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Improving traffic management through consideration of uncertainty and stochastics in traffic flow

S.C. Calvert*[a], H. Taale[a c], M. Snelder[a b] & S.P. Hoogendoorn[a]

*Corresponding author

[a] *Delft University of Technology, Department of Transport & Planning; PO Box 5048, 2600GA, Delft , The Netherlands*

[b] *TNO, Netherlands organisation for applied scientific research; PO Box 49, 2600AA, Delft, The Netherlands*

[c] *Rijkswaterstaat, Lange Kleiweg 34, 2288 GK Rijswijk, The Netherlands*

Email addresses: s.c.calvert@tudelft.nl; h.taale@tudelft.nl; maaike.snelder@tno.nl; s.p.hoogendoorn@tudelft.nl

Abstract- In a bid to cost-effectively tackle congestion, traffic management is often seen as a key option to utilise road capacity. Prior to the application of traffic management measures, a-priori analysis allows the effectiveness of measures to be judged and where necessary adapted. However, current approaches do this without considering the effects of stochastic uncertainty and fluctuations in traffic flow. These stochastic effects have been shown to substantially influence the evaluation of traffic management measures. In this contribution, a methodological framework is proposed and demonstrated in a multi-part case study, applying approaches that explicitly consider stochastic variations and applications for traffic management. The results of the case study demonstrate the effectiveness of the models and highlight the necessity to consider uncertainty and fluctuations when a-priori evaluating traffic management measures.

Keywords: *traffic flow; traffic management; uncertainty modelling; homogeneity*

1. Introduction

When congestion becomes a problem on a road or road network, there are generally three main solution areas available to tackle it: construction, pricing or traffic management. For a long time road authorities could reasonably keep up with increasing traffic demand through expansion of the road network. However, this is a finite solution as space and resources are limited (Mayor 2005). Traffic management became an increasingly preferred option towards the end of the twentieth century as an alternative to construction in many cases. Traffic management proves a more efficient alternative and focusses on influencing traffic flows such that the existing road and network capacity is more effectively utilised resulting in a reduction in congestion (Mayor 2005; Hoogendoorn et al. 2008). There are many different types of traffic management, but all have a goal to influence traffic flow to achieve an underlying goal. Many goals exist for the application of traffic management, such as congestion reduction, shorter travel times, improved throughput, reduction in emissions, and a combination of these (Hoogendoorn et al. 2012).

The effectiveness of traffic management is highly dependent on the ability to influence traffic flow. Unlike the flow of fluids, traffic consists of larger individual particles, namely the vehicles and their drivers, which can be influenced. The particles portray a relatively large amount of stochastic behaviour, which is connected in a large part to human driving behaviour. The fluctuations that occur in traffic flow due to this stochastic behaviour have a large effect on the effectiveness of traffic management (Abu-Lebdeh and Benekohal 2003). Homogeneity in traffic is important for optimal throughput and traffic flow, however fluctuations create heterogeneity and should be limited.

While heterogeneity in traffic influences the effectiveness of traffic management measures, the day-to-day uncertainty in traffic also plays an important role to give a fair representation of the effectiveness of measures. Presuming traffic to be ‘average’ and analysing a ‘representative’ day to estimate the potential effectiveness of a measure has shown to lead to bias outcomes (Calvert et al. 2012; van Lint et al. 2012). This is mainly due to the area of effectiveness for traffic management in the extreme traffic scenarios with large delays. However, these extremities are averaged out when considering a representative scenario and therefore the effectiveness of traffic management is underestimated when not considering uncertainty.

For a long time, traffic fluctuations and day-to-day uncertainty have not been sufficiently considered when applying models to evaluate traffic management measures (Hoogendoorn et al. 2008). In many cases, this has been caused by a lack of understanding of the underlying influence of stochastics in practice, but also due to a lack of appropriate tools that can be applied in practice (Hoogendoorn et al. 2008; Draaijers et al. 2010; TMIP 2013) and address the issues mentioned in the previous paragraph. This paper aims to offer a clear methodological framework to tackle these issues by adding to the current state of practice and to offer tangible tools that are demonstrated in the described case study. Application of the methodology in the case study combines the state-of-art modelling practices into a collective approach that will aid practical implementation, but also the further development of the methodologies applied in evaluation of traffic management and other traffic control tools.

In this contribution, two new modelling approaches are demonstrated in a joint case study to a-priori estimate the effectiveness of traffic management measures by considering the effects of uncertainty and fluctuations in traffic. The two levels of modelling stochastics that are considered: *uncertainty* is considered in macroscopic stochastics, which describes day-to-day uncertainties *between* traffic flows in scenarios, and *fluctuations* is considered in microscopic stochastics, which describe microscopic variability *in* the traffic flow between vehicles. This distinction is necessary as both have inherently different consequences and require different modelling approaches, even if the source of the stochasticity is the same.

We start by presenting the methodology of the applied framework in section 2. This is followed by the analysis setup of the presented methodology in sections 3, 4, & 5, for the network scan, measure selection and model application respectively. The results of the case study are given in section 6.The effectiveness of the models in the case study is evaluated and discussed in section 7, followed by the conclusions in section 8.

2. Methodology

The main aim of this case study is threefold: to demonstrate the correct workings of the developed approaches, to demonstrate the necessity to consider stochastics in the evaluation of traffic management measures, and to show the applicability for the case study network. To integrally demonstrate the developed approaches, the case is carried out on a network with the aim to derive weakly resilient locations, offer traffic management solutions for these locations and predict the positive effect that the traffic management measures are expected to have. These three steps are described in greater detail in the presented framework in this section. In section 7, we will also demonstrate the necessity of considering uncertainties and traffic flow fluctuations when estimating the future effects of specific traffic management measures.

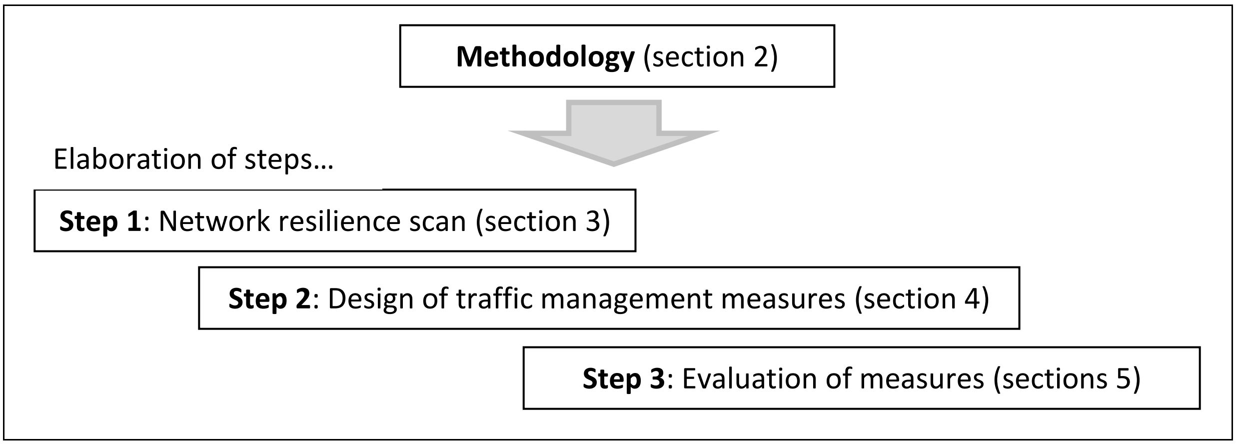

Figure 1: Overview of the methodological steps

The first part of the framework involves an evaluation of the network to detect vulnerable road sections where traffic management can be applied. This is further elaborated and applied in section 3. The second step involves the design of traffic management measures to tackle the main problem areas of the network, which is elaborated and applied in section 4. The third step evaluates the constructed measures for their effectiveness to reduce delays and is described in greater detail in section 5. These steps are displayed graphically in Figure 1.

Step 1: Network resilience scan

The first step involves a resilience scan of the considered network using the Link Performance Indicator for Resilience (LPIR) (Calvert and Snelder 2015) to determine vulnerable road sections. While some vulnerable locations in a network are obvious, others are less so, especially in complex networks with highly heterogeneous traffic flow. The LPIR evaluates the resilience level of individual road sections in relation to a wider road network. The focus of the methodology is on resilience as this considers the ability of road sections to recover from disturbances as well as considering the resistance of becoming congested. Resilience is found in many transportation related disciplines, such as transport networks, freight movements and logistics, but it not explicitly commonplace in traffic flow analysis (Cox et al. 2011; Chen and Miller-Hooks 2012; Ishfaq 2012). Also when considering the effect of stochastics in traffic flow for performance, resilience is considered relevant. The method is explicitly aimed at capturing the level of traffic heterogeneity. A distinction is made between a resistance part and a recovery part as part of the entire methodology with a focus on homogenous and volatile traffic, which plays an important role in resilience. The resilience is calculated in relation to the traffic flow characteristics at a flow level and the ability of road sections to maintain their predefined purpose to serve vehicles without overly experiencing congestion.

As resilience is defined in traffic flow as the combination of resistance and recovery, both elements are combined in the LPIR, given by:

$$LPIR = \frac{1}{T}\sum_{t=0}^{T}\left(\frac{\left[\frac{q+\psi^{q}}{v}\right]}{\left[\frac{q_{cap}(g,h)f+\psi^{cap}}{v_{crit}}\right]}1_{k\leq k_{crit}} + \frac{\left[\frac{q+\Delta q}{v_{eq}(q)}\right]}{\left[\frac{q_{cap}(g,h)f-q_{cd}}{v_{crit}}\right]}1_{k>k_{crit}}\right) \tag{1}$$

Here, for each time interval t, q is the traffic flow, v the traffic speed, v_{crit} the critical speed just before traffic breakdown, k the traffic density, q_{cap} the estimated capacity, f a temporal reduction factor for the capacity (i.e. due to incidents) and q_{cd} the estimated capacity drop. ψ^{q} and ψ^{cap} are volatility variables that give an indication of the extent of homogeneity for the traffic flow and capacity respectively. g and h represent the road and traffic characteristics and influence the capacity. Each variable is valid for a set time interval [*t*, *t*+*dt*). For readability, the notation of the dependence on *t* has been omitted from the equation. In Calvert and Snelder (2015) a more extensive explanation of the build-up of the equation is given. The LPIR gives an indication of the relative resilience of that road section compared to other road sections. A value of $LPIR \leq 1$ indicates that a road section is able to resist a significant drop in level-of-service and therefore remain uncongested and by definition must be considered resilient as well as robust. However, a road section that does suffer a drop in level-of-service, but can recover promptly should also be considered resilient as resilience considers the ability to recover from a disturbance or loss of service. However in the latter case, the road section may not be considered robust, as a failure event occurred.

The total LPIR score per road section is the average over all time intervals for the considered period. In this case, the considered period is a complete year of data for the A20 and A13 motorways in the year 2009, due to availability. The data is taken from an extensive collection of induction loops at a distance of approximately 300-500 metres. The induction loops relay one minute aggregated data on the traffic flow and the speed of traffic. The availability of a dense network of reliable induction loops allows this method to be efficiently applied, while we realise that this may not be present everywhere. For regions that do not have such a dense detection system, an adaptation of the LPIR to use the increasingly more available travel-times may be a good alternative. This would require extensive adjustments to the way the LPIR is calculated and does not fit in the scope of this paper and is therefore recommended for further research.

Step 2: Design of set of Traffic management measures

In the first decade of the 21st century, a coherent framework was developed in the Netherlands for the deployment and decision making processes surrounding traffic management (In Dutch: 'Gebiedsgericht benutten'). The framework is well regarded and is applied as the basis on which the majority of integral traffic management decisions are taken (Rijkswaterstaat 2003). In the framework, depicted in Figure 2, a distinction is made between services and measures for the application of traffic management. Services relate to the network wide objective that is being sought through traffic management for an identified problem. A service is in general a description of actions intended to achieve the desired effect for certain traffic flows, locations, or roads (e.g. limit the flow of incoming traffic, increase the capacity at the bottleneck). On the other hand measures relate to the physical application of an action that directly influences the traffic system. In general, measures are derived from services, where the measures are the actions that achieve the objectives set out in the services. The services categories are defined as: influencing throughput, redistribution of traffic flow, influencing demand, influencing capacity, and general network-services. In the past decade a number of additional services and measures may be added to the list, such as personalised in-car travel information and cooperative ITS.

Although there are many ways such an approach can be defined and implemented, we will focus here on the definitions and approach as described in (Landman et al. 2010; Hoogendoorn et al. 2014; Hoogendoorn et al. 2015). In this approach four main principles are applied:

- Spare capacity in the network is optimally utilised given the prevailing traffic conditions
- Capacity drop is prevented for as long a time as possible
- Traffic flows in the network should not be unnecessarily hindered (secondary congestion)
- A bottleneck should be resolved at the level at which it manifests.

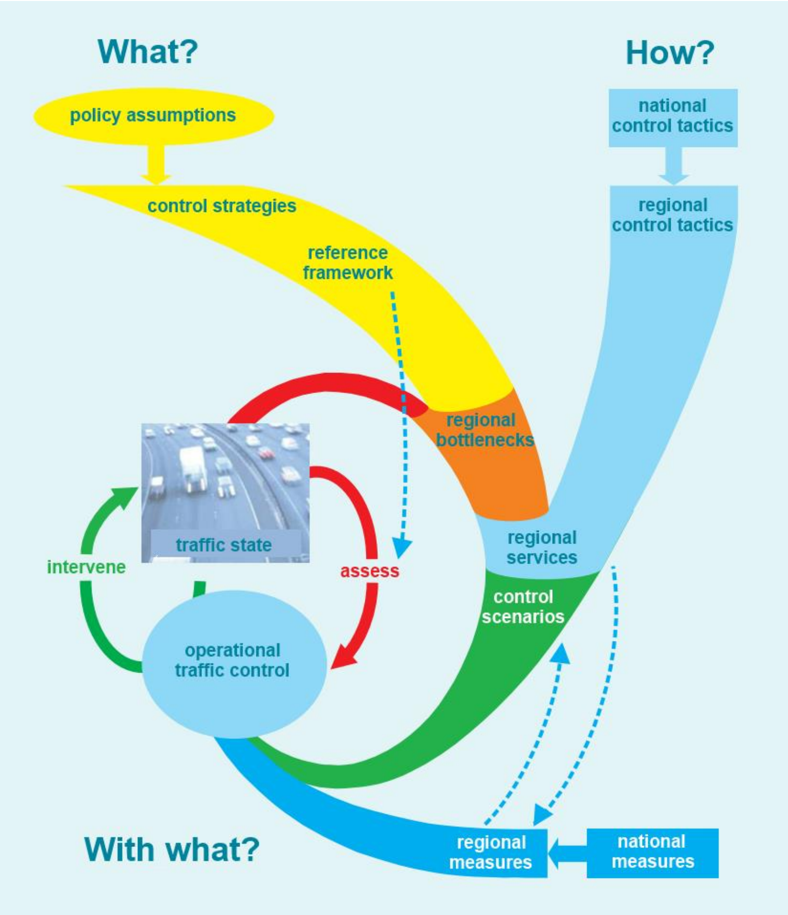

Figure 2: ‘Gebiedsgericht benutten’ (GGB) traffic management strategy framework (Rijkswaterstaat 2003)

These principles are applied in practice on a network such that multiple bottlenecks can be tackled without a solution at one location leading to secondary problems at another and also allowing multiple correlated bottlenecks to be simultaneously addressed. In most instances the measures that can be applied exist in the previously described framework; however the setup and application of the measures are controlled such that each one considers the setup of the other measures and such each measure does not work individually, but rather as part of an integrated system. In this case, a service solution will be defined for each identified location from which one or more traffic management measures can be selected. The selected measures will then be analysed for their effectiveness as described in the following step.

Step 3: Evaluation of measures

Forecasting the effect of traffic management measures for each carriageway is carried out using simulation models in which the stochastic character of traffic flow is considered on different levels. Two models are applied for this purpose: INDY-MonteCarlo and the First Order Model with Stochastic Advection (FOMSA). INDY-MonteCarlo is a dynamic macroscopic traffic model based on the LTM and enriched with advanced Monte Carlo sampling algorithms for uncertainty modelling (Yperman 2007; Calvert et al. 2014). INDY-

MonteCarlo is suited for use with uncertainty analysis and network scenarios. The considered scenario and uncertainties are given in the following subsection.

The FOMSA model is a Lagrangian based dynamic semi-macroscopic model based on first order traffic flow theory with additional invariant terms to consider stochastic driver behaviour (Calvert et al. 2015; Calvert et al. 2015). The use of Lagrangian coordinates allows vehicles and vehicle-groups to be individually followed and be assigned specific characteristics. Capturing micro-stochastic driving behaviour in a macroscopic model is important to accurately describe traffic flow phenomena on a macroscopic level. The model makes use of first order traffic flow theory in conjunction with an additional invariant term, the vehicle specific invariant, which describes the heterogeneous effect of vehicle behaviour and the level of aggressiveness of drivers and represents the vehicle specific change to a deterministic density value. The described model offers the advantages of including vehicle behaviour with an increased accuracy due to reduced diffusion effects, while doing this in a first order setting and therefore avoiding some of the complexity involved in second order model that are often applied to incorporate vehicle behaviour in macroscopic modelling (van Wageningen-Kessels et al. 2009). An analysis on the level of vehicles and vehicle interaction can give insight into the level of effectiveness of measures. This may be the case where there are multiple interacting traffic flows that cannot be captured as easily in a regular macroscopic model. In such a case, the FOMSA model is well suited and can be applied using a single vehicle platoon (therefore microscopically) or on a platoon basis.

As both models are setup for different types of analysis, the applied indicators differ and are applied as follows:

Sub-case 1: Congestion length and spillback:

$$L_{cong} = \max(L_{cong.end} - L_{cong.start}) \tag{2}$$

Travel time:

$$TT = \frac{\sum T_{B,i,t} - T_{A,i,t}}{N_{veh,i,t}} \tag{3}$$

The congestion length, L, is the largest distance from the start of congestion to the end of congestion or at a specific time. The travel time, TT, considers the average actual travel time of all vehicles between a two locations, A and B.

The results of the second analysis allow for a more in-depth qualitative analysis. This is carried out for the effects of congestion spillback over the various bottleneck locations for the three scenarios in this sub-case.

Sub-case 2: Total (network) delay:

$$TD = \sum_{t=0}^{t=e} \frac{veh(t)}{v_{free} - v_{obs}(t)} \tag{4}$$

Average peak travel time:

$$\overline{TT_{AP}} = \frac{\sum \frac{\sum T_{B,i,t} - T_{A,i,t}}{N_{veh,i,t}}}{\max(t)} \; for\ t = 1..4 \tag{5}$$

The total delay, *TD*, indicates the delay experienced by all vehicles during a specified time period, t=0..e, in relation to free-flow traffic conditions denoted by the speed, v_{free}. The average peak travel time, $\overline{TT_{AP}}$, considers the actual travel time for all vehicles during the main nominal peak period, $t = 1..4$.

3. Step 1: Network scan for weakness

The first step of the approach entails scanning the network for weak elements. The case study is performed for the A20 motorway, which forms the North Ring of Rotterdam motorway network. The network of greater Rotterdam is shown in Figure 3. The objective of the study is to evaluate the traffic operations on and throughput of the A20 motorway on the north ring of Rotterdam (see Figure 3) and consider traffic management improvements to improve traffic flow conditions on that corridor and the surrounding network. The A20 on the North Ring of Rotterdam has a number of bottleneck locations with spillback often reaching other bottleneck locations. There is a lot of intertwining traffic flows, both local and national. The congestion problems on the road have been a major concern for a while and continue to form a challenge, especially as there is very little space to expand the infrastructure to increase capacity. Therefore traffic management potentially has an important role to play.

Figure 3: Road network for Greater Rotterdam with the considered A20 motorway highlighted

The LPIR is calculated for the network and is performed using an aggregation time interval of 15 minutes. Data for the entire year of 2009 is used in the experiment. Road sections are defined as the section of road between two correct working loop detectors. In this case the critical density of traffic is assumed as 25 vehicles per kilometre per lane. Incidents are not explicitly considered, meaning that the incident reduction term is unused and has a value of 1. Upper bounds for the traffic flow are pragmatically estimated from data by taking the 99th percentile value of the flows for each road section. At bottleneck locations this will resemble the real capacity minus outliers, while at non-bottleneck locations the value will be less important as traffic flow will either remain uncongested (captured by the traffic speed) or will be influenced by an external bottleneck with a lower capacity value.

The LPIR results of the experiment are shown in Figure 3 on the considered network. Values generally vary between 0.0-1.4, with one section in particular reaching a LPIR value of 2.0. Road sections with higher values are sections that should be viewed in more detail and are the sections that should be most readily considered for improvement with traffic management to improve the traffic throughout and in turn the network performance. In Figure 4 road sections that appear with a red colour or darker are the least resilient. These are road sections that have a LPIR score equal to or above 1.2, with orange indicating values around 1.0, and yellow and green indicating values below 1.0, which are deemed to be road sections that have a lesser priority in comparison to the higher scoring road sections.

Using the results from the LPIR analysis a priority list can be drawn up that indicates which road sections should be addressed with priority by road authorities. This list is given in Table 1, with the numbered sections shown in Figure 4. A plausibility check based on expert judgement is performed to give an indication of the possible reasons of each section belonging to the list and the causality of the low resilience score. Causality can be added to the analysis by making use of the traffic characteristics and road characteristics terms from Equation (1). Data is added from other relevant variables, such as data on the road surface, infrastructure geometry, traffic composition, and many more. This more detailed analysis is not performed in this contribution, therefore causality is left to expert judgement.

Table 1: Locations with the highest LPIR values

Section nr (see Fig. 3)	**LPIR value**	**Location description**	**Section type**	**Initial estimation of problem**
1	2.0	A20L Terbregseplein		Joining flows after interchange and lane drop
2	1.9	A20R Centrum	Section with onramp	Narrow lanes, gradient and inflowing traffic on short onramp
3	1.7	A20L Kleinpolderplein	Weaving section	Weaving section
4	1.4	A20R Kleinpolderplein	Weaving section	Weaving traffic at interchange split
5	1.4	A20L Crooswijk	Weaving section	Weaving section

Figure 4: Network and results of the LPIR analysis

4. Step 2: Design of traffic management solutions

In the second step of the proposed framework, traffic management solutions are constructed for selected locations. The quick-scan resilience analysis of the network returned a number of locations that are found to be the least resilient (see Table 1). Using this analysis, a selection of feasible traffic management measures can be drawn up to tackle the problem locations. Two sub-cases are considered to allow both modelling techniques to be demonstrated. On the westbound carriageway (A20L) the FOMSA model is applied, as this corridor shows multiple interacting bottlenecks, which can be suitably analysed by this model. On this stretch it is ill advised to consider a single location as the occurrences of multiple bottlenecks do not stand alone, rather a coordinated traffic management approach is required. On the eastbound carriageway (A20R) advanced sampling Monte Carlo is applied using the INDY-MonteCarlo model. The second sub-case considers location 2 from Figure 4 at which will be referred to as location A20R31. The results from the LPIR analysis into the resilience of the motorway are considered to a focus on especially weak areas on the carriageways.

4.1 Sub-case 1: Westbound carriageway of the A20 Ring Rotterdam

The westbound carriageway of the A20 Ring Rotterdam has a long standing problem during peak periods due to multiple bottleneck locations. There is very limited space available for the realisation of extra capacity and many traffic management measures thus far have not eradicated the expansive congestion problems.

<u>Location: A20L from Terbregseplein interchange to Kleinpolderplein interchange</u>

Problem: Multiple bottlenecks in succession:

- Merging flows after an interchange (Terbregseplein interchange)
- Busy on-ramp (Crooswijk)

Solution: Coordinated traffic management with multiple solutions.

- Facilitate merging
- Maximize bottleneck capacity
- Limit traffic flow

Solution approach scenario 1: **Dynamic lane drop prior to merge location.**
(Terbregseplein interchange)

The first scenario considers the reduction of traffic flow into the motorway corridor. Due to congestion at the merging section at Terbregseplein, vehicle interaction can lead to congestion for both inflowing traffic flows (see Figure 5). This combined with the lane drop leads to an increased reduction of the local capacity. By moving the lane drop upstream before the merge, removes the necessity to merge over all lanes and means that any congestion resulting from the lane drop only affects one of the incoming flows. This measure is focussed on reducing the inflow of vehicles into the problem area and thus reducing the chance of secondary congestion on the Rotterdam Ring.

Figure 5: Terbregseplein motorway merge considered in scenario 1 of sub-case 1

Solution approach scenario2: **Construction of the considered A16-A13 bypass extension**

While creating extra capacity on the A20 is not possible, for a number of decades there have been plans to build a bypass extension to the A16 motorway, which would reduce the size of the traffic flow on the A20 (see Figure 6). The construction of the bypass diverts traffic from the A16 and A20 that have their destination north of Rotterdam away from the city ring road and therefore reduces the pressure on the North Ring (A20).

Figure 6: Planned A16/A13 bypass considered in scenario 2 of sub-case 1 (Rijkswaterstaat, 2015)

Solution approach scenario 3: **Ramp-metering (Crooswijk on-ramp)**

The third considered scenario involves focussing on the most significant downstream bottleneck location on the carriageway. As congestion moves in an upstream direction, the most downstream bottlenecks are of most significance as spillback will influence the greatest area. The on-ramp and weaving section at Crooswijk (location 5 in Figure 4) is one of the most downstream bottlenecks on the corridor, where the inflow of traffic is high during peak periods and has a disruptive effect on the main carriageway. Therefore ramp-metering is applied on the on-ramp to reduce the inflow and level of disruption on the main carriageway and therefore lead to a lower level of congestion and upstream spillback into other bottleneck locations. This scenario also requires network traffic management for the secondary roads that connect to the on-ramps due to limited space on the on-ramp for buffering and additional spillback onto the urban roads is undesired. This methodology has been previously described by (Hoogendoorn et al. 2014; Hoogendoorn et al. 2015). The secondary network traffic management is not modelled in the case and is presumed possible.

4.2 Sub-case 2: Eastbound carriageway of the A20 Ring Rotterdam

Similarly to the westbound carriageway, the eastbound carriageway also has extensive congestion problems with few options for capacity expansion. However there is one clear bottleneck location at which the majority of congestion occurs. This allows a more focussed approach to the problem.

Location: A20R Centrum (A20R31)

Problem: Narrow lanes, gradient and inflowing traffic on short onramp

Busy onramp with short merge distance onto a carriageway on a gradient with narrow lanes.

Solution: Restrict flow / Buffer traffic

Solution approach: Ramp-metering (Redesigned with coordinated traffic controls for secondary roads)

A ramp-metering installation is already present at the onramp, but not in use, partially due to the spillback onto the secondary road network and partially due to the limited effectiveness. The proposed measure will make use of the ramp-metering installation with an increased buffer-area. As the buffer area will still be insufficient and it is infeasible to allow traffic to buffer on the upstream roundabout, coordinated traffic control is proposed from traffic onto the roundabout for the directions heading to the onramp (Hoogendoorn et al. 2014; Hoogendoorn et al. 2015). The exact control setup will not be considered in the case, however the effect on the roundabout will. The effect of ramp-metering should delay the on-set of congestion, which has a positive effect through a reduction of the duration of the capacity drop and the reduction in secondary effects from the spillback from congestion on the motorway network. The effect of a reduced capacity-drop duration is estimated at 2% during the entire peak period on the upstream bottleneck link (Zhang and Levinson 2003).

Figure 7: Rotterdam Centrum onramp considered in sub-case 2

5. Step 3: Models and scenarios for evaluation

In this step, we will discuss the (experimental) set-up of the two models that will be used for the respective subcases. Also, the applied distributions and scenario application is discussed in this section. The correct choice and set-up of appropriate modelling tools is essential for the correct assessment of the measures that have been put forward in section 4. For subcase 1, we have opted to use FOMSA to model the interaction between bottlenecks, as the model considers microscopic fluctuations in traffic and therefore allows interactions between bottlenecks to be visible as trajectories are followed. For subcase 2, we elect to make use of INDY-MonteCarlo because of its ability to consider scenario based uncertainties that are present on the A20R at the considered bottleneck location.

5.1 FOMSA model setup (sub-case 1)

The first sub-case considers the westbound A20L carriageway over a distance of 11.6 km. The FOMSA model is setup with the correct number of lanes for each road section, including the presence of peak hour lanes. Vehicles can enter the corridor at on-ramps and exit it at off-ramps, positioned along the corridor. Traffic on the on- and off-ramps is only modelled upon entering and existing the main corridor, but is not explicitly modelled on the ramps themselves. Stochastic inflow of vehicles at the on-ramps is present and is derived directly from data and is therefore realistic. At on-ramp locations vehicles are forcibility added to the road and the surrounding vehicles on the carriageway have the opportunity to adjust their speed and headway to accommodate the new vehicle, as would be the case in reality. The basic setup of the traffic demand is derived from traffic data collected from the motorway at the relevant in- and outflow locations. During calibration of the model these values were adjusted in a conservative manner to create an accurate congestion pattern for the morning peak period, which is the dominant peak period. At locations on the carriageway where further capacity reductions are present, an additional capacity reduction is applied, which directly influences flow through the fundamental diagram. An example of this is at a location prior to the Crooswijk onramp where there is a sharp bend in the carriageway together with a gradient.

The applied fundamental diagram has a nominal jam density of 140 veh/km and a critical density of 25 veh/km. The maximum speed limit is 100 km/hr, the critical speed is set at 85 km/hr and the minimal spacing at standstill is 7.5 m. A time-step of 0.5s is applied to comply with the number of lanes and traffic density and the vehicle group size is 2 vehicles per group. A forced capacity drop value is applied of 10% for congestion, while the advection invariant is set at a value of 0.2 and the maximum acceleration bound is 1.0 m/s2.

The simulation is carried out for a time period of 60 minutes, in which the traffic flow is gradually increased up to the desired level and maintained for 15 minutes after which it is reduced to a lower level to allow congestion to dissipate. This is sufficiently long to demonstrate the build-up and dissipation of congestion. A short increase and decrease is chosen as a controlled way to demonstrate the effects of congestion and the performance of the motorway stretch. Use of real demand profile data proved complicated and overly time-consuming for the sake of the required demonstration and was not chosen. As different random sampling of the vehicle characteristics can lead to different results, a single identical sample is taken which is applied identically to each scenario for the sake of comparison.

5.2 INDY-MonteCarlo model setup (sub-case 2)

The applied network for sub-case 2 is shown in Figure 2. The network consists of 8200 links and 285 zones and is calibrated for an afternoon peak period between 2-8 PM. The network is derived from the Dutch national model and therefore has accredited speed and capacity values. The assignment model is INDY, which is a dynamic macroscopic model, which makes use of the Link Transmission Model (Yperman 2007). The network is calibrated for the evening peak period, which is the dominant period for the carriageway. The model is run with time steps of 10 seconds. The applied Monte Carlo routine makes use of Sobol numbers on the two input variables: demand and capacity, to construct a well distributed set of samples. Sobol quasi-random numbers were previously shown to give a good distribution of samples and 20 samples proved also sufficient to obtain and good distribution. Identical sample values are applied for both the reference and traffic management scenario for sake of comparison. The sampled distributions are shown later in this sub-section.

5.3 Scenarios and boundary conditions

Scenarios and stochastic fluctuations in traffic flows are considered in the analysis. Scenarios are defined as uncertainties on a day-to-day level or even on a greater time horizon, such as over multiple years and reflect the possibility of a set of conditions being present for a longer period of time during a day, such as weather conditions, the present of an incident or road works. Fluctuations are defined as inherent stochastic changes dynamically during a relatively short time period. Such fluctuations are often difficult to exactly predict in advance and are often the consequence of local conditions combined with external influences from the current scenario or scenarios. The main uncertainties can be reduced to variations of the traffic demand (on a day-to-day basis and for scenarios) and variations in capacities. Figure 8 gives an overview of how capacity and demand variations are influenced by scenarios and fluctuations in the traffic system. When considering day-to-day uncertainties, external temporal conditions play an important role, such weather effects, day of the week, etc. For stochastic fluctuations between vehicles, behavioural aspects are far more important, such as time-headways and level of aggressiveness. Traffic demand and infrastructure characteristics have a substantial effect on both uncertainty and fluctuations.

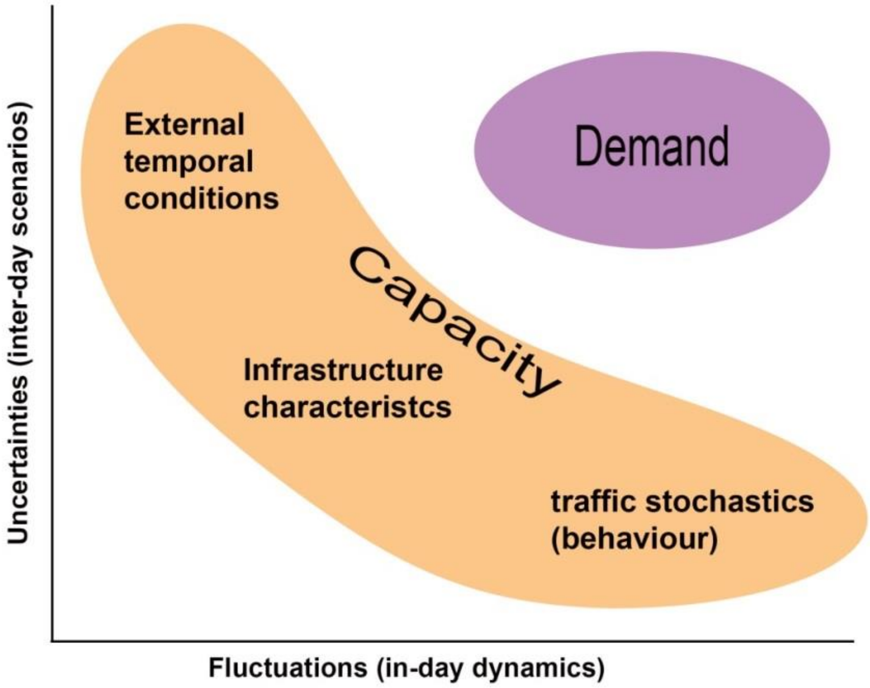

Figure 8: Relationship between uncertainty and fluctuations in traffic demand and capacity

In sub-case 2, a choice is made in relation to the scenarios to be considered for each location. The scenarios determine the demand profile for traffic and the base capacity levels for the network. For example, if a scenario is considered for a weekend day in wet weather, the traffic demand distribution will represent a set of feasible demand for a weekend day and the road capacity will represent a distribution of empirically obtained capacity values in wet weather. Dynamic in-day fluctuations of the traffic demand and actual capacity fluctuations are applied to the demand profiles and capacity values for sub-case 1. Doing this completes the distributions to be applied in the model analysis.

The goal of this case study is to evaluate the effect of traffic management on the A20, primarily during regular peak periods and demonstrate the applied models. For this reason the scenarios and the applied distributions are taken from non-holiday days. The demand distributions give an indication of the level of demand and the spread of the demand and is derived for each day of the week separately. The level of demand is derived through the selection and analysis of a set of five locations spread out across the network at a major motorway on which no or little congestion is present. The presence of congestion prohibits accurate demand estimation, as capacity is exceeded and therefore the measured levels do not resemble the true demand. The following assumptions are made for the data-processing to construct the distribution:

- Only week days are considered, outside of the holiday periods
- Capacity and demand data is captured for the month September through November 2014 as this is a coherent and continuous period with only a single holiday week.
- Both carriageways are analysed separately.
- Capacity variation is only applied locally to the considered bottleneck location
- Global capacity variations are not applied.

Demand distributions are applied to all Origin-Destination pairings equally as a generic indication of business.

Capacity scenarios and distributions

In this case only local capacity variation is applied and in particular for the bottleneck locations that are specifically considered. The capacity distributions are derived using an adapted Product Limit Method, described in Calvert et al (2015). As driving behaviour is a major factor that influences capacity, a distinction is also made for the day of the week for the bottleneck locations. The distributions for the local capacity variation are shown in Figure 9.

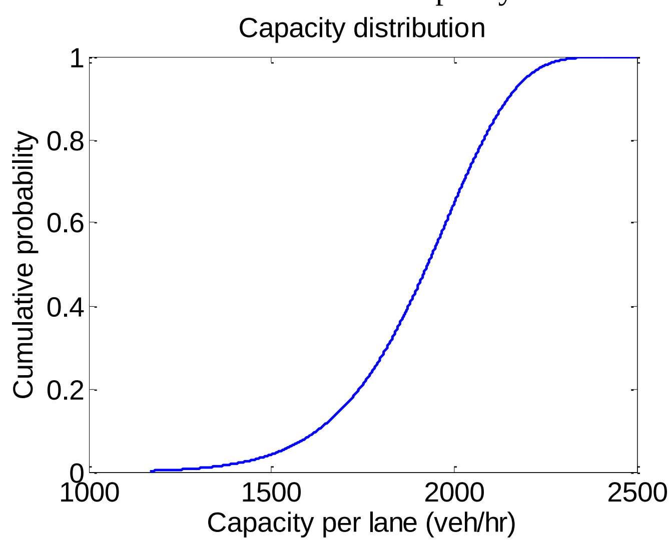

Figure 9: local capacity distribution

Traffic flow fluctuations

While uncertainties relate to scenarios, within scenario traffic flow remains a stochastic process with fluctuations that are often caused by differences between drivers. These fluctuations can lead to premature congestion and therefore an incomplete utilisation of capacity. Stochastic fluctuations are analysed for the two locations and a demonstration is given of improvements in homogeneity of traffic through the application of the proposed traffic measures. The initial parameter values are derived using the available data from the above distributions for the median day and calibration of the FOMSA model to represent the level of congestion. New parameter values are derived for the new situation with traffic management measure by sampling traffic stochasticity at a nearby reference location which has similar characteristics to the new situations. The parameter values are derived through comparison with the initial calibrated parameters prior to simulation.

Network adjustments

Implementation of the scenarios in the models requires adjustments of the network and to the traffic flows on the network compared to the reference scenario. These adjustments are given in Table 2.

Table 2: Model adjustments per scenario

Scenario	***Network/Flow changes***
Sub-case 1: Westbound (FOMSA)	
- Scenario 1 (lane drop)	Inflow reduced from 2+2 lanes to 2+1 lanes. Inflow rate from 3100 -> 2200 veh/hr on reduced road.
- Scenario 2 (bypass)	Inflow from A16 to A20L reduced from 3100 - > 1000 veh/hr
- Scenario 3 (ramp-metering)	Outflow from A20L to A16/A13 increased from 4100 -> 5100 veh/hr Inflow on on-ramp decreased from 1000 -> 500 veh/hr
Sub-case 2: Eastbound (INDY-MonteCarlo)	Capacity on-ramp reduced from 2052 -> 900 veh/hr Capacity weaving section increased from 5888->6006 veh/hr (+2%)

The first sub-case contains three scenarios, which influence two different locations. Scenario 1 and 2 are applied to the Terbregseplein interchange (see Figure 4, location 1). Scenario 1 reduces the inflow onto the Rotterdam ring through a lane drop prior to the lane merge and therefore aims to reduce secondary spillback and reduce traffic volume on the ring at the cost of possibly more congestion entering the ring from the east. Scenario 2 applies a planned bypass of the entire A20L north ring and results in a major reduction in the traffic volume. Scenario 3 applies ramp-metering at the Crooswijk on-ramp (Figure 4, location 5) to specifically target an important bottleneck. Scenarios 1-3 are implemented in the FOMSA model. The scenario for sub-case 2 also targets an on-ramp in the A20R direction eastbound. Implemented in INDY-MonteCarlo, the adjustments for the scenario involve a reduction of the inflow onto the main carriageway from the on-ramp located at the Centrum junction (Figure 4, location 2). A capacity increase of 2% is presumed on the main carriageway due to a reduction in weaving movements (Zhang and Levinson 2003).

6. Analysis and assessment of measures

The goal of the case is to demonstrate the effectiveness of the developed models in a real case for traffic analysis and effectiveness of traffic management. Additionally a further demonstration of the necessity of considering stochasticity in traffic flow for these analyses is sought. The latter goal is demonstrated in section 7 and the prior is addressed in this section. The traffic management scenarios are aimed at reducing congestion on the A20 motorway and increasing throughput. With this in mind, the total delay and travel time, as well as congestion length are considered as three relevant performance indicators. These were given in section 2 in Equations 2-5 and are applied in the analysis in this section.

6.1 Sub-case 1 (FOMSA)

The first sub-case considers three different scenarios to improve traffic flow on the westbound carriageway (A20L). The traffic speed diagrams for the three scenarios are given in Figure 10a-d. Comparison of the levels of congestion is made in relation to the reference scenario, shown in Figure 10a, for which no additional traffic management measures are taken. The numbers shown in Figure 10a represent the two locations where the traffic measures are applied, while in the scenario figures in the locations are given with an arrow. In the reference scenario, congestion occurs relatively early at the Crooswijk and propagates upstream. At Terbregseplein interchange congestion also occurs and is later exacerbated by the spillback from Crooswijk.

In scenario 1 (Figure 10b), the lane drop at Terbregseplein is moved upstream to before the merge with inflowing traffic from the adjoining motorway (A16). This has three consequences for the congestion pattern. Firstly the downstream activation of the Crooswijk bottleneck is avoided due to a reduction in the traffic flow that passes the merge point. The second consequence is that no congestion propagates upstream towards the A16 from the merge point, as congestion is triggered prior to the merge point. The third consequence is an increase in the severity of congestion on the upstream flow from the A20. However Figure 10b shows that the congestion remains limited due to the available upstream capacity to temporarily buffer the traffic.

Scenario 2 (Figure 10c) considers the presence of the A16/A13-bypass, substantially reducing the traffic flow onto the A20L. From Figure 10c it is clear that this has a large effect on the occurrence of congestion on the road. At all potential bottleneck locations, traffic flow is sufficiently reduced to prevent congestion occurring.

Scenario 3 (Figure 10d) focusses on the reduction of congestion at Crooswijk through ramp-metering. Inflowing traffic is reduced to 500 vehicles per hour at the onramp. This leads to a delay in the onset of congestion at the onramp and also leads to less severe congestion and a slower spread of congestion to upstream bottleneck locations. This also has a positive effect on the congestion that occurs at Terbregseplein, as can been seen in comparison to the reference. Further analysis of the bottleneck at Crooswijk showed that a reduction to approximately 200 vehicles per hour would be required to prevent congestion occurring at the on-ramp.

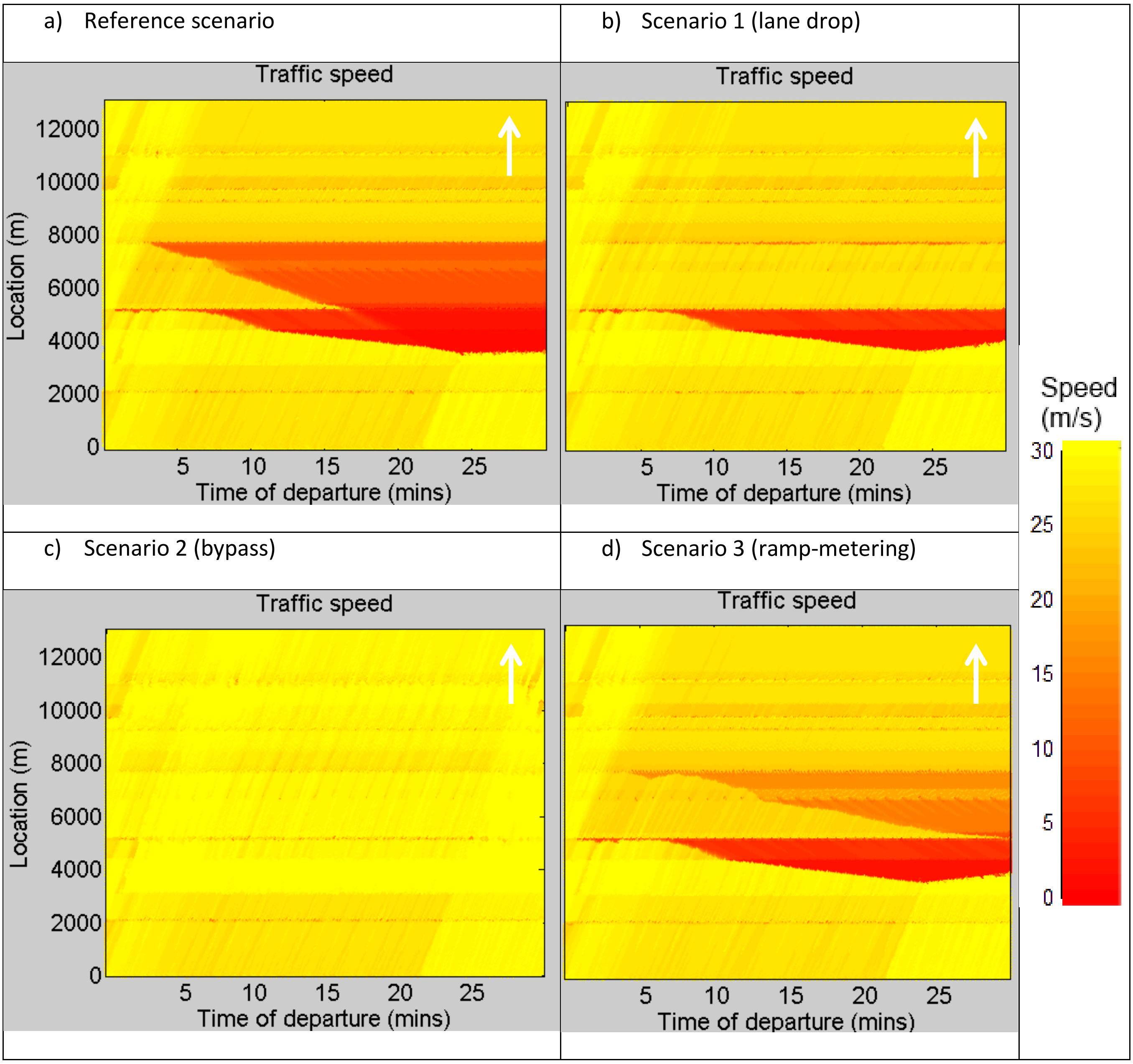

Figure 10: FOMSA model results given as the speeds for all scenarios of sub-case 1

A further analysis of the results of the scenarios is given by the developments of travel times and is shown in Figure 11. These are the actual travel times of vehicles that entered the motorway at the most upstream location and exited 11.5 km later at the most downstream location, which is not the case for all vehicles. The reference scenario shows an increasing travel time until the traffic demand is reduced and only a slight decrease in travel time once the inflow demand is reduced. This is due to the extensive congestion that occurs. The line for the reference scenario also finishes earlier as vehicles with later starting times spend too long in congestion to be able to exit the motorway stretch before the end of the 60 minute simulation. Scenario 1 (lane drop prior to merge) and Scenario 3 (ramp-metering) both show similar travel time patterns. For the higher inflow rate the travel time gradually grows as congestion increases, however at a much lower rate than the reference. Once the traffic inflow is reduced, congestion starts to dissipate and travel times quickly drops towards the free-flow travel time, which is approximately 6.5 minutes. Scenario 2 (Bypass) shows a slight increase in the travel time to 7 minutes when traffic is heavier, however as no congestion occurs, the travel-time remains low throughout.

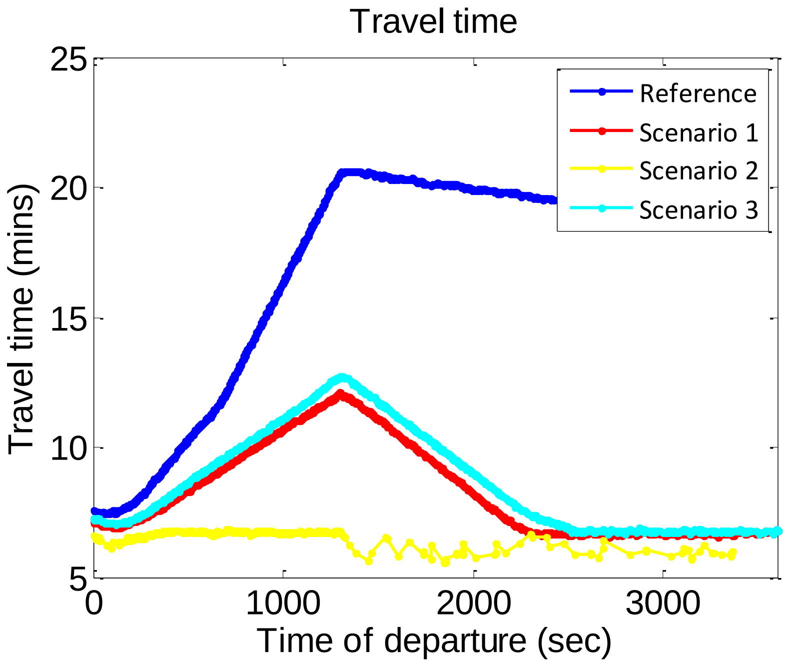

Figure 11: Travel-times in the sub-case 1 scenario's

6.2 Sub-case 2 (INDY-Monte Carlo)

The results of the total delay time of the 20 Monte Carlo simulations for the reference (blue) and scenario (red) are given in Figure 12. The results are in sorted ascendingly to give an indication of the distribution of the delay. The yellow bars show the percentage difference between the two. From this it is clear that there is an exponential distribution of the delay probability for the network. This means that in some extreme cases very high delays are present for certain traffic conditions, while in most cases there is some sort of an average delay, which corresponds to the extent of the traffic conditions. In the Monte Carlo simulations, two variables are applied, namely the global demand and the local capacity. Figure 13a-b shows the sampled demand and capacity factors respectively in comparison to the total delay time. The figures show that the effect of the traffic demand is much greater on the total delay than the change in capacity value. There is a very definitive increase for the demand samples, while the capacity samples shows a wider distribution with a small tendency for a higher total delay for lower capacity values, as may be expected. An explanation for this can be found in part by the fact that the demand factor is applied globally to the entire network, while the capacity factor is only applied to the analysed bottleneck location.

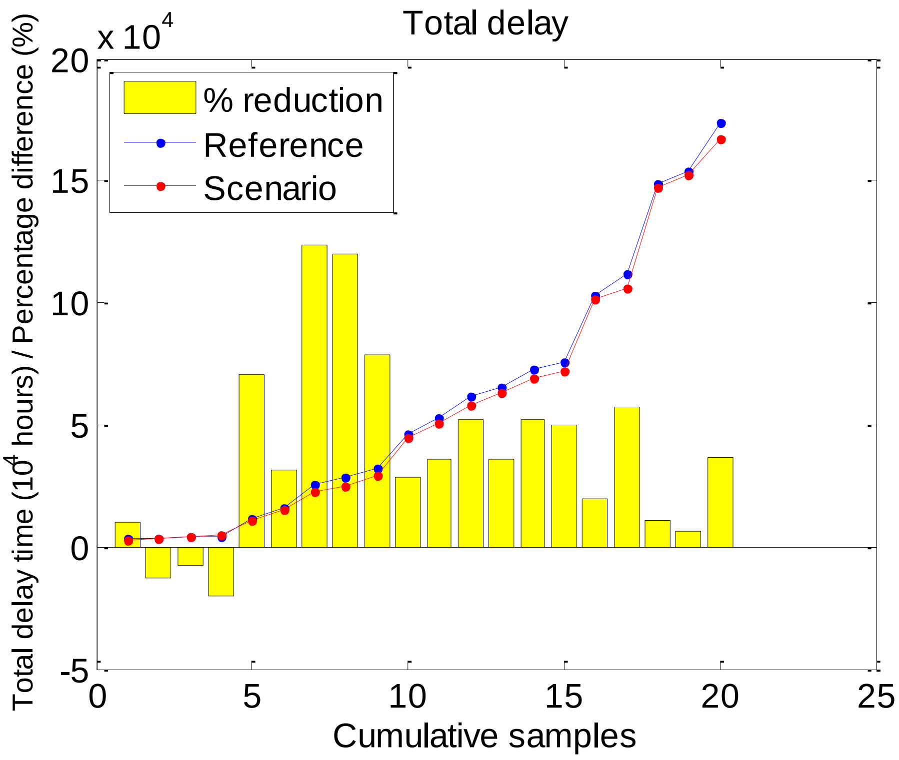

Figure 12. Total network delay for sub-case 2

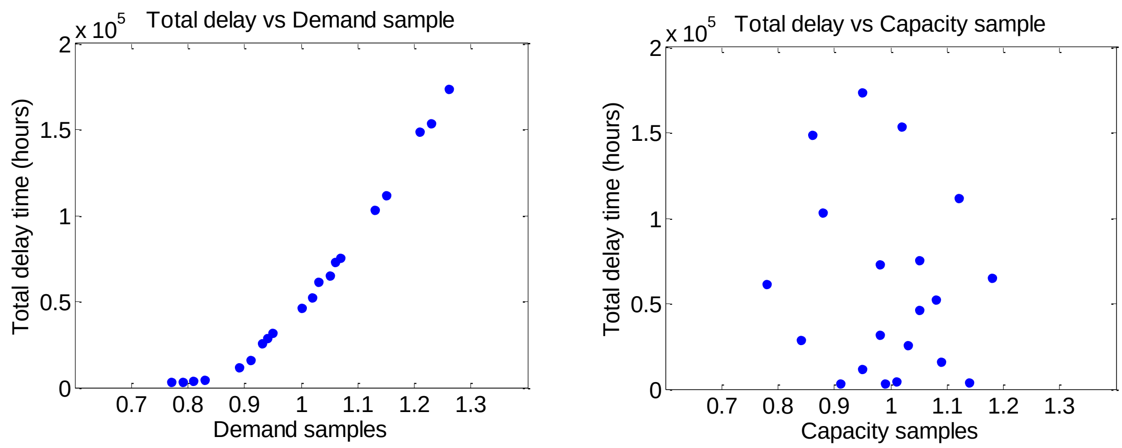

Figure 13 a-b. Total network delay versus demand sample for sub-case 2

The yellow bars in Figure 12 indicate the percentage difference between the reference scenario and considered scenario of the network delay. From this the effectiveness of the traffic management measure is indicated. The results show that the traffic management measure is effective with an improvement in the total delay of 2-12% for the majority of the samples, with a median improvement of 3.7%. The trend of the reduction in absolute terms is uniform over all samples, which results in a declining relative improvement for higher total delays. This can be expected as ramp-metering has a set bandwidth in which it is effective. Once traffic flow exceeds the upper bounds, congestion will occur and the improvement on traffic flow reaches its optimum.

The distribution of the travel times along the A20R motorway is given in Figure 14 for the reference (scenario) and scenario (red). The distribution of the travel times shows a much greater linearity than the delay time. This is due to the measurement of the travel time on the A20R only, while the total delay is calculated over the entire network. Therefore secondary delays, as a consequence of congestion on the A20R, are captured by the total delay time, which works exponentially for greater degrees of congestion at the considered bottleneck location.

The effect of the traffic management measure for the improvement in travel time is found to be in the range of 6-11% with a median value of 7.5%. The improvement in travel time along the considered road stretch is also more linear in relative terms, but does decrease slightly for the higher travel time samples.

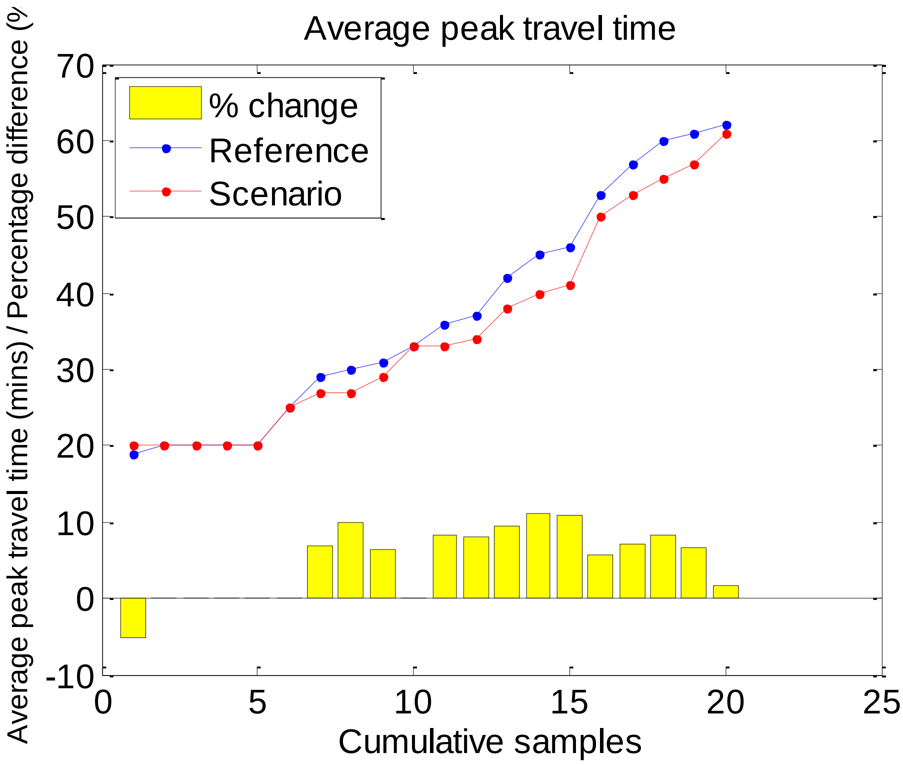

Figure 14. Average peak travel times for sub-case 2

In summary, the application of ramp-metering as a the traffic management measure for the Rotterdam-Centrum on-ramp is effective in reducing the network delay on the Rotterdam Ring (3.7%) and reducing the travel time on the A20R motorway (7.5%) and may be considered for implementation. The practical implementation of additional buffering and coordinated traffic signals on the connecting urban and provincial roads is not considered however. This should be reviewed before the ramp-metering can be applied to prevent secondary problems on the local road network.

7. Assessing the influence of stochastic characteristics

As part of step 3, consideration of the influence of variations in traffic flow is given to address the second goal of this case. The models in this case are designed and applied to consider uncertainty and stochastic fluctuations in traffic flow on different scales. The INDY-MonteCarlo model considers uncertainty in traffic flow and capacity values on a day-to-day level in which each individual day a different pattern is visible. The FOMSA model focusses on inter-vehicle stochastics in which each vehicle or vehicle group shows different behaviour and therein influences traffic flow. In this section the relevance of considering these stochastics is demonstrated by offering the alternative approach in which a deterministic approach is applied. When considering the real stochastic variations, one is considering the effects that are also present in reality on roads. Consideration of a non-existence average case as a deterministic calculation deviates from the real values which would be found in practice, which is shown in the next paragraphs. A discussion is also given on the outcomes.

7.1 Sub-case 1 (FOMSA)

The first sub-case, carried out with the FOMSA model, considers stochastic behaviour between vehicles, rather than their macroscopic day-to-day influence. Two parameters are adjusted to show their influence in the model, namely the advection invariant, which describes the following times, and the bounded acceleration rate. The case with no invariant value and no acceleration bound is shown Figure 15a. Figure 15b shows the same reference scenario with an invariant value of 0.4 and a bounded acceleration of 0.5 m/s2. The ‘stochastic’ case yields less congestion than the ‘deterministic’ case. Analysis of the results shows that this is mainly due to the ability of merging traffic to accommodate inflowing vehicles better when natural gaps are present, such as in the stochastic case. When all vehicles drive with identical gaps, inflowing vehicles force additional gaps when merging, which lead to a reduction of capacity. In numbers, there is little difference between both cases on the upstream bottleneck; however on the downstream bottleneck congestion in the deterministic case takes 904 seconds to reach the second upstream bottleneck. In the stochastic case this is 1143seconds, which is 26% longer. This causes the congestion spillback in the stochastic case to reach more than 200 metres further upstream than the deterministic case before stabilising and slowly dissipating. The effect on the travel time is found to be less than 1% during the congestion build-up. The influence of only considering the bounded acceleration is limited, as has been shown in previous research (Calvert et al. 2015). The individual characteristics of vehicles, when decelerating and accelerating, is not considered here and may change the outcome of the results as it can be hypothesised that it may lead to a quicker onset of congestion due to greater heterogeneity in the traffic flow at bottlenecks. Furthermore there may also be additional capacity drops effects which are limited here.

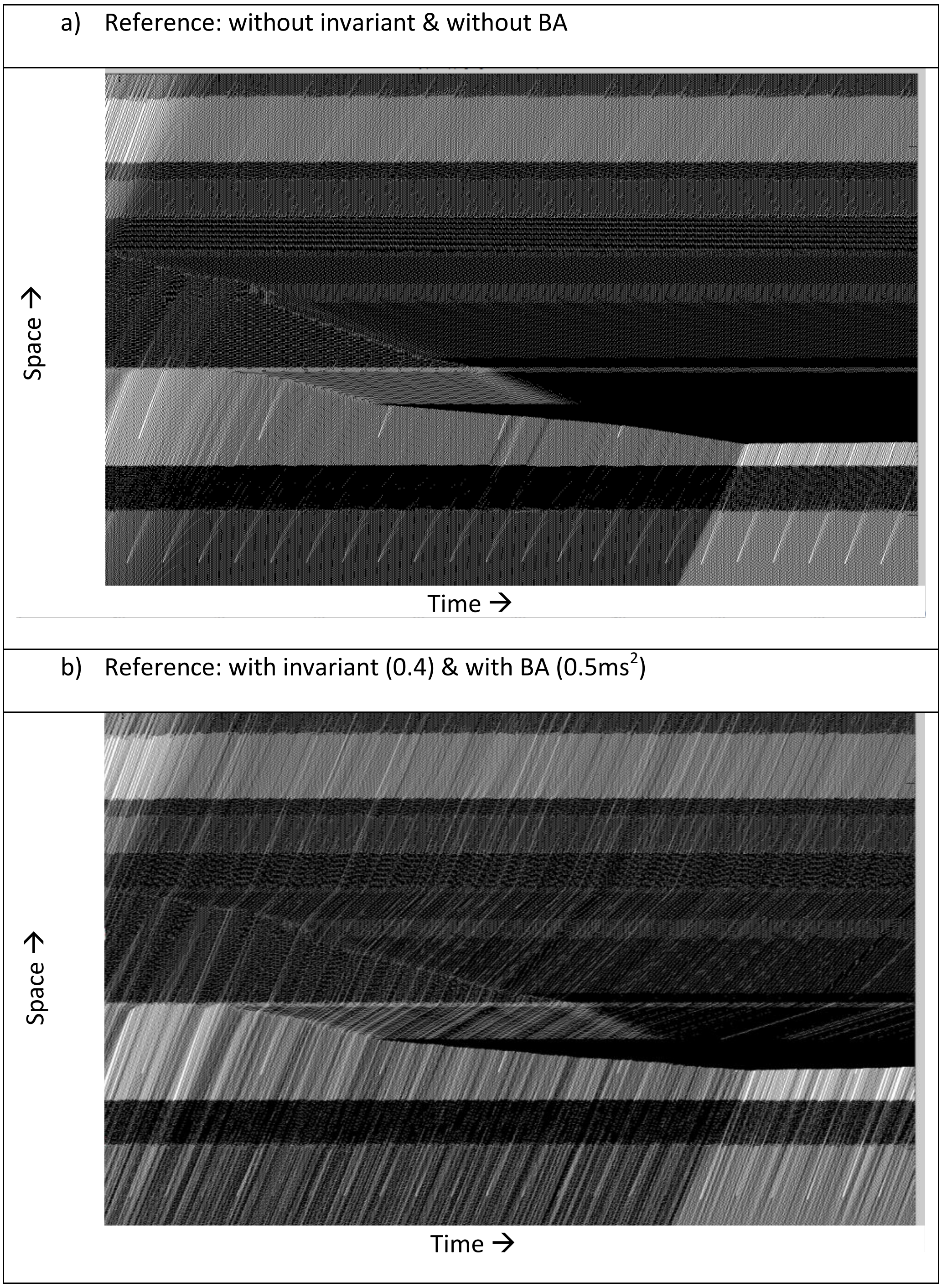

Figure 15a-b: Sub-case 1 comparison for modelling a) deterministically b) stochastically

7.2 Sub-case 2 (INDY-MonteCarlo)

A single INDY-MonteCarlo simulation is run for the median input values of the traffic demand and road capacity. This is the deterministic case and is performed for both the reference scenario and the traffic management scenario. The results of the deterministic runs are compared with the stochastic case in Figure 16. Here the deterministic results are given with dashed lines for both the reference and scenario for the total delay. The stochastic case here shows a reduction in total delay due to the traffic management measure to be 3.7%, while the reduction in the deterministic case is only 1.2%. From the results it is very clear that the deterministic case underestimates the improvements, which is due to the inability of an ‘average’ input value to consider the entire distribution of all possible values. This is especially the case for the more extreme ends of the distribution. This result demonstrates the importance to consider stochastic elements of traffic flow, especially when applying measures that are aimed at addressing extreme delays in traffic.

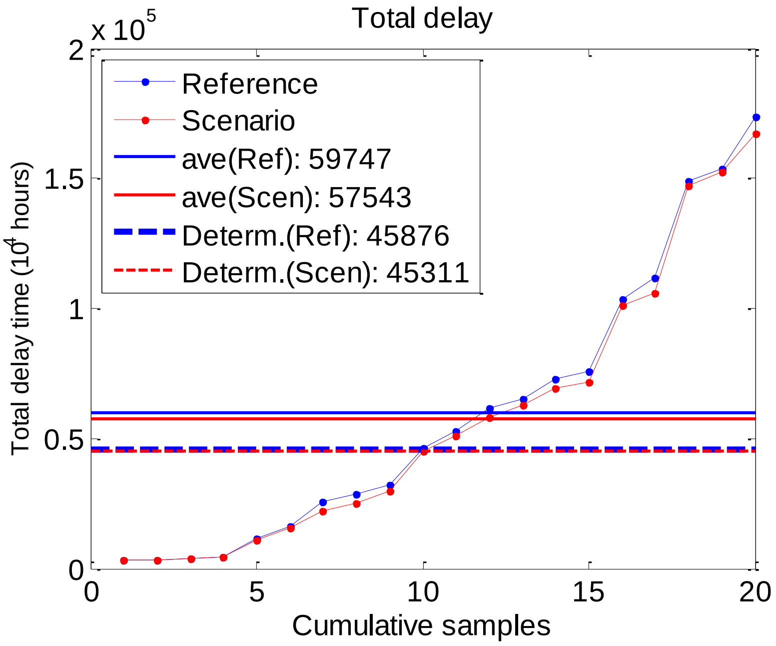

Figure 16: Comparison between stochastic and deterministic modelling for sub-case 2

7.3 Short discussion

It is not surprising that the models give different results when considering variations, either as uncertainty or fluctuations. It is important though to underline the relevance of the results and their implications. In practice, it is rare for uncertainty to be considered when analysing the potential gains for traffic management, as mentioned in the introduction. However, not doing so will lead to an underestimation of the positive effects of traffic management, for uncertainty, and to incomplete results for fluctuations. The underestimation for uncertainty scenarios lies in part in the flaw of averages. This states that when you put average input into a system, that the resulting output is not the same as the average of all results if they were to be processed separately (Savage 2009). In a traffic system, this is mainly due to secondary effects, such as congestion spillback into other flows, and the non-linearity of traffic flow and congestion, caused in part by the capacity drop. This is especially relevant for application is traffic management, traffic management measures aim to solve problems at the upper end of a distribution of flows, i.e. when traffic is (heavily) congested. If the upper end of the distribution of outcomes is underrepresented, then the outcome is always going to be an underrepresentation of the positive effects of traffic management.

In the case of fluctuations in traffic, there is something else afoot. Fluctuations in traffic lead to a greater heterogeneity in the traffic flow, which leads in turn to a lower operational capacity. Fluctuations will often be caused by traffic itself or by interaction with the infrastructure. When these fluctuations are not captured, an important connection is missing to explain why traffic may become congested at certain capacity or demand values on certain locations in a network. This can especially be relevant when dealing with multiple bottleneck locations, which interact with each other, as seen in sub-case 2 in this paper. Furthermore, many traffic management measures are geared to promote homogenous traffic flow. When the extent of heterogeneity in traffic is not considered in a modelling approach to estimate the effects of a measure, then the effect of the measure will be misrepresented. Of course there is something to be said on using microscopic models for this. However, these also have their limitations, such as requiring multiple runs for different seeds, limitation of network size and computation times, to name a few.

8. Conclusions

In this case study, a methodological framework for the application of stochastic effects in traffic modelling to aid the application of traffic management has been demonstrated. This was performed for the A20 motorway, the northern part of the Rotterdam Ring Road. The Link Performance Indicator for Resilience was first applied as a quick-scan method to indicate weak sections of a road network requiring attention. Weak sections on the network were identified and their sources were identified as possible locations to apply traffic flow improving traffic management measures. The application and selection of traffic management measures was applied and a set of measures were selected in two subcases. The first sub-case focussed on the eastbound A20L motorway using the FOMSA model to analyse the knock-on effects of individual vehicle behaviour on multiple interrelated bottleneck locations. The second sub-case focussed on the westbound A20R using the INDY-MonteCarlo model to consider day-today stochastic variations in the local capacity and global demand. The analysis in both cases is not related, but shows the application of the different models and why one model is more suited to one case, while another may be suited to another.

Ramp-metering at a critical location (Rotterdam Centrum onramp) on the westbound (A20R) carriageway was shown to be effective in reducing delays by 2-12%. On the eastbound (A20L) carriageway, ramp-metering at the Crooswijk onramp was found to have a positive effect on the reduction and delay of congestion. The most effective measures on this carriageway were found to be related to the reduction in traffic flow onto the North Ring. A change to the configuration of Terbregseplein merge between A20 and A16 traffic flows, showed that congestion on the A20L can be nearly eliminated by moving the lane drop prior to the merge. The construction of the A13/A16 bypass of the A20 Ring North was also considered and showed that such a measure would eradicate congestion as it would divert a sufficiently high amount of traffic from the A20L. However, it comes at a much greater financial cost and it not strictly a traffic management measure.

In the application of the case, the models showed they are able to perform well and demonstrated their value for their specific purposes and their ability to a priori evaluate potential traffic management measures for sensitive road sections and carriageways. The importance of consideration of the stochastic influence of traffic is further demonstrated for both day-to-day variations as well as intraday and inter-vehicle stochastics for the outcome of studies. Failure to consider the stochastic effects would of have resulted in a bias of 26% for the speed of congestion spillback in sub-case 1 and of 200% for the delay in the second sub-case. Finally, further research may be required to investigate how the method can be used for floating car data, such as travel-times, especially for regions that do not have sufficient macroscopic traffic data.

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