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1 **Extended Strip Model for slabs subjected to load combinations**

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## 1 **Abstract**

2           The loads that are used for the assessment of existing reinforced concrete slab bridges are  
3 the self-weight, superimposed loads, and distributed and concentrated live loads. As such, the  
4 shear capacity of reinforced concrete slabs under a combination of distributed and concentrated  
5 live loads is a topic of practical relevance. For slabs subjected to a single concentrated load, a  
6 plastic model for assessment exists: the Extended Strip Model, developed based on the Strip  
7 Model for concentric punching shear. A further adaptation of the model to assess slabs subjected  
8 to distributed and concentrated loads is presented in this paper. The proposed model is compared  
9 to experiments on slabs subjected to a single concentrated load and a line load. The conclusion of  
10 this comparison is that the Extended Strip Model results in a safe estimate of the maximum  
11 concentrated load on the slab, and that the method can be used for the assessment of existing  
12 bridges subjected to heavy truck loads.

13

## 14 **Keywords**

15 Assessment; Extended Strip Model; Flexure; Live loads; Plasticity-based model; Punching;  
16 Reinforced concrete; Slab bridges; Shear

## 1 **1. Introduction**

### 2 *1.1 Assessment of existing bridges in the Netherlands*

3           As the average age of the existing bridges in many parts of the world is increasing, the  
4 importance of methods for the assessment of these existing bridges is increasing as well. A  
5 common bridge type in the Netherlands [1] is the reinforced concrete solid slab bridge. Many of  
6 these slab bridges were built between the late 1950s and the early 1980s. The loads that are used  
7 for assessment in the Netherlands are the self-weight of the structure, the superimposed load, and  
8 the live loads. The live loads are given in NEN-EN 1991-2:2003 [2] and consist of a design  
9 tandem in each lane, combined with a distributed lane load. For shear assessment, the capacity of  
10 both reinforced concrete beams and slabs is taken as the one-way shear strength given in NEN-  
11 EN 1992-1-1:2005 [3]. Typically, the evaluation is then expressed based on a Unity Check: a  
12 ratio of the resulting shear stress from the applied loads over the shear capacity. If the Unity  
13 Check is larger than 1, the evaluated bridge is considered as not fulfilling the requirements [4].  
14 For the existing reinforced concrete slab bridges, it is often found that the shear capacity is  
15 insufficient. Therefore, the shear capacity of reinforced concrete slab bridges has been a topic of  
16 research in the Netherlands for the past decade.

### 17 *1.2 Methods for one-way and two-way shear*

18           Reinforced concrete slab bridges subjected to concentrated loads such as the design  
19 tandem failing in shear are cases that are situated at the transition between one-way shear (beam  
20 shear) and two-way shear (punching shear) [5]. Traditionally, shear models are strictly  
21 subdivided into methods for one-way shear and two-way shear. The models for one-way shear  
22 are compared with experiments on beams in three- or four-point bending [6-8], whereas the  
23 models for two-way shear are compared with experiments on slab-column connections [9]. The

1 loading case of a reinforced concrete slab bridge subjected to the load combination used for  
2 assessment lies somewhere in between these situations.

3         The most commonly used models for one-way shear are semi-empirical formulas derived  
4 from analysing the existing beam shear experiments [6, 7]. The shear capacity prescribed by  
5 NEN-EN 1992-1-1:2005 [3] and ACI 318-14 [10] follows a semi-empirical formula. Another  
6 model that has a theoretical basis and that has been introduced into design codes is the Modified  
7 Compression Field Theory [11]. In this theory, cracked concrete is considered as a separate  
8 material with its own constitutive equations, derived from panel tests. A simplification of the  
9 theory [12] can be found in the AASHTO LRFD 2015 code [13] and the *fib* Model Code 2010  
10 [14].

11         For two-way shear, the most commonly used models are also semi-empirical formulas  
12 derived from the results of slab-column connection tests [9]. The punching shear capacity  
13 prescribed by NEN-EN 1992-1-1:2005 [3] and ACI 318-14 [10] is described by a semi-empirical  
14 formula. Improvements to the punching shear provisions from NEN-EN 1992-1-1:2005 have  
15 been suggested [15]. Another model that has a theoretical basis is the Critical Shear Crack  
16 Theory [16, 17]. This theory is the basis for the provisions in the Swiss Code SIA 262:2003 [18]  
17 and the *fib* Model Code 2010 [14]. Recently, a simplified punching shear model has proposed  
18 that is based on the Critical Shear Crack Theory [19].

19         A category of models that can be used for one-way and two-way shear are plasticity-  
20 based models, which can be subdivided in lower- and upper-bound methods. While plasticity-  
21 based methods for shear [20-22] are not directly found in design codes, plasticity-based methods  
22 are the basis of engineering tools such as strut-and-tie models for D-regions [23], the strip  
23 method for flexure [24, 25], and yield line analysis [26].

### 1 ***1.3 Experiments on slabs under a single concentrated load***

2 To study the behavior of reinforced concrete slabs under a single concentrated load close  
3 to the support, a number of laboratory experiments were carried out. This load configuration was  
4 chosen, as it represents the case with the design tandem close to the support, which results in the  
5 largest shear stress for assessment. The specimens were half-scale reinforced concrete slab  
6 specimens of 5 m × 2.5 m × 0.3 m with a span of 3.6 m, tested close to a simple and continuous  
7 support, to represent a continuous slab bridge. In total, 127 experiments on 18 specimens were  
8 carried out [27-30]. The parameters varied in these experiments were: the position of the load in  
9 the transverse direction, the position of the load in the longitudinal direction, the amount of  
10 transverse reinforcement, the effect of previous cracking, the size of the loading plate, the  
11 moment distribution at the support, the concrete compressive strength, the overall width (with  
12 2.5 m as a reference), the type of reinforcement (deformed bars as compared to plain bars), and  
13 the type of support (line supports as compared to elastomeric bearing blocks). The main  
14 conclusion of these experiments was that the three-dimensional load path in a reinforced  
15 concrete slab differs significantly from the two-dimensional load path in a reinforced concrete  
16 beam, and results in a larger shear capacity. This effect was also called the transverse load  
17 distribution capacity of slabs in shear [31]. This conclusion, and the experimental results, also  
18 led to the development of recommendations [1] for the assessment of reinforced concrete slab  
19 bridges when using the Eurocode provisions NEN-EN 1992-1-1:2005 [3] and NEN-EN 1991-  
20 2:2003 [2].

21

## 1 **2. Extended Strip Model for slabs under combinations of loads**

### 2 ***2.1 Extended Strip Model for slabs under a single concentrated load***

3           The Extended Strip Model for reinforced concrete slabs under a single concentrated load  
4 [32] is developed based on the Strip Model for concentric punching shear in slabs [33-35]. The  
5 Strip Model is a lower-bound plasticity-based model that describes a possible load path prior to  
6 failure. As such, it shares features with the Strip Method for designing slabs in flexure [24, 25].  
7 In slabs under concentrated loads, a complex loading situation of one-way shear, two-way shear,  
8 and flexure develops. This situation is reflected in the Strip Model by combining beam strips that  
9 work in arching action (an element of one-way shear) together with slab quadrants that work in  
10 two-way flexure. This principle is sketched in Figure 1, which shows a column with strips  
11 branching out from the column, and the resulting quadrants. The length of the strip  $l_{strip}$  is  
12 considered from the face of the column to a position of zero shear. The load path may function  
13 until a limiting one-way shear is reached at the interface between the strip and the quadrant. This  
14 limiting one-way shear is taken as the inclined cracking load given in ACI 318-14 [10]. The  
15 maximum load is then achieved by summing the capacities of the four strips, assuming that the  
16 limiting one-way shear is achieved on the interface between the strip and the quadrant. The  
17 maximum load that can be carried in the quadrants is thus  $w_{ACI}$ , the inclined cracking load given  
18 in ACI 318-14, see Figure 1.

19           The Extended Strip Model [32, 36, 37] extends the concepts of the Strip Model for  
20 application to slabs of a finite size, with a single concentrated load. This load can be placed at  
21 any position on the slab, so that the Extended Strip Model can study asymmetric loading  
22 situations. The model is well-suited to combine the effects of one-way shear, two-way shear, and  
23 flexure that govern the loading case of a reinforced concrete slab subjected to a concentrated

1 load. To take into account the finite dimensions of the slab, and possible asymmetric loading, it  
 2 is necessary to take into account the geometry of the slab, the bending moment and shear  
 3 diagrams, as well as the effect of torsion. The resulting Extended Strip Model is then as shown in  
 4 Figure 2. The effects of the geometry and asymmetry now influence the resulting one-way shear  
 5 at the intersection between the quadrants and strips. As a result, the capacity of each single strip  
 6 is different. Again, the maximum concentrated load is found by summing the capacities of the  
 7 strips.

8 Whereas the effect of torsion could be neglected in the original Strip Model that studied  
 9 only symmetric loading cases, it becomes more important for asymmetric loading cases. The  
 10 effect of torsion was studied in a series of linear finite element models in which the ratio between  
 11 bending moment and torsional moments was analyzed [38]. The result of this analysis is a  
 12 simplified expression for the relative effect of torsion:

$$13 \quad \beta = 0.8 \frac{a}{d_x} \frac{b_r}{b} \text{ for } 0 \leq \frac{a}{d_x} \leq 2.5 \text{ and } 0 \leq \frac{b_r}{b} \leq \frac{1}{2} \quad (1)$$

14 If the effect of torsion is at its largest, the value of  $\beta = 0$  and it is considered that all capacity is  
 15 used to resist the effects of torsion. If the effect of torsion is negligible, the value of  $\beta = 1$  and it  
 16 is considered that all capacity is available to develop the required load path to resist the shear  
 17 effects. When  $a/d_x > 2.5$ , the value of  $a/d_x$  in Eq. (1) is replaced by 2.5, and only the effect of the  
 18 position along the width direction on the torsional behavior remains. The strips influenced by  
 19 torsion carry the factor  $\beta$  in Figure 2.

20 For loads close to the support, the effect of direct load transfer between the load and the  
 21 support is taken into account by increasing the capacity of the strip between the load and the  
 22 support. For loads close to the free edge, the physical length of the strip  $l_{edge}$  needs to be

1 compared to the loaded length of the strip  $l_w$ . If the loaded length is longer than the actual strip  
 2 length, then the strip length instead of the loaded length should be used. This influence of the  
 3 geometry is called the edge effect.

4 The effect of the overall bending moment diagram is reflected in Figure 2 by using the  
 5 distance between the points of contraflexure  $L$  and the distance  $a_M$ , which is the smallest of the  
 6 distance between the load and the support, or the distance between the load and the point of  
 7 contraflexure. The effect of the self-weight of the slab, which becomes important for the  
 8 assessment of slab bridges, is taken into account on the shear diagram by considering the stress  
 9  $v_{DL}$  of the dead load caused at the position of the concentrated load. Additionally, the Extended  
 10 Strip Model includes the size effect in shear on the limiting shear stress  $w_{ACI}$ . This limiting shear  
 11 stress is calculated differently for the  $x$ - and  $y$ -directions of the slab, to take into account the  
 12 different value of the effective depth depending on the layer of reinforcement that is considered.  
 13 Therefore, Figure 2 uses  $w_{ACI,x}$  and  $w_{ACI,y}$  for the different directions.

14 In the Extended Strip Model, the total maximum concentrated load  $P_{ESM}$  is calculated as:

$$15 \quad P_{ESM} = P_x + P_{sup} + P_y + P_{edge} \quad (2)$$

$$16 \quad P_x = \sqrt{2(1+\beta)} M_{sag,x} w_{ACI,x} \quad (3)$$

$$17 \quad P_{sup} = \frac{2d_x}{a_v} \sqrt{2(1+\beta)} M_{s,x} w_{ACI,x} \quad (4)$$

$$18 \quad P_y = \sqrt{2 \left( \frac{L}{L - a_M} \right)} M_{s,y} (w_{ACI,y} - v_{DL}) \quad (5)$$

$$P_{edge} = \begin{cases} \sqrt{2\beta \left( \frac{L}{L-a_M} \right) M_{s,y} (w_{ACI,y} - v_{DL})} & \text{for } l_w < l_{edge} \\ \beta \left( \frac{L}{L-a_M} \right) (w_{ACI,y} - v_{DL}) l_{edge} & \text{for } l_w \geq l_{edge} \end{cases} \quad (6)$$

2 The loaded length of the strip is determined as:

$$l_w = \sqrt{\frac{2M_{s,y}}{\beta (w_{ACI,y} - v_{DL}) \frac{L}{L-a_M}}} \quad (7)$$

4 The moment capacities are determined as:

$$M_{s,x} = M_{sag,x} + \lambda_{moment} M_{hog,x} \quad (8)$$

$$M_{s,y} = M_{sag,y} + \lambda_{moment} M_{hog,y} \quad (9)$$

7 with:

$$\lambda_{moment} = \frac{M_{sup}}{M_{span}} \quad (10)$$

9 and  $M_{sup}$  and  $M_{span}$  follow from the moment diagram of the slab subjected to all loads. At a  
 10 simple support, the value of  $\lambda_{moment}$  becomes 0, and the moment capacities from Eqs. (8) and (9)  
 11 become the sagging moment capacities  $M_{sag,x}$  and  $M_{sag,y}$ .

12 The one-way shear capacity is calculated based on ACI 318-14 [10], but a correction for  
 13 the size effect has been added [39]:

$$w_{ACI,x} = 0.166 d_y \sqrt{f_{ck}} \left( \frac{100\text{mm}}{d} \right)^{\frac{1}{3}} \quad (11)$$

$$w_{ACI,y} = 0.166 d_x \sqrt{f_{ck}} \left( \frac{100\text{mm}}{d} \right)^{\frac{1}{3}} \quad (12)$$

1 In Figure 2, the resulting loads are shown when the effects of the geometry, torsion, the acting  
 2 dead load, the static equilibrium, the position of the point of contraflexure, and the size effect are  
 3 taken into account.

#### 4 **2.2 Application to slabs under combinations of loads**

5 When a slab is subjected to a combination of loads the Extended Strip Model can be used  
 6 as well. When only a single tandem is used, the Extended Strip Model can be used by taking the  
 7 perimeter of the four considered wheel prints, and considering this area as one large concentrated  
 8 load from which the strips and quadrants are developed. Based on a field experiment on the  
 9 Ruytenschildt Bridge, which was tested to failure [40], it was shown that this application of the  
 10 Extended Strip Model results in a safe prediction of the maximum load in the test [36].

11 When a slab is subjected to a combination of concentrated and distributed loads, for  
 12 example as used in the live load model from NEN-EN 1991-2:2003 [2], the Extended Strip  
 13 Model can be used as well. The effect of the distributed load can now be taken into account in  
 14 the span direction as a reduction of the shear capacity. This effect of the distributed load is  
 15 represented by the shear stress caused by the distributed load at the position of the concentrated  
 16 load,  $v_{dist}$ . As a result, the loading on the quadrants and strips becomes as shown in Figure 3.  
 17 Since the effect of the distributed load is only considered in the span direction, only the values of  
 18  $P_y$  and  $P_{edge}$  from Eqs. (5) and (6) are changed for this application of the Extended Strip Model:

$$19 \quad P_y = \sqrt{2 \left( \frac{L}{L - a_M} \right) M_{s,y} (w_{ACI,y} - v_{DL} - v_{dist})} \quad (13)$$

$$P_{edge} = \begin{cases} \sqrt{2\beta \left( \frac{L}{L-a_M} \right) M_{s,y} (w_{ACI,y} - v_{DL} - v_{dist})} & \text{for } l_w < l_{edge} \\ \beta \left( \frac{L}{L-a_M} \right) (w_{ACI,y} - v_{DL} - v_{dist}) l_{edge} & \text{for } l_w \geq l_{edge} \end{cases} \quad (14)$$

As a result, the loaded length of the strip between the load and the support is now determined as:

$$l_w = \sqrt{\frac{2M_{s,y}}{\beta (w_{ACI,y} - v_{DL} - v_{dist}) \frac{L}{L-a_M}}} \quad (15)$$

An overview of these changes to the model is represented by the loads on the strips and quadrants shown in Figure 3.

### 3. Experiments on slabs under combinations of loads

#### 3.1 Test setup

To assess the behavior of slabs under a combination of loads, representative of the load combination used for the assessment of reinforced concrete slab bridges, experiments were carried out [41]. The tested specimens were eight slabs in total, each with the same size of 5 m × 2.5 m × 0.3 m. In total, 23 experiments were carried out on these slabs, with two or four tests carried out per slab depending on the loading configuration. The load combination used for the assessment of reinforced concrete slab bridges consists of the self-weight, the superimposed dead load, and distributed and concentrated live loads. Since the application of a uniformly distributed load in a laboratory setting in combination with concentrated loads becomes complex, a simplified loading scheme was used for these experiments. A single concentrated load close to

1 the support (as used in the first series of experiments described in §1.3) was combined with a line  
2 load acting over the full width of the slab, as can be seen in Figure 4.

3 In the experiments, the line load was applied in force-controlled manner first. Then, the  
4 concentrated load was increased in a displacement-controller manner until failure of the slab.  
5 The maximum value of line load was 240 kN/m. This load was calculated as the load causing  
6 50% of the failure shear stress at the support as determined in experiments on wide beams [28].  
7 The basic assumption here was that the behavior of a slab subjected to a line load would be  
8 similar to the behavior of a beam subjected to a concentrated load [42]. However, the behavior of  
9 a slab subjected to a line load and a concentrated load was unknown when preparing these  
10 experiments.

11 Two types of supports were used for the experiments: steel bearings or elastomeric  
12 bearings. For some specimens, a steel strip of 100 mm wide was used. As a result, the value of  
13 the support width  $b_{sup}$  changes, see Table 1.

14 A test was carried out at the simple support (sup 1 in Figure 4) as well as at the  
15 continuous support (sup 2 in Figure 4) when the load was placed in the middle ( $b_r = 1250$  mm).  
16 Two tests were carried out at each support when the load was placed close to the edge ( $b_r = 438$   
17 mm). Whereas the slab specimen only had one span, it was built to represent continuous slab  
18 bridges. Therefore, prestressing bars coupled to the strong floor of the laboratory were used to  
19 create a moment over support 2, creating the moment distribution of a continuous slab, as shown  
20 in Figure 5. The moment diagram in Figure 5 is also used to show the difference between the  
21 distances  $a$ ,  $a_M$ ,  $L$  and  $l_{span}$ .

1           The standard span length is 3.6 m, as shown in Figure 4. For a limited number of  
2 experiments, a temporary support was used to test at the continuous support, as testing at the  
3 simple support had resulted in large damage to the slab.  
4

### 5 **3.2 Specimens**

6           The concrete used in the specimens was delivered by truck mixer. The concrete quality  
7 C28/35 was used. Glacial river aggregates with a maximum aggregate size of 16 mm were used.  
8 The concrete compressive strength was measured in the laboratory on cubes. For the conversion  
9 to the cylinder compressive strength, a factor 0.82 was used [43], as recommended for the  
10 assessment of reinforced concrete slab bridges in the Netherlands. The resulting concrete  
11 compressive strengths of the individual specimens can be found in Table 1.

12           The reinforcement layout of the slabs is shown in Figure 6. All bars were deformed bars  
13 of steel quality S500. The measured yield strength of the  $\emptyset = 20$  mm bars was 542 MPa and of  
14 the  $\emptyset = 10$  mm bars  $f_{ym} = 537$  MPa. For all specimens, the longitudinal reinforcement ratio was  
15  $\rho_{x,sag} = 0.996\%$  and the transverse reinforcement ratio was  $\rho_{y,sag} = 0.258\%$ .

### 16 **3.3 Results**

17           The results of the 20 experiments are given in Table 1. In this table, the position of the  
18 load is indicated with CS/SS (testing at the continuous or simple support),  $a$ , the center-to-center  
19 distance between the load and the support, and  $b_r$ , which equals 1.25 m when the concentrated  
20 load is applied in the middle of the width, or 0.438 m when the concentrated load is applied close  
21 to the free edge - see Figure 4 for the two positions of the load. The result of the experiment is  
22 expressed as  $P_{conc}$ , the maximum value of the concentrated load, and  $v_{line}$ , the distributed load  
23 applied by the line load. The failure mode is either “B”, a beam shear failure with a clear shear

1 crack on the side face of the slab, or “WB”, a wide beam shear failure for which the crack is  
 2 inside the slab, and inclined cracks indicating shear stress can be observed on the bottom face of  
 3 the slab. These failure modes are shown in Figure 7. For all experiments, a loading plate of 300  
 4 mm  $\times$  300 mm was used, except for S20T2b, where a loading plate of 200 mm  $\times$  200 mm was  
 5 used.

#### 6 **4. Comparison between experiments and Extended Strip Model**

7 To verify the proposed Extended Strip Model and its application to slabs subjected to  
 8 concentrated and distributed loads, the maximum concentrated load  $P_{conc}$  from experiments from  
 9 Table 1 are calculated with the Extended Strip Model,  $P_{ESM}$ . The value of  $P_{ESM}$  is determined as  
 10 given in Eq. (2), with  $P_y$  and  $P_{edge}$  as given in Eqs. (13) and (14). The results of all calculations,  
 11 with the formulas as outlined in §2.2, are given in Table 2. A beam diagram is used to find the  
 12 moment and shear diagrams along the span direction of the slab. Based on this moment diagram,  
 13 the value of  $\lambda$  is determined. For example, for S24T2 the support moment is 188 kNm and the  
 14 span moment at the position of the concentrated load is 695 kNm, as can be seen in Figure 5. As  
 15 a result,  $\lambda = 188\text{kNm}/695\text{kNm} = 0.27$ . The effect of torsion is taken into account with the factor  
 16  $\beta$ , see Eq. (1), which equals 1 if the effect of torsion is negligible and which approaches 0 as the  
 17 effect of torsion increases. The value of the loaded length of the strip  $l_w$  is determined as given in  
 18 Eq. (15). The capacity of the  $x$ -direction strip between the load and the support is determined as  
 19  $P_{sup}$ , according to Eq. (4). The capacity for the  $x$ -direction strip between the load and the position  
 20 of zero shear,  $P_x$  is not affected by the formation of a direct strut, and is determined according to  
 21 Eq. (3). The capacity of the  $y$ -direction strip between the edge and the load is affected by torsion  
 22 and the edge effect, and is determined as given in Eq. (14). The capacity of the  $y$ -direction strip

1 between the load and the far side of the slab is determined as given in Eq. (13). Then, the  
2 capacity of the four strips is determined, and summed to find  $P_{ESM}$ , see Eq. (2). It can be seen  
3 that, as a result of the direct strut that forms between the load and the support for concentrated  
4 loads close to the support, the value of  $P_{sup}$  is larger than the value of  $P_x$ . For the experiments  
5 with a concentrated load close to the free edge, the value of  $P_{edge}$  becomes significantly smaller  
6 than the value of  $P_y$ .

7 As can be seen in Table 2, all predicted values of the maximum concentrated load are  
8 conservative estimates; all values of  $P_{conc}/P_{ESM}$  are larger than one. The mean value (AVG) of  
9  $P_{conc}/P_{ESM}$  equals 1.47. The standard deviation (STD) is 0.18, which results in a coefficient of  
10 variation (COV) of 12.5%. Given the complexity of the problem, which is a combination of one-  
11 way shear, two-way shear, and two-way flexure, the obtained value of the coefficient of variation  
12 is acceptable, especially since the presented method allows for a quick estimate of the maximum  
13 load with a hand calculation. The characteristic value (5% lower bound, assuming a normal  
14 distribution) equals 1.17, as would be expected from a lower-bound method. It can thus be  
15 concluded that the method is suitable for design and assessment purposes.

16 The comparison between the tested and predicted results is shown graphically in Figure  
17 8. From this figure, it can be seen that the general trend of the data follows a line that is parallel  
18 to the 45° line that is drawn in Figure 8. From Figure 8, it can be concluded as well that the  
19 Extended Strip Model provides a safe lower bound estimate of the maximum concentrated load  
20 on a reinforced concrete slab subjected to a combination of a concentrated load and a distributed  
21 line load. The actual distribution of the tested to predicted results is shown in a histogram in  
22 Figure 9. From the cumulative distribution, it can be found that the 5% lower bound of  $P_{conc}/P_{ESM}$

1 equals 1.12, which is similar to the value that was found based on the assumption of a normal  
2 distribution.

3

#### 4 **5. Discussion**

5 Previous research [36] has shown that the Extended Strip Model can be used for  
6 reinforced concrete slab bridges subjected to a single tandem. The current research shows that  
7 the Extended Strip Model can be used for reinforced concrete slab bridges subjected to a  
8 concentrated load and a distributed load. Extrapolating the results from the previous research  
9 makes it likely that the Extended Strip Model can be applied to reinforced concrete slab bridges  
10 subjected to a single tandem and the distributed loads. For these distributed loads, the effect of  
11 the load on the strips would be taken into account for the  $y$ -direction strips in the same way  $v_{DL}$  is  
12 accounted for in Figure 2. As such, the proposed method can be used for the assessment of  
13 bridges with a limited width, for estimating the maximum load that can be used in proof load  
14 testing, and for the assessment of superloads. For bridges with a limited width of a single lane,  
15 the loading combination of a single tandem and the distributed loads is the load combination  
16 required for assessment. For proof load testing [44], a single tandem is applied during the proof  
17 load test, and the distributed loads of the self-weight and the superimposed dead loads remain  
18 acting on the structure. Similarly, for the assessment of superloads, the superload can be  
19 simplified into a large surface of a concentrated load. The bridge then is subjected to this  
20 concentrated load, and the distributed loads of the self-weight of the bridge and the  
21 superimposed dead load.

1           The currently proposed method gives a lower bound of the maximum concentrated load.  
2       Since the method is based on the lower-bound theorem of plasticity, conservative results are  
3       expected. Moreover, in the derivation of the effect of torsion and other loads, conservative  
4       approaches were used. The goal of the developed method is to be able to estimate a maximum  
5       load with a quick hand calculation. For more precise results, it is recommended to use more  
6       advanced methods, such as nonlinear finite element models.

7           Currently, the proposed method cannot yet be extended to the use of multiple tandems  
8       staggered in different lanes. For this application, further research is required to evaluate how the  
9       tandems can be joined in the Extended Strip Model. However, no experimental results are  
10      available to compare the Extended Strip Model to this loading type.

11

## 12   **6. Summary and conclusions**

13           For the shear assessment of reinforced concrete slab bridges, a load combination  
14      consisting of permanent loads and live loads is used. The permanent loads are distributed loads,  
15      whereas the live loads are a combination of distributed lane loads, sometimes with different  
16      values for the distributed load for each lane, and concentrated loads that represent concentrated  
17      truck loads. This loading case represents a complex case, combining one-way shear, two-way  
18      shear, and two-way flexure.

19           To safely estimate the maximum concentrated load that can be applied to a reinforced  
20      concrete slab, representing a reinforced concrete slab bridge, the Extended Strip Model was  
21      developed. The Extended Strip Model combines strips working in arching action (one-way

1 shear) with quadrants working in two-way flexure, and shows a possible load path prior to the  
2 collapse state of the slab. It is a lower-bound plasticity-based method.

3 In the presented research, the Extended Strip Model is extended further to estimate the  
4 maximum concentrated load for the case of a reinforced concrete slab subjected to a concentrated  
5 load and distributed loads. This loading situation was used, as experiments on reinforced  
6 concrete slabs, representing reinforced concrete slab bridges, subjected to a concentrated load  
7 close to the support and a line load acting over the full slab width are available for comparison.  
8 The main features of the test setup, properties of the eight specimens, and results of the twenty  
9 experiments are repeated in this paper for convenience.

10 To evaluate the performance of the proposed changes to the Extended Strip Model for the  
11 application to a combination of a concentrated load and a distributed load, the experimental  
12 results were compared to the predicted values with the Extended Strip Model. This comparison  
13 showed that the Extended Strip Model leads to conservative estimates for the maximum  
14 concentrated load. Given that the proposed method is an easy-to-use hand calculation, it can be  
15 used to have a quick estimate of the maximum concentrated load for bridges with a single lane,  
16 in the case of proof load testing, and for the passing of a superload.

## 17 **Acknowledgements**

18 The authors wish to express their gratitude and sincere appreciation to the Dutch Ministry of  
19 Infrastructure and the Environment (Rijkswaterstaat) for financing this research work. The  
20 discussions with Dr. S. Alexander are gratefully acknowledged.

## 21 **List of notation**

22  $a$  center-to-center distance between load and support

1	$a_M$	center-to-center distance between load and support or between load and point of
2		contraflexure, whichever is smaller
3	$a_v$	face-to-face distance between load and support
4	$b$	slab width
5	$b_r$	distance between free edge and center of load along the width direction
6	$b_{sup}$	width of the support
7	$d$	average of $d_x$ and $d_y$
8	$d_x$	effective depth to the $x$ -direction reinforcement
9	$d_y$	effective depth to the $y$ -direction reinforcement
10	$f_{ck}$	characteristic concrete compressive strength
11	$f_{cm}$	average concrete compressive cylinder strength
12	$f_{ym}$	average steel yield strength
13	$l_{edge}$	length of the strip between the load and the edge
14	$l_{span}$	span length
15	$l_w$	loaded length of the strip
16	mode	failure mode
17	$q_{self}$	distributed load caused by self-weight
18	$v_{dist}$	shear stress caused by the distributed load
19	$v_{DL}$	shear stress caused by the dead load
20	$v_{line}$	applied line load over the width of the slab
21	$w_{ACI}$	one-way shear capacity given by ACI 318-14
22	$w_{ACI,x}$	one-way shear capacity based on $d_x$ given by ACI 318-14
23	$w_{ACI,y}$	one-way shear capacity based on $d_y$ given by ACI 318-14

1	$x$	position along span length
2	B	beam shear failure
3	CS	continuous support
4	$F_{pres}$	load caused by prestressing bars coupling the slab to the strong floor of the laboratory
5	$L$	distance between points of contraflexure
6	$M$	bending moment
7	$M_{hog,x}$	hogging moment capacity in the $x$ -direction
8	$M_{hog,y}$	hogging moment capacity in the $y$ -direction
9	$M_{s,x}$	moment capacity in the $x$ -direction
10	$M_{s,y}$	moment capacity in the $y$ -direction
11	$M_{sag,x}$	sagging moment capacity in the $x$ -direction
12	$M_{sag,y}$	sagging moment capacity in the $y$ -direction
13	$M_{span}$	sagging moment in the span caused by all loads on the slab
14	$M_{sup}$	hogging moment over the support caused by all loads on the slab
15	$P_{conc}$	maximum load at the concentrated load in the experiments
16	$P_{edge}$	capacity of strip between load and free edge
17	$P_{ESM}$	maximum load according to the Extended Strip Model
18	$P_{line}$	resultant of line load, maximum value
19	$P_{sup}$	capacity of strip between load and support
20	$P_x$	capacity of a strip in the $x$ -direction
21	$P_y$	capacity of a strip in the $y$ -direction
22	SS	simple support
23	WB	wide beam shear failure

- 1  $\beta$  effect of torsion
- 2  $\rho_{x,sag}$  reinforcement ratio of the main flexural sagging moment reinforcement
- 3  $\rho_{y,sag}$  reinforcement ratio of the transverse flexural sagging moment reinforcement
- 4

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- 19

## 1 List of tables and figures

## 2 List of Tables

## 3 Table 1 – Overview of experimental results

Test		$l_{span}$ (m)	$f_{cm}$ (MPa)	$a$ (m)	$b_r$ (m)	$b_{sup}$ (m)	mode	$P_{conc}$ (kN)	$v_{line}$ (kN/m)
<b>S20T1</b>	SS	3.6	49.62	0.60	1.250	0.28	B	1542	241.2
<b>S20T2b</b>	CS	2.4	49.62	0.60	1.250	0.28	WB	1552	240.4
<b>S20T3</b>	CS	2.4	49.62	0.60	0.438	0.28	WB + B	1337	240.4
<b>S20T4</b>	CS	2.4	49.62	0.60	0.438	0.28	WB + B	1449	240.4
<b>S21T1</b>	CS	3.6	46.54	0.60	1.250	0.10	WB + B	1165	240.8
<b>S21T2</b>	SS	3.6	46.54	0.60	1.250	0.10	WB + B	1386	241.2
<b>S22T1</b>	CS	3.6	47.54	0.60	0.438	0.10	WB + B	984	240.8
<b>S22T2</b>	CS	3.6	47.54	0.60	0.438	0.10	WB + B	961	240.8
<b>S22T3</b>	SS	3.6	47.54	0.60	0.438	0.10	WB + B	978	241.2
<b>S22T4</b>	SS	3.6	47.54	0.60	0.438	0.10	WB + B	895	241.6
<b>S23T1</b>	CS	3.6	48.27	0.60	1.250	0.28	WB + B	1386	240.4
<b>S23T2</b>	SS	3.6	48.27	0.60	1.250	0.28	WB + B	1132	240.8
<b>S24T1</b>	CS	3.6	48.27	0.60	0.438	0.28	WB + B	1358	240.4
<b>S24T2</b>	CS	3.6	48.27	0.60	0.438	0.28	WB + B	1182	240.4
<b>S24T3</b>	SS	3.6	48.27	0.60	0.438	0.28	WB + B	995	240.8
<b>S24T4</b>	SS	3.6	48.27	0.60	0.438	0.28	WB + B	784	240.8
<b>S25T2</b>	CS	3.6	48.03	0.40	1.250	0.10	WB + B	1620	240.4
<b>S25T3</b>	CS	3.6	48.03	0.40	0.438	0.10	WB + B	1563	240.8
<b>S26T1</b>	SS	3.6	48.03	0.42	0.438	0.10	WB + B	1448	240.8
<b>S26T2</b>	SS	3.6	48.03	0.42	0.438	0.10	B	1324	240.8
<b>S26T3</b>	CS	3.6	48.03	0.40	1.250	0.10	WB + B	1555	240.8
<b>S26T4</b>	CS	3.6	48.03	0.40	0.438	0.10	B	1363	240.8
<b>S26T5</b>	CS	3.6	48.03	0.40	0.438	0.10	WB + B	1451	240.8

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1 **Table 2** – Comparison between test results and maximum load predicted with the Extended Strip

2 Model

Test	$P_{conc}$ (kN)	$\lambda$	$\beta$	$l_w$ m	$P_x$ kN	$P_{sup}$ kN	$P_y$ kN	$P_{edge}$ kN	$P_{ESM}$ kN	$P_{conc}/P_{ESM}$
<b>S20T1</b>	1542	0.00	0.91	0.877	294	503	61	58	917	1.682
<b>S20T2b</b>	1552	0.73	0.91	0.728	240	465	85	81	872	1.781
<b>S20T3</b>	1337	0.81	0.32	1.545	245	562	106	11	924	1.447
<b>S20T4</b>	1449	0.72	0.32	1.502	245	549	104	11	909	1.595
<b>S21T1</b>	1165	0.33	0.91	1.161	289	441	61	55	847	1.376
<b>S21T2</b>	1386	0.00	0.91	0.955	289	383	56	53	781	1.774
<b>S22T1</b>	984	0.37	0.32	1.949	242	375	64	5	685	1.436
<b>S22T2</b>	961	0.36	0.32	1.942	242	373	63	5	684	1.406
<b>S22T3</b>	978	0.00	0.32	1.582	242	320	57	6	625	1.565
<b>S22T4</b>	895	0.00	0.32	1.581	242	320	57	6	625	1.432
<b>S23T1</b>	1386	0.27	0.91	1.085	292	562	63	60	977	1.419
<b>S23T2</b>	1132	0.00	0.91	0.918	292	499	58	56	905	1.251
<b>S24T1</b>	1358	0.27	0.32	1.833	243	468	63	6	779	1.744
<b>S24T2</b>	1182	0.27	0.32	1.834	243	468	63	6	779	1.518
<b>S24T3</b>	995	0.00	0.32	1.553	243	415	58	6	722	1.378
<b>S24T4</b>	784	0.00	0.32	1.547	243	415	59	6	722	1.085
<b>S25T2</b>	1620	0.43	0.60	1.486	267	848	63	36	1215	1.333
<b>S25T3</b>	1563	0.43	0.21	2.512	232	736	63	3	1035	1.511
<b>S26T1</b>	1448	0.00	0.22	1.952	233	562	56	4	855	1.693
<b>S26T2</b>	1324	0.00	0.22	1.949	233	562	56	4	855	1.548
<b>S26T3</b>	1555	0.53	0.60	1.544	267	877	65	36	1245	1.249
<b>S26T4</b>	1363	0.62	0.21	2.685	232	783	67	3	1085	1.256
<b>S26T5</b>	1451	0.58	0.21	2.653	232	774	66	3	1076	1.349
									AVG	1.471
									STD	0.184
									COV	0.125

3

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1 **List of Figures**

2 **Figure 1** – Overview of strips and quadrants [33].

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