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Marinangeli, L.; Alijani, F.; Hossein Nia Kani, Hassan

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# A Fractional-order Positive Position Feedback Compensator for Active Vibration Control

L. Marinangeli \* F. Alijani \* S.H. HosseinNia \*

\* *Department of Precision and Microsystems Engineering,  
Delft University of Technology, Mekelweg 2, 2628 CD Delft,  
The Netherlands (l.marinangeli@student.tudelft.nl; f.aliyani@tudelft.nl;  
s.h.hosseinniakani@tudelft.nl)*

**Abstract:** In this paper a novel Active Vibration Control (AVC) strategy based on fractional-order calculus is developed. A fractional-order Positive Position Feedback (PPF) compensator is proposed to overcome the limitations of the commonly used integer-order PPF such as: frequency spillover, amplitude amplification in the quasi-static region of the closed-loop response, and difficult tuning in multi-mode control. Tuning parameters of the controller are obtained by optimizing both magnitude and phase response of the controlled plant. Results are shown by comparing performances of the standard integer-order PPF and the optimized fractional-order PPF, both on a simple 1-DOF plant and on measured frequency response data from a rectangular carbon fibre composite plate.

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*Keywords:* Active Vibration Control, Spillover, Positive Position Feedback, Fractional-order control, Fractional-order filter, Experimental testing

## 1. INTRODUCTION

In the past decades research on Active Vibration Control (AVC) has found increasing interest in control of flexible thin-walled structures. These structures are often prone to undesirable vibrations, making AVC necessary particularly in industries where a lightweight design is of great importance. Piezoelectric transducers are often selected as sensors and actuators for the active control of flexible structures because of their unique properties including: low cost, low mass, ease of integration and wide frequency range of control. These types of transducers, when used as sensors, measure strain which is proportional to the physical displacement. In fact, control schemes specifically designed to use position as feedback signal have been extensively studied and applied in this context.

The main objective of the controller is to provide active damping to the structure (plant), which results in an attenuation of the resonance peak in the dynamic amplification. The dynamics of flexible structures have very interesting properties: because of their flexibility, they have a large number of elastic modes resulting in very high order transfer functions that are rather difficult to control. Controllers are designed to target specific vibration modes in a restricted bandwidth of interest, and the fact that transfer functions are of high order means that there are out-of-bandwidth modes which are neglected, but whose effect might influence the closed-loop response. The effect of the uncontrolled, or out-of-bandwidth modes, is known in the literature as *spillover* (Balas, 1978). Another important aspect regarding the controller is that, apart from being able to reduce structural vibrations, it should ensure robustness and closed-loop stability for the controlled system. In this

sense, careful positioning of sensors and actuators can have a great influence. The majority of the controllers studied in literature use a *collocated* configuration, where sensors and actuators are related to the same Degree of Freedom (DOF) of the structure. The phase of the open-loop collocated transfer function is always between  $0^\circ$  and  $-180^\circ$ , meaning that poles and zeros interlace on the imaginary axis, where zeros and poles correspond to anti-resonances and resonances of the frequency response, respectively. Collocated systems have the property of being always closed-loop stable with respect to out-of-bandwidth dynamics (Preumont, 2011) and that is why most of the research involves collocation.

One of the most popular collocated modal control schemes is Positive Position Feedback (PPF), which has been first proposed in 1985 by Goh and Caughey (Goh and Caughey, 1985) to overcome the instability associated with finite actuator dynamics. This controller was applied for the first time in 1987 by Fanson (Fanson, 1987) to experimentally suppress vibrations in large space structures. PPF is a second order low-pass filter which rolls off quickly at high frequencies, making it very appealing against possible instability or performance losses due to out-of-bandwidth dynamics. Direct Velocity Feedback (DVF), Resonant Control (RC) (Moheimani, S.O.R., and Fleming, 2006) and Integral Resonant Control (IRC) (Aphale et al., 2007) are also collocated control techniques which are popular in literature, but they present some limitations when used with piezoelectric transducers. In the field of AVC in general, apart from the aforementioned methods, many other different control strategies have been applied for several types of applications. Fractional-order calculus has been found to be a very effective tool in control (see

e.g. HosseinNia et al. (2014, 2013); Monje et al. (2004, 2008); Podlubny (1999)), however it has never found much room in the context of AVC and little research is present (Butler and de Hoon, 2013). In this paper, a fractional-order version of the PPF compensator is proposed to further improve its already appealing properties and to overcome some of its limitations such as low-frequency spillover, quasi-static gain amplification and difficult tuning in multi-mode control.

In the next section DVF, RC, PPF and IRC are elaborated and compared to justify the choice of PPF. Moreover, spillover effect has been introduced. In section 3 fractional-order PPF is developed and its parameters have been tuned using an optimization algorithm. An illustrative example is given in section 4, whereas final conclusions and remarks follow in section 5.

## 2. PROBLEM DEFINITION

### 2.1 Direct Velocity Feedback

Direct Velocity Feedback of a 1-DOF system is defined as follows:

$$\ddot{\xi} + 2\zeta\omega\dot{\xi} + \omega^2\xi = \omega^2 f \quad (1)$$

where  $\xi$ ,  $\omega$ ,  $\zeta$  are modal coordinate, natural frequency and modal damping of the structure, respectively;  $f = -g\dot{\xi}$  is the modal control force and  $g$  is the feedback gain. Equation (1) can be rewritten in the following form:

$$\ddot{\xi} + (2\zeta\omega + g\omega^2)\dot{\xi} + \omega^2\xi = 0 \quad (2)$$

It can be noted that active damping in this case is achieved through direct velocity feedback signal with gain  $g$ . DVF does not prevent the occurrence of spillover effect, but unconditional closed-loop stability is guaranteed nevertheless for  $g > 0$  (Moheimani, S.O.R., and Fleming, 2006). Despite its stability properties, DVF shows important limitations that do not make it an appealing control scheme in the context of active control with piezoelectric transducers. First of all, piezoelectric sensors measure strain of the structure, which can be considered as a displacement signal that would need to be differentiated before being feedback to the velocity controller. Therefore, in using DVF a differentiator is required, but generally not preferred. Secondly, in order to make sure that the compensator rolls-off at high frequencies, extra dynamics needs to be added to it although this could potentially cause instability (Moheimani, S.O.R., and Fleming (2006); Goh and Caughey (1985)). Furthermore DVF has high control effort at all frequencies, and in this context of vibration control it is best to restrict the control effort in the frequency range of interest also to prevent actuator saturation.

### 2.2 Resonant Control

Resonant Control consists of the realization of an electrical dynamic vibration absorber. It is a second order high-pass filter compensator with negative feedback, where the numerator dynamics convert the position feedback (from the piezoelectric sensor) to acceleration feedback. In modal coordinates RC is defined as follows:

*System:*

$$\ddot{\xi} + 2\zeta\omega\dot{\xi} + \omega^2\xi = \omega^2 f \quad (3)$$

*Compensator:*

$$\ddot{\eta} + 2\zeta_f\omega_f\dot{\eta} + \omega_f^2\eta = \ddot{\xi} \quad (4)$$

where  $\eta$ ,  $\omega_f$ ,  $\zeta_f$  are modal coordinate, natural frequency and modal damping of the compensator, respectively;  $f = -g\eta$  is the modal control force representing the negative position feedback. The frequency  $\omega_f$  is normally tuned to the structure's frequency of interest, and together with a proper choice for the parameters  $g$  and  $\zeta_f$  vibration reduction can be achieved. Here, closed-loop stability is guaranteed for  $g > 0$ . The high-pass filter characteristics prevent spillover at frequencies lower than the tuning frequency  $\omega_f$ , but not at higher frequencies where spillover causes changes both in magnitude and frequency of higher vibration modes in the closed-loop response. Therefore, when multiple modes shall be controlled at the same time, multiple compensators can be applied in parallel, but particular attention must be paid to tuning the different RC filters in order to limit the spillover effect. In fact, a compensator targeting a low frequency mode should be tuned prior to the compensator targeting a mode at a higher frequency. It is worth noting that RC is not appealing for practical implementation since actuators and sensors have generally high frequency dynamics which are not neglected by the high-pass filter of the RC and can therefore destabilize the closed-loop system.

### 2.3 Positive Position Feedback

Unlike RC, Positive Position Feedback is a second order low-pass filter with position signal which is positively feedback to the plant. In modal domain, the two equations for a single DOF system and PPF compensator are:

*System:*

$$\ddot{\xi} + 2\zeta\omega\dot{\xi} + \omega^2\xi = \omega^2 f \quad (5)$$

*Compensator:*

$$\ddot{\eta} + 2\zeta_f\omega_f\dot{\eta} + \omega_f^2\eta = \omega_f^2 \xi \quad (6)$$

where in this case  $f = g\eta$  is the modal control force representing the positive position feedback.

For this particular formulation, which is found in most of the literature, the closed-loop stability condition (Fanson, 1987) is simply given by:

$$g < 1 \quad \text{or equally} \quad \frac{K_f}{\omega^2} < 1 \quad (7)$$

where  $g = \frac{K_f}{\omega^2}$ , with gain  $K_f$ , is assumed to be positive since this method works with positive feedback. For a proper performance,  $\omega_f$  is tuned equal to  $\omega$  and  $\zeta_f$  is normally chosen to be bigger than  $\zeta$ . The low-pass filter characteristics cause the PPF to provide so-called active flexibility before the tuning frequency  $\omega_f$ , active damping around  $\omega_f$  and active stiffness for higher frequencies (Kwak and Heo, 2007). Therefore, it limits high-frequency spillover but it does not prevent low-frequency spillover which causes changes both in magnitude and frequency of lower vibration modes in the closed-loop response. When multiple modes need to be controlled at the same time, a compensator targeting a low frequency mode should be tuned after a compensator targeting a mode at higher frequency in order to account for the frequency shift caused by spillover effect.

DVF, RC and PPF are compared in Table 1.

Table 1: Controllers comparison.

Controller	Block diagram	Stability	Properties
DVF		Unconditional ( $g > 0$ )	<ul style="list-style-type: none"> <li>• Direct addition of active damping</li> <li>• Differentiator needed</li> <li>• No roll-off at high frequencies</li> <li>• High control effort at all frequencies</li> </ul>
RC		Unconditional ( $g > 0$ )	<ul style="list-style-type: none"> <li>• 2<sup>nd</sup> order HP filter</li> <li>• High frequency spillover</li> <li>• No roll-off at high frequencies</li> <li>• Difficult tuning in multi-mode control</li> </ul>
PPF		$0 < g < 1$	<ul style="list-style-type: none"> <li>• 2<sup>nd</sup> order LP filter</li> <li>• Low frequency spillover</li> <li>• Roll-off at high frequencies</li> <li>• Difficult tuning in multi-mode control</li> </ul>

System:  $\ddot{\xi} + 2\zeta\omega\xi\dot{\xi} + \omega^2\xi = \omega^2 f$  ;  $G(s) = \omega^2 / (s^2 + 2\zeta\omega s + \omega^2)$ .

#### 2.4 Integral Resonant Control

Another method that is often found in literature is Integral Resonant Control (Aphale et al., 2007). This method is a modified version of the Integral Force Feedback (IFF) method, which is developed for control systems where displacement actuators, and force sensors are used (Preumont, 2011). This implies that the transfer function of the collocated system to be controlled should represent the dynamic stiffness, and not the receptance which is used for the other controllers. Hence, a feed-through term is added to the collocated transfer function in order to convert its frequency response function from receptance to dynamic stiffness, allowing the use of integral feedback. This collocated transfer function also shows pole-zero interlacing property, but starting with a zero. The application of this technique however presents limitations: it generally requires a model of the structure; modes cannot be treated separately; the control gain decreases at higher frequencies causing it to be less effective for high frequency modes.

#### 2.5 Spillover

In general terms, spillover can be explained as the effect that modes which are outside the bandwidth of the controller have on the closed-loop system. Spillover is classified as *observation spillover* when sensor outputs are contaminated by the measured response of residual modes, and *control spillover* when residual modes are excited by the feedback control (Balas (1978)). Observation spillover can be eliminated by the use of collocated configuration,

whereas the effect of control spillover strongly depends on the feedback control scheme used. In this work, observation spillover is assumed not to be present due to the use of collocation, and only control spillover (here simply referred as *spillover*) is treated.

The closed-loop system can become degraded or even destabilized due to the presence of out-of-bandwidth modes. The desired behaviour of the controller would be to target a specific mode and leave the response for the uncontrolled modes ideally unchanged, which in practice never happens because of spillover. Uncontrolled modes can indeed present a change in magnitude and a shift in the resonance frequency making the tuning of the controller more difficult when multiple modes are controlled at the same time. The control action also causes a magnitude amplification in the quasi-static region of the closed-loop response.

Spillover effect is strictly related to the phase behaviour of the closed-loop: the more the phase of the closed-loop follows the phase of the plant, the less spillover is observed, at the expense of a smaller reduction in magnitude of the controlled resonance peak (see Figure 1). This relation between spillover and phase has not received much attention in literature (Niezrecki and Cudney, 1997) and it is intentionally highlighted by the authors. Therefore, in the next section, a novel controller that improves both phase and magnitude response of the close-loop is proposed using fractional-order calculus.

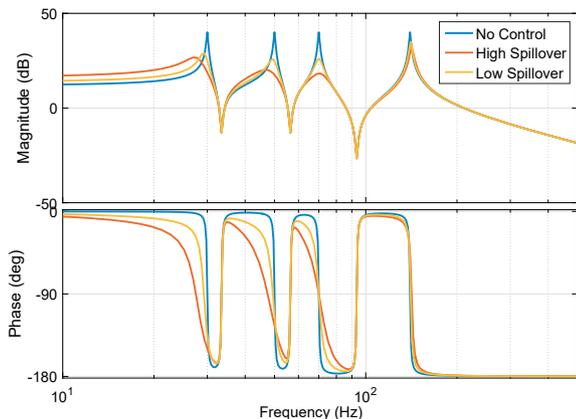


Fig. 1. Relation between spillover effect and phase of the closed-loop is shown by two differently tuned PPF compensators both used to suppress the 3<sup>rd</sup> mode of the plant: the more the phase of the closed-loop follows the phase of the plant the less spillover is observed, at the expense of a smaller reduction in magnitude of the controlled resonance peak. Quasi-static gain amplification is also caused by the control action.

### 3. PROPOSED METHOD

#### 3.1 Fractional-order systems

As stated by Monje et al. (Monje et al., 2010), 'Fractional calculus can be defined as the generalization of classical calculus to orders of integration and differentiation not necessarily integer'. Fractional-order dynamic systems can be expressed by fractional-order transfer functions whose simplest form in Laplace domain is  $s^\alpha$  where  $\alpha \in \mathbb{R}$  and  $s$  is the Laplace transform variable. The benefit of fractional-order transfer functions comes from the fact that they have magnitude and phase response which are not representative of any integer-order transfer functions thereby providing in-between characteristics. In other words, fractional calculus allows for a trade-off between the phase lag of an integrator and the high frequency gain of a differentiator. Control engineering applications have found increasing interest in this concept, which has then been applied to different control systems like the classical PID (Podlubny, 1999; Monje et al., 2004; Tejado et al., 2012).

#### 3.2 Fractional-order PPF

As seen in section 2, standard controllers for AVC present several limitations such as: quasi-static gain amplification, low and high frequency spillover, and difficult tuning in multi-mode control. Based on the already appealing properties of the PPF, a fractional-order PPF compensator is proposed to improve on these limitations. The integer-order PPF transfer function  $C(s)$  shown in Table 1 is rewritten in Equation (8) as

$$C_F(s) = \frac{1}{\left(\left(\frac{s}{\omega_f}\right)^{2\alpha} + 2\zeta_f \left(\frac{s}{\omega_f}\right)^\alpha + 1\right)} \quad (8)$$

where  $1 < \alpha < 2$ ,  $\alpha \in \mathbb{R}$  represents the fractional order. For  $\alpha = 1$  the standard integer-order PPF is recovered.

In Figure 2 the bode plots of both integer and fractional-order PPF filters are compared.

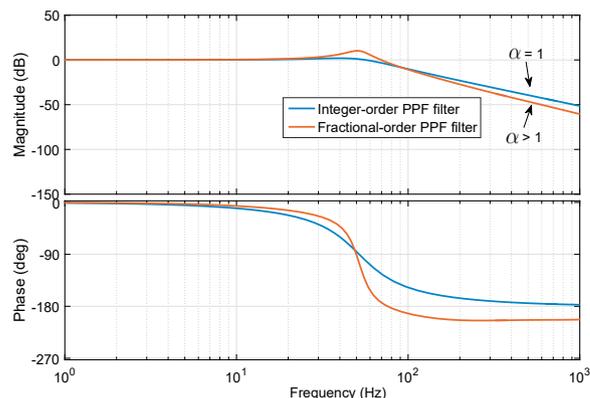


Fig. 2. Bode plots of integer and fractional-order PPF filters: fractional-order PPF with order  $1 < \alpha < 2$  has a steeper roll-off after the tuning frequency  $\omega_f$ .

The addition of another tunable parameter such as the fractional-order  $\alpha$ , allows for the improvement of the limitations of the integer-order PPF, by providing a different magnitude and phase response of the filter  $C_F(s)$ , as seen in Figure 2. For example, the steeper roll-off after the tuning frequency  $\omega_f$  indicates a greater filtering action at higher frequencies, thus limiting even more the high frequency spillover. Low frequency spillover is improved by the different phase change around  $\omega_f$ , which allows the phase of the closed-loop response to be closer to the phase of the plant, as already highlighted in Figure 1. The quasi-static gain amplification  $S_G$  in the closed-loop  $T(s)$  instead depends only on the controller gain  $g$  as shown in (9):

$$S_G = T(0) = \frac{G(0)}{1 - gC_F(0)G(0)} = \frac{1}{1 - g} \quad (9)$$

However, the fractional-order  $\alpha$  allows the use of smaller values of  $g$  for the tuning of the controller, resulting in a lower closed-loop gain in the quasi-static region.

As a consequence, having less spillover and lower quasi-static gain allows for an easier tuning of multiple filters in case of multi-mode control, and an overall improved control performance.

The addition of  $\alpha$  as a design parameter makes a full mathematical derivation of the effects of the fractional-order PPF very challenging to conduct. That is why an optimization approach is proposed to find the optimal filter parameters to actually improve on the aforementioned limitations. In section 4 an example is presented to show the expected performance of the new fractional-order controller.

#### 3.3 Stability Analysis

The stability analysis for fractional-order PPF controller  $C_F(s)$  can be explained as follows.

The roots of the denominator of  $C_F(s)$  should all lie on the complex left-half plane to have a stable controller. By

mapping  $\lambda = (\frac{s}{\omega_f})^\alpha$ , the characteristic equation of (8) can be rewritten as

$$\lambda^2 + 2\zeta_f \lambda + 1 = 0 \quad (10)$$

with roots

$$\lambda = -\zeta_f \pm \sqrt{\zeta_f^2 - 1} \quad (11)$$

Therefore, the condition to have a stable controller  $C_F(s)$  is

$$\Re\{s = \omega_f \lambda^{\frac{1}{\alpha}}\} < 0 \quad (12)$$

or equally

$$|\arg(\lambda)| < \alpha \frac{\pi}{2} \quad (13)$$

where  $\Re$  indicates the real part of a complex number.

Controller stability for different values of  $\alpha$  and  $\zeta_f$  is depicted in Figure 3.

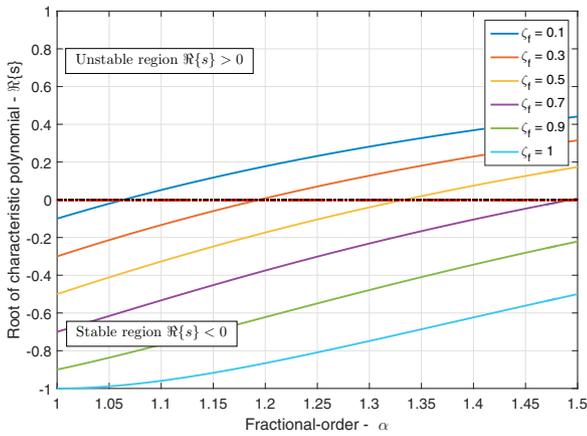


Fig. 3. Stability and instability regions for fractional-order PPF controller  $C_F(s)$  for different values of  $\zeta_f$ .

A closed-loop stability analysis is instead more difficult to conduct. Unlike the integer-order PPF, where the stability condition is simply given by  $g < 1$ , for the fractional-order PPF stability does not depend only on  $g$ , but also on the other parameters  $\zeta_f$  and  $\alpha$ . Thus, a full mathematical derivation of the stability condition is very extensive to obtain. Therefore, an alternative approach based on Nichols stability criterion is proposed.

The Nichols criterion states that closed-loop stability is guaranteed if the Nichols plot of the stable open-loop transfer function  $L(s)$  does not intersect the line where  $\angle L(s) = -180^\circ$  and  $|L(s)| \geq 0$  dB. Therefore, by imposing this condition to the open-loop  $L(s) = -gC_F(s)G(s)$ , stability and instability regions for the closed-loop transfer function  $T(s)$  can be depicted for different values of  $\alpha$ ,  $g$  and  $\zeta_f$  (see Figure 4).

As represented in Figure 4, for a fixed value of  $\zeta_f$  the stability region is defined by the relation between the fractional-order  $\alpha$  and the gain  $g$ , where the curve indicates the stability limit. Values of  $g = 0$  indicate that the controller  $C_F(s)$  becomes unstable, as seen in Figure 3. For different values of  $\zeta_f$  the stability region changes: as  $\zeta_f$  increases, the stable region increases towards higher values of  $\alpha$ ; moreover for  $\alpha = 1$  the stability condition for the integer-order PPF is recovered ( $g < 1$ ).

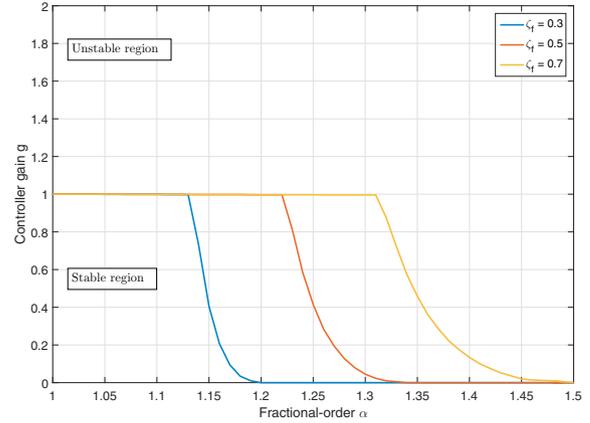


Fig. 4. Stability and instability regions for the closed-loop  $T(s)$  for different values of  $\zeta_f$ .

It is important to notice that the tuning frequency of the filter  $\omega_f$  does not affect the stability analysis, since  $\omega_f$  is assumed to be always tuned around the resonance frequency of the plant  $\omega$ , whose value lies in the stable region of the closed-loop.

### 3.4 Filter Optimization

The fractional-order PPF filter of (8) has an additional design parameter with respect to the standard integer-order PPF, that is the fractional order  $\alpha$  which provides more freedom for the choice of the other tuning parameters  $\omega_f$ ,  $\zeta_f$  and  $g$ . Therefore, an optimality problem is defined to find the mentioned parameters for a fractional-order PPF filter which can limit spillover effect and quasi-static gain amplification. This is done by improving both phase and magnitude response of the closed-loop because of their direct relation with spillover, as already seen in Figure 1.

The objective function  $h$  to be minimized is:

$$h = w_1(P_{\max} - S_G) + w_2 \sum (\angle T(s) - \angle G(s)) \quad (14)$$

$$+ p \left( \max \left( 0, \frac{P_1}{P_{\max}} - 1 \right) \right)^2$$

$$+ p \left( \max \left( 0, \frac{P_2}{P_{\max}} - 1 \right) \right)^2$$

$$+ p \left( \max \left( 0, \frac{S_G}{P_1} - 1 \right) \right)^2$$

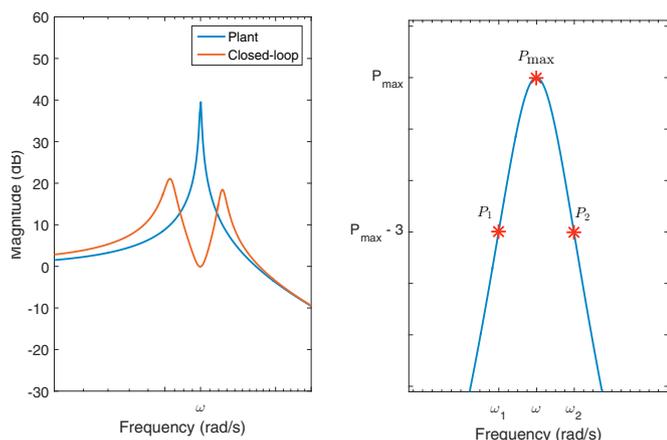
$$+ p \left( \max \left( 0, \frac{S_G}{P_2} - 1 \right) \right)^2$$

$$+ p (\max(0, \zeta_T))^2$$

where the magnitude of the closed-loop  $T(s)$ , where  $T(s) = \frac{G(s)}{1 - gC_F(s)G(s)}$ , is optimized by the difference between the magnitude  $P_{\max}$  at the resonance  $\omega$  and static gain  $S_G$  of the plant  $G(s)$ , so to impose maximal peak reduction; the phase is optimized instead by the minimization of the difference between phase of the closed-loop  $T(s)$  and phase of the plant  $G(s)$ ; two weights  $w_1$  and  $w_2$  are

used to give more importance either to the magnitude or phase optimization.

The objective is also enhanced by the addition of penalization functions, in order to make sure that the resonance peak is always reduced without creating two additional peaks around  $\omega$  due to the presence of extra zeros in the closed-loop (Kwak, Moon K. and Han, Sang-Bo and Heo, 2004) (see Figure 5a).  $P_1$  and  $P_2$  are the so-called *half-power* points at 3 dB down from the resonance peak and are used to compute the closed-loop damping  $\zeta_T$  at resonance with the Half-power Bandwidth method (see Figure 5b);  $p$  is the penalization factor. Closed-loop stability is also added as equality constraint by setting the number of unstable poles equal to 0.



(a) Closed-loop zero causing two extra peaks around resonance. This effect is limited by the addition of proper penalization functions.  
(b) Half-power points  $P_1$  and  $P_2$  at 3 dB down from the resonance peak. These points are used to compute the closed-loop damping  $\zeta_T$  by the Half-power Bandwidth method:  $\zeta_T = \frac{\omega_2 - \omega_1}{2\omega}$ .

Fig. 5. Conditions for closed-loop damping  $\zeta_T$ .

A Global-Search algorithm using a constrained non-linear optimization is chosen to solve the optimality problem because the objective  $h$  is highly non-linear and presents several local optima. The algorithm is implemented in MATLAB by means of the 'GlobalSearch' and 'fmincon' functions. The plant  $G(s)$  is chosen to represent a simple 1-DOF system as in Equation (5), and the outcome of the optimization gives the four tuning parameters for the filter  $C_F(s)$ . Optimization results depend mainly on the choice of the two weights  $w_1$  and  $w_2$ , from which a filter  $C_F(s)$  that has either a stronger effect on the magnitude or on the phase of the closed-loop can be obtained. In the next section some examples showing the potential benefit of using a fractional-order PPF rather than an integer-order PPF are presented.

## 4. ILLUSTRATIVE EXAMPLE

### 4.1 Simulation Results

Performances of the standard integer-order PPF and the optimized fractional-order PPF are compared first on a simple plant representing a 1-DOF system and then on

a plant representing a multi-DOF system. Comparison is made by tuning the integer-order PPF such that both compensators provide the same magnitude reduction of the resonance peak to be controlled. Tuning parameters for both filters are listed in Table 2. It is important to highlight that controllers are normally not tuned to achieve 100% peak reduction, and that is mainly done to avoid performance losses in multi-mode control.

Table 2: Tuning parameters for Integer and Fractional-order PPF filters, both providing same magnitude reduction of the resonance peak at  $\omega$ .

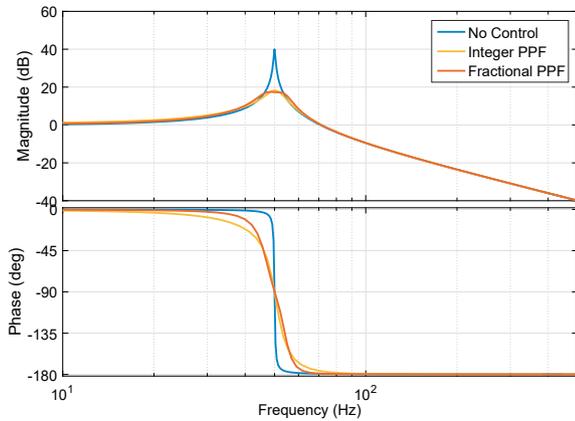
Parameter	Integer PPF	Fractional PPF
$g$	0.1	0.0365
$\omega_f$	$\omega$	$1.0366\omega$
$\zeta_f$	0.45	0.4227
$\alpha$	1	1.1844

In Figure 6a, a simple plant with single resonance at 50 Hz is controlled by both integer-order and fractional-order PPF compensators. The same magnitude reduction is achieved for the two closed-loop responses, but the phase response is closer to the phase response of the plant when fractional-order PPF is used. In Figure 6b, the corresponding step response is shown, where it is clearly seen how the vibration is quickly damped out by both controllers although the steady state gain is closer to 1 in case of fractional-order PPF. This corresponds to a magnitude in the frequency response closer to the ideal behaviour at 0 dB.

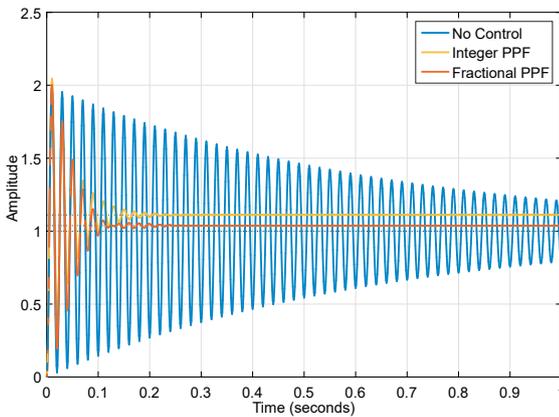
The real benefit of an improved phase behaviour is more evident in case of a multi-mode plant as shown in Figure 7. In Figure 7a, the 3<sup>rd</sup> mode is controlled and less spillover is present both at lower and higher frequencies than the controlled resonance when fractional-order PPF is used, since it maintains the phase response much closer to the plant response rather than what the integer-order PPF does. In Figure 7b, the 2<sup>nd</sup> and 4<sup>th</sup> modes are controlled simultaneously using two parallel PPF filters: again the spillover effect is much less in case of fractional-order PPF since the uncontrolled 1<sup>st</sup> and 3<sup>rd</sup> modes are less affected by the control action. Uncontrolled modes indeed present a smaller shift in frequency caused by spillover together with a largely reduced magnitude amplification in the quasi-static region, thus ensuring a better control performance and easier tuning of the filters especially when multiple modes are controlled at the same time.

### 4.2 Experimental Results

The same performance comparison of the two PPF filters is done on measured frequency response data. An experimental collocated transfer function has been obtained by performing modal tests on a rectangular carbon fibre composite plate. The plate was hung from four corners by nylon cords to simulate all edge free boundary conditions and an electrodynamic shaker was used to provide random excitation to obtain frequency response curves. An impedance head measures the force applied by the shaker and a Laser Doppler Vibrometer measures the velocity vibration response on the other side of the plate with respect



(a) Single-mode plant controlled.



(b) Step Response.

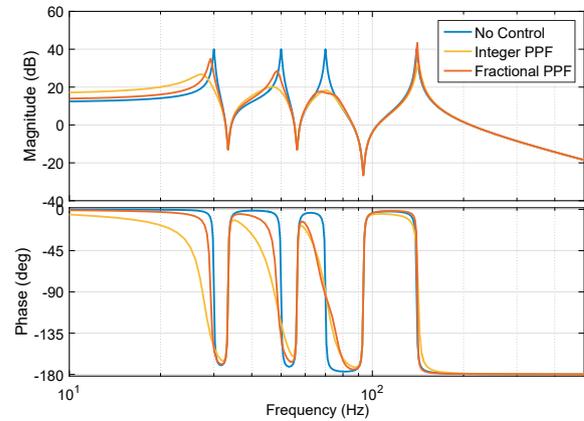
Fig. 6. Single-mode plant controlled by both integer and fractional-order PPF.

to the shaker (see Figure 8). The measured data is then imported into MATLAB as 'frd' object, velocity is converted to position, and PPF filters are applied similarly to what was done for the previous section. Tuning parameters of Table 2 are kept the same except for the gain  $g$  which has been adjusted according to the overall magnitude of the measured plant which is different from the one used in the previous section. In Figure 9, the measured plant response together with the controlled transfer functions are shown. In Figure 9a, the mode at 37 Hz is controlled and in Figure 9b, modes at 37 Hz and 108 Hz are controlled simultaneously. In both cases the spillover effect caused by the fractional-order PPF is found to be less both at low and high frequencies with respect to the standard integer-order PPF.

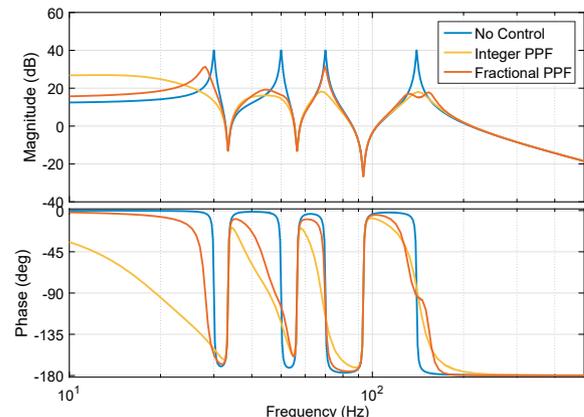
The experimental vibration setup of Figure 8 is proposed with the aim to eventually extend this work and implement the controller with piezoelectric sensors and actuators.

### 5. CONCLUSIONS

In this paper, a novel fractional-order compensator based on Positive Position Feedback has been proposed to limit the spillover effect caused by the dynamics of uncontrolled vibration modes. The strict relation between spillover ef-



(a) 3<sup>rd</sup> mode controlled.



(b) 2<sup>nd</sup> and 4<sup>th</sup> mode controlled.

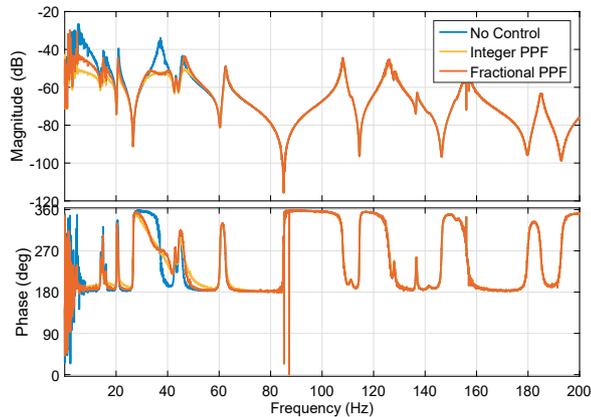
Fig. 7. Simple plant representing a multi-DOF system controlled by both integer and fractional-order PPF.



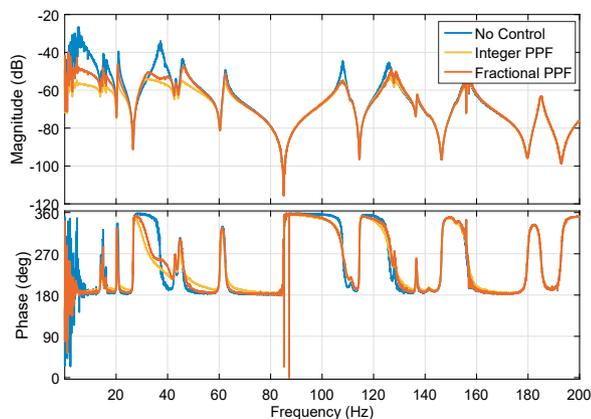
(a) Laser Doppler Vibrometer (b) Shaker and impedance setup

Fig. 8. Experimental vibration setup.

fect and closed-loop phase response has been highlighted and used for the controller optimization where both magnitude and phase response are optimized. A Global-Search algorithm using a constrained non-linear optimization has been used to obtain the tuning parameters. Performance of the fractional-order PPF filter has been verified by comparison with an integer-order PPF tuned such that



(a) Mode at 37 Hz controlled.



(b) Modes at 37 Hz and 108 Hz controlled.

Fig. 9. Effect of PPF on measured frequency response data.

both compensators provided the same magnitude reduction of the resonance peak. Results obtained both on simple plants, representing 1-DOF and multi-DOF systems, and experimental frequency response data have shown that fractional-order PPF improves the overall control performance with respect to the standard integer-order PPF by limiting the spillover due to uncontrolled modes both at high frequencies, by providing a steeper roll-off, and at low frequencies, by providing a better phase behaviour and limiting the magnitude amplification in the quasi-static region. It can thus be concluded that the use of fractional-order transfer functions is very promising to improve the performance of commonly used active vibration control strategies since they can provide in-between response characteristics that would not be achievable by standard integer-order transfer functions, and they allow to maintain a very simple design and implementation for the controller, as seen in this specific case. Furthermore, an experimental implementation of the proposed fractional-order compensator needs to be performed to better validate its behaviour in a practical application.

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