-free curve for γ, we see that the noise is the limiting factor for the identification of γ in the low-power regime. Interestingly, the identification of α and β is affected much less by the noise (as the errors for the noisy case are already close to zero). At optimal transmit powers, the KIM is always able to identify γ with relative errors not exceeding 5.25%. We can thus conclude that the KIM is suited to identify the fiber parameters jointly for the considered span lengths.

We note that at 40 km, the relative error for the noise-free identification of γ is no longer able to reach zero. We suspect that beyond this distance, the span length is getting too large for the KIM.

Conclusion

We investigated the suitability of a Koopman-based parameter identification method for optical fiber spans. The method does not require spatial derivatives, but nevertheless deteriorates as the span length increases. It was however not clear for which span length that would happen. We investigated the span lengths 5, 10, 20 and 40 km. For the considered transceiver, the method was always able to identify α and β_2 with negligible error. It was also able to identify γ for all link lengths, with low relative errors, as long as the transmit power is not too low. Our results also suggest that the method might fail for spans longer than 40 km. Future research should investigate if the reach of the method can be improved, and if it can be extended to links with multiple spans.

References

[1] B. Batagelj, “Review of so far proposed fiber n/sub 2/measurement schemes”, in *Proceedings of 2002 4th International Conference on Transparent Optical Networks (IEEE Cat. No. 02EX551)*, IEEE, vol. 1, 2002, pp. 103–106.

[2] S. L. Brunton, J. L. Proctor, and J. N. Kutz, “Discovering governing equations from data by sparse identification of nonlinear dynamical systems”, *Proceedings of the national academy of sciences*, vol. 113, no. 15, pp. 3932–3937, 2016.

[3] S. H. Rudy, S. L. Brunton, J. L. Proctor, and J. N. Kutz, “Data-driven discovery of partial differential equations”, *Science advances*, vol. 3, no. 4, e1602614, 2017.

[4] P. De Koster and S. Wahls, “Dispersion and nonlinearity identification for single-mode fibers using the nonlinear Fourier transform”, *Journal of Lightwave Technology*, vol. 38, no. 12, pp. 3252–3260, 2020.

[5] P. De Koster, J. Koch, O. Schulz, S. Pachnicke, and S. Wahls, “Experimental validation of nonlinear Fourier transform-based kerr-nonlinearity identification over a 1600 km SSMF link”, in *Optical Fiber Communication Conference*, Optica Publishing Group, 2022, W2A–39.

[6] C.-Y. Lin, A. Napoli, B. Spinnler, *et al.*, “Adaptive digital back-propagation for optical communication systems”, in *Optical fiber communication conference*, Optica Publishing Group, 2014, pp. M3C–4.

[7] M. Piels, E. P. da Silva, D. Zibar, and R. Borkowski, “Performance emulation and parameter estimation for nonlinear fibre-optic links”, in *2016 21st European Conference on Networks and Optical Communications (NOC)*, IEEE, 2016, pp. 1–5.

[8] A. Mauroy, “Koopman operator framework for spectral analysis and identification of infinite-dimensional systems”, *Mathematics*, vol. 9, no. 19, p. 2495, 2021.

[9] A. Mauroy and J. Goncalves, “Koopman-based lifting techniques for nonlinear systems identification”, *IEEE Transactions on Automatic Control*, vol. 65, no. 6, pp. 2550–2565, 2019.

[10] B. O. Koopman, “Hamiltonian systems and transformation in Hilbert space”, *Proceedings of the National Academy of Sciences*, vol. 17, no. 5, pp. 315–318, 1931.

[11] A. Mauroy, Y. Susuki, and I. Mezić, *Koopman operator in systems and control*. Springer, 2020.

[12] S. L. Brunton, M. Budišić, E. Kaiser, and J. N. Kutz, “Modern Koopman theory for dynamical systems”, *SIAM Review*, vol. 64, no. 2, pp. 229–340, 2022.

[13] G. P. Agrawal, *Fiber-optic communication systems*. John Wiley & Sons, 2012.

[14] M. Brehler, C. Mahnke, S. Chimmalgi, and S. Wahls, “NFDMLab: Simulating nonlinear frequency division multiplexing in Python”, in *2019 Optical Fiber Communications Conference and Exhibition (OFC)*, IEEE, 2019, pp. 1–3.