Stability of in-plane vibration of an elastically supported rotating thin ring, revisited

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Stability of in-plane vibration of an elastically supported rotating thin ring, revisited

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Abstract

Stability of the in-plane free vibration of a rotating thin ring elastically mounted to an immovable axis is revisited in this study. We aim to demonstrate theoretically that the ring can be unstable in contrast to a commonly accepted belief that the instability cannot occur [1].
The inner surface of the ring is assumed to be connected to an immovable axis by means of distributed radial and circumferential springs with stiffness (per unit length) k_r and k_c. The outer surface is stress-free.

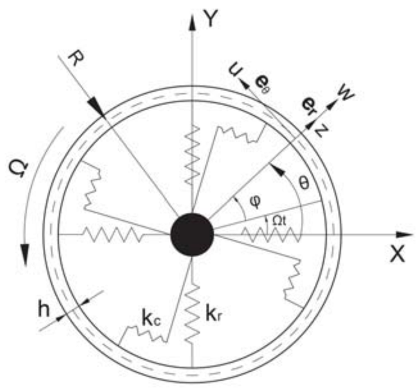


Figure 1. A rotating thin ring on an elastic foundation.

The ring parameters and coordinate systems are defined in Figure 1. A space-fixed coordinate system (r,θ) is employed. An auxiliary coordinate z is introduced as $z = r - R$. Other parameters, which are not shown in Figure 1, are: b the width; E the Young's modulus; ρ the mass density; A the area and I the cross-sectional moment of inertia of the ring. It is assumed that the radial and circumferential displacements $w(z,\theta,t)$ and $u(z,\theta,t)$ of a differential element on the ring are defined by

$$\begin{aligned} w(z,\theta,t) &= w_0(\theta,t) + z w_1(\theta,t) + z^2 w_2(\theta,t), \\ u(z,\theta,t) &= u_0(\theta,t) \end{aligned} \tag{1}$$

where $w_0(\theta,t)$ and $u_0(\theta,t)$ are the radial and circumferential displacements of the middle surface, respectively; $w_1(\theta,t)$ and $w_2(\theta,t)$ are the higher order corrections of the radial displacement. These corrections enable us to take a linear distribution of the through-thickness variation of the radial stress into account. The radial stress at the inner surface of the ring is not zero because of the presence of the radial springs. If the stiffness of the radial springs is large enough, the radial stress at the inner surface cannot be neglected. Thin rings are considered,

thus, the transverse shear deformation and rotatory inertia are not incorporated in the formulation of the governing equations.
The boundary conditions for the inner and outer surfaces of the ring must be satisfied. The material of the ring is considered to be linearly elastic. For the outer surface of the ring, the radial stress should be zero, which implies that

$$\sigma_r\big|_{h/2} = E\varepsilon_r\big|_{h/2} = E(w_1 + hw_2) = 0. \tag{2}$$

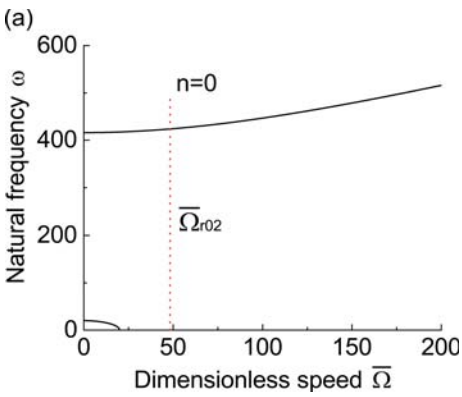
The inner surface is connected to the springs, thus

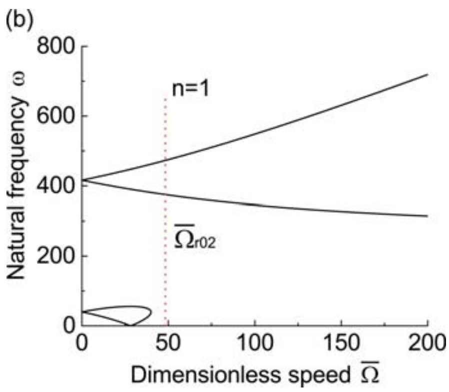
$$b\sigma_r\big|_{-h/2} = Eb\varepsilon_r\big|_{-h/2} = Eb(w_1 - hw_2) = k_r w\big|_{-h/2}. \tag{3}$$

Combining Eqs. (2) and (3), $w_1(\theta,t)$ and $w_2(\theta,t)$ can be expressed in terms of $w_0(\theta,t)$. To account for the rotation-induced hoop tension, the same nonlinear strain-displacement relation as in Ref. [2] is applied. Using the Hamilton's principle, the nonlinear equations which govern the radial and circumferential motions of the ring are derived in a space-fixed reference system. The linearised governing equations are obtained about the axisymmetric static equilibrium. The latter is derived from the governing equation in the radial direction. Details of the derivations can be found in [3].
To analyse the problem, it is convenient to introduce the following dimensionless parameters and variables

$$t_0^2 = \rho AR^4/(EI),\ \tau = t/t_0,\ \bar{\Omega} = \Omega t_0,\ \chi = EAR^2/(EI),\ \bar{K}_r = k_r R^4\big/(EI),\ \bar{K} = k_c\big/k_r. \tag{4}$$

Since the radial expansion of the ring grows with the increasing rotational speed, there should be an upper limit of the rotational speed above which the prestresses due to rotation may exceed the allowable strength of the materials. There are two kinds of prestresses which should be examined beforehand. These are: the maximum hoop stress which occurs at the outer surface of the ring and the maximum radial prestress which appears at the inner surface of the ring. To avoid a discussion of the physical behaviour of different materials, the maximum prestress $\sigma^0_{\max} \le 0.2E$ is chosen to define the regime in which the ring material is assumed to behave linearly.
To illustrate that instability can occur prior to material failure, the following dimensionless parameters are chosen: $\chi = 1200, \bar{K}_r = 4\times10^5, \bar{K} = 0.001$. These parameters correspond to a thin ring with small bending stiffness and stiff foundation.





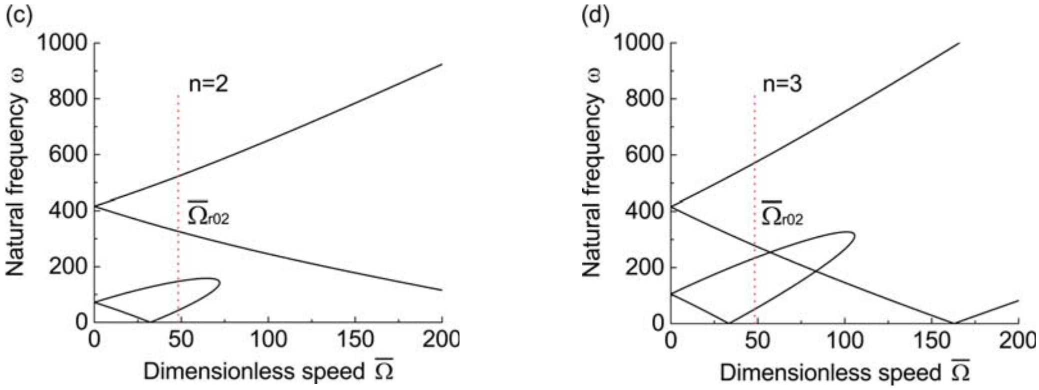


Figure 2. Dimensionless natural frequencies versus rotational speeds.

Fig. 2 illustrates the relationship between the rotational speed and the natural frequencies of the first four modes in the space-fixed reference system. We count the modes starting form $n=0$, the latter implying an angle-independent deformation. Note that in this mode the rotating ring can and does deform elastically. The vertical dotted line in Fig.2 corresponds to the speed which is determined by $\sigma^0_{\max}=0.2E$. For mode numbers greater than zero, both the lower and higher natural frequencies split into two branches which result in four distinct natural frequencies per mode. However, for the $n=0$ modes, the natural frequencies do not bifurcate. The upper branch of the $n=0$ mode increases monotonically as the speed of rotation grows. The lower branch first descends and then crosses the horizontal axis at a certain rotational speed. It can be shown that above this speed, the natural frequency becomes purely imaginary which indicates the onset of instability of the divergence type. The ring displacement increases exponentially in time in the circumferential direction. For modes $n\geq 1$ one can see that the lower set of natural frequencies first branches into two curves at $\overline{\Omega}>0$, then the two collide with each other and disappear from the real plane (become complex-valued) after a certain speed. Since the characteristic polynomial has real-valued coefficients, the complex roots appear in conjugate pairs and one may say that flutter occurs after the collision speed. The lowest speed at which instability occurs is for the mode $n=0$. Divergence instability of the 0th mode always occurs before flutter could happen.

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