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## ABSTRACT

Metro disruptions due to unexpected events reduce transit system reliability, resulting in significant productivity loss and long passenger delays. Bus bridging strategy is often used to connect stations affected by metro disruptions such that passengers could continue their journey. The literature usually designed bridging routes and then allocated buses to designed routes with specific frequencies. The restriction that each bus can only operate on a route greatly limits the service flexibility and decreases operation efficiency. We propose a flexible bus bridging strategy to deal with the disruptions of metro networks. The proposed strategy optimizes a tailored bridging path for each bus. The path dictates the stations that a bus should visit in sequence once it is dispatched from the depot. A two-stage model that balances the needs of transit agency and passengers is developed to optimize the tailored bridging paths based on affected metro stations, reserved buses, bus capacity, passenger demands and bus travel times. The Stage I model produces schematic bridging paths by minimizing the maximum bus bridging time. The Stage II model further details the paths by minimizing average passenger delay. The superiority of the proposed strategy to a traditional strategy is demonstrated in a case study in Rotterdam, The Netherlands.



***Keywords***: Bus bridging, Metro network disruptions, Tailored bridging paths, Two-stage model, Integer linear programming

## 1. INTRODUCTION

Metro systems serve as a major carrier in many metropolises to support the mobility needs of passengers, owning to its large capacities, high operating speeds and reliability. Nevertheless, due to unexpected events, such as infrastructure malfunctions, accidents and extreme weather conditions, metro disruptions frequently occurred in recent years throughout the world. For instance, severe metro disruptions in Barcelona in August 2008, London in August 2010, Shanghai in September 2011, Singapore in December 2011 and Beijing in August 2016 interrupted the travel plans of many passengers. In some cities, the frequency of metro disruptions is surprisingly high. The number of Mass Transit Railway (MTR) disruptions in Hong Kong ranged from 166 to 344 between 2005 and 2014 (*1*). 15,549 unplanned disruptions were recorded on metropolitan rail services in Melbourne, Australia, in the first half of 2011, which range from small delays to full service closures (*2*).

Metro disruptions lead to unacceptable service affecting a large number of commuters. Transit agencies have adopted various approaches in response to unplanned metro disruptions. Based on the surveys within 71 international transit agencies, parallel transit systems and bus bridging have been recognized as two main strategies to deal with metro disruptions (*3*). Parallel transit systems make use of an existing parallel public transport system that mirrors part of or entire corridor where disruption occurs. However, many cities do not have parallel transit systems in the area of metro disruption or the extra capacities of parallel transit systems are not enough for the stranded passengers (*3*).

Compared with parallel transit systems, bus bridging is more widely used during metro disruptions. Bus bridging strategy connects the disrupted metro system with buses dispatched from depots. It has not received enough attention until recently. Kepaptsoglou and Karlaftis (2009) proposed methodology to design temporary bus services to restore the connectivity of disrupted metro system (*4*). Their methodology framework consists of three steps performed sequentially: generation of candidate bridging routes, selection of optimal bridging routes and allocation of buses to the routes. The bridging routes are generated using a shortest path algorithm and then modified using a heuristic algorithm. Jin et al. (2015) and van der Hurk et al. (2016) made improvements to develop integrated models to optimize route selection and bus allocation simultaneously after the generation of candidate bridging routes (*5, 6*). Candidate bridging routes are generated using a column generation algorithm in Jin et al. (2015) and using a path generation method together with a path reduction method in van der Hurk et al. (2016).

Existing bus bridging studies assumed that buses operate on predetermined bridging routes with specific frequencies. With limited bus resources, the resulting bus bridging service may not be able to handle the outbursts of passenger demand efficiently given the frequency requirement and the constraint that one bus could only operate on one route. Optimizing a tailored bridging path for each bus to follow may result in more efficient bus bridging service. We may consider the Bus Bridging Problem (BBP) from the perspective of Vehicle Routing Problem (VRP).

The VRP is generally defined as the problem of designing least-cost delivery routes from a depot to a set of geographically scattered customers, subject to side constraints (*7*). One classical VRP is capacitated VRP, in which vehicles have capacity limitation (*8*). BBP differs from capacitated VRP in that: (1) BBP does not have to consider the process that buses return to the bus depots (open VRP (*9*)); (2) BBP could use buses from multiple bus depots (multi-depot VRP (*10*)); (3) BBP considers passengers with various origins and destinations (VRP with pick-up and delivery (*11*)). Thus, we formulate the BBP as an open, multi-depot, capacitated VRP with pick-up and delivery of passengers. To the best of our knowledge, it has not been studied in the VRP literature.

We develop a two-stage integer linear programming formulation to optimize a tailored bridging path for each bus to follow. A path depicts the stations that a bus should visit in sequence once it is dispatched from the depot. The affected metro stations, reserved buses, bus capacity, passenger demands and bus travel times are considered in the optimization. The objective of the model considers the needs of metro agency and passengers. The first priority is to minimize the maximum bus bridging time, which is the time when all stranded passengers are transported to their destination stations or a turnover station. The second priority is to minimize average passenger delay to reduce the negative impacts of disruptions on passengers. The advantage of the proposed model is demonstrated in a case study based on the metro network in Rotterdam, The Netherlands.

Our approach has the potential for real-life application with the rapidly growing usage of new technologies. For example, transit agency could get the information of passenger demands via Automated Fare Collection (AFC) data or mobile phone data so that they can make decisions for the bus bridging operation. They could also obtain real-time bus locations via automatic vehicle location technology and give instructions to buses via wireless communication technologies. The introductions could be displayed on on-board screens for bus drivers to follow. Passengers could obtain real-time information of the buses they could take via apps on smartphones or variable message signs at stations. Then they can decide to either use the bridging service or continue their journey by other means.

The remainder of this paper is organized as follows. In Section 2, the Bus Bridging Problem is described. In Section 3, the novel bus bridging model is formulated. In Section 4, the results of the applications of the proposed model to a hypothetical case study are discussed, compared with a traditional strategy. Concluding remarks are offered in Section 5.

## 2. PROBLEM DESCRIPTION

Consider a part of a metro network in Figure 1, where part of the network around station S4 is out of service due to infrastructure malfunctions. The influence of the disruption extends to the nearest turnover stations for each direction, where track crossover is available. Only beyond the turnover stations can the metro line operate in short routing mode. Therefore, the whole metro network is disrupted, including both the metro line segments from station S1 to station S6 and from station S7 to station S10. Passengers are stranded at affected stations. There are two bus depots D1 and D2 with buses reserved nearby.

The BBP is to provide bus service for stranded passengers in disrupted metro area with limited bus resources from bus depots such that they could continue their journey. Passenger demands are described by origin-destination (OD) flow matrix, including demands between turnover stations, between turnover and disrupted stations and between disrupted stations. The demands originated from or destined to a turnover station are actually an aggregation for all stations beyond the turnover station.

To simplify the problem, two assumptions are made: (1) passenger demands and bus travel times are known and constant; (2) buses have the same and fixed capacity. Instead of predetermining bridging routes and assigning buses to routes with given frequencies like previous studies, we propose a flexible bus bridging strategy to assign tailored bridging paths to buses. Take Bus 2 in Figure 1 as an example, the tailored bridging path for it is D2→S8→S5→S6→S9. Tailored bridging paths are often non-intuitive as shown in Figure 1. The bridging service is completed when all buses complete their respective bridging paths.

A bus is assumed to only upload passengers destined to its next arriving station when it arrives at a station. The loading rule is applicable since passengers ought to be informed of the next destined station of a coming bus, rather than the whole bridging path. For each bus, dispatching station is defined as the metro station it is dispatched to from the depot and a trip is defined as the movement from one metro station to another.

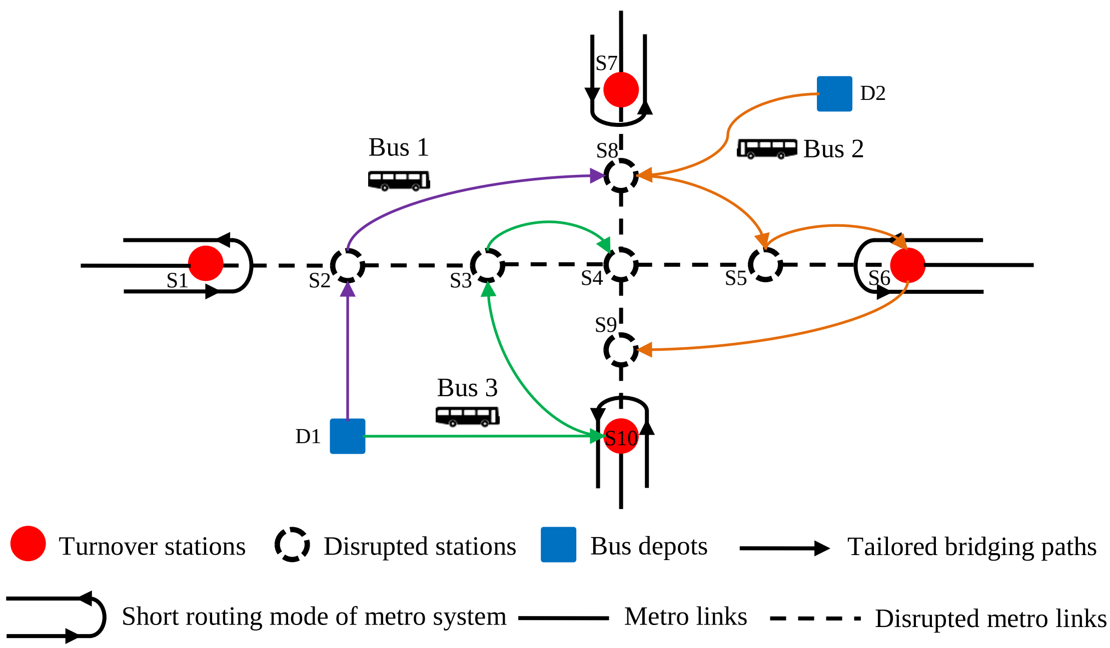


**FIGURE 1 Description of the Bus Bridging Problem.**

## 3. MODEL FORMULATION

Notations of the inputs, parameters and variables are summarized in Table 1.

**TABLE 1 Notations Used in this Study.**

| **Input Sets and Parameters** | |
|---|---|
| $\mathbf{S}=(1,2,\cdots,S)$ | Set of metro stations in the disrupted area, $s\in\mathbf{S}$ |

| | |
|---|---|
| $\mathbf{B}=(1,\cdots,B)$ | Set of buses, $b\in\mathbf{B}$ |
| $Q(o,d)$ | Passenger demand from station $o$ to station $d$ |
| $f_{s,b}$ | Travel time from the depot of bus $b$ to station $s$ |
| $t_{o,d}$ | Bus travel time from station $o$ to station $d$ |
| $C$ | Bus capacity |
| **Stage I Model** | |
| **Intermediate Sets and Variables** | |
| $\mathbf{N}$ | Set of all subsets of metro stations in the disrupted area, $\mathbf{N}\subseteq\mathbf{S}$, $2\leq\lvert\mathbf{N}\rvert\leq S$-1 |
| $T_b$ | Bridging time for bus $b$ |
| $T_{\max}$ | Maximum bus bridging time |
| **Decision Variables** | |
| $y_{s,b}$ | A binary variable indicating whether bus $b$ will be dispatched to station $s$. If so, $y_{s,b}$ =1, otherwise $y_{s,b}$ =0 |
| $x_{o,d,b}$ | An integer variable indicating the number of trips bus $b$ travel from station $o$ to station $d$ |
| **Stage II Model** | |
| **Parameters** | |
| $\mathbf{P}=(1,\cdots,P)$ | Set of passenger types, $p\in\mathbf{P}$ |
| $Pax_p$ | Number of passengers for the $p^{\text{th}}$ type of passenger batch |
| $H_p$ | Bus travel time for the $p^{\text{th}}$ type of passenger batch |
| $E_p$ | Total number of trips the $p^{\text{th}}$ type of passenger batch needed to be served |
| $O_p$ | Origin station of the $p^{\text{th}}$ type of passenger batch |
| $D_p$ | Destination station of the $p^{\text{th}}$ type of passenger batch |
| $R_b$ | Number of total trips of bus $b$ under condition of Stage I model |
| **Intermediate Variables** | |
| $CT_{b,r}$ | The time that bus $b$ finishes its $r^{\text{th}}$ trip in the bus bridging process |
| $TD_{b,r}$ | Total delay for passengers transported in the $r^{\text{th}}$ trip of bus $b$ |
| $w_{p,b,r}$ | Introduced decision variables for linearization. If bus $b$ take the $p^{\text{th}}$ type of passenger batch at its $r^{\text{th}}$ trip in the bus bridging process, $w_{p,b,r}$= $CT_{b,r}$, otherwise $w_{p,b,r}$ =0 |
| **Decision Variables** | |
| $z_{p,b,r}$ | A binary variable indicating whether bus $b$ will take the $p^{\text{th}}$ type of passenger batch at its $r^{\text{th}}$ trip in the bus bridging process. If so, $z_{p,b,r}$ =1, otherwise $z_{p,b,r}$ =0 |

We define the bridging time for a bus as the time that the bus completes the bridging service since it is dispatched from the depot. The bridging time includes the time from the depot to one of the stations in the disrupted metro area and the time for traveling between stations in the disrupted area. Let $y_{s,b}$ represent the dispatching station of buses. $y_{s,b}$=1 if bus $b$ is dispatched to station $s$, and 0 otherwise. Let $x_{o,d,b}$ represent the number of trips bus $b$ travel from station $o$ to station $d$. The bridging time for bus $b$ is given by:

$$T_b = \sum_{s\in\mathbf{S}}\left(f_{s,b}\times y_{s,b}\right)+\sum_{o\in\mathbf{S}}\sum_{d\in\mathbf{S}}\left(t_{o,d}\times x_{o,d,b}\right) \tag{1}$$

As shown in Equation (1), the variables $y_{s,b}$ and $x_{o,d,b}$ have uniquely determined the bus bridging time. Nevertheless, different sets of bridging paths could reproduce the same set of $x_{o,d,b}$ and the same $T_b$. For example, suppose $x_{1,2,b}$=1, $x_{1,3,b}$=1, $x_{2,3,b}$=1 and $x_{3,1,b}$=1, the bridging paths of S1→S2→S3→S1→S3 and of S1→S3→S1→S2→S3 would result in the same $T_b$. Additional objectives could be considered to optimize the bridging path for each bus.

In this study, we propose a two-stage integer linear programming model to determine tailored bridging paths for buses. The objectives of the two stages are constructed from the perspectives of metro agency and passengers, respectively. Stage I determines key components of the tailored bridging paths with the objective of minimizing the time to transport all stranded passengers to their destination stations or turnover stations, which is equivalent to minimizing the maximum bus bridging time. Decision variables for each bus include the dispatching station and number of trips it travels from one station to another. Table 2 presents an illustration of number of trips between stations for a bus. For instance, it travels from S3 to S6 for three times. To reduce passenger costs incurred by the disruption, Stage II further details the tailored bridging paths with the objective of minimizing average passenger delay. Decision variables for each bus include the stations that a bus should visit in sequence, as illustrated in Figure 1.

**TABLE 2 Illustration of number of trips between stations for a bus.**

| Destination / Origin | S1 | S2 | S3 | S4 | S5 | S6 |
|---|---|---|---|---|---|---|
| **S1** | / | 0 | 0 | 1 | 0 | 0 |
| **S2** | 0 | / | 2 | 0 | 0 | 0 |
| **S3** | 0 | 0 | / | 0 | 0 | 3 |
| **S4** | 0 | 1 | 0 | / | 0 | 0 |
| **S5** | 0 | 1 | 0 | 0 | / | 0 |
| **S6** | 1 | 0 | 1 | 0 | 0 | / |

## 3.1 Stage I Model

Stage I is formulated as linear integer programming model as follows:

$$\min\ T_{\max} \tag{2}$$

s.t.

$$T_b \le T_{\max} \quad \forall b \in \mathbf{B} \tag{3}$$

$$C \times \sum\nolimits_{b\in\mathbf{B}} x_{o,d,b} \ge Q(o,d) \quad \forall o \in \mathbf{S}, \forall d \in \mathbf{S}, o \ne d \tag{4}$$

$$\sum\nolimits_{s\in\mathbf{S}} y_{s,b} \le 1 \quad \forall b \in \mathbf{B} \tag{5}$$

$$\sum\nolimits_{o\in\mathbf{S}} \sum\nolimits_{d\in\mathbf{S}} x_{o,d,b} \le M \times \sum\nolimits_{s\in\mathbf{S}} y_{s,b} \quad M \text{ is a large number} \quad \forall b \in \mathbf{B} \tag{6}$$

$$y_{s,b} = \max(0, \sum\nolimits_{d\in\mathbf{S}} x_{s,d,b} - \sum\nolimits_{o\in\mathbf{S}} x_{o,s,b}) \quad \forall s \in \mathbf{S}, \forall b \in \mathbf{B} \tag{7}$$

$$M \times \left( \sum\nolimits_{d\in\mathbf{N}} y_{d,b} + \sum\nolimits_{o\in\mathbf{S}-\mathbf{N}, d\in\mathbf{N}} x_{o,d,b} \right) \ge \sum\nolimits_{o,d\in\mathbf{N}} x_{o,d,b} \quad \forall b \in \mathbf{B}, \mathbf{N} \subseteq \mathbf{S} \text{ and } 2 \le |\mathbf{N}| \le S-1 \tag{8}$$

$$y_{s,b} \in \{0,1\} \quad \forall s \in \mathbf{S}, \forall b \in \mathbf{B} \tag{9}$$

$$x_{o,d,b} \in N \quad \forall o \in \mathbf{S}, \forall d \in \mathbf{S}, o \neq d, \forall b \in \mathbf{B} \tag{10}$$

$$x_{o,d,b} = 0 \quad \forall o \in \mathbf{S}, \forall d \in \mathbf{S}, o = d, \forall b \in \mathbf{B} \tag{11}$$

Where $T_{\max}$ represents the maximum bus bridging time. Constraints (3) restrict the bridging times of all buses to be no larger than $T_{\max}$. Constraints (4) make sure all passenger demands to be satisfied. Constraints (5) guarantee that one bus can be dispatched to at most one station. Constraints (6) ensure that a bus could travel between metro stations only after it is dispatched from depot to one of the stations in disrupted area. That is, $x_{o,d,b}$ could be nonzero only if one of $y_{s,b}$ is nonzero. Constraints (7) are the common "flow conservation" constraints. Constraints (8) are bridging path elimination constraints which ensure that each trip includes a depot. Constraints (8) prevent the occurrence of illegal bridging paths. Figure 2 illustrates the illegal and legal bridging paths. A bridging path without the depot is illegal and cannot be assigned to a bus since every bus departs from a depot.

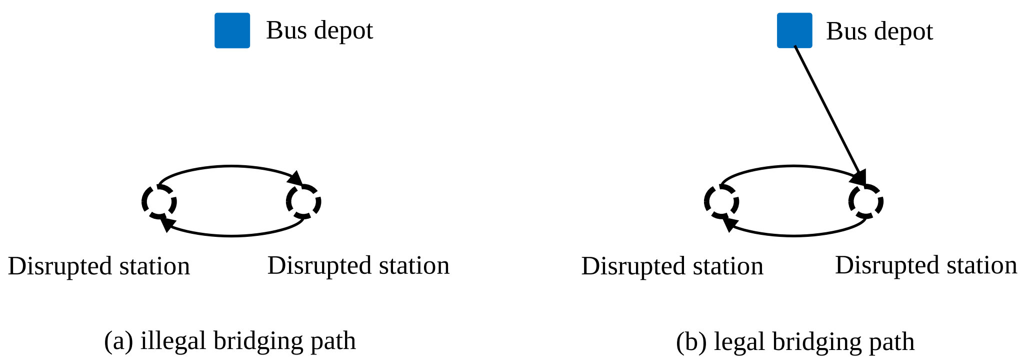


**FIGURE 2 Illustration of illegal and legal bridging paths.**

### 3.2 Stage II Model

After Stage I model, the dispatching destinations and numbers of trips to travel from one station to another are determined for buses. But the station sequence that a bus should visit still need to be determined in Stage II model based on the results of Stage I model. The arrival of a bus at a given station results in the decrease of passenger demand at the station. The number of passengers carried in each trip of a bus could be affected by the station sequence of another bus. To model the dynamic change of passenger demand and its interactions with the station sequences of buses, we decompose passenger demand for each station pair into different types of passenger batch. Each type of passenger batch is characterized by $F(p)=(Pax_p, H_p, E_p, O_p, D_p)$, where $Pax_p$ represents number of passengers; $H_p$ represents bus travel time; $E_p$ represents total number of trips each passenger batch needed to be served; $O_p$ and $D_p$ represent the origin station and the destination station, respectively.

$O_p$ and $D_p$ can be obtained from the origin and destination of passengers, respectively. $H_p$ equal to bus travel time from station $O_p$ to station $D_p$. For each station pair $o$ and $d$, there may exist three types of passenger batch with different numbers of passengers: (1) $Pax_p = C$, this type of passenger batch exists when passenger demand between station pair $o$ and $d$ is not smaller than bus capacity; (2) $Pax_p = Q(O_p, D_p) - C \times \text{floor}(Q(O_p, D_p)/C)$, where function floor($x$) rounds $x$

to the nearest integer not larger than $x$, this type of passenger batch exists when passenger demand is not integer multiple of bus capacity; (3) $Pax_p$ =0, this type of passenger batch exists when there are trips without picking up passengers between station pair $O_p$ and $D_p$, *i.e.*, $\sum_{b\in\mathbf{B}} x_{O_p,D_p,b}$-ceil($Q$($O_p$, $D_p$)/$C$)>0, where function ceil($x$) rounds $x$ to the nearest integer not smaller than $x$. $E_p$ is related to $Pax_p$ and can be obtained by:

$$E_p = \begin{cases} floor(Q(O_p,D_p)/C) & \text{if } Pax_p = C \\ 1 & \text{if } 0<Pax_p<C \\ \sum_{b\in\mathbf{B}} x_{O_p,D_p,b} - ceil(Q(O_p,D_p)/C) & \text{if } Pax_p = 0 \end{cases} \quad (12)$$

The number of total trips, $R_b$, bus $b$ makes in the bus bridging process is given by:

$$R_b = \sum_{s\in\mathbf{S}} y_{s,b} + \sum_{o\in\mathbf{S}} \sum_{d\in\mathbf{S}} x_{o,d,b} \quad \forall b\in\mathbf{B} \quad (13)$$

For each bus, the first trip in the bus bridging process is to be dispatched from the depot to one of the stations in disrupted metro area. Let $CT_{b,1}$ represent the time bus $b$ finishes its first trip, which can be obtained by:

$$CT_{b,1} = \sum_{s\in\mathbf{S}} \left( f_{s,b} \times y_{s,b} \right) \quad \forall b\in\mathbf{B} \quad (14)$$

Let $z_{p,b,r}$ represent the station sequence of buses. $z_{p,b,r}$=1 if bus $b$ will take the $p^{th}$ type of passenger batch at its $r^{th}$ trip, and 0 otherwise. The time that bus $b$ finishes its second trip in the bus bridging process is given by:

$$CT_{b,2} = CT_{b,1} + \sum_{p\in\mathbf{P}} \left( H_p \times z_{p,b,2} \right) \quad \forall b\in\mathbf{B} \quad (15)$$

Similarly, the time that bus $b$ finishes its $r^{th}$ trip in the bus bridging process is given by:

$$CT_{b,r} = CT_{b,r-1} + \sum_{p\in\mathbf{P}} \left( H_p \times z_{p,b,r} \right) \quad \forall b\in\mathbf{B}, \forall r = 2,\cdots,R_b \quad (16)$$

Then the total delay for passengers transported in the $r^{th}$ trip of bus $b$ can be obtained by:

$$TD_{b,r} = \sum_{p\in\mathbf{P}} (Pax_p \times z_{p,b,r} \times CT_{b,r}) \quad \forall b\in\mathbf{B}, \forall r = 2,\cdots,R_b \quad (17)$$

Based on the analysis above, Stage II model that minimizes average passenger delay in the bus bridging process, which is to minimize the loss of all passengers, could be formulated as nonlinear integer programming problem as follows:

$$\min \sum_{b\in\mathbf{B}} \sum_{r=2}^{R_b} TD_{b,r} \Big/ \sum_{o\in\mathbf{S}} \sum_{d\in\mathbf{S}} Q_{o,d} \quad (18)$$

s.t.

$$\sum_{p\in\mathbf{P}} z_{p,b,r} \le 1 \quad \forall b\in\mathbf{B}, \forall r = 2,\cdots,R_b \quad (19)$$

$$\sum_{b\in\mathbf{B}} \sum_{r=2}^{R_b} z_{p,b,r} = E_p \quad \forall p\in\mathbf{P} \quad (20)$$

$$\sum_{O_p=o,D_p=d} \sum_{r=2}^{R_b} z_{p,b,r} = x_{o,d,b} \quad \forall o\in\mathbf{S}, \forall d\in\mathbf{S}, \forall b\in\mathbf{B} \quad (21)$$

$$\sum_{D_p=s} z_{p,b,r} \ge \sum_{O_p=s} z_{p,b,r+1} \quad \forall s\in\mathbf{S}, \forall b\in\mathbf{B}, r = 2,\cdots,R_b - 1 \quad (22)$$

$$y_{s,b} \ge \sum_{O_p=s} z_{p,b,2} \quad \forall s\in\mathbf{S}, \forall b\in\mathbf{B} \quad (23)$$

$$z_{p,b,r} \in \{0,1\} \quad \forall p\in\mathbf{P}, \forall b\in\mathbf{B}, \forall r = 2,\cdots,R_b \quad (24)$$

$$z_{p,b,1} = 0 \quad \forall p \in \mathbf{P}, \forall b \in \mathbf{B} \tag{25}$$

Constraints (19) ensure that each bus can transport at most one type of passenger batch every time it takes a trip. Constraints (20) ensure total number of times each type of passenger batch needed to be served. Constraints (21) ensure that total number of trips for each station pair to be satisfied under conditional of Stage I model. Constraints (22-23) maintain routes continuity for each bus (each bus can depart from a station only after it arrives at the station).

Objective function (18) is the only nonlinear ingredient of Stage II model. It can be linearized by Objective function (26) and Constraints (27-28). New decision variables $w_{p,b,r}$ are introduced for the linearization and they can be any arithmetic number.

$$\min \ \sum_{p\in\mathbf{P}}\sum_{b\in\mathbf{B}}\sum_{r=2}^{R_b}(Pax_p \times w_{p,b,r}) \Big/ \sum_{o\in\mathbf{S}}\sum_{d\in\mathbf{S}} Q_{o,d} \tag{26}$$

s.t.

$$w_{p,b,r} + M \times \left(1 - z_{p,b,r}\right) \geq \sum_{s\in\mathbf{S}}\left(f_{s,b} \times y_{s,b}\right) + \sum_{p\in\mathbf{P}}\sum_{n=2}^{r}\left(H_p \times z_{p,b,n}\right)$$
$$\forall p \in \mathbf{P}, \forall b \in \mathbf{B}, \forall r = 2,\cdots,R_b \tag{27}$$

$$w_{p,b,r} \geq 0 \quad \forall p \in \mathbf{P}, \forall b \in \mathbf{B}, \forall r = 2,\cdots,R_b \tag{28}$$

## 4. CASE STUDY

The proposed strategy is validated in a hypothetical case based on the metro network of Rotterdam, The Netherlands. The integer linear programs of the proposed two-stage model are solved with the MIP solver in CPLEX (*12*) with the YALMIP interface (*13*) running on a PC with a 3.70 GHz Intel Core CPU and 4.0 GB of memory. In most cases, the proposed model can be solved efficiently within a few minutes. A traditional strategy often used by transit agencies in response to such disruptions is used for comparison purpose. In the traditional strategy, first a shortest route is found to connect all affected stations. Then each bus is dispatched to the nearest station in the disrupted area and travels along the shortest route, *i.e.*, makes roundtrips between the first and the last stations of the shortest route. The bus visits each station to unload and load passengers. The traditional strategy is evaluated based on mean values of 100 simulation runs. In each run, after a given bus unloads passengers at a station, passengers who have the same travel direction as the given bus are randomly selected to board the bus until it is full. The maximum bridging time for traditional strategy is defined as the time when all passengers reach their destination stations or turnover stations.

### 4.1 Case settings

The case settings are described as follows. Six stations were shut down due to disruption (see Figure 3). Stations 1, 2, 3 and 6 are turnover stations for crossover. Buses reserved in two surrounding depots which match the reality are dispatched to provide bus bridging service. We used an agent-based multimodal dynamic network simulation tool based on (*14*) to count the number of passengers that use the considered metro segments during a period of one hour in case of no disturbance – and, assuming no rerouting, would thus strand in a disruption lasting one hour – constructing an OD matrix for bus bridging from those counts. From the same simulation, we also recorded the travel times in the road network between each pair of stations and from the bus depots to each station, using the road links shown in Figure 4. In the simulation, we include signalized intersections, configured with the Webster method, and fundamental diagrams with subcritical delays and capacity drops (*15*). The multimodal network, including train/metro/tram/bus timetables, are derived from the static model of the municipality for the year 2015; the demand data originates from the activity-based Albatross model (*16*) for a working day in the year 2004, with correction factors to match household and trip counts for 2015.

Table 3 presents passenger flows between stations and travel times between stations or from depots to stations for the hour 17:00 to 18:00, part of the evening peak. The numbers outside and inside of the bracket represent passenger flows and bus travel times (unit: minute), respectively. One minute is added for stopping at a station to unload and load passengers. Bus capacity is 98 passengers.


**FIGURE 3 Disrupted area in metro network of Rotterdam, The Netherlands.**

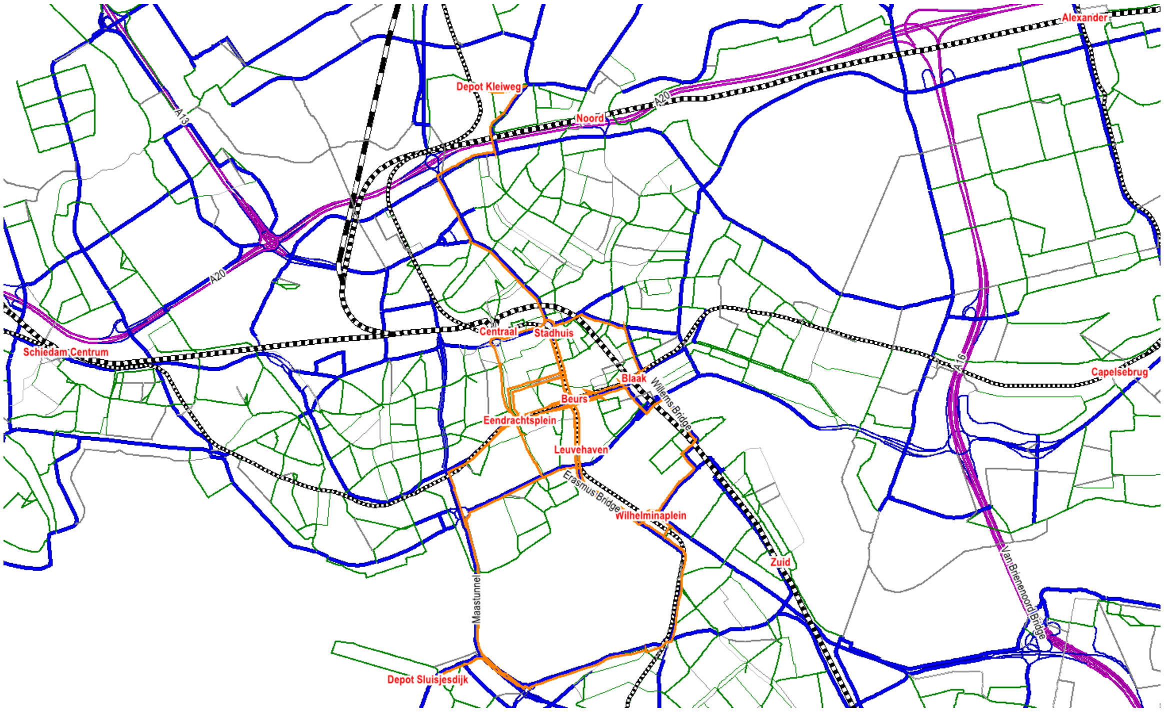


**FIGURE 4 Excerpt of the simulated multimodal network with the road links for bus bridging highlighted (orange).**

**TABLE 3 Passenger flows between stations and travel times between stations or from depots to stations in the case study.**

| **Destination** / **Origin** | **Station 1** Eendrachtsplein | **Station 2** Stadhuis | **Station 3** Blaak | **Station 4** Beurs | **Station 5** Leuvehaven | **Station 6** Wilhelminaplein |
|---|---|---|---|---|---|---|
| **Station 1** Eendrachtsplein | / | 215 (5) | 1259 (2) | 86 (7) | 0 (5) | 135 (5) |
| **Station 2** Stadhuis | 317 (4) | / | 542 (10) | 21 (6) | 9 (8) | 1446 (10) |
| **Station 3** Blaak | 1311 (8) | 291 (12) | / | 182 (8) | 0 (9) | 589 (7) |
| **Station 4** Beurs | 156 (8) | 141 (3) | 483 (1) | / | 0 (2) | 264 (4) |
| **Station 5** Leuvehaven | 0 (8) | 43 (7) | 2 (3) | 0 (4) | / | 114 (2) |
| **Station 6** Wilhelminaplein | 113 (16) | 1712 (16) | 325 (7) | 60 (12) | 31 (8) | / |
| **Depot 1** Kleiweg | (26) | (18) | (28) | (24) | (22) | (27) |
| **Depot 2** Sluisjesdijk | (11) | (37) | (31) | (34) | (15) | (10) |

## 4.2 Determining the bus fleet size

Sensitivity analysis is used to explore the tradeoff between bus bridging performance and bus fleet sizes. Figure 5 reports bridging times and passenger delays achieved with various fleet sizes for the proposed strategy and traditional strategy. As can be seen, the proposed strategy could achieve similar performance as the traditional strategy using fewer buses. For instance, the proposed strategy requires 12 buses to transport all passengers within 105 minutes while the traditional strategy requires 30 buses. What's more, it can be observed that increasing the number of buses reduces bridging times and passenger delays rapidly first and then slowly. The results can be used to help transit agency determine the required fleet size for bus bridging service to achieve a certain level of response effectiveness.

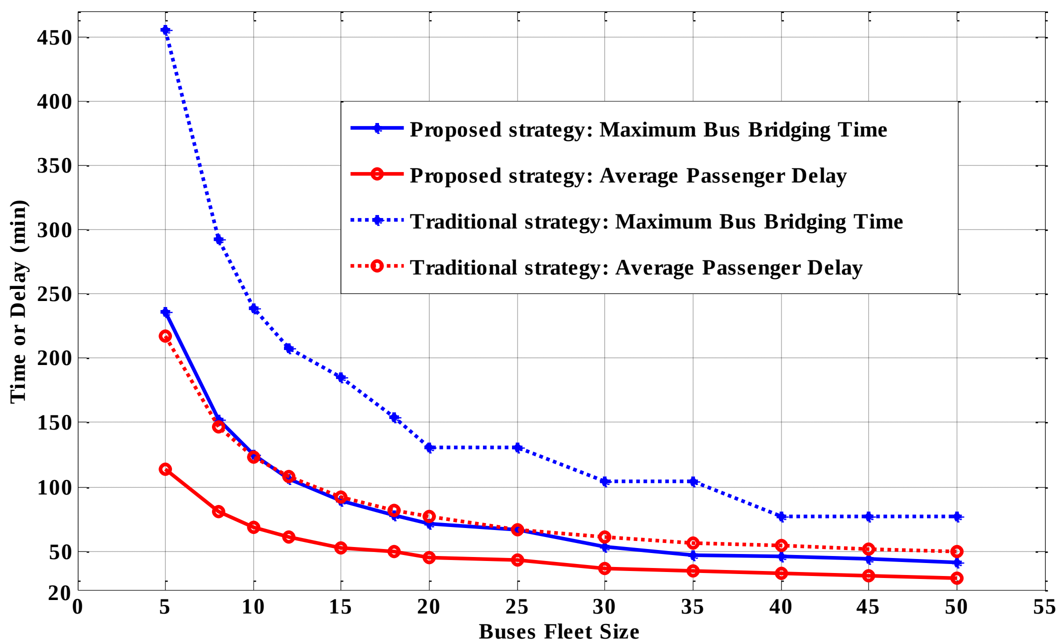


**FIGURE 5 Performance measures under different bus fleet sizes for both our proposed strategy and the traditional strategy.**

## 4.3 Results and analyses

It can be observed from Figure 5 that a reasonable balance between bus amounts and performance measures of the bus bridging operation appears when bus fleet size is within the range from 12 to 20 in the case study. In this section, we use the case with 12 buses to analyze our proposed strategy, compared with the traditional strategy.

Advantage of our proposed strategy exists not only in its aggregated level but also in each station. Figure 6 presents bridging time and average delay at each station. It can be shown that the proposed strategy outperforms the traditional strategy at every station. The maximum bridging time and average delay for all stations of the proposed strategy are 106 min and 70.6 min, respectively. They are even smaller than minimum bridging time and average delay for all stations of the traditional strategy, which are 127 min and 81.3 min, respectively.

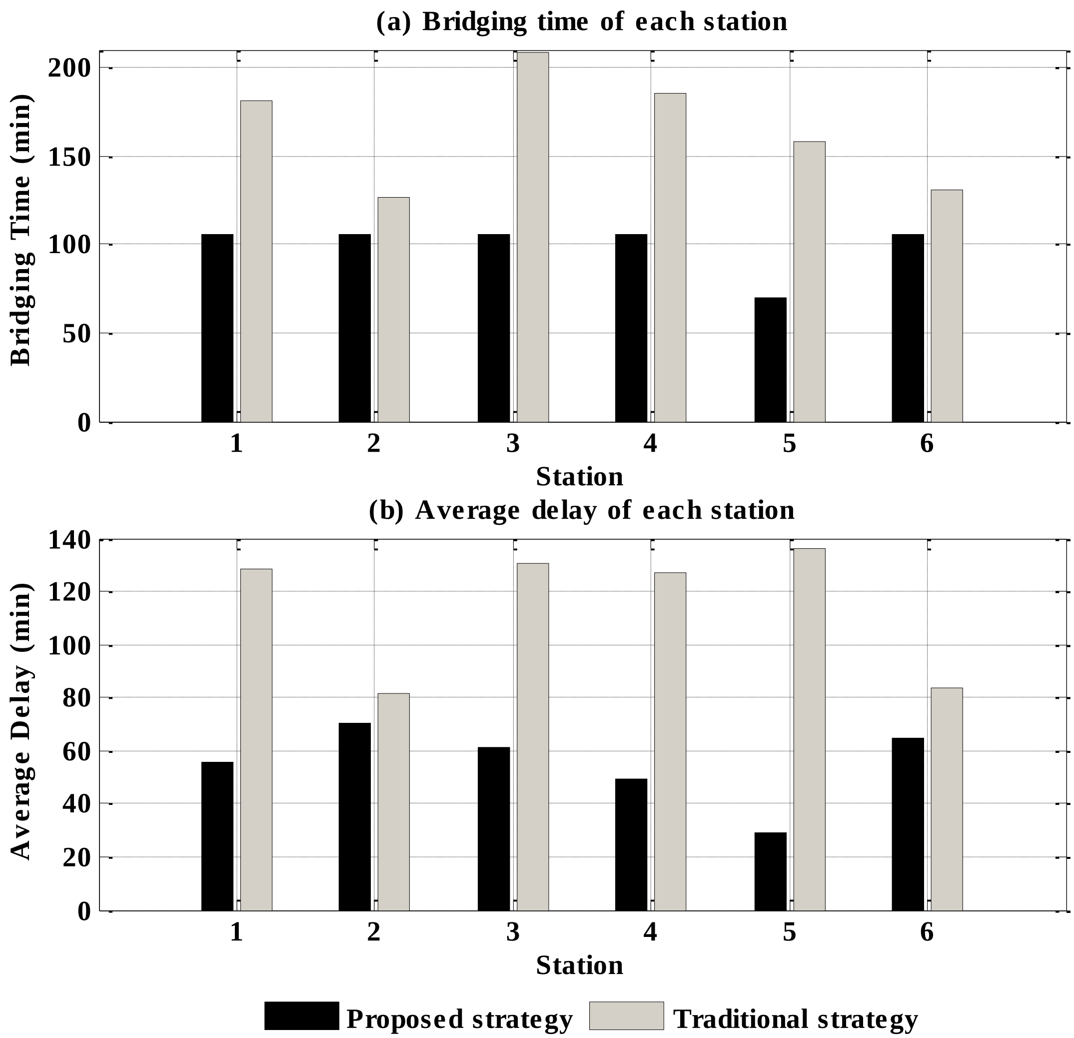


**FIGURE 6 Bridging time and average delay at each station.**

The analysis of each OD group also demonstrates the advantage of proposed strategy, as shown in Figure 7. Passenger delays are more evenly distributed in the proposed strategy. The range of average passenger delays in OD groups for proposed strategy is 68 min. It is much smaller than that for traditional strategy, which is 94.6 min. Similar result is observed for bridging times of passengers in each OD group.

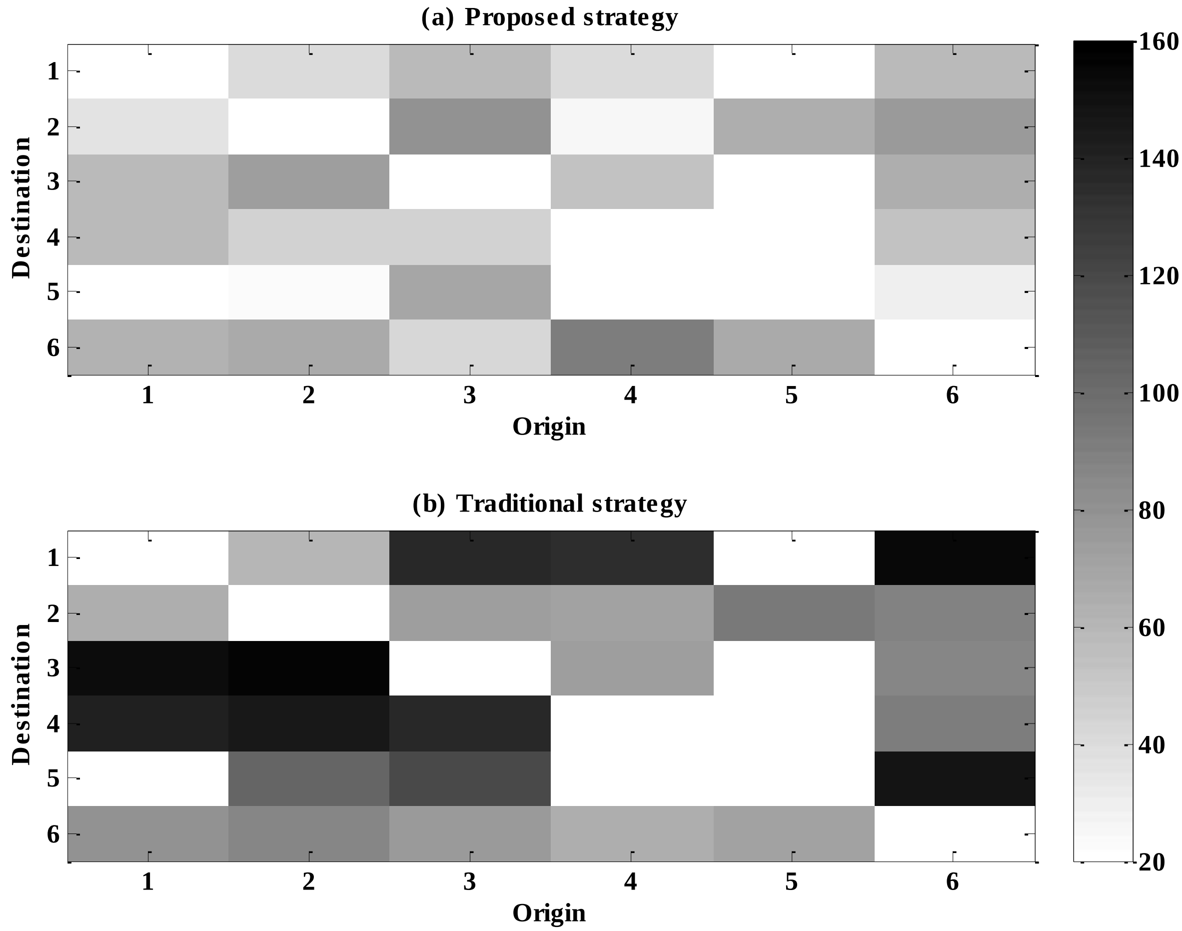


**FIGURE 7 Average passenger delay in each OD group (unit: min).**

Better performance of our proposed strategy stems from better patterns to transport passengers. Figure 8 presents the cumulative plots for completing the transportation of passengers from station 6 to other stations. In our proposed strategy, the patterns to transport passengers are adjusted according to different passenger demands. When passenger demand is small, buses transport passengers in several trips within a short time; when passenger demand is large, buses arrival at stations more regularly to transport stranded passengers. For instance, passengers from station 6 to station 1, 3, 4, 5 are transported within 2, 4, 1, 1 bus trips respectively since the passenger demands are small; while there will be buses arriving at station 2 almost every 10 minutes to drop off passengers from station 6 since the passenger demand is large. In contrast, passengers are always transported regularly in the traditional strategy. Passengers from different OD groups are treated equally regardless of different demands. For instance, there will always be buses arriving at a station every 20-30 minutes to drop off passengers from station 6. Similar results are observed for other stations.

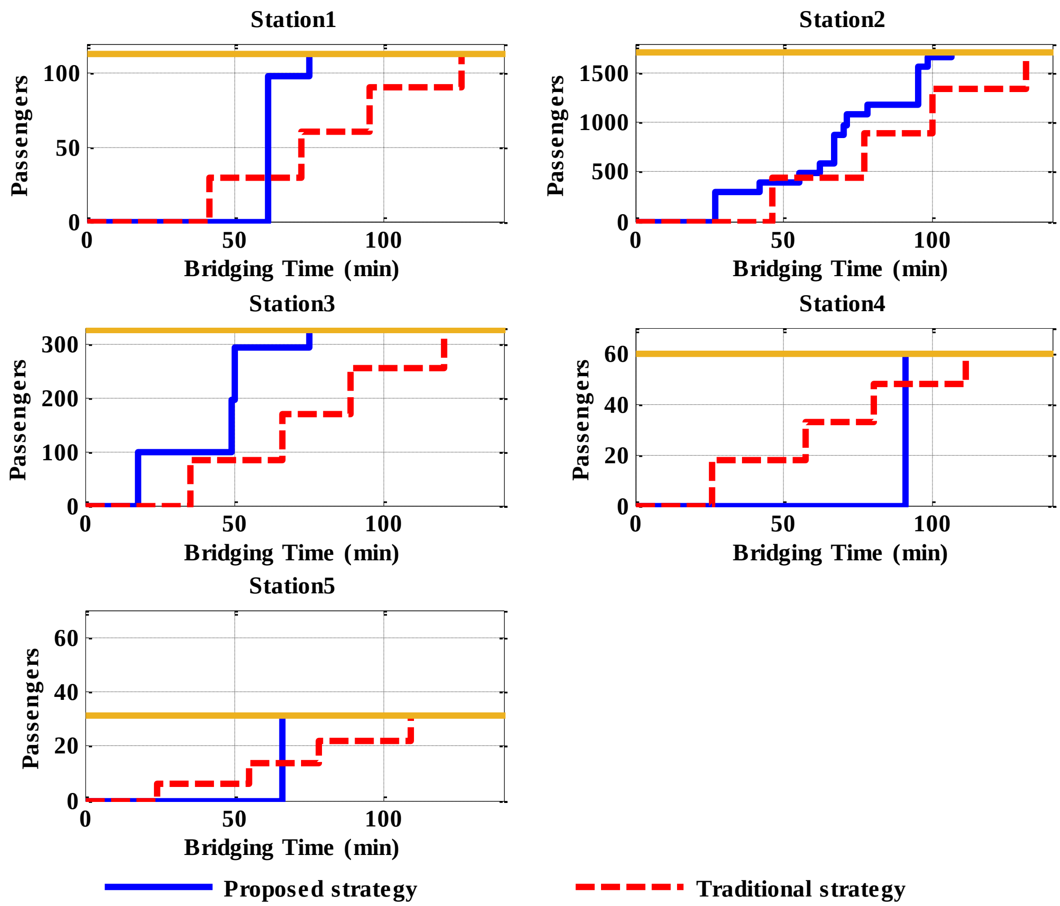


**FIGURE 8 Cumulative plots for completing the transportation of passengers from station 6 to other stations.**

## 5. CONCLUSION AND DISCUSSION

In this study, we propose a flexible bus bridging strategy to maintain passengers' journey in the affected stations during disruptions of metro networks. Unlike existing literatures to design bus routes and then allocate buses to predefined routes with specific frequencies, a novel bus bridging model is formulated to optimize a tailored bridging path for each bus. The proposed bus bridging strategy is formulated as a two-stage model to balance operational priorities of both transit agency and passengers. The Stage I model minimizes maximum bus bridging time while the Stage II model minimizes average passenger delay.

The proposed strategy is evaluated in a case study of the metro network in Rotterdam, The Netherlands. The results indicate that: (1) our proposed strategy outperforms the traditional strategy from the perspectives of both transit agency and passengers; (2) inconvenience of the disruption is distributed more evenly over passengers; (3) sensitivity analysis can be used to determine bus fleet size for the bus bridging service to achieve a certain level of response effectiveness; (4) patterns to transport passengers of the proposed strategy can be adjusted according to passenger demands.

The proposed model is somewhat limited by the assumption that passenger demands and travel times are not time-dependent. Further research could focus on extending the model to handle dynamic arrivals and departures of passenger as well as dynamic travel times. Also, other realism improvements such as stochastic elements in passenger demands and travel times can be considered in further research.

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