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Unambiguous Detection of Migrating Targets with Wideband Radar in Gaussian Clutter

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***Abstract*—In this paper the problem of unambiguous moving target detection with wideband radar is considered. Doppler ambiguities present in low pulse repetition frequency mode are resolved using range migration phenomenon, which however results in strong ambiguous sidelobes of both clutter and targets. Utilization of standard detection algorithm leads to generation of false alarms by the ambiguous sidelobes of real targets. Assuming Gaussian distribution of clutter, two new detectors exploiting high resolution spectral estimation to remove these false detections are developed in this paper. These detectors are based on spectral estimation obtained with non-parametric Iterative Adaptive Approach from spectrum analysis. The benefits of the proposed detector are demonstrated via numerical simulations showing improvement over existing techniques.**

***Index Terms*—Radar Detection, Wideband radar Range Migration, Velocity Ambiguity, Iterative Adaptive Approach (IAA).**

I. Introduction

Recently detection of moving targets with wideband (WB) radars have attracted significant attention due to advantages for target detection and classification resulting from high range resolution. One of WB radars disadvantages is that fast moving targets migrate from one range cell to another during coherent processing interval (CPI). This phenomenon however can be exploited to overcome velocity ambiguities in low pulse repetition frequency (PRF) mode [1].

A matched filter technique for unambiguous estimation of range-velocity map, taking into account target migration, has been proposed and called wideband coherent integration (CI) [1], [2]. This processing suffers from strong residuals at aliased velocities (called ambiguous sidelobes) limiting its ability to extract moving targets unambiguously.

Further research in the area was aimed at finding an efficient estimator capable to deal with migrating targets in presence of clutter. Among the proposed techniques, the most promising results are obtained with Bayesian sparse estimator [3] and Iterative Adaptive Approach (IAA) from spectrum analysis [4].

However, all the aforementioned techniques only perform unambiguous amplitude estimation instead of detection. In particular IAA is not followed by any detector [4], also Bayesian sparse estimator [3] provides sparse estimation, but not detection. Moreover, the design of compressed sensing detector is a rather complicated problem itself.

Designing of a detector for IAA output is also not straightforward task because of non-linear nature of the algorithm. Some attempts to solve this problem were done in [5], [6] with application to ground moving target detection. Hence the discussion in these papers allows the data to be non-uniformly sampled, the problem of false alarms generated by strong sidelobes of targets is not considered there.

In this paper we present an efficient detector exploiting IAA under the assumption of complex multivariate Gaussian distribution of clutter, commonly used in narrowband radars. This assumption can be reasonable for wideband surveillance radars with high range resolution, but moderate azimuth resolution. In this case, integration of clutter is obtained from large width of range-azimuth cell at long ranges, which allows to use central limit theorem and assume clutter to follow complex multivariate Gaussian distribution. Therefore we will show that well-known Kelly's test [7] and AMF detector [8] can be extended for unambiguous moving target detection by exploiting IAA amplitude estimation.

This paper is organized as follows. In Section II the models of target and clutter are introduced. Then in Section III we briefly recall the derivations of the Kelly's test and AMF, these results are further exploited in Section IV to develop IAA-based detectors and analyze their performance. The conclusions are drawn in Section V.

II. Clutter and target models

A signature of moving point scatterer observed by wideband radar has been shown [1], [3] to have the following signature in fast-frequency / slow-time domain as following:

$$\mathbf{T}^{tt}_{k,m} = \exp\left(j2\pi\left(-\frac{\tau_0 B}{K}k + \frac{2v_0 f_c T_r}{c}\left(1+\frac{B}{Kf_c}k\right)m\right)\right), \tag{1}$$

where $m = 0...M-1$ is the pulse (sweep) number, $k = 0...K-1$ is the fast-time index, T_r is the pulse repetition interval (PRI), f_c is the carrier frequency and B is the waveform bandwidth, so the signal occupies frequencies from f_c to $f_c + B$. The point target has an initial time delay $\tau_0 = 2R_0/c$ depending on the initial target range (R_0), and constant velocity (v_0). The last term in (1) is specific for the

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wideband waveform, it models range migration of moving target and depends only on its radial velocity v_0 and allows to measure target velocity unambiguously.

The same target signature can be expressed in slow-time/fast-time domain:

$$\mathbf{T}^{tt}_{l,m} = \exp\left(j2\pi\frac{2v_0 f_c}{c}T_r m\right) \operatorname{sinc}\left(\pi\left(l-\left(l_0-\frac{v_0 T_r}{\delta_R}m\right)\right)\right), \tag{2}$$

where $l = 0..K-1$ is fast time index, l_0 is the initial range cell of the target and δ_R is the radar range resolution.

Unambiguous estimation of range-velocity map can be obtained by coherent summation of target amplitude in several range cells. Due to migration effect the matched filter should be applied on the low resolution segment (LRRS) containing K range cells, such that the condition on maximal target velocity (V_{max}) is hold:

$$K \geq V_{max} M T_r/\delta_R. \tag{3}$$

The following assumptions on clutter are made in this study:

- Clutter can be modeled in each range cell separately, since it does not migrate. Therefore, the migration term in target model (1), (2) is negligible for clutter scatterers and can be ignored;
- In each range cell clutter is a realization of stationary random process having multivariate Gaussian distribution with zero mean and covariance matrix (CM) $\mathbf{M_v}$.

Under these assumptions, clutter CM in LRRS (bi-dimentional data) has the following block-diagonal structure:

$$\mathbf{M} = \begin{bmatrix} \mathbf{M_v} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{M_v} & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{M_v} \end{bmatrix}, \tag{4}$$

where $\mathbf{M_v}$ is $M \times M$ clutter CM in slow-time usually used in the narrow band case.

III. Adaptive detectors in Gaussian clutter

The received data in LRRS under test can be given in matrix notations by [4]:

$$\mathbf{y} = \mathbf{A}\mathbf{x} + \mathbf{n}, \tag{5}$$

where $\mathbf{y}$ and $\mathbf{n}$ stand for KM-long vectors of data and noise in LRRS respectively, $\mathbf{x}$ is vectorized counterpart of unknown range-velocity map and $\mathbf{A}$ is the sensing matrix of size $KM \times N_t N_v$ including vectorized targets signatures corresponding to N_t different range and N_v different velocity hypothesis (2) as columns $\mathbf{a}_{t,v} = vec(\mathbf{T}^{tt}_{t,v})$.

The generalized likelihood ratio test (GLRT) proposed by Kelly [7] is an asymptotically optimum test in Gaussian clutter. To obtain a test capable to work with ambiguous targets, we recall the derivations of Kelly and extend it with IAA-based detector. The GLRT is given by:

$$\Lambda(\mathbf{Z},\mathbf{y}) = \frac{f_1(\mathbf{Z},\mathbf{y})}{f_0(\mathbf{Z},\mathbf{y})} \underset{H_0}{\overset{H_1}{\gtrless}} T, \tag{6}$$

where $f_0(\mathbf{Z},\mathbf{y})$ is joint PDFs of LRRS under test and reference LRRSs under hypothesis of target absence (H_0) and $f_1(\mathbf{Z},\mathbf{y})$ is the counterpart in the case of target presence (H_1). Matrix $\mathbf{Z}$ is built of vectorized data in reference LRRSs ($\mathbf{z}_i$) as columns.

The clutter in L reference LRRSs is assumed to be i.i.d. zero mean complex Gaussian noise with covariance matrix $\mathbf{M}$: $\mathbf{z}_i \sim CN[0,\mathbf{M}]$. The joint PDFs under hypothesis H_1 is:

$$\begin{aligned} f_1(\mathbf{Z},\mathbf{y}) &= f_1(\mathbf{y})\prod_{i=1}^{L} f(\mathbf{z}_i) \\ &= \frac{1}{\pi^N|\mathbf{M}|}\exp\left(-(\mathbf{y}-\alpha\mathbf{a})^H\mathbf{M^{-1}}(\mathbf{y}-\alpha\mathbf{a})\right) \\ &\quad \prod_{i=1}^{L}\frac{1}{\pi^N|\mathbf{M}|}\exp\left(-\mathbf{z}_i^H\mathbf{M}^{-1}\mathbf{z}_i\right), \end{aligned} \tag{7}$$

and joint PDF under H_0 is obtained from f_1 assuming zero amplitude of the target i.e. $f_0(\mathbf{Z},\mathbf{y}) = f_1(\mathbf{Z},\mathbf{y})|_{\alpha=0}$. In the presented expressions for PDFs the covariance matrix of clutter $\mathbf{M}$ and target amplitude α are unknown.

If we now substitute the inner product of matrices in PDFs with the form of matrix trace $\mathbf{z}_i^H\mathbf{M}^{-1}\mathbf{z}_i = tr\big(\mathbf{M^{-1}Z}\big)$, we can obtain:

$$f_{0,1}(\mathbf{Z},\mathbf{y}) = \left(\frac{1}{\pi^N|\mathbf{M}|}\exp\left(-tr\big(\mathbf{M^{-1}T_{0,1}}\big)\right)\right)^{L+1}, \tag{8}$$

and $\mathbf{T}_0$ and $\mathbf{T}_1$ being defined as:

$$\mathbf{T}_1 = \frac{1}{L+1}\left((\mathbf{y}-\alpha\mathbf{a})(\mathbf{y}-\alpha\mathbf{a})^H + \sum_{i=1}^{L}\mathbf{z}_i\mathbf{z}_i^H\right) \tag{9}$$

and $\mathbf{T}_0 = \mathbf{T}_1|_{\alpha=0}$.

Matrices $\mathbf{T}_0$ and $\mathbf{T}_1$, being the sample covariance matrices (SCM), are the MLE of clutter CM. The second summand in (9) is a SCM $\mathbf{S}$, estimated from the reference LRRSs and scaled by a factor $L/(L+1)$. By inserting the MLE of SCM into densities one can obtain:

$$\max_{\mathbf{M}} f_{0,1} = \left(\frac{1}{\pi^N|\mathbf{T}_{0,1}|}\right)^{L+1}, \tag{10}$$

The determinants of matrices $\mathbf{T_0}$ and $\mathbf{T_1}$ can be written in terms of SCM $\mathbf{S}$ using matrix determinant lemma. It yields the following expressions, subject to maximization with respect to unknown amplitude of target α:

$$|\mathbf{T_1}| = \left(\frac{L}{L+1}\right)^N|\mathbf{S}|\left(1+\frac{1}{L}(\mathbf{y}-\alpha\mathbf{a})^H\mathbf{S^{-1}}(\mathbf{y}-\alpha\mathbf{a})\right) \tag{11}$$

and under hypothesis H_0: $|\mathbf{T_0}| = |\mathbf{T_1}|\,|_{\alpha=0}$.

Inserting these values into GLRT (6) gives:

$$\Lambda(\mathbf{Z},\mathbf{y}) = \left(\frac{1+\frac{1}{L}\mathbf{y}^H\mathbf{S^{-1}}\mathbf{y}}{1+\frac{1}{L}(\mathbf{y}-\alpha\mathbf{a})^H\mathbf{S^{-1}}(\mathbf{y}-\alpha\mathbf{a})} \right)^{L+1}. \quad (12)$$

Maximization of (12) with respect to target amplitude α with known steering vector $\mathbf{a}$ is obtained by minimization the denominator of (12) by completing the squares. Amplitude estimation is then obtained via whitening matched filter:

$$\hat{\alpha} = \alpha_W = \frac{\mathbf{a}^H\mathbf{S^{-1}}\mathbf{y}}{\mathbf{a}^H\mathbf{S^{-1}}\mathbf{a}}. \quad (13)$$

Then GLRT reduces to the well-known Kelly's test [7], optimal for single target detection in Gaussian clutter:

$$\Lambda = \left(1 - \frac{1}{L}\frac{|\mathbf{a}^H\mathbf{S}^{-1}\mathbf{y}|^2}{(\mathbf{a}^H\mathbf{S}^{-1}\mathbf{a})\left(1+\frac{1}{L}\mathbf{y}^H\mathbf{S}^{-1}\mathbf{y}\right)}\right)^{-(L+1)} \quad (14)$$

which depends only on statistics

$$\gamma_{Kelly} = \frac{|\mathbf{a}^H\mathbf{S}^{-1}\mathbf{y}|^2}{(\mathbf{a}^H\mathbf{S}^{-1}\mathbf{a})\left(1+\frac{1}{L}\mathbf{y}^H\mathbf{S}^{-1}\mathbf{y}\right)} \underset{H_0}{\overset{H_1}{\gtrless}} T_{Kelly}. \quad (15)$$

The second term in the denominator can be interpreted as the loss due to substitution of clutter CM by its estimation. The expected value of this factor depended on the size of the problem and the number of reference cells, but not on the data itself, thus it can be moved to the threshold. Then the test reduces to the Adaptive Matched Filter (AMF) [8]:

$$\gamma_{AMF} = \frac{|\mathbf{a}^H\mathbf{S}^{-1}\mathbf{y}|^2}{\mathbf{a}^H\mathbf{S}^{-1}\mathbf{a}} \underset{H_0}{\overset{H_1}{\gtrless}} T_{AMF}. \quad (16)$$

The thresholds for both tests is regulated by the required probability of False Alarm (P_{FA}). As shown in [7], [8], the thresholds are defined using $P = L+1-N$ [9]:

$$T_{Kelly} = L\left(1 - P_{fa}^{\frac{1}{P}}\right), \quad (17)$$

and expected value of the loss factor $\rho = (L+2-N)/(L+1)$:

$$T_{AMF} = T_{Kelly}/\rho. \quad (18)$$

IV. Adaptive detectors exploiting Iterative Adaptive Approach for amplitude estimation

Both the aforementioned detectors suffer from the ambiguous sidelobes of strong targets in the scene and therefore can produce many false alarms in presence of multiple targets in the scene. Sidelobes-free amplitude estimations of a few independent targets in the scene can be obtained with Iterative Adaptive Approach (IAA) [10]. Then we can rewrite GLRT (12) for each hypothesis with the steering vector $\mathbf{a}$:

$$\Lambda(\mathbf{Z},\mathbf{y}) = \left(\frac{1+\frac{1}{L}\mathbf{y}^H\mathbf{S^{-1}}\mathbf{y}}{1+\frac{1}{L}\left(\mathbf{y}-\alpha_I\mathbf{a}\right)^H\mathbf{S^{-1}}\left(\mathbf{y}-\alpha_I\mathbf{a}\right)} \right)^{L+1} \quad (19)$$

where α_I represents the amplitude estimation obtained iteratively by IAA:

$$\alpha_I = \frac{\mathbf{a}^H\mathbf{R^{-1}}\mathbf{y}}{\mathbf{a}^H\mathbf{R^{-1}}\mathbf{a}}, \quad (20)$$

TABLE I: Parameters of simulated data

Waveform		
Carrier frequency	f_0	10GHz
Bandwidth	B	1 GHz
PRI	T_r	1 ms
Pulses	M	32
LRRS	K	4
Processing parameters		
Number of ambiguities	N_a	3
IAA iterations	I	15
Maximum velocity	V_{max}	15 m/s
Range oversampling	k_t	2
Velocity oversampling	k_v	2
Migration per ambiguity	μ_a	3.2

using covariance matrix of clutter plus targets in the LRRS $\mathbf{R}$ estimated via IAA [10].

To process further, we use the following simplification:

$$\begin{aligned} &\left(\mathbf{y}-\alpha_I\mathbf{a}\right)^H\mathbf{S^{-1}}\left(\mathbf{y}-\alpha_I\mathbf{a}\right) \\ &= \mathbf{y}^H\mathbf{S^{-1}}\mathbf{y} + \mathbf{a}^H\mathbf{S^{-1}}\mathbf{a}\left(\left|\alpha_I-\alpha_W\right|^2 - \left|\alpha_W\right|^2\right) \\ &= \mathbf{y}^H\mathbf{S^{-1}}\mathbf{y} - \frac{\left|\mathbf{a}^H\mathbf{S^{-1}}\mathbf{y}\right|^2}{\mathbf{a}^H\mathbf{S^{-1}}\mathbf{a}}\left(1 - \frac{\left|\alpha_I-\alpha_W\right|^2}{\left|\alpha_W\right|^2}\right). \end{aligned} \quad (21)$$

Then the GLRT (19) can be rewritten as following:

$$\begin{aligned} \Lambda(\mathbf{Z},\mathbf{y}) = \Bigg(1 - \frac{1}{L}&\frac{\left|\mathbf{a}^H\mathbf{S^{-1}}\mathbf{y}\right|^2}{(\mathbf{a}^H\mathbf{S^{-1}}\mathbf{a})\left(1+\frac{1}{L}\mathbf{y}^H\mathbf{S^{-1}}\mathbf{y}\right)} \\ &\left(1 - \frac{\left|\alpha_I-\alpha_W\right|^2}{\left|\alpha_W\right|^2}\right)\Bigg)^{-(L+1)} \underset{H_0}{\overset{H_1}{\gtrless}} T. \end{aligned} \quad (22)$$

Therefore, the test statistics is given by:

$$\gamma_{Kelly-IAA} = \frac{\left|\mathbf{a}^H\mathbf{S^{-1}}\mathbf{y}\right|^2}{(\mathbf{a}^H\mathbf{S^{-1}}\mathbf{a})\left(1+\frac{1}{L}\mathbf{y}^H\mathbf{S^{-1}}\mathbf{y}\right)} \quad (23)$$

$$\left(1 - \frac{\left|\alpha_I-\alpha_W\right|^2}{\left|\alpha_W\right|^2}\right) \underset{H_0}{\overset{H_1}{\gtrless}} T_{Kelly}, \quad (24)$$

and it differs from the previous test (15) by the second term, which is a 'gain' factor being always less or equal to one. This term exploits sidelobe-free amplitude estimation obtained by IAA and thus prevents strong sidelobes to pass through the detection chain. On the other hand, for the correct position of the target, both estimations tends to the correct value $\alpha_W \approx \alpha_I$ and thus the introduced term in braces (23) tend to 1. Also, by using the threshold from the original test (17), the false alarm probability of the new test would not exceed the set value P_{FA}, hence the expected value can be slightly less, than required.

Similarly to the previous derivations, the AMF test exploiting IAA estimation can be obtained by:

$$\gamma_{AMF-IAA} = \frac{|\mathbf{a}^H\mathbf{S}^{-1}\mathbf{y}|^2}{\mathbf{a}^H\mathbf{S}^{-1}\mathbf{a}}\left(1 - \frac{\left|\alpha_I-\alpha_W\right|^2}{\left|\alpha_W\right|^2}\right) \underset{H_0}{\overset{H_1}{\gtrless}} T_{AMF}. \quad (25)$$

The number of reference cells can be reduced by using the assumptions on clutter given in Section II. In particular, instead of obtaining clutter CM in LRRS $\mathbf{M}$, clutter CM in one range

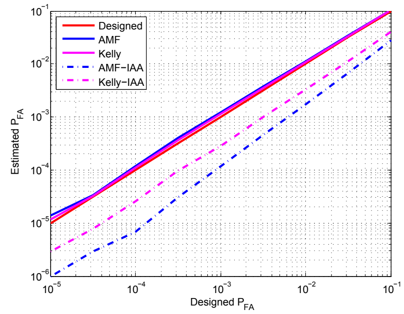

Fig. 1: False alarm regulation in Gaussian noise

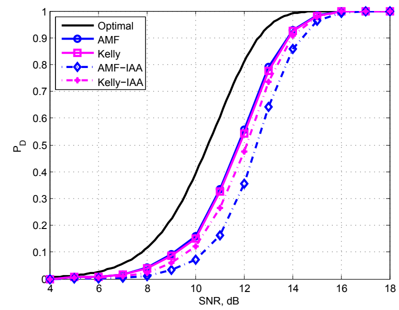

Fig. 2: Probability of detection in Gaussian noise, $P_{FA} = 10^{-6}$

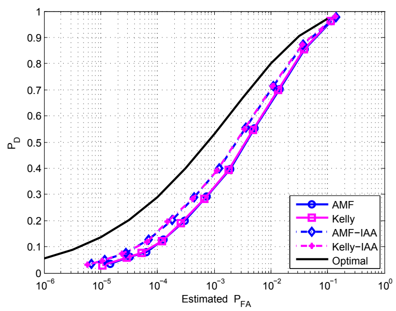

Fig. 3: ROC curves for a target with SNR=7dB

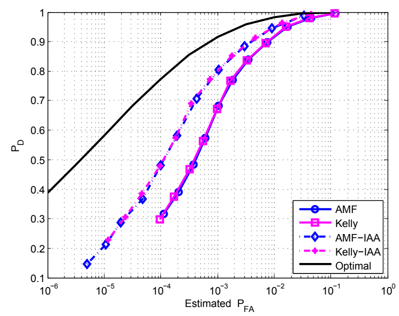

Fig. 4: ROC curves for a target with SNR=10dB

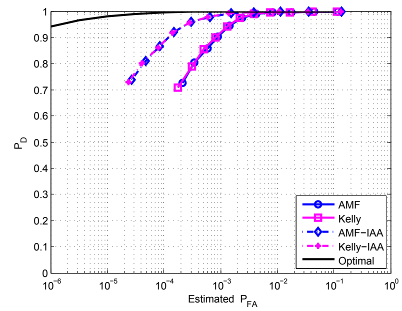

Fig. 5: ROC curves for a target with SNR=13dB

cell $\mathbf{M_v}$ can be estimated. Then the loss factor will correspond to the virtual number of averaged LRRS, proportional to the square of the number of range cells in LRRS, thus $L_2 = K^2 L$ and $\rho = (L_2 + 2 - N)/(L_2 + 1)$.

Performance of the proposed detectors in terms of their ability to hold the designated false alarm, probability of detection and ROC curves for targets with $SNR = 7\,dB, 10\,dB, 13\,dB$, (after coherent integration) are shown in Fig. 1 - 5 accordingly for radar and processing parameters mentioned in Table I.

Fig. 1 shows that both the proposed detectors provide lower false alarm, than desired, using the thresholds from the original tests, while the loss in probability of detection (P_D) resulting from this mismatch is rather negligible (less than $1dB$, see Fig. 2). The improvement achieved by the proposed detectors is shown in Fig. 3 - 5. All the curves are almost equal for large values of P_{FA}, and weak targets (see Fig. 3). On the other hand, the proposed techniques overcome Kelly's test and AMF for typical for radar values of P_{FA} and strong targets, so when the issue of sidelobes becomes more crucial. Accordingly, the detection improvement for strong targets is more substantial.

V. Conclusions

In this paper the problem of unambiguous detection of moving targets by wideband radar is discussed and range migration phenomenon is used to resolve velocity ambiguities present in low PRF mode. Unambiguous amplitude estimation is obtained with high resolution Iterative Adaptive Approach (IAA) from spectral analysis and exploited in the proposed detectors. IAA - based detectors overcome their classical analogs in terms of ROC curves. *The problem of correct threshold for IAA-detectors will be solved in the final paper.*

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