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Aeroelastic optimization of composite wings subjected to fatigue loads

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An analytical model to predict the fatigue life of a composite laminate is discussed. It is based on the method developed by Kassapoglou to predict fatigue failure. The analytical model calculates stresses in each ply using classical lamination theory, degrades the residual strength using the linear degradation law and predicts failure based on Tsai Wu failure theory. The cycles to failure are predicted using the updated cycle-by-cycle probability of failure. The predictions are validated for both a constant amplitude and a variable amplitude loading on a Glass/Epoxy laminate. Furthermore the analytical model is extended to work with laminates described using lamination parameters instead of ply angles and stacking sequence. The analytical fatigue model is then integrated in the TU Delft aeroelastic and structural optimization tool PROTEUS. A thickness and stiffness optimization of the NASA Common Research Model (CRM) wing has been carried out. Results show that fatigue may play an important role in the aeroelastic optimization of a composite wing.

I. Introduction

Structural weight reduction is one of the key approaches to reduce the fuel consumption and hence increase the performance of an aircraft. With high specific strength, the use of composite materials can be beneficial in terms of weight saving. A further advantage of the composite materials, is their inherent anisotropic behavior, which can be tailored to achieve beneficial aeroelastic deformations and hence improved performance during the flight, thus providing a greater efficiency with minimum weight penalty.

In majority of the studies [1] dealing with aeroelastic tailoring of the composite wings, the main design drivers are stiffness, static strength, buckling and static and dynamic aeroelastic stability. In order to avoid fatigue problems, a conservative knockdown factor is applied to the allowable material strain levels. With this design philosophy, failure under repeated loading is avoided since, the loads acting on the structure are too low to initiate or propagate any existing damage. As composite wing designs become more optimized for improved aeroelastic behavior and further weight savings, the difference between the magnitude of typical fatigue loads and ultimate static strength of a design becomes smaller. As a result, fatigue loading which, historically, was not a design driver for composite structure, now becomes more important and may impact the design. This means that using conservative knock-down factors would be too conservative and weight-inefficient. There is, therefore, a need for a fatigue analysis approach which is sufficiently accurate and efficient so it can be used in a design and optimization framework.

Fatigue damage in composites is a complex process involving different mechanisms, such as matrix cracking, fiber breakage, delamination and fiber/matrix debonding. As a result, fatigue life prediction becomes much more complicated as compared to metals. A nice overview of the different fatigue prediction algorithms exists in the literature [2–4]. Fatigue models can be divided mostly into three categories, empirical, phenomenological and progressive damage models. Empirical models such as Miner rule [5] use the information from S-N curve or Goodman diagrams to define a damage parameter which keeps track of the fatigue life. The parameter starts with 0 and increases until the value equals 1 which indicates the final failure. Phenomenological models calculate the fatigue life by measuring the degradation in the macroscopic material property. This property can be the stiffness or the strength of the material. In the progressive damage models, deterioration of the composite materials using the different damage mechanism is modelled. These models are quite complex due to the various damage mechanisms occurring in composites and the extensive simulations needed to track damage evolution.

For the current study, the focus is on using residual strength phenomenological models for predicting fatigue life of a composite. The residual strength wear out model has an inherent failure criterion: failure occurs when residual

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strength degrades to the maximum applied stress. There has been significant research on applying different residual strength degradation theories under a variety of loading conditions. Philippidis and Passipoularidis [6] give a nice overview of the different theories available in the literature and their validity with respect to experiments. The majority of the models requires empirical parameters that are determined by using experiments and curve-fitting. This makes the universal application of fatigue models quite difficult. Based on the models developed by Broutman and Sahu [7] , Kassapoglou [8–10] formulated a residual strength wear out model without the need of any curve-fitting or parameters obtained from fatigue tests.

In the present study, an analytical model based on Kassapoglou method has been formulated to determine the fatigue life of a composite laminate. The model is validated for a constant amplitude as well as a variable amplitude fatigue test by comparing the prediction on a Glass/Epoxy laminate subjected to New Wisper spectrum [11]. The experimental data of the Glass/Epoxy laminate is obtained from the OPTIMAT blades database [12]. This database includes a large enough number of specimens which gives accurate values of the experimental scatter which is a variable needed in the method by Kassapoglou. The analytical fatigue model is then integrated into an aeroelastic optimization framework and the effect of including fatigue as one of the design constraints along with strength, buckling and aeroelastic stability is investigated.

II. Fatigue Model

The analytical fatigue model combines the Kassapoglou method [8–10, 13] with the first ply failure theory to determine the fatigue life under a constant amplitude as well as a spectrum loading. For the sake of completeness, the steps involved in assessing the fatigue life using the Kassapoglou method have been listed below.

1) For the given loads, stresses in each ply of the laminate can be determined using the classical laminate theory.
2) Based on the applied stress σ, the probability p that the residual strength of the ply is not higher than the applied stress in the ply is calculated. For the 1st cycle, the residual strength σ_r is equal to the static failure strength σ_{sf}. The value of p will depend on the type of statistical distribution and stress ratio R. Generally, the static strength allowables for a composite material follow a normal distribution, two parameter Weibull or log normal distribution.
3) Once the probability of failure is determined, the number of cycles to failure [8], N, if the failure mode doesn't change, is determined by
$$N = -\frac{1}{ln(1-p)} \tag{1}$$
4) Residual strength of the ply after n cycles of the applied stress is determined through a degradation model [13] given by
$$\sigma_r = \sigma_{sf}(1-(1-\frac{\sigma}{\sigma_{sf}})\frac{n}{N-1} \tag{2}$$
5) If the applied stress in any ply is higher than the residual strength of the ply, the laminate fails. Otherwise, the statistical parameters of the static strength, such as mean and standard deviation in case of a normal distribution are degraded and the process continues until the maximum number of cycles has been reached or the laminate has failed.

For the values of stress ratio other than 0, the value of p, needs to be modified to account for the fact that cyclic stress doesn't start from 0 but some finite value. This is taken into account by modifying the statistical distribution of the residual strength. The figure 1 depicts the assumed change in the distribution for the stress ratio greater than 0. The 1% value of the distribution is shifted towards the mean of the distribution by a factor r which is given by

$$\begin{aligned} r &= 1-R, 0<R<1 \\ r &= 1-\frac{1}{R}, R>1 \end{aligned} \tag{3}$$

The mean and the 99 % values are kept constant. The resulting distribution is assumed to be two parameter Weibull distribution. The shape and the scale parameter can be obtained by solving the equation 4 iteratively.

$$\begin{aligned} \beta(1-\frac{1}{\alpha}) &= X \\ e^{-\frac{x_1^\alpha}{\beta}} - e^{-\frac{x_2^\alpha}{\beta}} &= 0.98 \end{aligned} \tag{4}$$

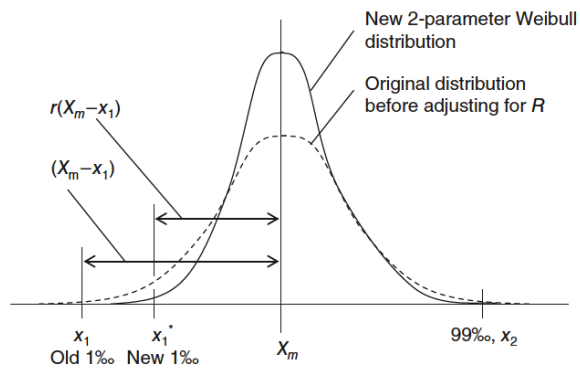


Fig. 1 Modification of the probability distribution for stress ratio other than 0 [13].

where x_1 is the 1% value of the new distribution, x_2 is the 99 % value of the original distribution, α is the shape parameter of the Weibull distribution, β is the scale parameter of the Weibull distribution and X is the mean of the original distribution.

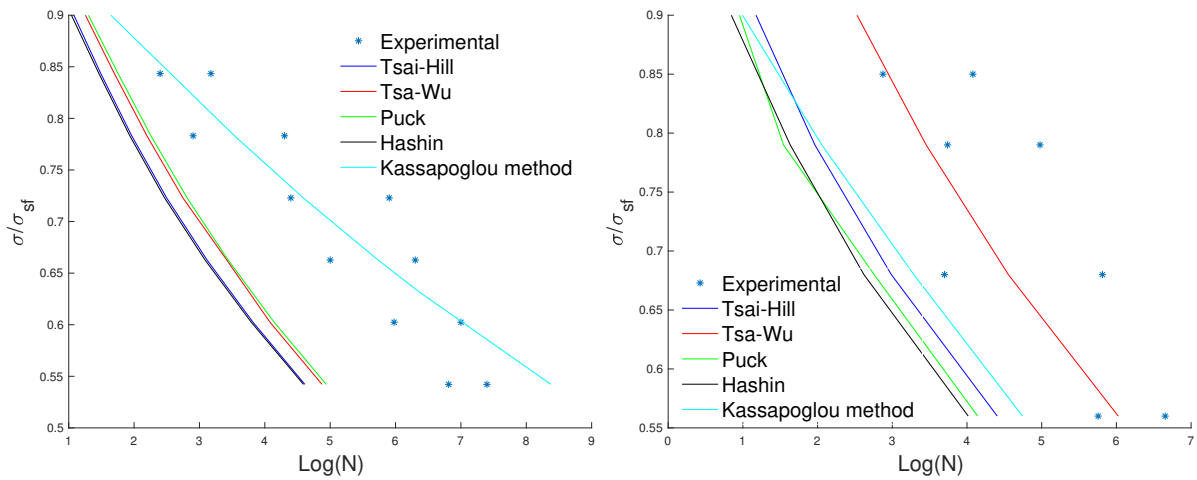
In the current analytical model, instead of comparing the applied stress to the residual strength of the ply, the first ply failure theory is used to determine if there is a failure in the structure due to the degradation of the residual strength. Four first ply failure theories, namely Tsai-Hill, Tsai-Wu, Hashin and Puck were individually combined with the Kassapoglou method and their accuracy is compared to the results from experimental and the original Kassapoglou method [8] for different stress ratio in figure 2. $[(\pm 45/0_2)2]s$ T800/5245 bismaleimide (BMI) laminate is used for the comparison. Table 1 shows the strength allowables as well as the mean and the standard deviation of the material used. Results show that the degradation model combined with all the failure theories is conservative with respect to the experimental results. Keeping in mind the accuracy along with the ease of implementation, Tsai-Wu was chosen as the most suitable failure theory to predict the fatigue life. However, it is understood that the Tsai-Wu failure criterion does not always match test results, especially under biaxial compression, but it gives a good indication of first ply failure for other loading situations. Furthermore, the approach presented here can easily be modified to include a different failure criterion as necessary.

Table 1 Material property of T800/5245 ply [14].

Property	**E1** (GPa)	**E2** (GPa)	**G12** (GPa)	ν_{12}	X_t (MPa)	X_c (MPa)	Y_t (MPa)	Y_c (MPa)	**S** (MPa)
Mean	147	10.3	7	0.27	3460	1730	50	165	75
Standard Deviation					318.32	171.27	4.6	16.335	7.425

To extend the model to predict failure under the spectrum loading, let us assume that the starting strength of the ply follows a normal distribution with s as the standard deviation and μ as the mean. At the first load step, for the applied stress, steps 1-4 are followed and the degradation in the static strength along with the degradation in distribution parameters is calculated. If the resulting Tsai-Wu index does not indicate failure, then the next load step is applied. In this step, since the load doesn't start from 0, the distribution is modified from a normal distribution to a Weibull distribution as was explained before. Based on the applied stress, the degradation in the static strength, mean, shape and scale parameters are calculated. If the laminate doesn't fail based on the Tsai Wu failure theory, the next load step is applied. Since the stress ratio changes, the distribution will change as well. To get the Weibull distribution corresponding to the current stress ratio, the normal distribution for the degraded static strength that corresponds to the stress level of 0 must be known. Since the mean is assumed to be constant for the both distributions, to get the normal distribution, the standard deviation has to be calculated for the degraded static strength. Using equation 4, the standard deviation which results in the scale and the shape parameter of the previous load step is determined iteratively. Then the new shape and the scale parameter are calculated and based on that, the degradation in the strength and the distribution parameters along with the Tsai Wu failure index are calculated. If the specimen has not failed, the similar process is repeated for the next step till the laminate has failed.

With composite materials, there is a high scatter in fatigue life due to anisotropic heterogeneous characteristics, such



(a) R=0.1 **(b) R=10**

Fig. 2 Comparison of 4 failure theories with results from the experimental and original Kassapoglou model [8].

as lay-up, manufacturing defects and imperfections, test complications, and environment [15]. To account for scatter in the calculation of the fatigue life, the B-basis reliability is used. The definition of a B value is that at least 90% of the population of values is expected to be equal to or exceed a particular property with a confidence of 95%. On an S-N diagram, the reduction corresponding to the difference between B-Basis and mean life is approximately 20% on stress (vertically). Thus in the analytical model, the calculated fatigue life is knocked down by 20%, to incorporate the effects of scatter.

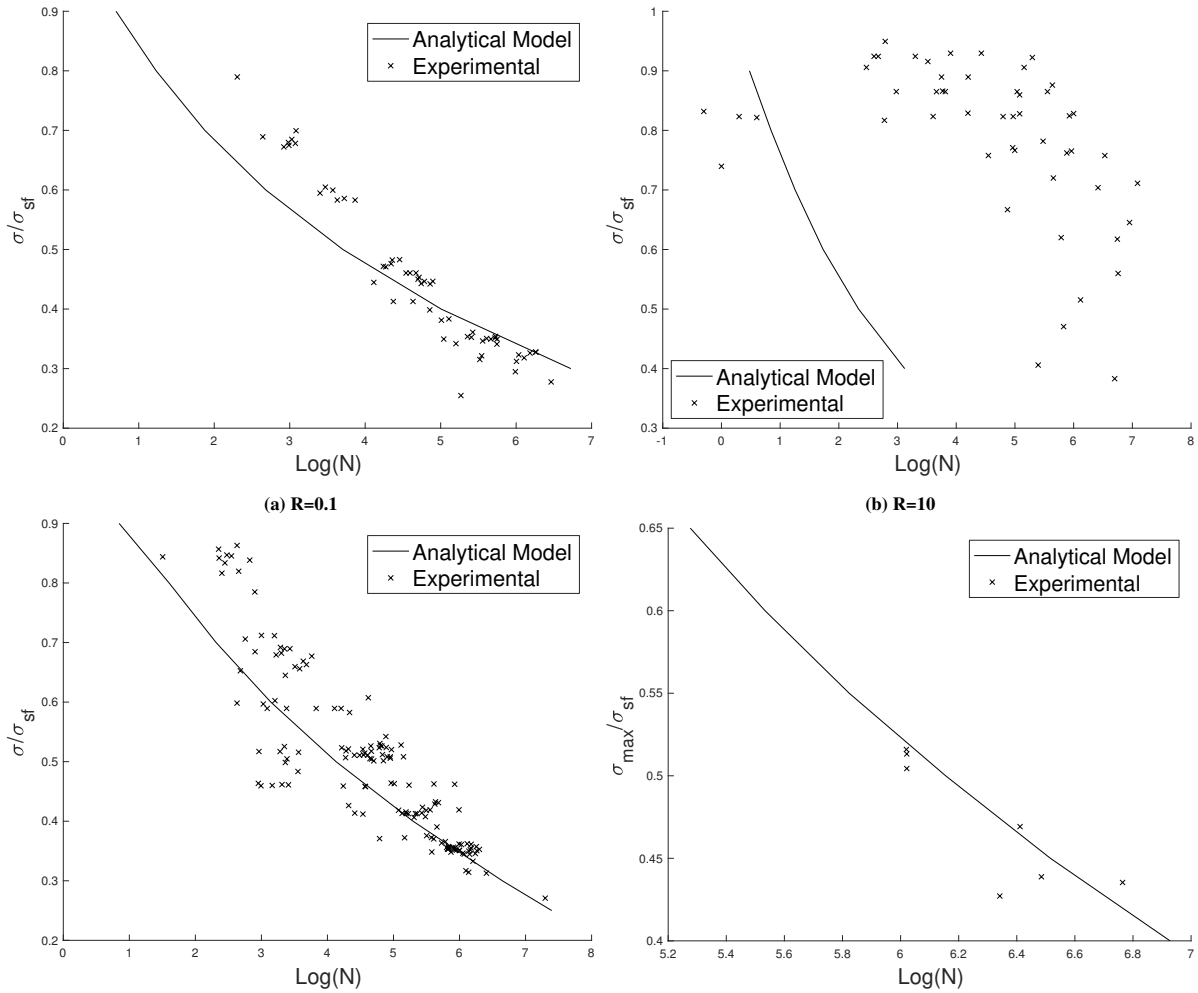
The analytical fatigue model is validated for spectrum loading by comparing it to the test results from the Glass/Epoxy laminate for the New Wisper Spectrum [11]. The experimental data was obtained from the 'OPTIMAT BLADES' project [12] in which extensive experimental campaign on a Glass/Epoxy material used in the Wind Turbine Rotor Blades industry was performed. Two types of lay up were considered in the tests; a unidirectional (UD) layup $[0_4]$ and a multidirectional (MD) $[\pm 45/0]_4/\pm 45]$ lay-up. Table 2 shows the static strength allowables and the respective mean and standard deviation. It is important to note that the complete set of data points from the OPTIMAT database [12] was used for each property accounting for batch-to-batch variation. Small changes in the coefficient of variation can have significant effect on the fatigue life prediction [8].

Table 2 Properties of Glass/Epoxy laminate used in OPTIMAT project [12].

Laminate	**Property**	**E1** (GPa)	**E2** (GPa)	**G12** (GPa)	ν_{12}	X_t (MPa)	X_c (MPa)	Y_t (MPa)	Y_c (MPa)	**S** (MPa)
UD	Mean	41.5	15	8.6	0.36	828.75	530	55	161	49.78
	Standard Deviation					83.2	79.23	4.76	8.9	1.14
MD	Mean	28.7	15.15	7.6	0.47	552.63	465	143.89	198	130.7
	Standard Deviation					55.16	35.34	12.47	5.81	18.91

Figure 3 compares the fatigue life of the UD ply predicted by the analytical model to the experimental results for the single stress ratio as well as the spectrum load. There is a good agreement observed between the analytical model and the experimental results for the stress ratio of 0.1 and -1. With respect to the stress ratio of -10, the prediction of the analytical model is a bit on the conservative side. For the spectrum loading, a reasonable agreement is observed between the analytical and the experimental model.

For the MD laminate, since the properties for the ±45 ply were not available, the MD laminate is modelled as laminate consisting of single lamina having fiber direction of 0 degrees with the laminate properties described in table 2. As can be seen in the figure 4, there is a quite a good agreement on the fatigue life with the experimental results for the stress ratio of 0.1 and 10. However for the stress ratio of -1, the predictions do not match the experiments. One possible reason for the nonconservative nature of the result, is the assumption of representing the MD ply with the single lamina.



(a) R=0.1 **(b) R=10**

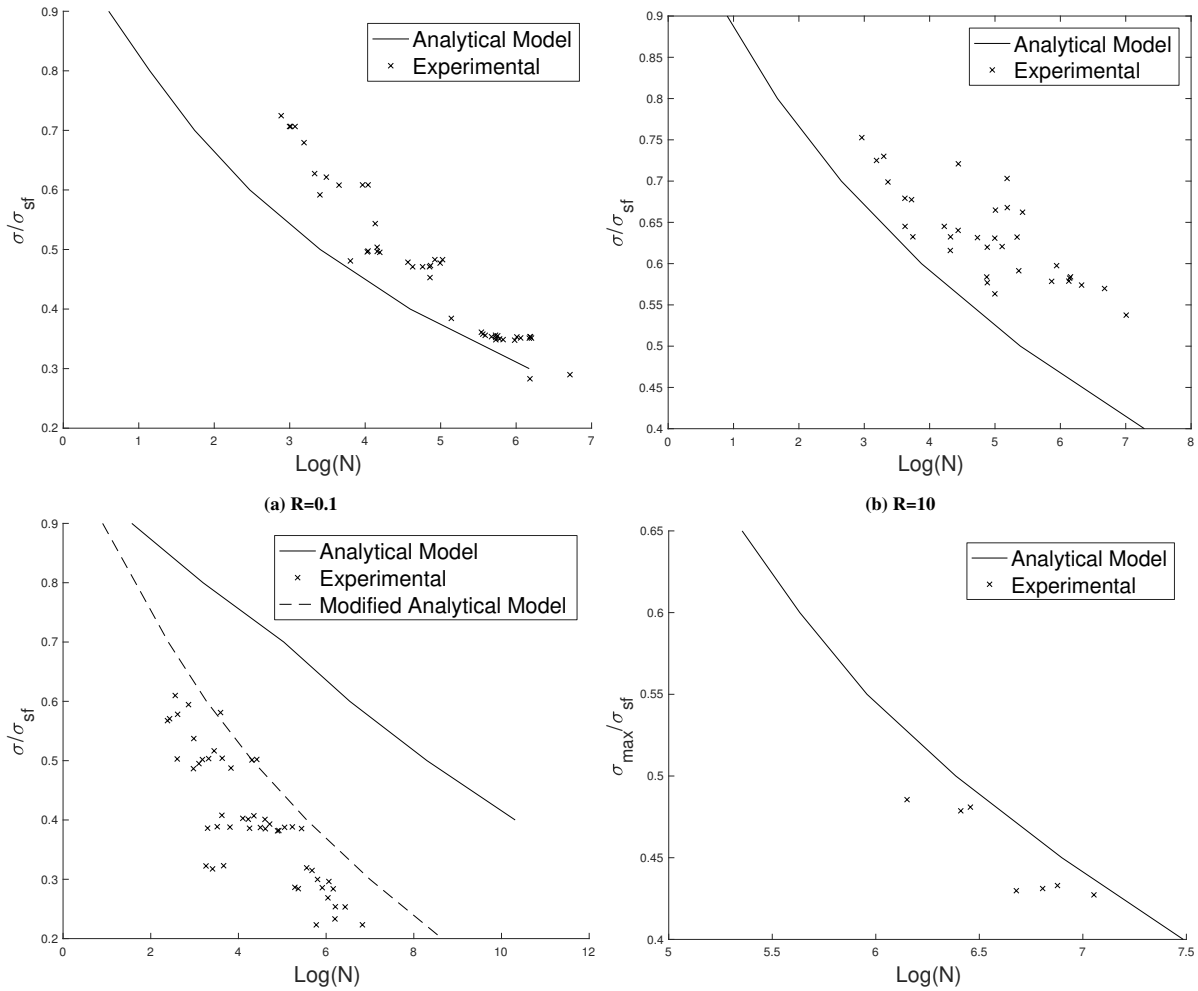
(c) R=-1 **(d) New Wisper Spectrum**

Fig. 3 Comparison of fatigue life predicted by analytical model and experimental results for the UD laminate.

While this makes the model very efficient, it does not capture differences in failure modes that the different plies would exhibit. This will cause some inaccuracies. For the stress ratio of -1, the dominant failure mechanism is shear in the ±45 laminate. For a laminate with only a 0 degree ply, a stress ratio of -1 doesn't produce any shear degradation and hence the fatigue life predictions are nonconservative. This effect is also seen in the prediction of the spectrum loading where the analytical model is slightly nonconservative compared to the experiments.

To see how the predictions for this case can be improved, one can modify the analysis model to use the coefficient of variation from the dominant failure mode for this laminate which was shear in the 45/-45 plies. This means that instead of the tension and compression specific values which the model requires for stress ratio of -1, one can use the shear specific coefficient of variation 14% and apply it to both the tension and compression parts of the $R = -1$ cycle. This would yield a standard deviation of 79.57 MPa for the tension part of the cycle and a standard deviation of 66.96 MPa for the compression part of the cycle. With this modification, the dashed curve shown in Figure 4c is obtained which is much closer to experimental data. This modified curve was not used in the subsequent analysis. It only serves as an indication of the direction of further work to improve the accuracy of the model.

The process to predict failure by analyzing each step of a spectrum load until the laminate fails can become computationally expensive. For example, on a Intel i5 2.7 GHz single core processor, it took 3 hours to analyze the UD ply for the New Wisper spectrum with maximum applied stress of 0.5 times the static failure stress. If an entire wing needs to be analyzed, the computational time would be in the range of 100 hours, which becomes quite expensive for an optimization process. Hence 2 assumptions have been made which can decrease the computational time quite significantly. To understand the basis of these assumptions, consider a simple spectrum shown in figure 5. The first



(a) R=0.1 **(b) R=10**

(c) R=-1 **(d) New Wisper Spectrum**

Fig. 4 Comparison of fatigue life predicted by analytical model and experimental results for the MD laminate.

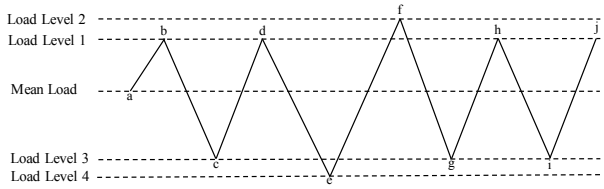


Fig. 5 Simple spectrum with 4 load levels.

assumption is that the ratio of degradation remains constant for given load cycle. Looking at the figure 5, the second load cycle consisting of points c and d, is similar to 4th and 5th load cycle consisting of points g, h, i and j. Instead of repeating the analysis at the individual points, the model parameters can directly be degraded using the ratio calculated at the first instance. For example, the residual strength, if the combination of c and d repeats n times is given by equation 5.

$$\sigma_{r_2} = \sigma_{r_1} * d^n \tag{5}$$

where σ_{r_1} is the residual strength before the first point of the load cycle, σ_{r_2} is the residual strength after the n similar load cycles and d is the residual strength degradation ratio calculated during the first instance of the particular load cycle. Thus, for the 4th and the 5th load cycle, the degradation ratio can be applied directly to the residual strength and the distribution parameters.

The second assumption is the extension of the first assumption over the entire spectrum. The ratio of degradation remains constant over the entire spectrum. This assumption can become nonconservative, as the ratio of the degradation increases with the decrease in the residual strength. Thus to be on the conservative side, the entire spectrum needs to be analyzed and the ratio of the degradation needs to be updated again after every 3rd sequence. For example, if the spectrum described in figure 5 is repeated 7 times, the degradation ratio for different parameters is calculated during the first sequence and the effect of second and third sequence is directly calculated by the degrading the model parameters by their respective ratios calculated for the first sequence. The degradation ratio is updated by analyzing the fourth sequence and is then used to degrade the properties for the fifth and the sixth sequence and finally the seventh sequence is analyzed. Thus, the entire spectrum is analyzed only thrice instead of 7 times resulting in reduced computational effort.

Table 3 compares the residual strength after the application of the New Wisper spectrum obtained from the full analysis and from the analysis with the assumptions included. The maximum applied stress is 0.5 times the static failure stress. The second column lists the maximum change in the residual strength, and the third column shows the difference in the computational time required to the perform the analysis. As can be seen, by implementing both the first and the second assumption, there is a good agreement with the original model with the error being less than 1%. With the use of these assumptions, the analytical model becomes faster by 92%.

Table 3 Validity of the assumptions made to predict the fatigue life.

Assumption Type	ΔX (%)	Δt (%)
1	0.0013	85.2
2	0.92	56.4
Both 1 and 2	0.97	92.8

Typically a composite laminate is described by the ply angles and the stacking sequence. An alternative way to represent the composite laminate is through the use of lamination parameters. The lamination parameters describe the in-plane and out-of-plane behavior of the composite laminates and were first introduced by Tsai and Pagano [16]. In an optimization process, the use of lamination parameters is advantageous as compared to the use of ply angles and stacking sequence. The first advantage is that fixed number of lamination parameters describe the entire laminate irrespective of number of plies, thus reducing the number of design variables. The second advantage is that as the lamination parameters are continuous, a gradient-based optimizer can be used, making the optimization process more efficient. For the current aeroelastic optimization process, lamination parameters are used to represent the composite laminate.

With the current fatigue model, only the laminates described by ply angles and stacking sequence can be analyzed. With the lamination parameters, there is no information on the number of the plies or the angle of the plies. As a result, the fatigue model is modified such that it can be applied to the lamination parameter domain.

From hereon, the analytical model to predict failure in the laminate described by ply angle and stacking sequence will be called the original model. To determine if the laminate has failed, the Tsai Wu failure criterion has been implemented in the original model. The failure criterion in its original form explicitly depends on the ply angles and the stacking sequence. To adapt it for the lamination parameters, Ijsselmuiden et al. [17] formulated a failure envelope based on the conservative approximation of Tsai-Wu failure criterion that does not explicitly depend on the ply angle. The approach assumes that all ply angles could be present at any location through the thickness of the laminate and thus is safe regardless of the ply angle. Khani et al. [18] modified the failure envelope such that it can be used with principal strains. In the current fatigue model, for the failure criterion, this modified Tsai Wu failure envelope is implemented.

As the failure envelope works with the principal strains, the fatigue model is modified to work on principal strains rather than lamina stresses. To determine the probability of failure, instead of comparing the lamina stresses in each ply to their residual strength, the value of the modified Tsai Wu criterion of the entire laminate is compared to the failure index, which at the start is equal to 1. For this purpose, the statistical distribution of the Tsai Wu criterion needs to be calculated. Based on the distribution of the static strength allowables, an array of the input to the Tsai Wu failure criterion is randomly generated. A deterministic computation of the failure criterion on the randomly generated input is performed and the distribution of the modified Tsai Wu is estimated.

In the original model, the degradation in the residual strength of the ply is calculated with equation 2. In the case of modified model, the stress terms in the equation are replaced by principal strains. The equation for the degradation in residual strength can then be expressed as

$$\sigma_r = \sigma_{sf}(1 - (1 - \frac{\epsilon_i}{\epsilon_{sf}})\frac{n}{N-1}$$
$$\epsilon_r = \epsilon_{sf}(1 - (1 - \frac{\epsilon_i}{\epsilon_{sf}})\frac{n}{N-1} \tag{6}$$

where ϵ_i represent the principal strains of the laminate based on the applied stress and ϵ_{sf} represent the principal strains at which the laminate will fail.

The distribution parameters of the modified Tsai Wu criterion are degraded by R_{tw}, which is expressed as

$$R_{tw} = \frac{f}{f_r} \tag{7}$$

where f is the value of the modified Tsai Wu criterion before the degradation and f_r is the value of the modified Tsai Wu criterion after the degradation.

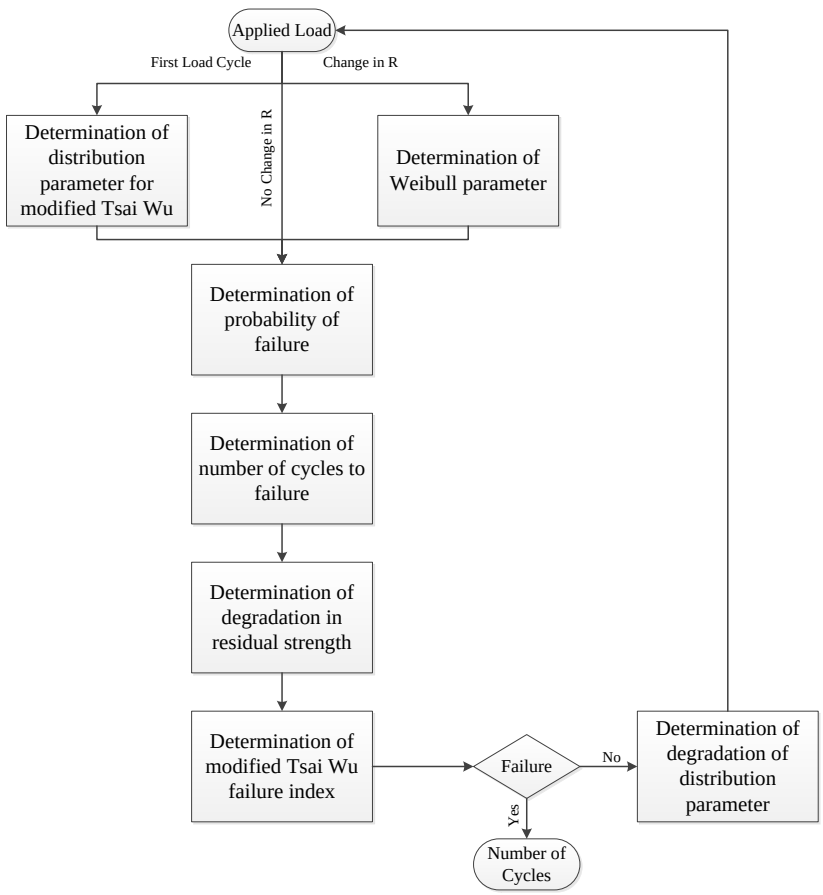


Fig. 6 Algorithm for the modified fatigue model.

The figure 6 depicts the flowchart of the fatigue model to determine failure for a composite defined by the lamination parameters. The figure 7 compares the result by the original model and the modified model for the OPTIMAT UD ply $[90_4]$ subjected to spectrum loading. As can be seen, because of the conservative nature of the modified Tsai Wu criterion [18], the fatigue predictions of the modified fatigue model are conservative compared to the original model. The $[90_4]$ stacking sequence was chosen for the comparison because the modified Tsai Wu criterion was found to be most conservative for a $[0_4]$ stacking sequence and least conservative for $[90_4]$ stacking sequence. With this it can be concluded that the modified fatigue model will always be conservative compared to the original model.

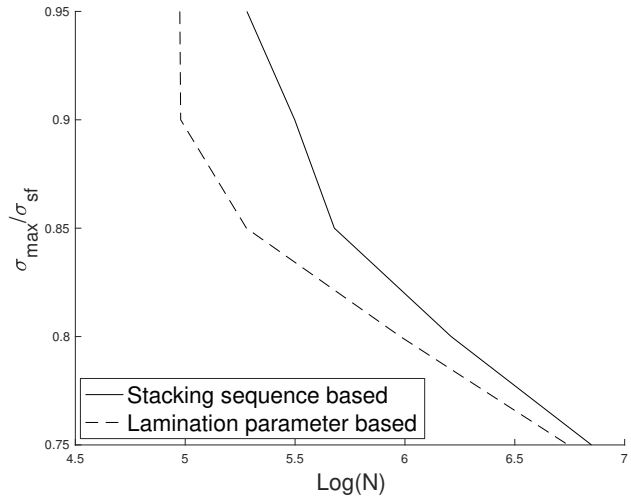


Fig. 7 Comparing life prediction of model with lamination parameter and model with stacking sequence.

III. Optimization Studies

To understand the effect of the proposed fatigue analysis over the traditional knockdown factor in the design of the wing, the analytical fatigue model is integrated into PROTEUS [19], which is an in-house aeroelastic tool, developed at the Delft University of Technology, to carry out optimization of a composite wing. Figure 8 depicts the schematic representation of the framework of the PROTEUS. To start with, the wing is first divided into multiple spanwise sections, where each section is defined by laminates in the chord wise direction. The cross sectional modeller uses the laminate properties and the cross-sectional geometry to generate the Timoshenko cross-sectional stiffness matrices. A non linear aeroeasltic analysis is carried out for multiple load cases by coupling the geometrically nonlinear Timoshenko beam model to a vortex lattice aerodynamic model. A linear dynamic aeroelastic analysis is carried out around the nonlinear static equilibrium solution. In the post processing, the cross sectional modeller is used to retrieve the strains in the three-dimensional wing structure. Based on the applied strains in the structure, strength and buckling properties of the wing are calculated and fed to the optimizer as constraints. In the current study, fatigue properties of the wing will also be calculated using the analytical fatigue model and fed as a constraint to the optimizer.

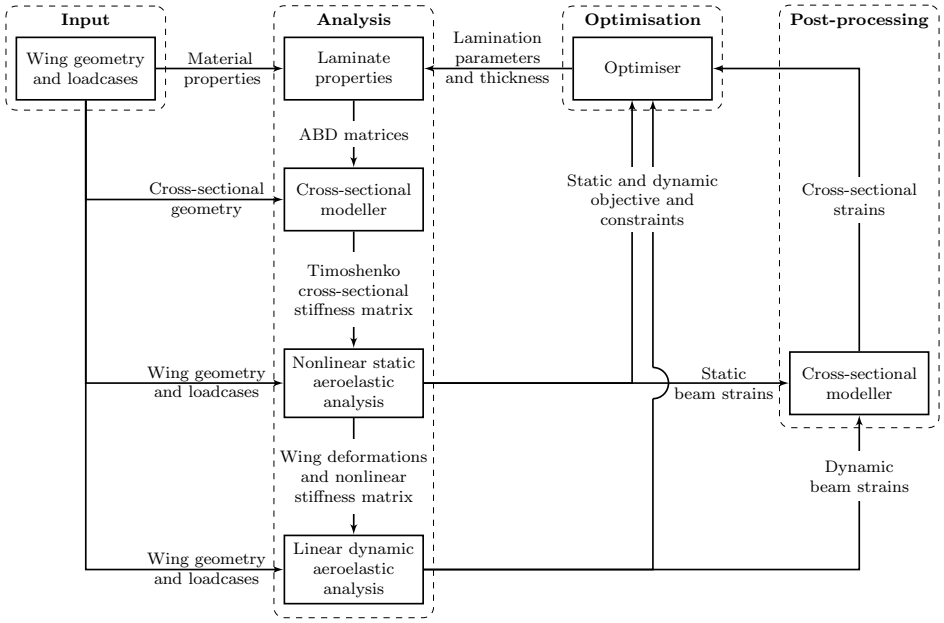


Fig. 8 Framework of PROTEUS [19].

To analyze the wing for fatigue, a shortened version of the TWIST spectrum (Mini-TWIST) [20] is used as the load spectrum. The Mini-TWIST spectrum consists of ten flight levels where each flight level has ten different levels of stress ratio. The stress ratio has been normalized to stress level at cruise condition. At each stress ratio, the number of cycles is different. The ten flight levels are repeated in a random manner to make a block of 4000 flights which is equal to 58,442 cycles. This block is repeated 10 times, equivalent to 40,000 flights which is considered as the maximum life of the aircraft. A fatigue failure parameter F is calculated for every laminate by subjecting it to Mini-TWIST spectrum for 10 times. F is defined as

$$F = r\frac{N_t}{N_f} \tag{8}$$

where r is the modified Tsai Wu value at the time of failure, N_t represents the total cycles the structure has to withstand and N_f represents the total cycles at the failure. If the laminate doesn't fail after the 40,000 flights, r is then the maximum modified Tsai Wu value calculated in the spectrum.

For each laminate, running the fatigue model for 40,000 flights can become time consuming. Looking at the process to calculate the fatigue life in the lamination parameter domain, calculation of the Weibull parameters for different stress ratios is computationally the most expensive. In an effort to speed up the process, in addition to the 2 aforementioned assumptions, a third assumption is made. The ratio of change in the Weibull parameters for all the laminates remains constant. Thus, Weibull parameters at a new stress ratio is determined only for the single laminate and based on the ratio of change in the parameters, the Weibull parameters for all the other laminates are determined. Figure 9 depicts the delta plot of percentage change in the residual strength value of the laminates with and without the 3rd assumption. The overall difference is less than 0.5%. In terms of computational efficiency, without the 3rd assumption it takes 104 minutes to run a block of 4000 flights and with the 3rd assumptions it takes 1.5 minutes. Thus, with the 3rd assumption, 98.5% saving in the computational effort is achieved without sacrificing too much in terms of accuracy.

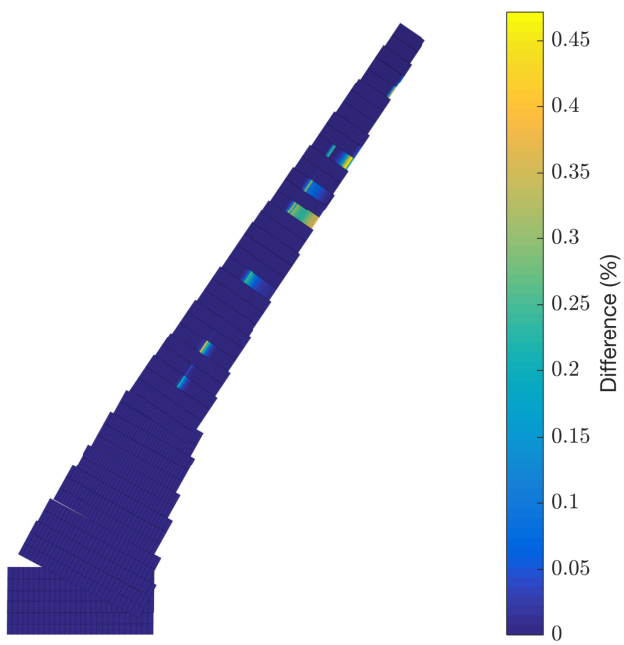


Fig. 9 Delta plot of residual strength with and without 3rd assumption.

A. Optimization approach

The NASA CRM [21], originally developed for the 4th AIAA drag prediction workshop, is used as a case study for the current analysis. The main characteristics of the CRM are summarized in Table 4. Figure 10 depicts the wing planform. Ribs, engine, leading edge devices and trailing edge devices are taken into account as concentrated masses. Stringers are also modelled to strengthen the top and bottom skin of the wing.

Two optimization studies were run: one, an aeroelastic optimization without fatigue as a constraint, and two, an aeroelastic optimization with fatigue as a constraint. In the first one, the material properties are knocked down by 68%

Table 4 Characteristics of the CRM wing.

Parameter	Value
Span	58.769 m
Leading edge sweep angle	35 deg
Wing aspect ratio	8.4
Taper ratio	0.275
Planform wing area	412 m^2
Cruise Mach	0.85
Design Range	14,300 km
Design Payload	45,000 kg
Maximum takeoff weight	296,000 kg

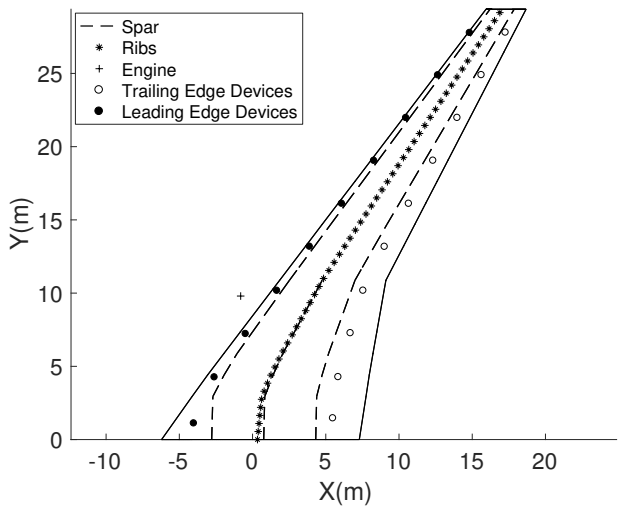


Fig. 10 CRM wing planform.

to account for fatigue, damage, material scatter and environment, whereas in the second case, the material properties are knocked down by 48% to account for the effects of damage and environment. Additionally, as was mentioned before, in the analytical model, the fatigue life is also reduced by 20% to account for fatigue scatter. The UD AS4/3501 carbon/epoxy composite was used as the reference material. Table 5 shows the material properties along with the statistical deviations [22]. The optimization problem is shown in the table 6. The objective of the study is to minimize the structural weight of the wing. The wing is divided into 10 sections along the spanwise direction. For the top skin and the bottom skin, each spanwise section consists of two laminates in the chord-wise direction and for the spars, each spanwise section has only one laminate in the chord-wise direction. This results in 64 unique laminates. Laminates are symmetric and unbalanced. Every laminate is described by eight lamination parameters and one thickness variable, resulting in a total of 576 design variables. Figure 11 depicts the laminate distribution along the top skin of the wing. It also shows the stiffness for each laminate, where the wing stiffness distribution is represented by the polar plot of thickness normalized modulus of elasticity $\hat{\mathbf{E}}_{11}(\theta)$ which is given by

$$\hat{\mathbf{E}}_{11}(\theta) = \frac{1}{\hat{\mathbf{A}}_{11}^{-1}(\theta)} \tag{9}$$

where $\hat{\mathbf{A}}$ is the thickness normalized membrane stiffness matrix and θ ranges from 0 to 360 degrees [23].

To ensure that lamination parameters represent a realistic ply distribution, feasibility equations formulated by Hammer et al. [24], Raju et al. [25] and Wu et al. [26] are applied. The aforementioned modified Tsai Wu failure envelope is used to assess the static strength of the laminate. The stability of the panel in buckling is based on idealized buckling model formulated by Dillinger et al. [23]. To guarantee the static and dynamic aeroelastic stability of the wing,

Table 5 Material Properties of AS4/3506.

Property	Value (GPa)	Standard Deviation (MPa)
E_{11}	147	
E_{22}	10.3	
G_{12}	7	
ν_{12}	0.27	
X_t	2.28	127.6
X_c	1.72	106.6
Y_t	0.057	5.7
Y_c	0.23	142.6
S	0.076	3.36

Table 6 Optimization Setup.

Type	Parameter	# responses
Objective	Minimize Wing Mass	1
Design Variables	Lamination Parameter Laminate Thickness	576
Constraints	Laminate Feasibility	384
	Static Strength	1024/loadcase
	Buckling	4096/loadcase
	Fatigue	3875
	Aeroelastic Stability	10/loadcase
	Local Angle of Attack	22/loadcase

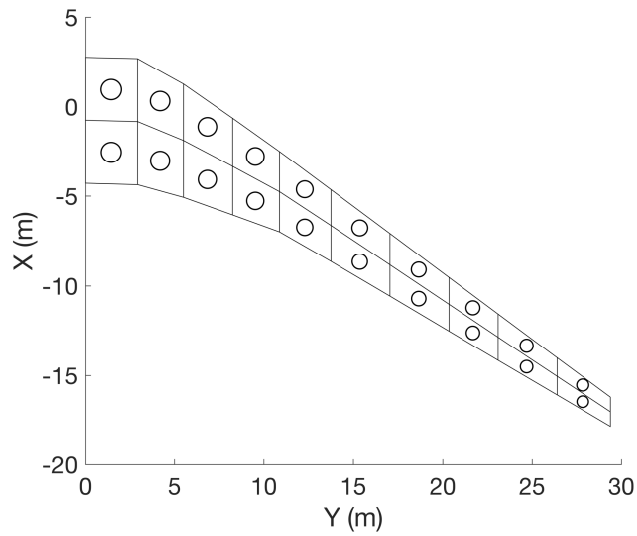


Fig. 11 Laminate Distribution of the top skin of CRM.

the real part of the eigenvalues of the state matrix should be less than zero. The local angle of attack is constrained to a maximum of 12 degrees and a minimum of -12 degrees. For the current optimization, the Globally Convergent Method of Moving Asymptotes (GCMMA) developed by Svanberg [27] is used as a gradient-based optimizer.

Table 7 gives the information on the loadcases which are used for the current study. These loadcases which were provided by NASA, represent the cruise condition, 2.5g symmetric pull up manoeuvre and -1g symmetric push down manoeuvre. The loadcase 2 and 3 represent the limit load which is defined as the maximum loads the wing expects in service. To satisfy the requirements for the ultimate load, an additional safety factor of 1.5 is applied to the strength and buckling values calculated for the limit loads.

B. Results

Figure13 plots the critical constraints for both the optimization studies, one with and one without the analytical fatigue model. For the case of simplicity, from hereon, the case without the fatigue model will be referred as the first study and the case with the fatigue model will be referred as the second study. For the first study, the wing is critical in both strain and buckling. The inner part of the top skin and spar is buckling critical whereas the outer part is critical in strain. The bottom skin is mainly driven by strain. In the case of the second study, similar trend can be observed. However, the wing is also critical in fatigue. The outer part of both top and bottom skin and the inner part of the front spar are driven by fatigue.

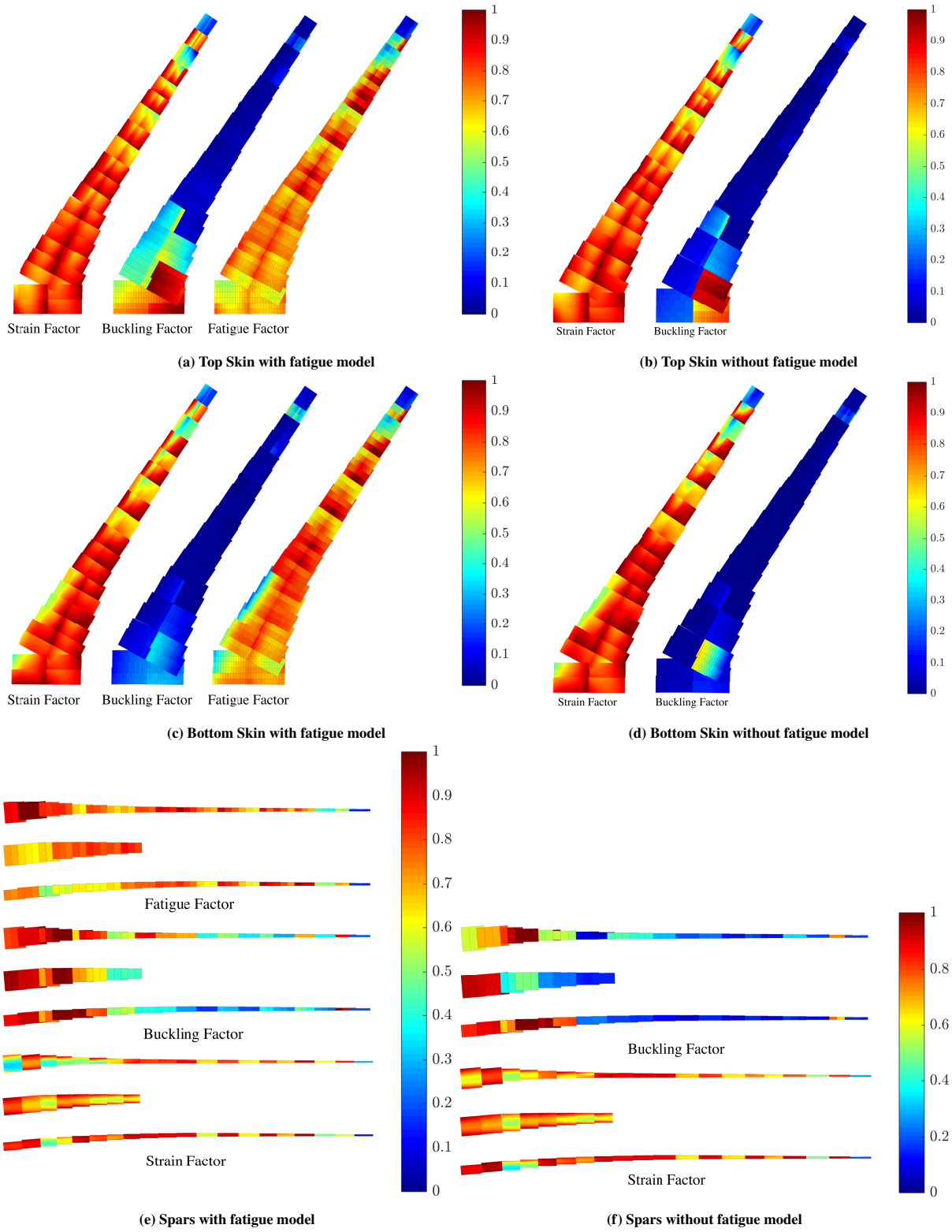
Figure 12 shows the stiffness and the thickness distribution of both the studies. The region near the wing root is

Table 7 List of Loadcases.

Loadcase ID	V_{EAS} (m/s)	Altitude (m)	Load Factor	Fuel level/Max fuel (%)
1	136	11,000	1	70
2	240	3,000	2.5	80
3	198	0	-1	80

dominated by buckling and as a result the out of plane stiffness properties are more pronounced as compared to the rest of the wing in both the studies. With respect to middle and outer part of the wing, the in plane stiffnesses in the top skin are oriented forward to increase the nose down twist and shift the load inboard. The optimized wing in the second study has in plane stiffness oriented slightly more forward than the first study. One reason for this could be the thickness distribution along the chordwise direction in the outer part. In the case of the first study, the difference in the thickness between the front and the aft part is bigger than the second study. As a result the forward shift in the elastic axis is much more pronounced which leads to higher wash out twist. For the second study, tailoring is mainly achieved by the in plane stiffness direction and hence more forward than the first study. With respect to the bottom skin, as the outer part of the wing is critical in fatigue for the second study, the in plane stiffness direction is along the wing axis to maximize the load carrying capabilities of the wing, whereas in the other study, the in plane stiffness is oriented a bit backward.

Looking at the thickness distribution, as expected, the optimized wings in the first study where a knockdown of 68% is applied to the material allowables, are thicker compared to the wings in the second study. The weight of the wing in the first study is 11994 kg, whereas in the second study, the wing weighs at 9146 kg. Thus, by including analytical fatigue model, the weight of the wing can be reduced by approximately 23.7%. Additionally, the maximum fatigue factor for the wing in the first study is 0.75. Thus, application of knockdown factor to incorporate the effect of fatigue, which was used in the first study can lead to a conservative result.



(a) Top Skin with fatigue model

(b) Top Skin without fatigue model

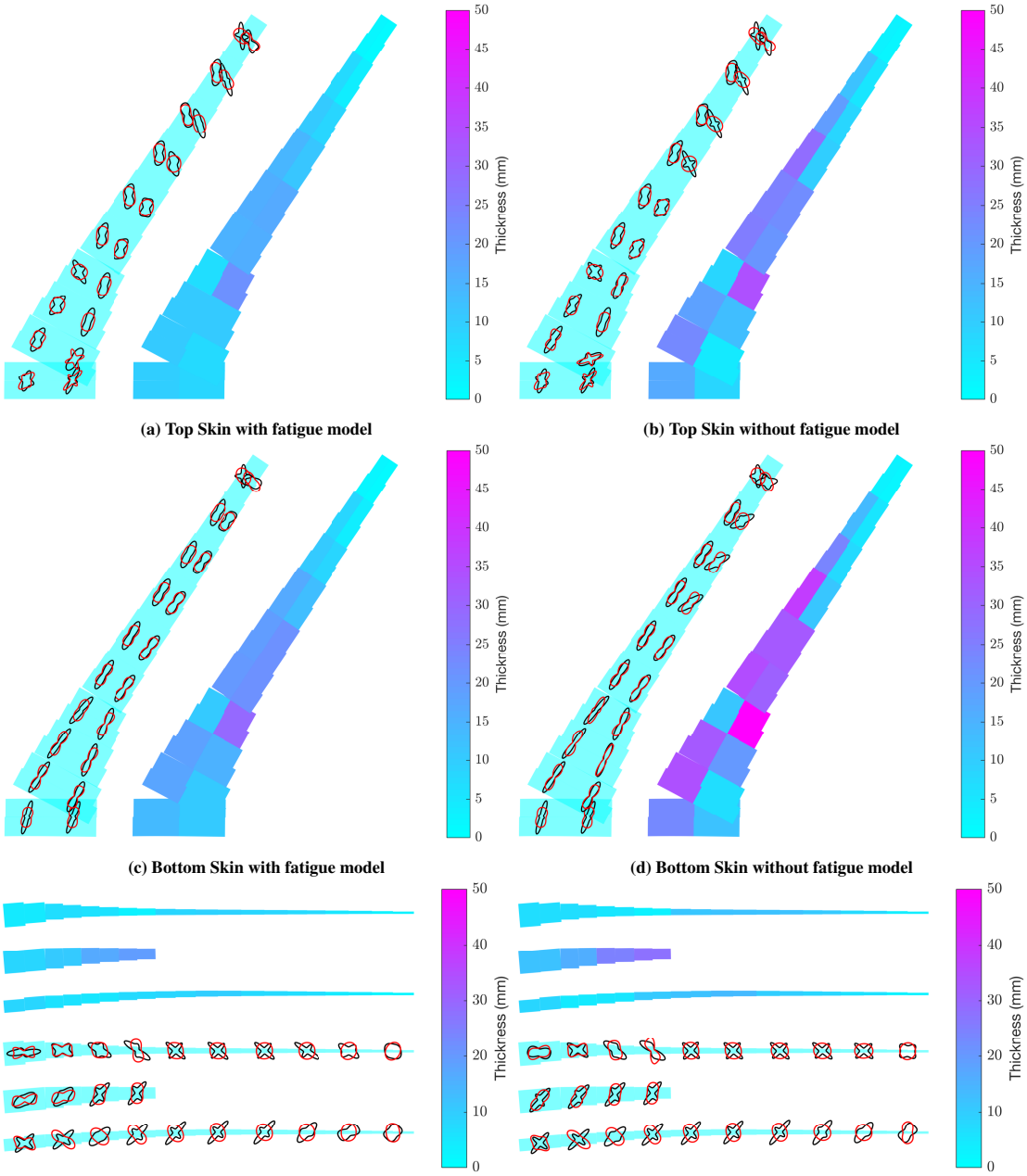
(c) Bottom Skin with fatigue model

(d) Bottom Skin without fatigue model

(e) Spars with fatigue model

(f) Spars without fatigue model

Fig. 12 Value of the critical constraints on the optimized CRM wing.



(a) Top Skin with fatigue model

(b) Top Skin without fatigue model

(c) Bottom Skin with fatigue model

(d) Bottom Skin without fatigue model

(e) Spars with fatigue model

(f) Spars without fatigue model

Fig. 13 Stiffness and thickness distribution for the optimized CRM wing (In-plane stiffness: black, out-of-plane stiffness: red.)

IV. Conclusions

The fatigue model developed earlier by Kassapoglou was extended by adding a first ply failure theory to determine the failure of the ply. Different first ply failure theories were compared to the experimental data and Tsai Wu was found to have the best mix of accuracy and ease of implementation. The predictions of the model were compared to the test results of glass/epoxy laminate for the New Wisper spectrum. Reasonable agreements were found between the predicted and the experimental results. Additionally, the analytical model was also extended to work with laminate described by lamination parameter instead of ply angles and stacking sequence. Since the failure criterion in the lamination parameter domain was conservative in nature, the life predictions were also conservative compared to prediction for laminate defined by ply angles.

The developed analytical fatigue model was integrated in the aeroelastic and optimization tool PROTEUS. Two optimization studies of the CRM wing were carried out, one with fatigue as a constraint using the analytical fatigue model and another was the traditional one by using knockdown factors on material allowables to account for fatigue. Results show that a composite wing is not only critical in strength and buckling but also in fatigue when accounted for explicitly. Accordingly, a different optimum in terms of stiffness and thickness distribution is obtained when fatigue is included as an additional constraint on top of strength and buckling. Furthermore, by including a mathematical fatigue model instead of conservative knockdown factor, the weight of the wing is reduced by 23.7%.

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