Augmented Full Wavefield Modeling - An Iterative Directional Modeling Scheme for Inhomogeneous Media

Hammad, Hussain; Verschuur, D.J.

Publication date
2017

Document Version
Final published version

Published in
Augmented Full Wavefield Modeling - An Iterative Directional Modeling Scheme for Inhomogeneous Media

Th A4 03

Augmented Full Wavefield Modeling: An Iterative Directional Modeling Scheme for Inhomogeneous Media

H.I. Hammad* (Delft University of Technology), D.J. Verschuur (Delft University of Technology)

Summary

We derive a representation theorem for modeling directional wavefields using reciprocity theorem of the convolution-type. A Neumann series expansion of the representation yields a series that is similar to that of Bremmer. A generalized Neumann series is also derived similar to that used for solving the non-directional Lippmann-Schwinger representation. An example shows how the series can model each scattering order separately for inhomogeneous media. This could potentially be useful in imaging and inverse problems.

Introduction

Full wavefield modeling is a method that incorporates transfer operators, reflection and transmission operators, as well as, propagation operators, in order to model wave propagation (Wapenaar, 1996; de Hoop, 1996; Berkhout, 2014). Hammad and Verschuur (2016a) have implemented full wavefield modeling for homogeneous media and have also shown the transfer operators for such media (Hammad and Verschuur, 2016b). They have also shown propagation operators and compared them to the ones obtained for locally inhomogeneous media. However, handling strongly inhomogeneous media requires the actual direct arrival rather than an approximate one.

This paper shows the derivation of a representation theorem that can handle such media. The representation theorem is similar to that derived by Corones (1975). A Neumann, as well as, a generalized Neumann expansion of the method is also presented. An example illustrates the implementation aspects of the theorem.

Differential system of equations

We start with the partial differential equations for directional wavefields in the frequency domain (e.g. Wapenaar and Grimbergen, 1996), which state that

$$\partial_3 \boldsymbol{p} = \boldsymbol{B}\boldsymbol{p} + \boldsymbol{s}, \tag{1}$$

where $\boldsymbol{p}$ is the power-flux-normalized wavefields given by $\boldsymbol{p} = \begin{pmatrix} p^+ \\ p^- \end{pmatrix}$, where the plus and minus signs denote upgoing and downgoing wavefields respectively. The source, $\boldsymbol{s}$, is also composed of two directional parts such that $\boldsymbol{s} = \begin{pmatrix} s^+ \\ s^- \end{pmatrix}$. The operator $\boldsymbol{B}$ is defined in terms of the transfer operator, $\boldsymbol{\theta}$, and the generalized slowness operator, $\boldsymbol{\Lambda}$, such that

$$\boldsymbol{B} = -j\omega\boldsymbol{\Lambda} + \boldsymbol{\theta}. \tag{2}$$

The transfer operator is composed of reflection, $\mathcal{R}^{\pm}$ and transmission operators, $\mathcal{T}^{\pm}$, from above and below such that $\boldsymbol{\theta} = \begin{pmatrix} \mathcal{T}^+ & \mathcal{R}^- \\ \mathcal{R}^+ & \mathcal{T}^- \end{pmatrix}$ and $\boldsymbol{\Lambda} = \begin{pmatrix} \Lambda^+ & \mathbf{0} \\ \mathbf{0} & -\Lambda^- \end{pmatrix}$.

Augmented Full Wavefield Representation Theorem

In order to derive a representation theorem for Augmented Full Wavefield Modeling, we start with the directional reciprocity theorem of the convolution type (Wapenaar and Grimbergen, 1996), which states that

$$\int_{\partial\mathcal{V}} \boldsymbol{p}_a^T \mathbf{N} \boldsymbol{p}_b d^2 n_3 d^2 \boldsymbol{x}_h = \int_{\mathcal{V}} \boldsymbol{p}_a^T \boldsymbol{N} \Delta\mathbf{B} \boldsymbol{p}_b d^3\boldsymbol{x} + \int_{\mathcal{V}} \left[\boldsymbol{p}_a^T \boldsymbol{N} \boldsymbol{s}_b + \boldsymbol{s}_a^T \boldsymbol{N} \boldsymbol{p}_b\right] d^3\boldsymbol{x}, \tag{3}$$

where $\Delta\mathbf{B} = \mathbf{B}_b - \mathbf{B}_a$, $\mathbf{N} = \begin{pmatrix} \mathbf{0} & \mathbf{I} \\ \mathbf{I} & \mathbf{0} \end{pmatrix}$, where $\mathbf{I}$ is the identity matrix and n_3 is the normal to surface along the vertical direction. Taking the volume, $\mathcal{V}$ equal to $\mathbb{R}^3$ and substituting the states in Table 1, where

Table 1: States A and B for deriving the representation theorem.

	State A	State B
Wavefield	$\boldsymbol{V}(\boldsymbol{x},\boldsymbol{x}')$	$\boldsymbol{p}(\boldsymbol{x})$
Operator	$-j\omega\boldsymbol{\Lambda} + \boldsymbol{\mathcal{T}}$	$-j\omega\boldsymbol{\Lambda} + \boldsymbol{\theta}$
Source	$\boldsymbol{I}\delta(\boldsymbol{x}-\boldsymbol{x}')$	$\boldsymbol{s}(\boldsymbol{x})$

the Green's function or augmented propagator, $\mathbf{V}$($\mathbf{x}$,$\mathbf{x}$'), which includes transmission is defined as

$\boldsymbol{V}(\boldsymbol{x},\boldsymbol{x}') = \begin{pmatrix} H(x_3 - x_3')V^+(\boldsymbol{x},\boldsymbol{x}') & 0 \\ 0 & -H(x_3' - x_3)V^-(\boldsymbol{x},\boldsymbol{x}') \end{pmatrix}$, whereas $\boldsymbol{\mathcal{T}}(\boldsymbol{x}) = \begin{pmatrix} \mathcal{T}^+(\boldsymbol{x}) & 0 \\ 0 & \mathcal{T}^-(\boldsymbol{x}) \end{pmatrix}$ and

$\boldsymbol{\mathcal{R}}(\boldsymbol{x}) = \begin{pmatrix} 0 & \mathcal{R}^-(\boldsymbol{x}) \\ \mathcal{R}^+(\boldsymbol{x}) & 0 \end{pmatrix}$, we arrive at

$$\boldsymbol{p}(\boldsymbol{x}') = \boldsymbol{p}_0(\boldsymbol{x}') + \int_{\mathbb{R}^3} d^3\boldsymbol{x}\ \boldsymbol{V}(\boldsymbol{x}',\boldsymbol{x})\boldsymbol{\mathcal{R}}(\boldsymbol{x})\boldsymbol{p}(\boldsymbol{x}) \tag{4}$$

where

$$\boldsymbol{p}_0(\boldsymbol{x}') = \int_{\mathbb{R}^3} d^3\boldsymbol{x}\ \boldsymbol{V}(\boldsymbol{x}',\boldsymbol{x})\boldsymbol{s}(\boldsymbol{x}). \tag{5}$$

Note that equation 4 is different from that given in de Hoop (1996) and Wapenaar (1996) since the propagator includes transmission, and the equation is more similar to that of Corones (1975).

We can derive the propagator in a similar manner to that derived in Wapenaar and Berkhout (1989). The resulting propagator, which includes transmission is

$$V^{\pm}(\boldsymbol{x},\boldsymbol{x}') \approx I + |x_3 - x_3'|(-j\omega\Lambda^{\pm} + \mathcal{T}), \tag{6}$$

where I is the identity operator.

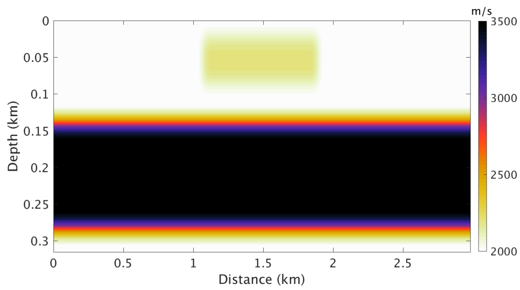

Figure 1: A laterally-inhomogeneous velocity model used for modeling. Although the method can handle strongly inhomogeneous media, the model is kept relativity simple for illustration purposes.

Iterative Solutions

We can write equation 4 in matrix-vector form such that

$$\mathbf{p} = \mathbf{p}_0 + \mathbf{VRp}, \tag{7}$$

where $\mathbf{p}_0 = \mathbf{Vp}$. We can also use the discretization scheme of Berkhout (1984) where each column of $\mathbf{P}$ contains one monochromatic shot record such that

$$\mathbf{P} = \mathbf{P}_0 + \mathbf{VRP}. \tag{8}$$

Using Neumann series, equation 7 can be solved iteratively such that

$$\mathbf{p}_k = \mathbf{p}_0 + \mathbf{VRp}_{k-1}, \tag{9}$$

where $\mathbf{p}_0 = \mathbf{Vp}$. Although Neumann series might theoretically converge, it may not numerically converge in practice with finite precision. So, therefore we seek a generalized Neumann series, where we

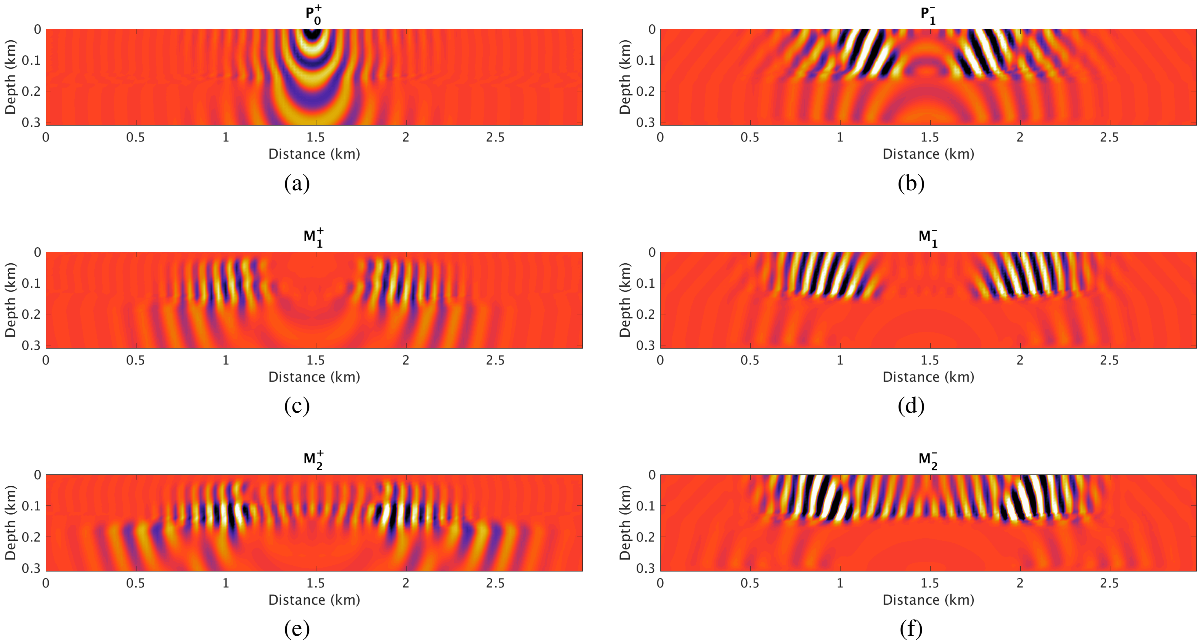

Figure 2: The real part of monochromatic wavefields, 15 Hz, for different scattering orders. (a) The downgoing direct arrival. (b) The upgoing primary. (c,d) The downgoing and upgoing first-order multiples, respectively, as well as, (e,f) the upgoing and downgoing second order multiples. Note that not only the upgoing and downgoing wavefields are obtained, but also the ones for each scattering order.

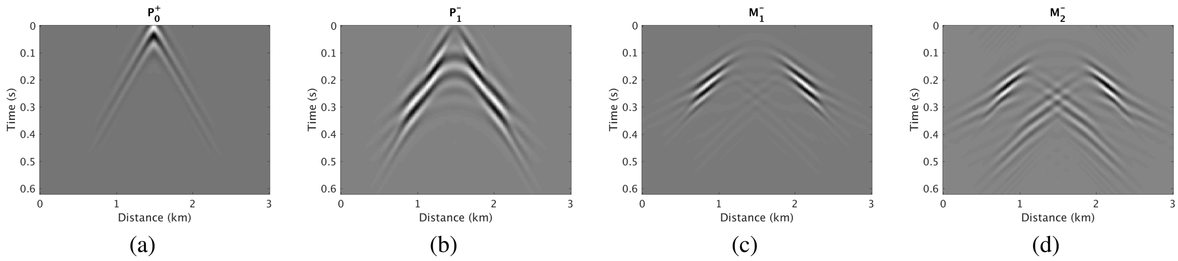

Figure 3: A time-domain shot record for different scattering orders. (a) Direct arrival. (b) Primary wavefield. (c) First-order multiples. (d) Second-order multiples.

adopt the so called overrelaxation method (Kleinman and van den Berg, 1991), which was applied to the two-way Lippmann-Schwinger integral equation. After adopting such scheme, we arrive at

$$\mathbf{p}_k = \alpha \mathbf{p}_0 + [\mathbf{I} - \alpha \mathbf{I} + \alpha \mathbf{VR}]\mathbf{p}_{k-1}, \tag{10}$$

where the the step length, α, is expressed as

$$\alpha = \frac{\mathbf{r}_0^H(\mathbf{I} - \mathbf{VR})\mathbf{r}_0}{\|(\mathbf{I} - \mathbf{VR})\mathbf{r}_0\|}, \tag{11}$$

where the initial residual $\mathbf{r}_0 = \mathbf{VRp}_0$ and the residual for each iteration, $\mathbf{r}_k$, is expressed as

$$\mathbf{r}_k = [\mathbf{I} - \alpha \mathbf{I} + \alpha \mathbf{VR}]\mathbf{r}_{k-1}. \tag{12}$$

Numerical Example

We demonstrate the theorem using the model shown in Figure 1. In this example, we assume the absence of a free surface, in addition to the absence of an upgoing component of the source. Figure 2 shows the downgoing wavefields and upgoing wavefields for different orders of scattering. Figure 2a shows the downgoing direct arrival, which also includes the transmission. Note also that the direct arrival is different from that computed using the so-called generalized Bremmer series. A more accurate direct arrival, in which the transmission effects are included, initiates the recursive process and, therefore, impacts the rest the of the iterations.

The upgoing primary is shown in Figure 2b. The downgoing, as well as, the upgoing multiples are shown in the rest of the plots (Figure 2c-f). Since this example does not include a free surface, nor does it include an upgoing component of the source, the multiples, $\mathbf{m}_n$ for a scattering order $n > 0$ can be computed from the difference between the even and odd-numbered wavefieds such that $\mathbf{m}_n^+ = \mathbf{p}_{2n}^+ - \mathbf{p}_{2n-2}^+$ and $\mathbf{m}_n^- = \mathbf{p}_{2n+1}^- - \mathbf{p}_{2n-1}^-$, as demonstrated in the original work of Bremmer (1951). The time-domain shot records are shown in Figure 3 for each scattering order. Note that the amplitudes are thresholded so that later multiples are also visible.

Conclusion

We have presented a representation theorem for Augmented Full Wavefield Modeling using directional reciprocity theorem of the convolution type. The resulting expression is similar to that of Corones (1975) and the Neumann series expansion represents a generalization of the Bremmer series. A numerical example demonstrates the capability of the method in not only directional wavefields but also those of each scattering order.

Acknowledgment

We would like to thank the Delphi Sponsors of their support. The first author would like to thank Saudi Aramco of supporting his research at Delft University of Technology.

References

Berkhout, A.G. [2014] Review Paper: An outlook on the future of seismic imaging, Part I: forward and reverse modeling. *Geophysical Prospecting*, **62**(5), 911–930.

Berkhout, A.J. [1984] *Seismic migration, imaging of acoustic energy by wave field extrapolation: A. Theoretical Aspects*. Elsevier Science Publishers B.V., Amsterdam, The Netherlands.

Bremmer, H. [1951] The W.K.B. approximation as the first term of a geometric-optical series. *Communications on Pure and Applied Mathematics*, **4**(1), 105–115.

Corones, J. [1975] Bremmer series that correct parabolic approximations. *Journal of Mathematical Analysis and Applications*, **50**(2), 361 – 372.

Hammad, H. and Verschuur, D. [2016a] Joint Migration Inversion for Laterally Varying Media. In: *78th EAGE Conference and Exhibition 2016*.

Hammad, H. and Verschuur, D.J. [2016b] *Towards forward modeling beyond Zoeppritz's: Full-wavefield modeling for inhomogeneous media*, chap. . 3960–3965.

de Hoop, M.V. [1996] Generalization of the Bremmer coupling series. *Journal of Mathematical Physics*, **37**(7), 3246–3282.

Kleinman, R.E. and van den Berg, P.M. [1991] Iterative methods for solving integral equations. *Radio Science*, **26**(1), 175–181.

Wapenaar, C. [1996] One-way representations of seismic data. *Geophysical Journal International*, **127**(1), 178–188.

Wapenaar, C.P.A. and Berkhout, A.J. [1989] *Elastic Wave Field Extrapolation: Redatuming of Single- and Multi-component Seismic Data*. Elsevier Science Publishers B.V., Amsterdam, The Netherlands.

Wapenaar, C.P.A. and Grimbergen, J.L.T. [1996] Reciprocity theorems for one-way wavefields. *Geophysical Journal International*, **127**(1), 169–177.