The Arched Strut
a Tool for Modelling Column-Slab Connections

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Publication date
2018
Document Version
Accepted author manuscript
Published in
Proceedings of the 40th IABSE Symposium

The Arched Strut – a Tool for Modelling Column-Slab Connections

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Abstract

The arched strut is an addition to the strut-and-tie (STM) tool kit. It models the combination of disturbed behavior in one direction with slender behavior in the perpendicular direction. Common applications for the arched strut are in the design of connections between a reinforced concrete slab and its supporting columns or punching of bridge decks.

The arched strut can be applied to any combination of shear and moment at a column-slab connection. The designer is given clear guidance on anchorage requirements for the flexural reinforcement and the expected ductility of the connection. The method does not model a particular failure criterion; rather, it defines an acceptable load path that meets design objectives.

The paper outlines the basis for the arched strut and presents examples illustrating its use in design.

Keywords: slab-column connections; slabs; punching shear; moment transfer; strut-and-tie modelling

1 Background

The Strip Model [1] is a general approach for the analysis of load transfer at a column-slab connection. The key element of this model is an arched strut to transfer shear between the slab and column. The curvature of the strut is the result of a transverse tension field generated by slender flexural behaviour in a direction perpendicular to the arch. The magnitude of the tension field is limited by the one-way shear strength of the slab.

The arched strut does not model a particular failure model. It provides a load path that is consistent with static constraints and does not exceed material capacities.

1.1 Layout of arch strips

As is usually the case with any type of strut and tie modelling, the global statics of the region should be established prior to developing the strut and tie model itself. In the case of a slab-column connection, this means choosing the global slab design moments and corresponding tributary areas for the load case being considered.

With the global statics of the slab established, the designer chooses an array of slab strips, called arch strips here, to transfer load between slab and column. Each arch strip is supported by the

column at one end and extends to a position of zero shear at the other. The width of the strip is defined by the supporting column.

The designer is free to choose any arrangement of arch strips for a particular load case. Different load cases may call for different configurations of arch strips.

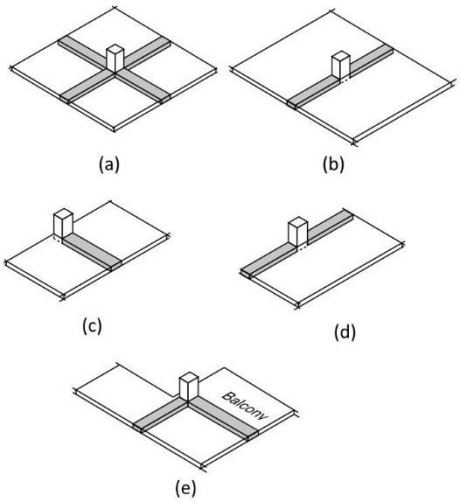

Figure 1. Possible arch strip configurations

Figure 1 illustrates a few possible configurations of arch strips. Figure 1(a) shows the configuration typical of most interior column-slab punching tests while 1(b) illustrates the arrangement that is more practical in design. The arch strip arrangement in Fig. 1(c) is typical of an edge connection under gravity load or a gravity plus lateral load where the lateral load adds to the negative moment about an axis parallel to the free edge. Figure 1(d) is appropriate for a load case where lateral load is reducing or even eliminating the negative moment at the free edge. Figure 1(e) shows an unusual connection with a balcony slab and a re-entrant corner.

Any column face that does not support an arch strip may be assumed to carry at most one-way shear. For example in Fig. 1(c), there is only one arch strip but there are three column faces that engage the slab. The two side faces not supporting an arch strip can be loaded in one-way shear.

A basic principle of the method is that support can be given only to load tributary to that support. Tributary areas are defined by column centre lines and midspan locations. The corresponding available supports are defined by column centre-lines and the selected configuration of arch strips. Figure 2 illustrates this concept in the context of an edge connection with a single arch strip perpendicular to the free edge of the slab.

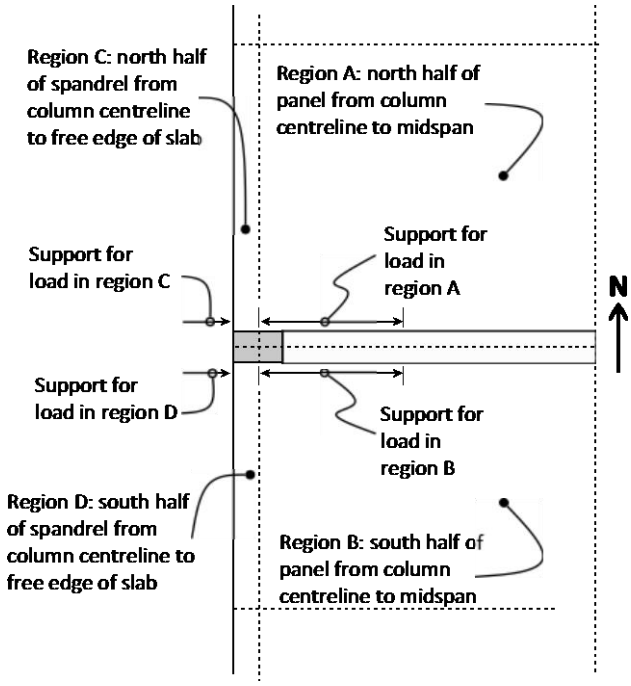

Figure 2. Tributary load and support

1.2 Limit analysis

Figure 3 shows the design loading of a typical arch strip. The arched strut itself acts at the column end of the strip amd is loaded in shear on each side by the adjacent slab. This side shear, q_c, cannot exceed the one-way shear capacity of the slab, however this is defined by the governing design standard. The parameter χ $(0 \leq \chi \leq 1)$ accounts for the strip being loaded more heavily on one side than the other. Where an arch strip along the free edge of a slab, $\chi = 0$; where an arch strip supports equal loads on each side, $\chi = 1$.

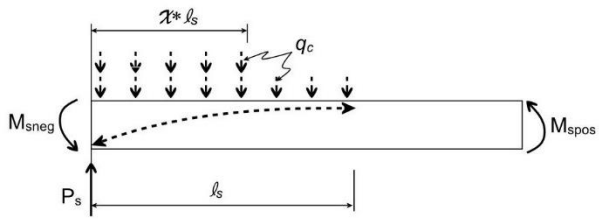

Figure 3. Design loading of arch strip

The length of the arched strut, l_s, comes from flexural equilibrium of the strip:

$$l_s = \sqrt{\frac{2M_s}{q_c(1+\chi^2)}} \quad (1)$$

where $M_s = M_{sneg} + M_{spos}$

The stepped nature of the loading diagram in Fig. 3 is not simply a calculation convenience; it is also a reasonable description of post-cracking conditions when the flexural reinforcement through the column begins yielding. Local yielding of reinforcement initiates the development of torsional moments that redistribute load within the strip.

Figure 3 does not include load applied directly to the strip. Direct load applied beyond l_s is part of the load that is redistributed by torsion. It is already part of the stepped loading diagram. Direct load applied within l_s, is more appropriately modelled with a conventional straight-line strut-and-tie rather than an arched strut. Ignoring this direct load errs on the safe side. Where the intensity of the uniformly distributed load is very high, as in mat foundations, ignoring the effect of direct loading may be excessively conservative.

The capacity, P_s, of a single arch strip, is given by:

$$P_s = q_c \times l_s(1+\chi) = \sqrt{\frac{2M_s q_c(1+\chi)^2}{1+\chi^2}} \quad (2)$$

1.3 Shear capacity

Equations (1) and (2) require design estimates of the one way shear strength of the slab, q_c. Here q_c will be based on the Canadian design standard [3] but any valid estimate of one-way shear capacity could be used instead.

$$q_c = \phi_c \beta \sqrt{f_c'} \times d_v \quad (3)$$

where $\phi_c = 0{,}65$ is a partial safety factor, $\beta = \frac{230mm}{1000mm+d_v}$ is a shear factor that accounts for size effect, $d_v = 0{,}9d$ is the effective depth of the slab for shear, and f_c' is the design strength of the concrete.

1.4 Flexural support of strip

The flexural support, M_s, provided to the arch strip is the design moment acting within a pre-defined band, b_{as}. In most cases at an interior connection, the designer will choose two collinear arch strips with b_{as} taken as c_2 plus $1{,}5h$ on either side, where c_2 is the column dimension perpendicular to the strip and h is the thickness of the slab. Figure 4 illustrates this case.

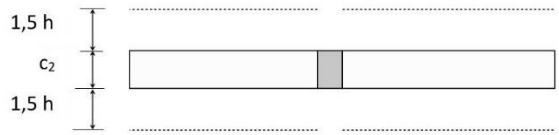

Figure 4. Width of flexural support for interior column with collinear arch strips

Where two perpendicular arch strips meet at a column, the band of flexural support is reduced to eliminate "overlap." Figure 5 illustrates the concept of an overlapping band of flexural support. It follows that the width of flexural support for each strip shown in Fig. 1(a) would be the column dimension perpendicular to the strip.

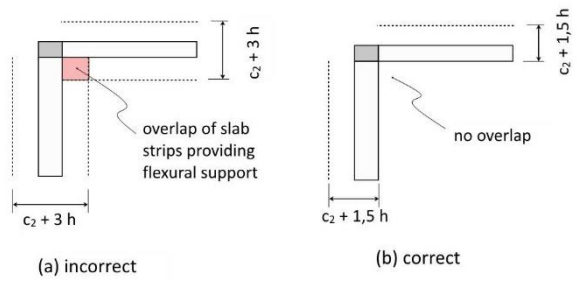

Figure 5. Flexural support for orthogonal strips

Note that flexural support of a strip is based on design moment and not the resistance of the reinforcement provided. The value of M_s must be consistent with the design moments of the load case being considered.

Design process

The arched strut concept leads to a design process that is very different from traditional shear design in two-way slabs. Rather than focus on a mechanism of shear failure, the designer checks that the lateral distribution of design bending

moments will provide sufficient flexural support to the proposed arch strips.

1.4.1 Lateral distribution of design moment

In most design cases, the total design load is known which means the required minimum design capacity for each strip, P_s, is also known. Rearranging Eq.(2) produces an estimate of the design moment that must be banded through the column.

$$M_s = \frac{P_s^2(1+\chi^2)}{2q_c(1+\chi)^2} \tag{4}$$

The design question becomes whether there is a lateral distribution of design bending moment that will satisfy both local requirements of the arched strut and the more global serviceability and strength requirements of the slab.

1.4.2 Proportioning reinforcement

With a satisfactory distribution of bending moment, it remains to select the appropriate reinforcement and to detail this reinforcement for development.

A simplified approach is to treat the arched strut as a beam segment of width b.

$$b = \min(b_{as}, c_1 + c_2, 2c_2) \tag{5}$$

where c_1 is the column dimension parallel to the arch strip.

The maximum permissible moment for the section, $M_{s,max}$, is calculated in terms of balanced strain conditions, defined as that point where yielding of tension reinforcement and crushing of concrete occur simultaneously. Most design standards contain provisions to limit flexural reinforcement based on balanced strain, from which the calculation of $M_{s,max}$ is straightforward.

Figure 6 shows the limit condition for an arched strut supported by both negative and positive moments. Here the moments are replaced by equivalent force couples. At the column, the nodal zone supports the compression block associated with negative moment and anchors the bottom steel associated with positive moment. To account for this, the total moment M_s rather than only the negative moment portion must be less than $M_{s,max}$.

Figure 6 also provides an indication of appropriate extension of top reinforcement that provides moment support for the arched strut. In the limit, this reinforcement should be extended at least a development length beyond the distance l_s from the face of the column support.

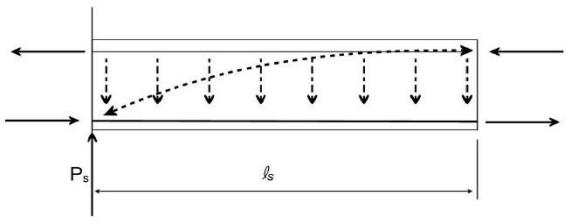

Figure 6. Limit condition for arch strip

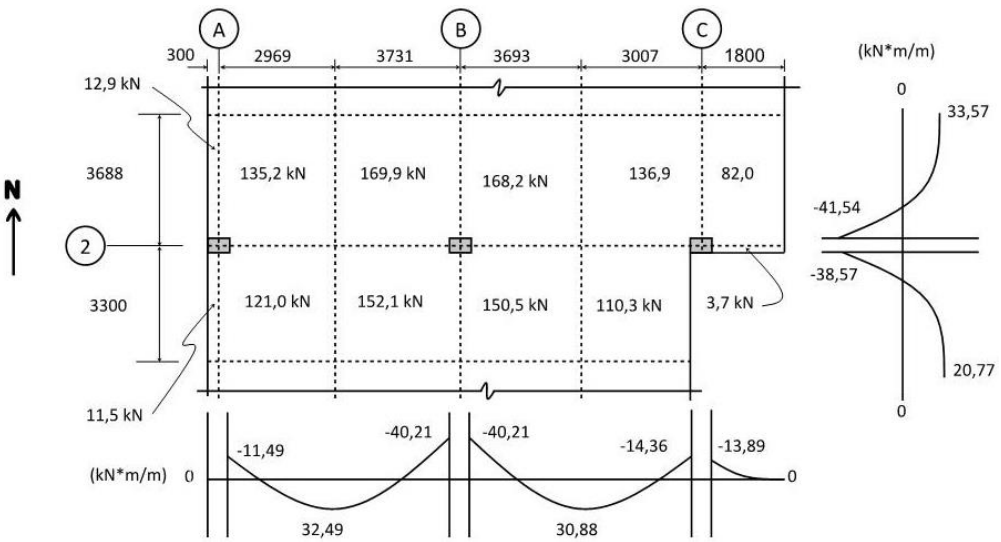

Figure 7. Example slab: average design moments and tributary loads

2 Example

Figure 7 shows tributary loads and average design moments for a design strip from a two-way slab and column structure. Consistent with North American practice, the design moments are based on clear span dimensions. All columns measure 600 mm by 400 mm. The slab is 250 mm thick and carries a total factored design load of 12,35 kPa. The average flexural depth, d, of the slab is 210 mm with an effective depth for shear, d_v, of 189 mm. The design strength of the concrete is 35 MPa.

Consistent with the Canadian standard [3], the one-way shear value, q_c, is taken as 140,6 kN/m. The limiting moment based on balanced strain conditions is approximately 350 kN·m per metre of beam width

2.1 Column-slab connection at B2

Figure 8 shows a configuration of arch strip at column B2. The total negative and positive cantilever moments for the design strip parallel to gridline B are calculated from the average design moments shown in Fig. (7). The side faces of the column that do not support arch strips are free to carry one-way shear. This accounts for 56,2 kN (2 x 28,1) assigned to each column side face. The remaining load must be carried by arch strips "a" and "b."

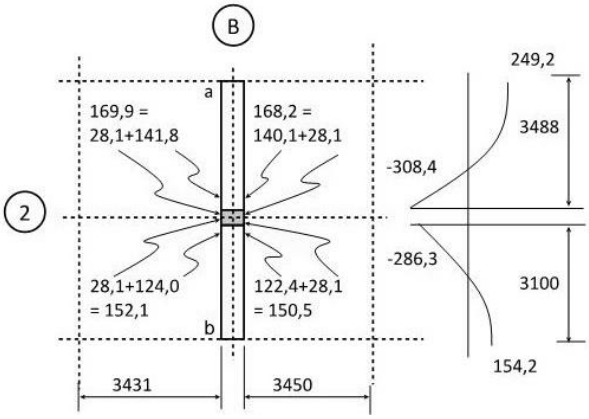

Figure 8. Interior column-slab connection at B2

2.1.1 Arch strip a

For arch strip a:

$$\chi = {}^{140,1}/_{141,8} = 0{,}988$$

$$P_s = 140{,}1\ kN + 141{,}8\ kN = 281{,}9\ kN$$

$$M_s = 141{,}3\ kN \cdot m \qquad \text{(by Eq. (4))}$$

The total cantilever moment for the half-panel tributary to arch strip "a" is $249{,}2 + 308{,}4 = 557{,}6\ kN \cdot m$.

Since the arch strips are collinear and there are no slab edges or perforations to consider, the width of slab providing flexural support to arch strip "a" is taken as:

$$c + 3h = 600 + 3 \times 250 = 1350\ mm$$

The question now is whether it is reasonable to assume a value of $M_s = 141{,}3\ kN \cdot m$ acts within a slab strip that is 1350 mm wide given the corresponding total panel cantilever moment of $557{,}6\ kN \cdot m$ acts within a design strip that is 7481 mm wide.

Typical lateral distribution rules for flexural design moment would call for an approximately uniform distribution of positive moment across the design strip. Negative moment is banded over the column. Among other requirements, the Canadian standard calls for 1/3 of the design negative moment to be within a band 1.5 h on either side of the column.

The average intensity of positive design moment, $33{,}57kN \cdot {}^{m}/_{m}$, can be taken directly from Fig. 7. The design positive moment contributing to M_s is:

$$33{,}57kN \cdot {}^{m}/_{m} \times 1{,}35m = 45{,}3\ kN \cdot m$$

The design negative moment contributing to M_s is:

$$\frac{1}{3} \times 308{,}4kN \cdot m = 102{,}8kN \cdot m$$

The flexural support for the arch strip mobilized under the current load case is:

$$45{,}3 + 102{,}8 = 148{,}1\ kN \cdot m$$

The flexural support provided, 148,1 kN·m, exceeds the moment of 141,3 kN·m required by Eq.(4), albeit not by a large margin. This means the proposed lateral distribution of design moment is sufficiently concentrated over the column for the shear load to be transferred with the proposed configuration of arch strips.

From Eq. (5), the width of the arched strut is 1000mm. From the balanced strain condition [3] $M_{s.max} = 350kN \cdot m$, well in excess of the 141,3 kN·m required to support the arched strut. This means that it is feasible to provide reinforcement for this moment and have acceptably ductile behaviour.

2.1.2 Arch strip b

For arch strip b:

$$\chi = {}^{122{,}4}/_{124{,}0} = 0{,}987$$

$$P_s = 122{,}4\ kN + 124{,}0\ kN = 246{,}4\ kN$$

$$M_s = 108{,}0\ kN \cdot m \qquad \text{(by Eq. (4))}$$

Using the same lateral distribution rules as before, the average intensity of positive design moment (from Fig. 7) is $20{,}77kN \cdot {}^{m}/_{m}$

The design positive moment contributing to M_s is:

$$20{,}77kN \cdot {}^{m}/_{m} \times 1{,}35m = 28{,}0\ kN \cdot m$$

The design negative moment contributing to M_s is:

$$\frac{1}{3} \times 286{,}3\ kN \cdot m = 95{,}4\ kN \cdot m$$

The flexural support for the arch strip mobilized under the current load case is:

$$28{,}0 + 95{,}4 = 123{,}4\ kN \cdot m$$

The flexural support provided, 123,2 kN·m, exceeds the moment of 108,0 kN·m required by Eq.(4). The proposed lateral distribution of design moment is sufficiently concentrated over the column for the shear load to be transferred with the proposed configuration of arch strips. As before, the required value of M_s is less than $M_{s,max}$.

2.2 Edge column at A2

Figure 9 shows a load distribution for the edge column at A2 using a single arch strip perpendicular to the free edge of the slab. This configuration is appropriate for most gravity load design cases at edge columns.

Since no arch strip acts along the free edge of the slab, the shear that can be transferred on the two side faces of the column is limited to the one-way capacity.

The total shear capacity of the side faces is:

$$2 \times 0{,}6\ m \times 140{,}6\frac{kN}{m} = 168.4\ kN$$

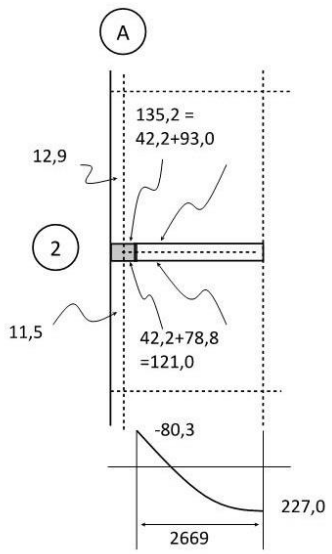

Figure 9. Edge column-slab connection at A2

Half of this capacity $(2 \times 42{,}2\ kN)$ is applied to support load from the span. The other half is available to support load applied to the edge, in this case limited to distributed load outside the column centre-line.

For the single arch strip shown in Fig. 9:

$$\chi = {}^{78{,}8}/_{93{,}0} = 0{,}847$$

$$P_s = 78{,}8\ kN + 93{,}0\ kN = 171{,}8\ kN$$

$$M_s = 52{,}8\ kN \cdot m \qquad \text{(by Eq. (4))}$$

By inspection the proposed design moments shown in Fig. 9 are more than sufficient. By itself, the negative design moment of *80,3 kN·m* exceeds that required. Detailing of the reinforcement perpendicular to the free edge of the slab to develop this moment, beyond the scope of this paper, is an important question that is not well addressed in design standards. Alexander [1] presents a method, not included here, for assessing this development.

2.3 Column-slab connection at C2

The column-slab connection at C2 is awkward to assess by conventional means. Globally, it is an edge column with gridline C defining the outside line of column supports. Locally, the designer must account for the cantilevering balcony slab on one side combined with the re-entrant corner on the other. Figure 9 shows a proposed layout of arch strips.

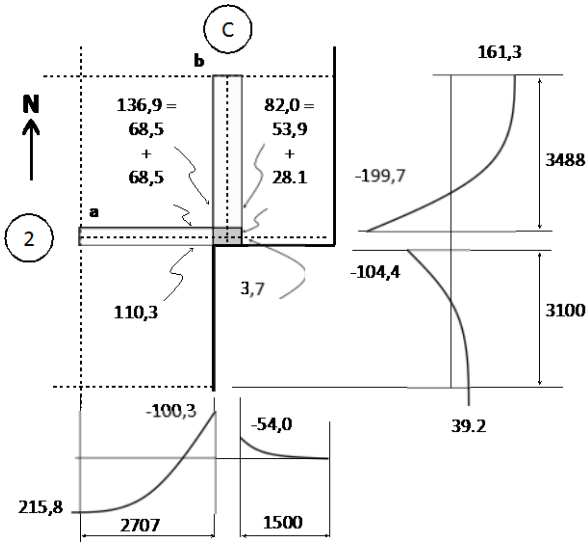

Figure 10. Column-slab connection at C2

2.3.1 Arch strip a

The load on the slab panel bounded by gridlines 2 and C is equally divided between arch strips a and b. By necessity, the entire load from the slab panel on the south side of gridline 2 must be carried by arch strip a. As a result, for arch strip a:

$$\chi = {}^{68{,}5}/_{110{,}3} = 0{,}621$$

$$P_s = 68{,}5\ kN + 110{,}3\ kN = 178{,}8\ kN$$

$$M_s = 60{,}0\ kN \cdot m \qquad \text{(by Eq. (4))}$$

The minimum negative moment associated with arch strip a will be the larger of 1/3 of the total negative moment $(100{,}3/3 = 33{,}4)$ or 100% of the net negative moment $(100{,}3 - 54{,}0 = 46{,}3)$ to be transferred. In this case, the negative moment to be transferred, *46,3 kN·m*, governs.

The average positive moment from Fig. 7 is 30.88 kN·m/m. For this strip:

$$b_{as} = c_2 + 1.5h = 775mm$$

The positive design moment associated with arch strip a is:

$$30{,}88kN \cdot {}^{m}/_{m} \times 0{,}775\ m = 23{,}9\ kN \cdot m$$

The total flexural support to arch strip a is:

$$46{,}3 + 23{,}9 = 70{,}2\ kN \cdot m$$

This meets the flexural support requirement determined for arch strip a of *60,0 kN·m*.

At the column support, the re-entrant corner limits the width of the compression block to $c_2 = 400\ mm$. Balanced strain conditions give a limiting value of $M_{s,max} = 0{,}4m \times 350kN = 140kN \cdot m$, in excess of that required.

2.3.2 Arch strip b

Arch strip b is loaded on its west side by half of the load on the northwest panel and on its east side by the cantilevering balcony. Part of the balcony load (28,1 kN + 3,7 kN) is carried by one-way shear on the east face of the column. The remaining load, 53,9 kN, is carried by arch strip b. For arch strip b:

$$\chi = {}^{53{,}9}/_{68{,}5} = 0{,}787$$

$$P_s = 68{,}5\ kN + 53{,}9\ kN = 122{,}4\ kN$$

$$M_s = 27{,}0\ kN \cdot m \qquad \text{(by Eq. (4))}$$

The minimum negative moment associated with arch strip b is the larger of 1/3 of the total negative moment or 100% of the net negative moment to be transferred. The negative moment to be transferred, *95,3 kN·m*, governs.

As was the case for the edge column at A2, the negative moment to be transferred to the column provides more than enough support for the arched strut. Anchorage of this reinforcement (see [1]) is likely to be a greater design issue.

From balanced strain conditions, $M_{s,max} = (0{,}6\ m + 1.5 \cdot 0{,}25\ m) \times 350kN = 341\ kN \cdot m$, well in excess of that required.

3 Discussion

Shear transfer in reinforced concrete is characterized as deep or slender beam behaviour. Deep beam behaviour is modelled using strut and tie. Slender beam behaviour is reasonably modelled using a limiting shear stress. The arched strut describes two-way shear transfer at a concentrated load or reaction as a combination of these, with deep beam behaviour in one direction interacting with slender beam behaviour in a perpendicular direction. This basic description of behaviour is consistent with test observations and produces reasonable predictions of strength [1, 2].

The arched strut concept easily accommodates any combination of shear and moment transfer between a slab and column. Some designers might be surprised to see that there is no need to calculate properties of critical sections and that the shear strength limit in a two-way system is no different than in a one-way. The apparent higher shear stress is a result of local arching action and not some magical property of two-way slabs.

Some will see a striking similarity to the corner supported element of Hillerborg's Strip Method [4,5], a lower bound plasticity approach for the flexural design of two-way slabs.

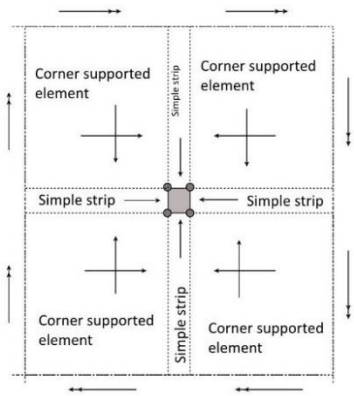

Figure 11. Hillerborg's Strip Method

Figure 11 shows the typical layout for the tributary area to a single column using the Strip Method. Each quadrant of slab is modelled with a corner supported element. A simple one-way strip of slab frames into each column face.

Hillerborg's corner supported element is a lower bound solution for bending only; the point support reaction at the corner does not satisfy material limits. Note that Hillerborg's method assumes no interaction between the column supported elements and the adjacent simple strips. The simple strips framing into the column faces carry only the load applied to the strips.

Compare the layout in Fig. 11 with that in Fig. 12, which shows the layout of arch strips used to model concentric punching tests. The arched struts assume there is interaction between strips that framing into the side of the column and the adjacent quadrants of two-way slab. This interaction defines a load path for shear transfer that does satisfy material limits.

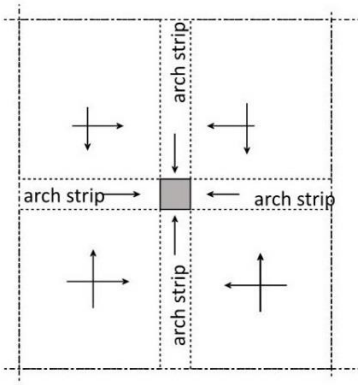

Figure 12. Arch strips in two directions

4 Conclusions

The arched strut models load transfer between a two-way slab and a concentrated load or support. It explains two-way shear behaviour using a combination of strut and tie behaviour in one direction with slender beam behaviour in a perpendicular direction. No particular failure mechanism is considered. Instead, a load path is assumed and the consequences of that load path are dealt with.

5 References

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