Lumen maintenance predictions for LED packages

van Driel, W.D.; Schuld, M.; Jacobs, B.; Commissaris, F.; van der Eyden, J.; Hamon, B.

DOI
10.1016/j.microrel.2016.03.018

Publication date
2016

Document Version
Accepted author manuscript

Published in
Microelectronics Reliability

Lumen Maintenance Predictions for LED Packages

W.D. van Driel[1,3], M. Schuld[2], B. Jacobs[1], F. Commissaris[1], J. van der Eyden[1], B. Hamon[4]

[1]Philips Lighting, High Tech Campus, Eindhoven, The Netherlands

[2]CQM, Eindhoven, The Netherlands

[3]Delft University of Technology, DIMES-ECTM, The Netherlands

[4]Philips Lighting, Rue des Brotteaux - 01708, Miribel, France

Email: willem.van.driel@philips.com

Abstract

Commercial claims for LED-based products in terms of lumen maintenance are fully based on TM-21 extrapolations using LM-80 data. This paper indicates that there may be a risk in doing this as TM-21 only relies on the behavior of the average LED degradation, instead of taking into account the degradation of all individual LEDs. Therefore, we propose a more profound statistical analysis in order to make the appropriate step from TM-21 extrapolation to lumen maintenance on a product level. This is needed as some commercial claims are based on 10 years of warranty and some service bids provide periods of 20 to 25 years of operation. This paper reviews the different approaches currently available to perform lumen maintenance extrapolations. We propose a new method to analyze and extrapolate LM-80 data using a more profound statistical approach.

Highlights

The main highlights of the presented research are:

- A new statistical method to extrapolate LM-80 data
- The method outperforms the currently available ones as it is statistically founded
- Five cases were executed and benchmarked with the TM-21 method
- A full statistical acceleration model is now available for lifetime assessment of LEDs

1. Introduction

Solid State Lighting (SSL) refers to a type of lighting that uses semiconductor light-emitting diodes (LEDs), organic or polymer light-emitting diodes (OLED / PLED) as sources of illumination rather than electrical filaments, plasma (used in arc lamps such as fluorescent lamps), or a gas. SSL applications are now at the doorstep of massive market entry into our offices and homes. This penetration is mainly due to the promise of an increased reliability with an energy saving opportunity: a low cost reliable solution [1].

Per today, commercial claims for LED-based products in terms of lumen maintenance are fully based on LM-80 data [2] and TM-21 extrapolations [3, 4, 5]. IES LM-80-08 is an approved method for measuring lumen maintenance of LED lighting sources. The IES standard TM-21-11 provides a guideline for lifetime prediction of LED devices. It uses average normalized lumen maintenance data coming from LM-80 measurements and performs non-linear regression for lifetime modeling. It cannot capture the dynamic and random variation of the degradation process of LED devices. The lumen maintenance life is defined as the time when the maintained percentages of the initial light output fall below a failure threshold. There may be a risk in doing this as TM-21 only relies on the behavior of the average LED degradation, instead of taking into account the degradation of all individual LEDs. A more profound statistical analysis is required to make the step from TM-21 extrapolation to lumen maintenance on a product level. In this paper we investigate the different approaches that are able to perform lumen maintenance extrapolations. For that, we have analyzed several LM-80 data sets from a statistical point of view.

2. Problem Formulation

Lumen maintenance is the basis for commercial claims of LED-based products [6, 7, 8]. As such, it is extremely vital to perform projections that are statistically sound and correct. Being an industry agreement, TM-21 flaws in this respect and alternative approaches are needed. Such an alternative approach should encompass the following nature:

- Use all the raw data, per setting, per LED and per time point.
- Provide statistically sound results in terms of prediction stability.
- Provide a true value for the lumen life of the LED technology.

Chapter 3 describes the current agreed methods and provides an alternative statistical approach.

3. Statistical Methods

3.1 Current Agreed Methods

Per today, all LED suppliers deliver LM-80 datasets typically at three currents and three temperatures. A typical data set is depicted in Figure 1 [6]. This relative data is then used for the TM-21 extrapolation tool to create a prediction that is listed in Figure 2. The result is truncated using the so-called 6x rule, where one can only claim a value that is six time the LM-80 time (e.g. with 6khrs test time, one can only claim 36khrs lumen maintenance).

Within the TM-21 committee, an initial approach to the problem of projecting lumen maintenance life was the consideration of multiple mathematical models [4, 5]. These ranged from 1-parameter exponential decay until 3-parameter multi-exponential decay.

Note that there is a risk in accepting lifetimes that are predicted far beyond the LM-80 testing time because of the significant effect of measurement errors. In order to prevent that, it is important to use either golden samples or to use (agreed) censoring data points.

2.0 TEST CONDITION 1: 55 °C 0.200 A

TABLE 2.1 - LUMEN MAINTENANCE RESULTS
TEST CONDITION 1: 55 °C 0.200 A

Photometric test drive current: 0.200 A
Photometric test ambient temperature: 25 ± 2 °C
Failures observed: none

Load board ID	Device number	Zero hour measurements Flux (lm)	V_F (V)	Lumen Maintenance (%) 168	500	1000	2000	3000	4000	5000	6000	7000	8000	9000
A800000B9457031C	1	515.10	25.79	100.4	100.5	99.9	99.7	100.0	99.9	99.9	99.2	99.1	98.9	98.1
	2	518.67	26.40	99.9	99.9	99.4	99.1	99.3	99.2	98.9	98.5	98.4	98.3	97.6
	3	518.57	25.75	100.1	100.3	99.8	99.6	100.1	100.0	100.0	99.4	99.3	99.2	98.5
	4	511.77	26.23	100.7	100.9	100.3	100.3	100.7	100.8	100.7	100.3	100.1	100.0	99.1
	5	517.49	26.12	99.9	99.9	99.4	99.0	99.7	99.8	99.7	99.2	99.0	99.2	98.4
	6	516.60	27.17	100.2	100.2	99.6	99.6	99.9	99.8	99.8	99.2	99.1	99.0	97.7
	7	522.52	27.56	100.3	100.4	100.0	99.9	100.3	100.3	100.3	99.4	98.8	97.7	96.0
	8	512.03	26.30	100.4	100.4	99.8	99.7	100.1	100.0	99.9	99.3	99.1	99.1	98.5
	9	516.71	25.83	100.5	100.3	99.9	99.8	100.2	100.1	100.1	99.6	99.6	99.5	98.5
	10	516.33	25.70	100.3	100.3	99.8	99.9	100.4	100.4	100.5	99.9	99.7	99.5	98.6
	11	520.62	25.94	100.2	100.1	99.6	99.5	99.9	99.9	99.9	99.4	99.3	99.2	98.4
	12	515.88	26.51	100.1	100.1	99.6	99.6	100.2	100.2	100.3	99.7	99.6	99.3	98.5
F600000B9E7D031C	1	524.22	26.27	100.0	100.2	99.6	99.7	99.8	99.8	99.6	99.0	98.9	98.7	97.8
	2	516.25	27.44	99.9	100.0	99.4	99.5	99.6	99.4	99.4	98.6	98.2	97.4	95.8
	3	514.76	26.05	99.3	99.3	98.8	98.8	98.8	98.7	98.5	97.7	97.9	97.7	96.8
	4	515.71	25.99	99.5	99.7	99.1	99.1	99.3	99.2	99.2	98.8	99.0	99.0	98.3
	5	517.33	25.99	99.9	100.1	99.6	99.8	100.0	99.8	99.9	99.3	99.4	99.3	98.4
	6	512.51	25.96	99.9	100.0	99.8	100.1	100.4	100.3	100.3	99.8	99.8	99.4	98.0
	7	514.99	25.59	99.8	99.8	99.1	99.2	99.1	99.1	98.9	98.4	98.6	98.5	97.6
	8	514.99	26.45	99.9	100.1	99.7	99.9	100.0	100.1	100.0	99.6	99.6	99.3	98.2
	9	520.72	25.88	99.7	99.9	99.4	99.7	99.8	99.7	99.7	99.2	99.3	99.3	98.6
	10	517.69	25.52	99.8	100.1	99.6	99.8	99.9	99.9	99.9	99.5	99.6	99.6	98.9
	11	507.07	26.24	100.5	100.9	100.6	100.7	101.1	101.0	101.0	100.5	100.5	100.4	99.3
	12	522.09	25.92	99.8	100.0	99.4	99.5	99.8	99.7	99.6	99.1	99.1	99.1	98.2
			n	24	24	24	24	24	24	24	24	24	24	24
			mean	100.0	100.1	99.6	99.6	99.9	99.9	99.8	99.3	99.2	99.0	98.1
			median	100.0	100.1	99.6	99.7	100.0	99.9	99.9	99.3	99.2	99.2	98.4
			std. dev.	0.3	0.3	0.4	0.4	0.5	0.5	0.6	0.6	0.6	0.7	0.9
			min	99.3	99.3	98.8	98.8	98.8	98.7	98.5	97.7	97.9	97.4	95.8
			max	100.7	100.9	100.6	100.7	101.1	101.0	101.0	100.5	100.5	100.4	99.3

Figure 1:Typical LM-80 data set showing lumen decay per LED as function of measurement time [6].

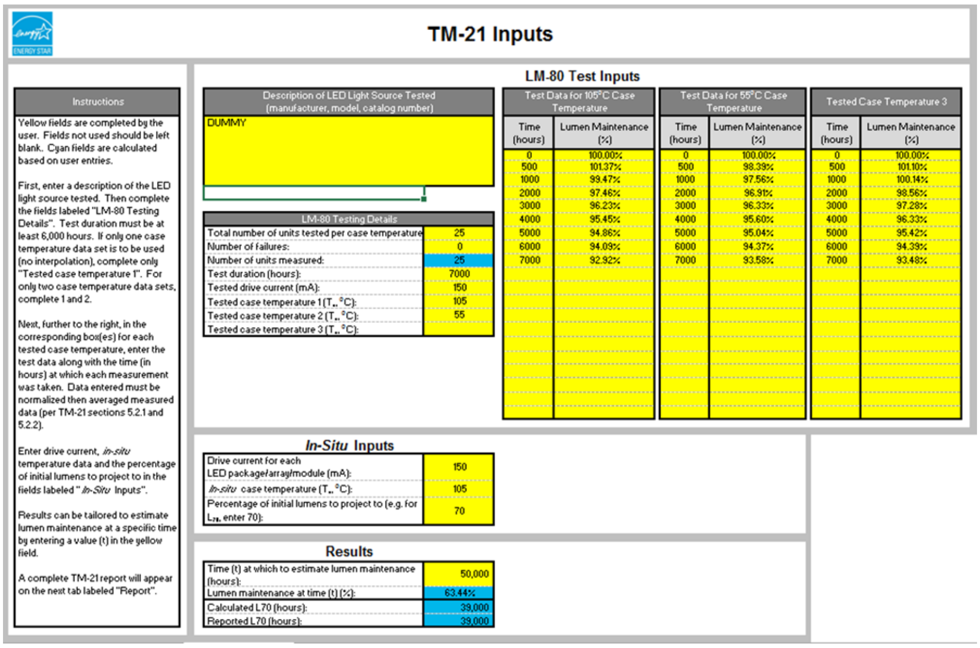

Figure 2: TM-21 report example.

Within TM-21, being an industry agreement, finally, the simplest possible form was chosen as:

$$\Phi(t) = \exp\left(-\alpha t^{\beta}\right) \qquad (1)$$

where:

- T is time in hours;
- $\Phi(t)$ is the <u>averaged</u> normalized luminous flux output at time t;
- α is the decay rate constant derived by a least squares curve-fit;
- β is the shape parameter.

For each separate temperature and/or current L70, that is $\Phi = 0.7$, can then be calculated using averaged normalized luminous flux output:

$$L_{70} = \left(-\ln(0.7)/\alpha\right)^{1/\beta} \qquad (2)$$

Estimates of (α, β) can be easily obtained by applying the least squares method. Temperature acceleration, within the measured temperatures, is allowed and supposed to follow the Arrhenius equation:

$$\alpha = C \exp\left(\frac{-E_\mathrm{a}}{k_\mathrm{B} T_s}\right) \tag{3}$$

where:

- C is a pre-exponential factor;
- E_a is the activation energy (in eV);
- T_s is the in-situ absolute temperature (in K);
- k_B is the Boltzmann's constant (8.617385x10-5 eV/K).

This model for α can be easily extended by using the inverse power law model that takes into account the effect of current:

$$\alpha = C \exp\left(\frac{-E_\mathrm{a}}{k_\mathrm{B} T_s}\right) I^n \tag{4}$$

where:

- I is the current;
- n is a life-stressor slope.

If applicable the interaction between temperature and current can be added easily.

The TM-21 method has become quite a standard way of working within the Lighting industry. In the issue of LEDs Magazine from December 2014, Hansen and Davis [9] used the approach to assess LM-80 data across a variety of packaged LEDs in an effort to determine the effects of different LED platform designs and materials on performance, light quality metrics, and cost.

Alternative approaches are rare as only few other publications build upon the TM-21 method. An exemption is the VDE standard VDE-AR-E2715-1 [10] currently published in Germany only. Here, the authors describe the so-called Border Function method (In German: Grenzfunction). This Border Function (BF) method is based on the assumption that an exponential model is a conservative estimation (worst-case scenario) of the actual long term luminous flux maintenance as it is expected that most LED packages will show a long-term luminous flux maintenance which is better than the assumed exponential function. Fan et al. from the CALCE institute of technology [11] have used the degradation-data-driven method (DDDM) which is based on the general degradation path model. They use it to predict the reliability of HP LEDs through analyzing the lumen maintenance data collected from the IES LM-80-08 lumen maintenance test standard. Their method is capable of getting much more reliability information out of the data (e.g., mean time to failure, confidence interval, reliability function). In an accompanying paper, Fan et al. [12] describe a particle filter-based (PF-based) prognostic approach based on both Sequential Monte Carlo (SMC) and Bayesian techniques. These techniques are used to predict the lumen maintenance life of LED light sources. Also here the alternative approach achieves better prediction performance, with an error of less than 5% in predicting the long-term lumen maintenance life of LED light sources. Lall et al. [13] follow up on this approach by using Bayesian Probabilistic Models for the assessment of the onset of degradation in solid state luminaires. The failure threshold decay rate has been calculated using an Arrhenius model, neglecting the effects of current density and humidity. The statistical approach is quite valid but also seen as complicated. Quan et al. [14] describe an in-situ method to monitor the lumen degradation of LED packages. They conclude that the luminous flux of the LEDs show a steady and slow depreciation but no proper statistical analysis was performed on their measured data. Huang et al. [15, 16, 17] investigated the degradation mechanisms of mid-power white-light LEDs. In their studies, a modified Wiener process was employed for the modeling of the LED devices' degradation, following the earlier work of Tsai et al. [18]. Using this method, the dynamic, random variation, as well as the non-linear degradation behaviors of the LED devices was described. They applied the Hallberg-Peck's model to describe the effects of temperature and humidity on LED degradation thereby ignoring the crucial effects of the current density on this degradation. Other studies devote lumen decay to silicone degradation and/or crack formation [19, 20]. In these investigation, silicone degradation was quantitatively evaluated using finite element analysis and used to estimate the LED package lifetime depending on the operation conditions. Buffolo et al. [21] present the results of a reliability investigation performed on four different groups of commercially available mid-power white LEDs. Their data gathered all along the 4000 h of stress accumulated suggest the presence of multiple degradation mechanisms that may limit the useful lifespan of the LED packages. This study lacks a proper statistical analysis of the experimental data, nor proposes an alternative method.

3.2 Alternative for Model Fitting

An alternative approach is to study the "degradation" data of each LED individually. It means that for each individual LED a model as stated in equation (1) is fitted. Then, we can predict L70 values for each LED, and turn degradation values into failure times. The question is whether the differences between predicted lifetimes are due to production variation only, or due to operating variation, such as temperature and current as well. Such an experiment is called an accelerated degradation test (ADT). In order to conduct an ADT efficiently, there are several aspects that need to be considered. These aspects are termination time, the number of stress factors, the number of stresses, the choice of stress levels, and the sample size for each stress level. For instance Nelson [22], and Meeker and Escobar [23] already addressed those aspects.

Besides the mentioned aspects, one of the most important questions arising from a degradation experiment is how many hours (or cycles) an accelerated degradation experiment should last for gathering proper data to allow one to make inference about the product lifetime under the normal use condition. In this paper we focus on the convergence of the quantile estimators (such as B10 or B50) to decide whether we are able to make this inference. Therefore, determination of the termination time cannot be decided upfront. Yu and Tseng [24] proposed to combine the outcomes of an ADT with a known accelerate life test (ALT) model. They showed that the termination time of a degradation experiment has a huge impact on the precision of estimating a product's lifetime. It appeared that the Mean Time to Failure (MTTF) estimates oscillate severely at the beginning; however, as the termination time t_i (with i=1,2, …n) increases, more degradation data are collected, the MTTF estimate converges. It is obvious that B10 and B50 behave similarly. Our intuitive approach to determine the termination time for an LM-80 experiment is based on the work of Yu and Tseng [24]. In this paragraph we will explain the mentioned approach.

The approach for determining the termination time for an ADT has three steps:

1. Use the degradation paths to estimate the lifetimes of LEDs under specific temperatures and currents up to the testing time t_i. So for each LED the parameters (α, β) of equation (1) needs to be estimated, such that L_{70} can be calculated.
2. Find a suitable life-stress model and use a Maximum Likelihood (ML) procedure to estimate B10 (50) under certain use conditions (T, I). Lognormal and Weibull distributions are both appropriate models to fit the (estimated) lifetime data. Check the distribution assumptions by making probability plots, and study the patterns of the parallel lines (for different values of T and I).
3. Investigate the behavior of B10 (50) for different times t_i (i=1,2,..n) and propose an appropriate termination time. B10 (or B50) at time t_i often oscillates severely at the beginning, but will converge in time. Yu and Tseng [24] mention three types of convergence patterns: monotonically increasing to a value, monotonically decreasing to a value, and slightly oscillating around a value. To derive an appropriate termination time they also propose an algorithm that considers the relative rate of change of the asymptotic mean lifetime by using the 3-period moving average. In this study we focus mainly on the question whether we have sufficient testing data in time to show convergence at all using the mentioned LM-80 data.

In the next paragraph we will demonstrate this alternative method for several LM-80 data sets coming from high-power (HP) and mid-power (MP) LEDs Mignot, Nicolas <nicolas.mignot@philips.com>.

4. Analysis of the selected use cases

In order to assess the applicability of our proposed statistical approach we have gathered 5 use cases of long term lumen maintenance data. These are:

- Case 1a: HP LED technology, 14khrs LM-80 data at 4 currents and 4 temperatures
- Case 1b: HP LED technology, 10khrs LM-80 data at 3 currents and 4 temperatures
- Case 2a: MP LED technology, 10khrs LM-80 data at 3 currents and 3 temperatures
- Case 2b: MP LED technology, 8khrs LM-80 data at 3 currents and 3 temperatures
- Case 2c: MP LED technology, 12khrs LM-80 data at 3 currents and 3 temperatures

For cases 1a and 1b the target application settings are 85°C and a forward current of 1A. For cases 2a, 2b and 2c the target application settings are also 85°C but using a forward current of 150 mA.

All 5 data sets are subjected to the alternative method. For that, all data points with a sufficient level of degradation are used. Figure 3 shows the predicted B50L70 as function of the LM-80 measurement time (or degradation time). The following is observed:

- Case 1a: the predicted B50L70 value gradually increases as function of time to reach almost stable values around 80khrs.
- Case 1b: the predicted B50L70 value keeps on increasing and a stable value is not insight yet. A value of 180khrs seems to be reached, but the curve itself could start decreasing after that.
- Case 2a: the predicted B50L79 value keeps on increasing, stable value seems to hit 65khrs.
- Case 2b: the predicted B50L70 value gradually decreases as function of time to reach a stable value around 60khrs.
- Case 2c: the predicted B50L70 value first increases after which it gradually decreases as function of time to reach a stable value around 60khrs.

From a test termination point of view, cases 2a / 2b / 2c have reached stable values and, thus, the LM-80 test can be stopped. For 1a and 1b however, stable values are not in reached yet meaning the test cannot be stopped.

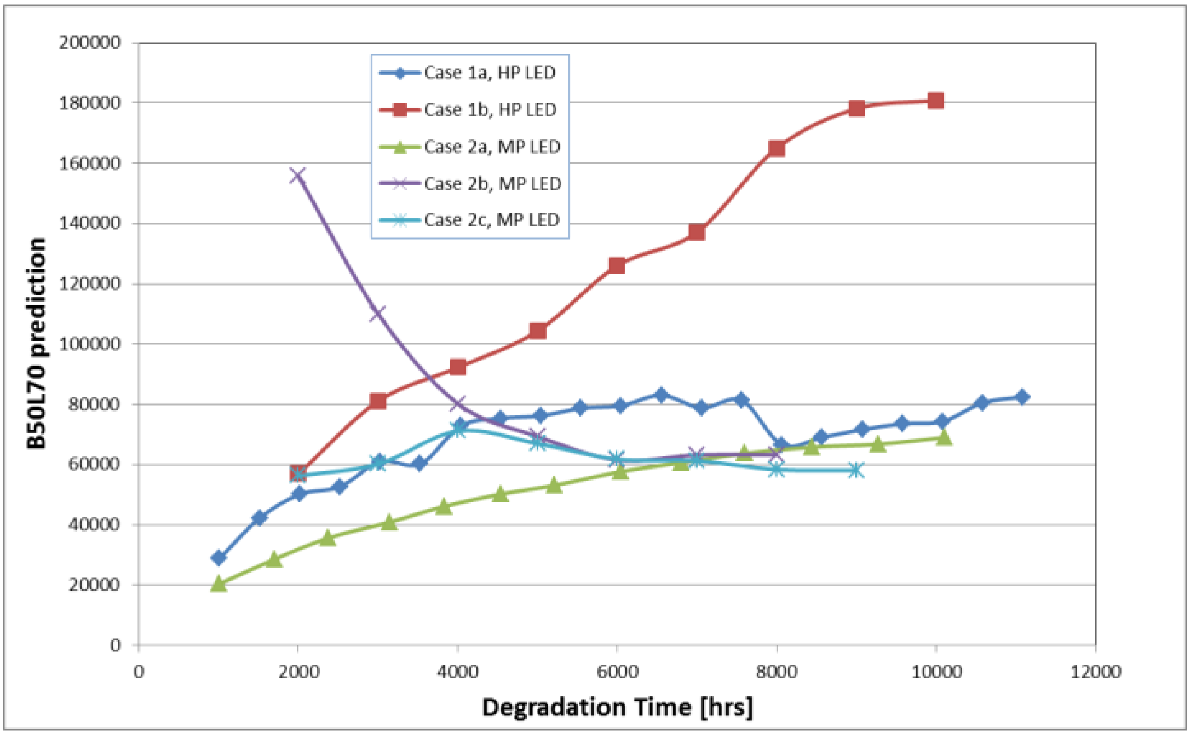

Figure 3: Predicted B50L70 as function of the LM-80 measurement time for the 5 use cases.

Table 1 gives all the predicted acceleration model parameters following equations (3) and (4). The activation energy is in the range 0.1 – 0.4eV, which is believed to be the correct values for this failure mode. The standard deviation is quite reasonable. The effect of the current, parameter n, is quite different and a large spread is found. A negative value indicates that with a higher current, the degradation is worse. A positive value is not reasonable (higher current improves the degradation level).

The fitted parameters listed in Table 1 uniquely describe the lumen maintenance performance for the 5 LED packages under any application condition.

Table 1: Resulting fitted parameters following eq. [3] and [4]. σ is the lognormal standard deviation.

Case	C	n	Ea	σ
1a	7.82	-0.50	0.11	0.35
1b	9.98	-0.19	0.07	0.93
2a	0.55	0.15	0.34	0.27
2b	1.92	-2.89	0.11	0.25
2c	4.52	-0.72	0.16	0.68

With the fitted parameters available, Table 2 depicts the comparison of the proposed statistical method with the TM-21 prediction. In general the deviation is found to be in the order of 0% to 14%. The main reason for this deviation is due to the fact that TM-21 only predicts the B50L70 values based on extrapolating the given test data. It does not take the other conditions into account whereas the proposed method searches for congruency in the full dataset using all tested conditions. Taking this into account, the comparison is quite reasonable.

Table 2: Comparison of the proposed statistical method with the existing TM-21 method for B50L70 values.

Case	Reference	TM-21 prediction [hrs]	Proposed method [hrs]	Difference [%]
1a	1A, 85C	85000	83005	-2%
1b	1A, 105C	142000	161669	14%
2a	150mA, 85C	65000	68981	6%
2b	150mA, 85C	63000	63227	0%
2c	150mA, 85C	45000	51355	14%

5. Conclusions and Discussion

In this paper we describe the different approaches currently available to perform lumen maintenance extrapolations for LEDs. We proposed an alternative statistical approach to estimate lumen depreciation of LED’s. In order to demonstrate this approach, we have analyzed 5 LM-80 data sets from a statistical point of view. A reasonable comparison with the existing TM-21 extrapolated values was found. The analysis of these data sets shows the strength of the described method as the resulting unique fitted parameters describe the lumen maintenance of the LED over a long period. In principle there is also no need for a limitation based on the so-called 6x rule from TM-21.

Other than the approach as described in section 3.2, different – more complex - stochastic models are used to describe the degradation path. As stated, the big challenge is to get accurate estimates of a product’s lifetime. The performance of an ADT, obviously, strongly depends on the appropriateness of the modeling of its degradation path. A typical degradation path consists of mean degradation curve and its error term (measurement error). There are two approaches available in the literature. First, the mixed effects model is one of the most popular approaches in degradation analysis. In order to describe the unit-to-unit variations of the test units, the unknown parameters of the mean degradation path are described in terms of the mixed (or random) effects. Often the mixed effects formulations do not take the time-dependent error structure into consideration. Therefore, the stochastic process formulation, or Gauss-Markov method can be an alternative approach to model the product's degradation path. Dealing with those more complex models, to find the maximum likelihood estimates (MLEs) of the unknown parameters, the mixed effects model is computationally intensive. STATA (see: http://www.stata.com) or R (https://www.r-project.org) can be used. However, on-hand procedures do not always guarantee that the precise parameter estimations can be obtained.

Acknowledgments

The research leading to these results has received funding from the European Unions Seventh Framework Program (FP7/2007-2013) for DEWI Dependable Embedded Wireless Infrastructure Joint Undertaking under grant agreement 621353 and from specific national programs and/or funding authorities.

References

1. W.D. van Driel, X.J. Fan, Solid State Lighting Reliability: Components to Systems, ISBN 978-1-4614-3066-7, 31 August 2012, Springer, 617 pages.
2. IES LM-80-08: Approved method for measuring maintenance of Led light sources.
3. IES TM-21-11: Projecting Long Term Lumen Maintenance of LED Light Sources.
4. Hong A. Qiao, Trenton C. Pulsipher, John E. Hathaway, Eric E. Richman, Emil Radkov, A Statistical Method to Analyze LED Lumen Depreciation and Project Useful LED Product Life, IES Annual Conference, November 9, 2010, Toronto, Ontario.
5. IES Conference, Development of a Standard Method for Projecting Lumen Maintenance of LED products (TM-21).
6. R. Hechfellner and S. Landau, Understanding LED performance, Led Lighting 2009.
7. USA department of Energy, Lifetime and Reliability, fact sheet available at: ssl.energy.gov
8. Led Luminaire Lifetime: Recommendations for Testing and Reporting, Solid State Lighting Product Quality Initiative, Next Generation Lighting Industry Alliance with the U. S. Department of Energy, 3rd edition September 2014.
9. M. Hansen and L. Davis, Using LM-80 measurement methodology to determine the true value of packaged LEDs, Leds Magazine, December 9, 2014.

10. VDE-AR-E 2715-1, Measurement and prediction of reduction in luminous flux of LEDs, German Standard, November 2012.
11. J. Fan, Kam-Chuen Yung, M. Pecht, Lifetime Estimation of High-Power White LED Using Degradation-Data-Driven Method, IEEE Transactions on Device and Materials Reliability Volume 12 / Issue 2, p 470 – 477, 2012.
12. J. Fan, Kam-Chuen Yung , M. Pecht, Predicting long-term lumen maintenance life of LED light sources using a particle filter-based prognostic approach, Expert Systems with Applications, Volume 42, Issue 5, Pages 2411–2420, 2015.
13. Pradeep Lall, Junchao Wei, Peter Sakalaukus, Bayesian Models for Life Prediction and Fault-Mode Classification in Solid State Lamps, 16th International Conference on Thermal, Mechanical and Multi-Physics Simulation and Experiments in Microelectronics and Microsystems, 2015.
14. Chen Quan, Luo Xiaobing, Chen Qi, Wang Kai, Liu Sheng, Li Jingyana, Research on lumen depreciation related to LED packages by in-situ measurement method, Microelectronics Reliability 55 2269–2275, 2015.
15. Huang JL, Golubović DS, Koh S, Yang DG, Li XP, Fan XJ, and Zhang GQ, Degradation mechanisms of mid-power white-light LEDs under high temperature-humidity conditions, IEEE Transactions on Device and Materials Reliability. vol. 15, no. 2, pp. 220–228. 2015.
16. Huang JL, Golubović DS, Koh S, Yang DG, Li XP, Fan XJ, and Zhang GQ, Degradation modeling of mid-power white-light LEDs by using Wiener process, Optical Express. vol. 23, no. 15, DOI:10.1364/OE.23.00A966. 2015.
17. Huang JL, Golubović DS, Koh S, Yang DG, Li XP, Fan XJ, and Zhang GQ, Optical degradation mechanisms of mid-power white-light LEDs in LM-80-08 tests, Microelectronics Reliability 55, Issue 12, Part B, Pages 2654–2662, 2015.
18. T.-R. Tsai, C.-W. Lin, Y.-L. Sung, P.-T. Chou, C.-L. Chen, and Y. Lio, Inference from lumen degradation data under Wiener diffusion process, IEEE Trans. Reliab. 61, 710-718, 2012.
19. S. Watzke, P. Altieri-Weimar, Degradation of silicone in white LEDs during device operation: a finite element approach to product reliability prediction, 15th International Conference on Thermal, Mechanical and Multi-Physics Simulation and Experiments in Microelectronics and Microsystems, EuroSimE 2014.
20. Sung-Uk Zhang, Quantification of silicone degradation for LED packages using finite element analysis, Microelectronics Reliability 55, pp. 2678–2684, 2015.

21. M. Buffolo, C. De Santi, M. Meneghini, D. Rigon, G. Meneghesso, E. Zanoni, Long-term degradation mechanisms of mid-power LEDs for lighting applications, Microelectronics Reliability 55, pp. 1754–1758, 2015.
22. Accelerated Testing: Statistical Models, Test Plans, and Data Analysis. Wayne B. Nelson. ISBN: 978-0-471-69736-7. 624 pages. September 2004.
23. Meeker and Escobar, A Review of Accelerated Test Models, Statist. Sci. Volume 21, Number 4, 552-577, 2006.
24. Hong-Fwu Yu and Sheng-Tsaing Tseng, On-line procedure for terminating an accelerated degradation test, Statistica Sinica 8, 207-220, 1998.