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DOI

[10.3997/2214-4609.201700680](https://doi.org/10.3997/2214-4609.201700680)

Publication date

2017

Document Version

Accepted author manuscript

Published in

79th EAGE Conference and Exhibition 2017

Citation (APA)

Holicki, M., Drijkoningen, G., & Wapenaar, K. (2017). Snapshot wavefield decomposition for heterogeneous velocity media. In *79th EAGE Conference and Exhibition 2017: Paris, France, 12-15 June 2017* Article Th-P1-13 EAGE. <https://doi.org/10.3997/2214-4609.201700680>

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Snapshot wavefield decomposition for heterogeneous velocity media

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January 13, 2017

Summary

We propose a novel directional decomposition operator for wavefield snapshots in heterogeneous-velocity media. The proposed operator demonstrates the link between the amplitude of pressure and particle-velocity plane waves in the wavenumber domain. The proposed operator requires two spatial Fourier transforms (one forward and one backward) per spatial dimension and time slice. To illustrate the operator we demonstrate its applicability to heterogeneous velocity models using a simple velocity-box model and a more heterogeneous velocity model, based on real data, from close to the Annerveen gas field, The Netherlands.

Introduction

An important part of many seismic imaging steps is the decomposition of wavefields according to their propagation direction; examples include noise removal, redatuming and inversion techniques. In the industry it is common to decompose wavefields entering and exiting a surface, allowing for up-down wavefield decomposition on receiver arrays. More recently it has become an integral part in subsurface imaging using Reverse Time Migration (RTM), as it can reduce artifacts introduced by the RTM migration operator (Díaz and Sava, 2015). This led to the introduction of Poynting decomposition, by Yoon and Marfurt (2006), as an alternative to the computationally expensive plane-wave decomposition. Holicki et al. (2016) developed a novel exact decomposition alternative for homogeneous media, that circumvents Poynting decomposition's inaccuracies due to wavefield interference. This work now generalizes the previous work to arbitrarily heterogeneous velocity models, illustrating its usefulness for wavefield decomposition.

Theory

The idea behind wavefield decomposition is to scale wavefield quantities to each other such that their addition or subtraction perfectly removes the undesired part of the wavefield. For directional wavefield decomposition this means scaling the pressure and particle velocity fields to each other and adding them such that the only waves remaining propagate in the desired direction. Historically this was done in the wavenumber-frequency or space-frequency domain. However we now wish to derive space-time and wavenumber-time expressions, such that the decomposition can be applied on time slices.

To derive time-space domain acoustic wavefield decomposition operators for inhomogeneous velocity but constant density acoustic models, we begin with the source-free isotropic inhomogeneous linearized equations of continuity and motion, in the time-space domain (Aki and Richards, 2002):

$$\partial_t p = -\rho c^2 \vec{\nabla} \cdot \vec{v} \quad (1a)$$

$$\partial_t \vec{v} = -\frac{1}{\rho} \vec{\nabla} p \quad (1b)$$

where ∂_t is the temporal differentiation operator, p is the pressure, ρ is the density, c is the heterogeneous propagation velocity, $\vec{\nabla}$ is the vector of spatial derivatives and \vec{v} is the particle-velocity vector.

The above expressions contain temporal derivatives ∂_t . To allow us to decompose wavefields on snapshots we wish to remove these derivatives by expressing ∂_t in terms of spatial operators. To this end let us derive the acoustic wave equation for pressure by inserting Equation 1b into Equation 1a, while assuming constant density:

$$\left(\frac{1}{c}\partial_t\right)^2 p = \Delta p \quad (2)$$

where $\Delta = \vec{\nabla} \cdot \vec{\nabla}$ is used to denote the Laplacian. We now define the square root of the Laplacian Δ as:

$$\Delta = \sqrt{\Delta}\sqrt{\Delta} \quad (3)$$

where $\sqrt{\Delta}$ is a pseudo-differential operator. Inserting this definition into Equation 2 we can define an equivalent temporal differentiation operator ∂'_t when acting on the pressure p as:

$$\partial'_t = c\sqrt{\Delta} \quad (4)$$

We now have the necessary operator to derive decomposition operators. To derive expressions for up-down wavefield decomposition we now wish to interrelate the pressure and vertical particle velocity. To that end let us express p in terms of v_z using Equation 1b:

$$p = -\partial_z^{-1} \rho \partial_t v_z \quad (5)$$

Note that ∂_z^{-1} takes the vertical primitive of the function it acts on, assuming the constant of integration to be zero. Furthermore, in this work operators are assumed to operate on the entire following expression. This is a valid assumption as equivalent assumptions were made when deriving Equation 1.

Inserting Equation 5 into the right-hand side of Equation 2, integrating both sides with respect to time and substituting for $c^2\Delta$ via Equation 4 we find:

$$\partial_t p = -\partial_t'^2 \partial_z^{-1} \rho v_z \quad (6)$$

Analogous to Wapenaar and Berkhout (1989) we can combine Equation 6 with Equation 1b to find the following linear set of equations:

$$\partial_t \begin{pmatrix} p \\ v_z \end{pmatrix} = \begin{pmatrix} 0 & -\partial_t'^2 \partial_z^{-1} \rho \\ -\frac{1}{\rho} \partial_z & 0 \end{pmatrix} \begin{pmatrix} p \\ v_z \end{pmatrix} \quad (7)$$

We can eigenvalue decompose the above; we choose to pressure-normalize the decomposition:

$$\partial_t \begin{pmatrix} p \\ v_z \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ \frac{1}{\rho} \partial_z \partial_t'^{-1} & -\frac{1}{\rho} \partial_z \partial_t'^{-1} \end{pmatrix} \begin{pmatrix} -\partial_t' & 0 \\ 0 & \partial_t' \end{pmatrix} \frac{1}{2} \begin{pmatrix} 1 & \partial_t' \partial_z^{-1} \rho \\ 1 & -\partial_t' \partial_z^{-1} \rho \end{pmatrix} \begin{pmatrix} p \\ v_z \end{pmatrix} \quad (8)$$

It can be shown that the last matrix term, including the $1/2$ scale factor, corresponds to matrix decomposition along the vertical z . We may thus write:

$$\begin{pmatrix} p^\downarrow \\ p^\uparrow \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 1 & \partial_t' \partial_z^{-1} \rho \\ 1 & -\partial_t' \partial_z^{-1} \rho \end{pmatrix} \begin{pmatrix} p \\ v_z \end{pmatrix} \quad (9)$$

To better understand the decomposition we consider the operator term in the time-wavenumber (t, \vec{k}) -domain governed by the wavenumber vector \vec{k} . We may then write:

$$\mathfrak{F} \left\{ \frac{1}{c} \partial_t' \partial_z^{-1} v_z \right\} = \sqrt{\frac{\vec{k} \cdot \vec{k}}{k_z^2}} \tilde{v}_z = \sqrt{1 + \frac{\vec{k}_H \cdot \vec{k}_H}{k_z^2}} \tilde{v}_z \quad (10)$$

Note that tildes above symbols are used to indicate the wavenumber domain, \mathfrak{F} is the spatial Fourier transform and \vec{k}_H is the horizontal wavenumber vector. Both wavenumber domain expressions have instabilities as $k_z \rightarrow 0$. To derive a stable operator we interrelate wavenumbers and particle velocities via Equation 1b. We can solve Equation 1b in the (t, \vec{k}) domain for the pressure \tilde{p} in terms of any component i or j of $\tilde{\vec{v}}$, which allows us to relate ratios of components of $\tilde{\vec{v}}$ to corresponding wavenumber ratios:

$$\frac{k_i}{k_j} = \frac{\tilde{v}_i}{\tilde{v}_j} \quad (11)$$

Before inserting the above into Equation 10, we express $k_i k_i$ as $k_i k_i^*$, using the fact that k is real, allowing us to write:

$$\sqrt{\frac{\vec{k} \cdot \vec{k}}{k_z^2}} \tilde{v}_z = \sqrt{\frac{\vec{k} \cdot \vec{k}^*}{k_z k_z^*}} \tilde{v}_z = \sqrt{\frac{\tilde{\vec{v}} \cdot \tilde{\vec{v}}^*}{\tilde{v}_z \tilde{v}_z^*}} \tilde{v}_z = \frac{|\tilde{\vec{v}}|}{|\tilde{v}_z|} |\tilde{v}_z| e^{i \arg(\tilde{v}_z)} = |\tilde{\vec{v}}| e^{i \arg(\tilde{v}_z)} \quad (12)$$

This expression is unconditionally stable.

Equation 12 can easily be generalized to decomposition along any direction, via a rotation of the coordinate system, e.g. in two dimensions rotation of the coordinate system counterclockwise by ϕ with respect to the horizontal axis:

$$|\tilde{\vec{v}}| e^{i \arg\{\cos(\phi)\tilde{v}_x - \sin(\phi)\tilde{v}_z\}} \quad (13)$$

Now the decomposition operator decomposes in the direction prescribed by ϕ , e.g. for $\phi = 0$ the operator decomposes into left-right-going pressure normalized wavefields. Now that we understand how to interpret the decomposition operators let us write the up-down pressure- and vertical-particle-velocity-normalized decomposed wavefields as:

$$p^\downarrow = \frac{1}{2} \left[p + \rho c \mathfrak{F}^{-1} \left\{ |\tilde{\vec{v}}| e^{i \arg(\tilde{v}_z)} \right\} \right] \quad (14a) \quad v_z^\downarrow = \frac{1}{2} \left[v_z + \frac{1}{\rho c} \mathfrak{F}^{-1} \left\{ \frac{|\tilde{v}_z|}{|\tilde{\vec{v}}|} \tilde{p} \right\} \right] \quad (15a)$$

$$p^\uparrow = \frac{1}{2} \left[p - \rho c \mathfrak{F}^{-1} \left\{ |\tilde{\vec{v}}| e^{i \arg(\tilde{v}_z)} \right\} \right] \quad (14b) \quad v_z^\uparrow = \frac{1}{2} \left[v_z - \frac{1}{\rho c} \mathfrak{F}^{-1} \left\{ \frac{|\tilde{v}_z|}{|\tilde{\vec{v}}|} \tilde{p} \right\} \right] \quad (15b)$$

Note that the particle-velocity-normalized decomposition is also unconditionally stable, assuming realistic densities and medium velocities, as the particle-velocity fraction varies between zero and one.

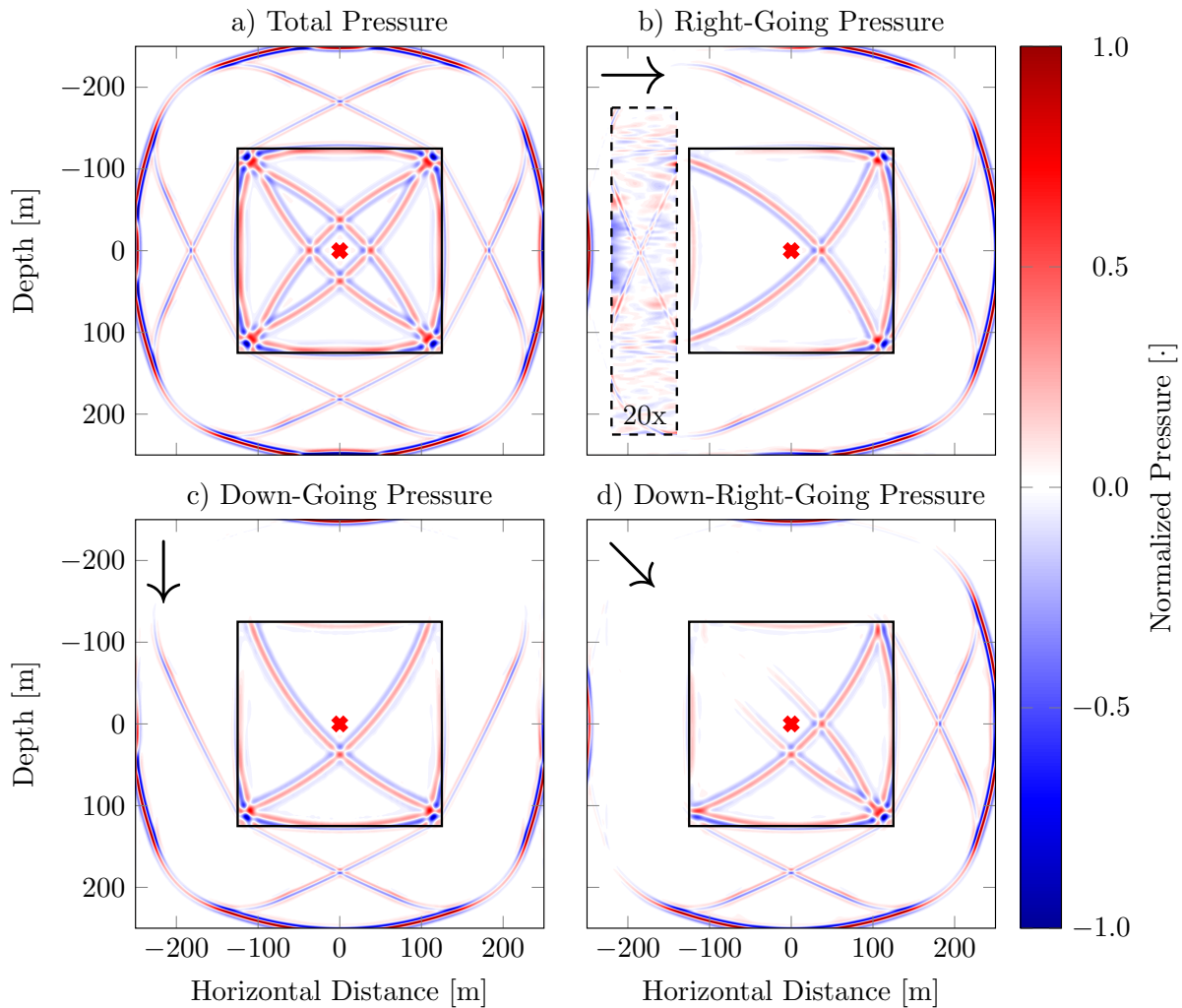


Figure 1 Decomposed pressure wavefields due to a source at the center of a 2 km/s box (black box) embedded in a 1 km/s constant-density background medium with free-surface boundaries at model edges. a) Total wavefield. b) Right-going wavefield amplified 20 times inside dashed square to illustrate errors. c) Down-going wavefield. d) Down-right-going wavefield computed using Equation 13 for $\phi = 45^\circ$.

Synthetic Examples

To illustrate the advantages and limitations of our snapshot decomposition operators we will consider a simple constant-density model with a centered high-velocity box and a heterogeneous velocity model based on seismic and well data near the Annerveen gas field (Vidal et al., 2014). The data were decomposed using Equation 9 and Equation 13.

Figure 1 shows a pressure wavefield due to a pressure source excited at the center of a 2 km/s box (indicated by a black square), surrounded by a 1 km/s background constant-density medium. We see some numerical wavefield leakage Figure 1b, because we did not correct for the fact that the finite difference grid was staggered in time. We also see low-vertical wavenumber errors that are attributable to operating and tapering in the wavenumber domain. Tapering has however proved to be valuable for removing strong horizontal or vertical artefacts due to low-wavenumbers from the decomposition operator.

Figure 2 shows a decomposed snapshot of an acoustic wavefield excited by a pressure source at the center of the constant-density Annerveen model. We see how nicely the operator decomposes the wavefield. Again we did not account for the time shift between the pressure and particle velocity fields. This demonstrates that the decomposition operators remain applicable when there is a small time shift be-

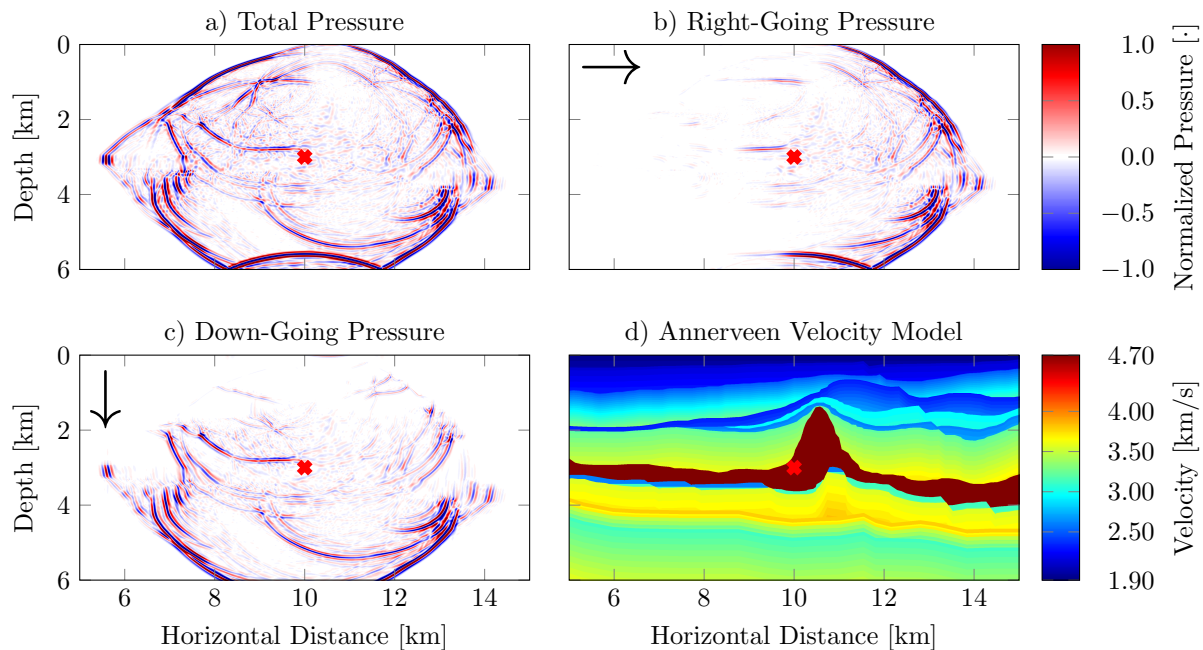


Figure 2 Decomposed pressure wavefields due to a source at the center of the constant-density Annerveen velocity model (d). a) Total wavefield. b) Right-going wavefield. c) Down-going wavefield.

tween the pressure and particle-velocity wavefields. This is an important result as it greatly improves performance in RTM schemes because one does not have to correct for the time shift before decomposing wavefields. Correcting for the spatial staggered grid however remains imperative for acceptable results.

Conclusions

In this work, we generalized our previously presented decomposition operators (Holicki et al., 2016), laying the mathematical foundation for the previously introduced homogeneous-velocity-model decomposition operator. We also generalized the operator to decompositions into arbitrary directions via Equation 13. This new omni-directional operator is an invaluable tool for snapshot wavefield decomposition and complements the likes of Poynting decomposition in RTM when decomposing in heterogeneous velocity models.

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