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Phase detection of coherence singularities and determination of the topological charge of a partially coherent vortex beam

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In the theory of partial coherence, coherence singularities can occur in the spectral degree of coherence: in case the fields at two different points are completely uncorrelated, the phase of the spectral degree of coherence is undefined. For a partially coherent vortex beam, the detection of coherence singularities is linked to the measurement of topological charge, whose magnitude equals the number of ring dislocations in its far field amplitude. However, the phase distribution of coherence singularities is rarely mentioned in literature and the amplitude distribution can hardly reflect the sign of topological charge. In this letter, we present a phase-analysis method for measuring the coherence singularities by introducing a movable perturbation at a certain point in an illumination window of a finite size. Using the proposed method, we measure experimentally the coherence singularities of a partially coherent vortex beam in the focal plane. From the results, the magnitude and sign of the topological charge can be determined simultaneously from the phase distribution of the coherence singularities. Our results can find application in information transmission.

Singular optics is a branch of modern optics that has recently raised much interest in the scientific community. It deals with a wide class of effects occurring near points where certain parameters of the optical field are undefined, or "singular" [1-5]. In the past decades, various kinds of optical singularities have been introduced and identified, such as phase singularities[1], polarization singularities[5] and Poynting vector singularities. Especially optical phase singularities have become a hot topic due to their ability to carry orbital angular momentum (OAM)[6,7] and applications in high-dimensional Hilbert space[8]. However, for a vortex beam, the phase singularity will disappear as the degree of coherence decreases[9,10]. Thus in 2003, Schouten introduced coherence singularities of a partially coherent field in his Young's interference experiment[11] using the spectral degree of coherence (SDOC) function. Since then, coherence singularities have attracted much attention, and numerous works have been proposed both in theory and experiment[12-18]. However, the measurement of phase distribution for coherence singularities is rarely mentioned in the literature.

In order to extract phase information of a spatially partially coherent field, one needs to measure at least a two-dimensional slice of the complex-valued four-dimensional SDOC. In the past few years, a large amount of methods and techniques for measuring the SDOC have been introduced, such as the Hanbury Brown and Twiss method (HBT)[19-21], interferometry method,[22,23] diffraction method[24,25], and phase-space tomography method[26-28]. However, all of them have certain limitations when it comes to measurement complexity, dimensions, or applicability of light source. For example, using a non-parallel double slit[23], one can only measure a one-dimensional SDOC function, and the extended HBT method mentioned in Ref. 21 requires a light source that obeys Gaussian statistics.

In the theory of partial coherence, coherence singularities can occur in the SDOC when pairs of points are completely uncorrelated, in which case the phase of the SDOC is undefined. Such singularities are usually referred to as coherence singularities and have been found in partially coherent vortex beams[18]. The most direct application of detecting coherence singularities is the measurement of topological charge[4,30-33], which has proved to be highly significant in decoding and optical communication. For the partially coherent Laguerre-Gaussian (LG) beam[29,32], which is a typical kind of partially coherent vortex beam, it was found in theory and verified in experiment that the magnitude of the topological charge can be determined by measuring the far-field SDOC, based on the fact that the number of its ring dislocations equals $|l|$[31-33]. However, researches have always been focused on measuring the modulus of the SDOC with an on-axis reference point (i.e. only correlations with respect to the field at the center of the beam are considered), or cross-correlation functions which often give information about only the magnitude of the topological charge, but not the sign. More information contained in the SDOC was neglected, while in fact the sign of the topological charge also plays an important role in practical applications, for example as an additional degree of freedom for optical storage and communication[33-35]. Recently, a method for simultaneously determining the sign and magnitude of the topological charge of a partially coherent LG_{0l} beam was proposed by measuring the modulus of the SDOC with respect to an on-axis reference point after it propagates through a couple of cylindrical lenses[36]. However, the results were obtained from the amplitude of the SDOC not phase.

To sum up, the present work reports a direct measurement of coherence singularities of a partially coherent vortex beam. This is done by measuring the phase distribution of its SDOC with a reference point whose position can be controlled, and from this phase distribution the magnitude and sign of the topological charge can be obtained simultaneously. Besides its basic simplicity in practical implementation, this method can be also used for robust optical communication, so that the need for coherent sources with high phase stability is eliminated. Instead, partially coherent sources with variable topological charges can exploit an additional dimension for signal encoding with advanced modulation formats, e.g. pulsed amplitude modulation, or orthogonal frequency division multiplexing. This would allow for using low-cost systems for free-space optical communication, for example in visible light communication or outdoor direct-range communication[37].

As is mentioned above, the coherence singularity occurs in the SDOC of a partially coherent vortex beam which is defined as:

$$\mu(\boldsymbol{r}_1,\boldsymbol{r}_2,\omega) = W(\boldsymbol{r}_1,\boldsymbol{r}_2,\omega)/\sqrt{I(\boldsymbol{r}_1,\omega)I(\boldsymbol{r}_2,\omega)}. \quad (1)$$

Here, $W(\boldsymbol{r}_1,\boldsymbol{r}_2,\omega)$ denotes the cross spectral density function (CSDF) at frequency ω. It represents the correlation between the fields at $\boldsymbol{r}_1$ and $\boldsymbol{r}_2$, and $I(\boldsymbol{r},\omega)$ is the intensity distribution. Here we only consider monochromatic light, so we will ignore ω in the following theoretical derivation. Since we are now interested in a two-dimensional slice of the four-dimensional SDOC, we define a reference point at $\boldsymbol{r}_0$. Therefore, we need to measure only $W(\boldsymbol{r},\boldsymbol{r}_0)$ and $I(\boldsymbol{r})$. The intensity can be easily recorded by a camera, while the CSDF is a complex-valued function which is hard to measure directly.

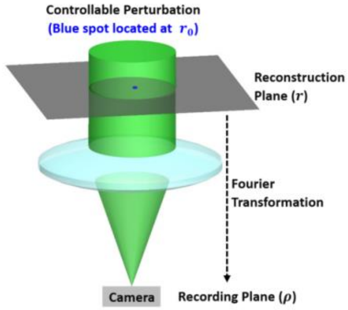

FIG. 1. The schematic diagram of the measurement method.

A schematic diagram of the setup that is used to measure the CSDF is shown in Fig. 1. We denote the coordinates of reconstruction plane and recording plane by coordinate vectors $\boldsymbol{r}$ and $\boldsymbol{\rho}$, respectively. By placing a lens after the reconstruction plane, and a camera in the Fourier plane of this lens, we can record the Fourier transformation intensity distribution of the to be reconstructed partially coherent field in the reconstruction plane. Let us denote an aperture function that defines the region of interest in the reconstruction plane as $A(\boldsymbol{r})$. The recorded intensity distribution can be written as:

$$I_0(\boldsymbol{\rho}) = \iint W(\boldsymbol{r}_1,\boldsymbol{r}_2)A(\boldsymbol{r}_1)A^*(\boldsymbol{r}_2) \times \exp[-i2\pi\boldsymbol{\rho}(\boldsymbol{r}_1-\boldsymbol{r}_2)]\mathrm{d}\boldsymbol{r}_1\mathrm{d}\boldsymbol{r}_2. \quad (2)$$

Then, we introduce an additional perturbation (i.e. we shift the phase, modulate the amplitude, or both) at a point $\boldsymbol{r}=\boldsymbol{r}_0$ (blue spot) in the reconstruction plane, for example using a spatial light modulator (SLM), whose pixels are sufficiently small. The transmission function becomes $A(\boldsymbol{r})+P\delta(\boldsymbol{r}-\boldsymbol{r}_0)$, where $\delta(\boldsymbol{r})$ is a Dirac function, and P is the complex-valued perturbation constant, and the perturbed intensity distribution can then be written as:

$$I(\boldsymbol{\rho}) = \iint W(\boldsymbol{r}_1,\boldsymbol{r}_2)\times[A(\boldsymbol{r}_1)+P\delta(\boldsymbol{r}_1-\boldsymbol{r}_0)] \times [A(\boldsymbol{r}_2)+P\delta(\boldsymbol{r}_2-\boldsymbol{r}_0)]^* \times \exp[-i2\pi\boldsymbol{\rho}(\boldsymbol{r}_1-\boldsymbol{r}_2)]\mathrm{d}\boldsymbol{r}_1\mathrm{d}\boldsymbol{r}_2. \quad (3)$$

Taking the inverse Fourier transformation of $I(\boldsymbol{\rho})-I_0(\boldsymbol{\rho})$, we obtain:

$$P\times\left[W(-(\boldsymbol{r}-\boldsymbol{r}_0),\boldsymbol{r}_0)A\big(-(\boldsymbol{r}-\boldsymbol{r}_0)\big)\right]^* + P^*\times[W(\boldsymbol{r}+\boldsymbol{r}_0,\boldsymbol{r}_0)A(\boldsymbol{r}+\boldsymbol{r}_0)]. \quad (4)$$

Now, we in turn assign two different perturbation constants $P_{+/-}$ at the perturbation point, and measure the intensities $I_{+/-}$ for each value of P. In this way, we can obtain a linear system of equations and easily solve the following expression:

$$W(\boldsymbol{r}+\boldsymbol{r}_0,\boldsymbol{r}_0) = \mathcal{F}^{-1}\left\{\frac{P_-[I_+(\boldsymbol{\rho})-I_0(\boldsymbol{\rho})]-P_+[I_-(\boldsymbol{\rho})-I_0(\boldsymbol{\rho})]}{{P_+}^*P_- - {P_-}^*P_+}\right\}. \quad (5)$$

Then, we can obtain $W(\boldsymbol{r},\boldsymbol{r}_0)$ by shifting $W(\boldsymbol{r}+\boldsymbol{r}_0,\boldsymbol{r}_0)$ by $-\boldsymbol{r}_0$, and calculate the SDOC using Eq. (1). Note that this correlation function is related to the location of perturbation point, which corresponds to the reference point. Our method allows us to move the perturbation point to any point in the reconstruction plane where the intensity is non-zero. As a result, the complete four-dimensional SDOC of the partially coherent vortex beam can be measured. Note that the perturbation point should not be moved beyond the beam range.

Now consider the example of a LG_{0l} beam whose electric field in the source plane can be expressed as

$$E(s,\varphi)=\left(\frac{\sqrt{2}s}{w_0}\right)^l \exp\left(-\frac{s^2}{w_0^2}\right)\exp(il\varphi), \quad (6)$$

where s and φ are the radial and azimuthal (angular) coordinates in the source plane, l is the topological charge of the partially coherent LG_{0l} beam, σ_0 represents the transverse spatial coherence width, and w_0 denotes the beam width of the fundamental Gaussian beam. Now we introduce a more general expression for the LG_{0l} beam, so that it can describe spatial partial coherence. The CSDF can be described as $W(s_1,s_2,\varphi_1,\varphi_2)=\langle E^*(s_1,\varphi_1)E(s_2,\varphi_2)\rangle$,[9] where angular brackets represent ensemble average and asterisk stands for complex conjugate. The intensity can be found from the CSDF as $I(s,\varphi)=W(s,\varphi,s,\varphi)$. For a Schell-model source, $W(s_1,s_2,\varphi_1,\varphi_2)=\sqrt{I(s_1,\varphi_1)I(s_2,\varphi_2)}\mu(s_1-s_2)$. Assuming the SDOC $\mu(s_1-s_2)$ is Gaussian profile, we can obtain[29,32]

$$\begin{aligned}&W(s_1,s_2,\varphi_1,\varphi_2)\\&=\left(\frac{2s_1s_2}{w_0^2}\right)^l \exp\left[-\frac{(s_1^2+s_2^2)}{w_0^2}\right]\exp[il(\varphi_1-\varphi_2)]\\&\times \exp\left[-\frac{s_1^2+s_2^2-2s_1s_2\cos(\varphi_1-\varphi_2)}{2\sigma_0^2}\right]. \quad (7)\end{aligned}$$

In the far field or focal plane, the CSDF will evolve into

$$\begin{aligned}W(\boldsymbol{r}_1,\boldsymbol{r}_2)=&\iint W(\boldsymbol{s}_1,\boldsymbol{s}_2)\\&\times\exp[-i2\pi(\boldsymbol{s}_1\cdot\boldsymbol{r}_1-\boldsymbol{s}_2\cdot\boldsymbol{r}_2)]\,\mathrm{d}\boldsymbol{s}_1\,\mathrm{d}\boldsymbol{s}_2. \quad (8)\end{aligned}$$

Here, for ease of calculation, the polar coordinates have been transformed to Cartesian coordinates. Substituting Eq. (7) and Eq. (8) into Eq. (1), the SDOC of a partially coherent LG_{0l} beam in the focal plane can be obtained.

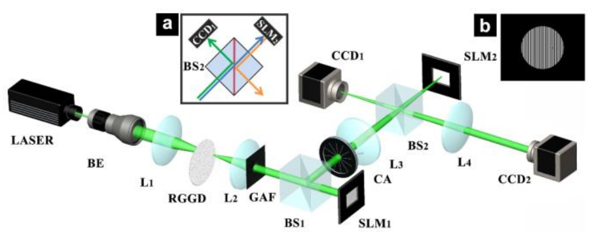

FIG. 2. Experimental setup for generating a partially coherent LG_{0l} beam and measuring its spectral degree of coherence. BE is a beam expander; L_1, L_2, L_3 and L_4 are thin lenses; RGGD is a rotating ground-glass disk; GAF is a Gaussian amplitude filter; BS_1 and BS_2 are beam splitters; SLM_1 and SLM_2 are spatial light modulators; CA is a circular aperture; CCD_1 and CCD_2 are charge-coupled detectors. Illustrations: (a) is the light path detail at BS_2 and (b) is an example for the design of aperture and perturbation function (added with a linear phase ramp).

Figure 2 shows the experimental setup we used to generate a partially coherent LG_{0l} beam and measure its SDOC. A coherent laser beam emitted by a solid-state laser at a wavelength of $\lambda=532\mathrm{nm}$ is expanded by the beam expander, and focused by lens L_1 onto a rotating ground glass disk, thereby producing a partially coherent beam with Gaussian statistics. After the beam is collimated by lens L_2 and transmitted by the Gaussian amplitude filter (GAF), the output beam becomes a Gaussian Schell-model (GSM) beam, whose intensity and SDOC both satisfy the Gaussian distribution. The GSM beam from the GAF goes to the first beam splitter (BS_1) and illuminates the first spatial light modulator (SLM_1), to which we assign the fork pattern as designed by the method of computer-generated holograms. The grating patterns of the holograms were obtained by adding a linear phase ramp to a phase vortex. The first diffraction order of the reflected field of SLM_1 is a partially coherent LG_{0l} beam, and it is limited by a circular aperture. A singularity occurs in the SDOC of the partially coherent LG_{0l} beam as we propagate it to the focal plane using L_3. We separate the beam by BS_2 into two paths (see Fig. 2a for details): in one path, we measure the focal intensity distribution using the first charge coupled device (CCD_1) which is placed at the focal plane of L_3, and in the other path, we determine the CSDF of the beam using SLM_2 (corresponding to the reconstruction plane in Fig. 1) and CCD_2 (recording plane in Fig. 1). The SDOC can be easily calculated with the CSDF and focal intensity in CCD_1 according to Eq. (1).

In detail, we put SLM_2 in the reconstruction plane and then assign to it an aperture function to select areas of interest, and a perturbation function, which is a phase shift in certain point (see Fig. 2b as an example). In our proposed method, this perturbation point corresponds to the reference point that defines the two-dimensional slice of the four-dimensional SDOC. Ideally, the perturbation should be a delta function. In experiment, the perturbation is approximated by varying the phase value of pixels in a small circular region on SLM_2. The detector CCD_2, which is put in the Fourier plane of SLM_2, measures the corresponding modulated intensity distributions for different values of the perturbation. In this experiment, only three intensity patterns are needed for calculating the CSDF. One pattern corresponds to applying no perturbation function to SLM_2, and the other patterns correspond to shifting the phase of the field in the perturbation region by different values, e.g. $\exp(2i\pi/3)$ and $\exp(-2i\pi/3)$. For all three patterns, SLM_2 applies an aperture function. These three intensities captured by CCD_2 and the intensity captured by CCD_1 are used to calculate the final SDOC. With this method, we can freely control the position of the reference point of the SDOC by moving the perturbation point in SLM_2 and calculate different distributions of coherence singularities.

Firstly, we set the reference point of the SDOC for a partially coherent LG_{0l} beam at the origin (i.e. at the center of the beam). The theoretical simulations and experimental results of the amplitude and phase distribution of a focused partially coherent LG_{0l} beam with beam width and spatial coherence width both being 0.8 mm are shown in Fig. 3 for different topological charges l. In this case, the coherence singularities show up as dark rings in the amplitude and circular dislocations in the phase, i.e. the phase is undefined on the ring. The number of dark rings of the amplitude of the SDOC equals $|l|$, which is the magnitude of the topological charge of the partially coherent LG_{0l} beam. This conclusion can also be obtained from the analytic expression for the focused CSDF as shown in Eq. (7) of Ref. [31]. From this equation, one can find that the number of zeros of the Laguerre polynomial with mode l is equal to the topological charge. Moreover, the magnitude of the topological charge $|l|$ also can be determined from the phase distribution, since $|l|$ equals the number of phase jumps. However, comparing the results of $l = 3$ and $l = -3$ in Fig. 3, one cannot identify the sign of topological charge without placing additional optical elements as described in Ref. [36].

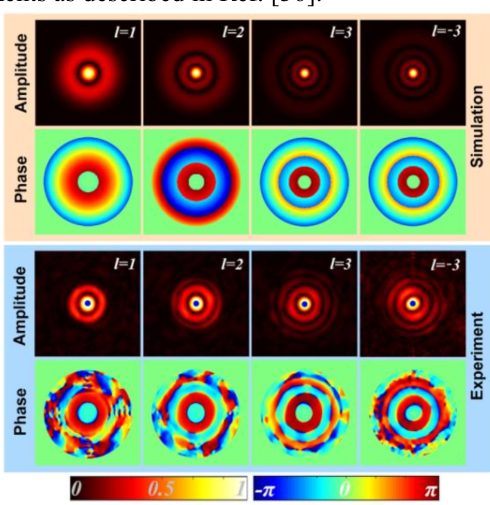

FIG. 3. Theoretical simulations and experimental results of the amplitude and phase distributions of spectral degree of coherence with an on-axis reference point for the focused partially coherent LG_{0l} beams with different topological charge.

Our method also allows us to measure the SDOC with off-axis reference points. Fig. 4 shows the simulations and experimental results of the amplitude and phase distribution of SDOC with an off-axis reference point for the focused partially coherent LG_{0l} beams with different topological charges. In other words, we move the reference points away from the origin of the coordinate system, while the beam waist and coherence width remain the same as that in Fig. 3. From the amplitude distribution of Fig. 4, one can see that the ring dislocations which are shown in Fig. 3 disappear. Dark areas appear near the horizontal axis, which are in fact coherence singularities. In this case, the coherence singularities show up as dark dots in amplitude and helical phases around the dots, i.e. the phase is undefined in these dots. From the amplitude distribution, we conclude that the number of coherence singularities (dark areas) is equal to $|l|$.[38] But we still cannot distinguish the sign of the topological charge. In contrast, from the measured phase patterns, we can not only obtain $|l|$ from the coherence singularities (which are defined as the points where the value of the SDOC equals zero, while the corresponding phase is undefined), but also determine the sign of the topological charge from the direction in which the phase rotates. A phase that is rotating counterclockwise corresponds to a positive sign, whereas a phase that is rotating clockwise represents a negative sign.

The experimental figures in Fig. 3 & Fig. 4 are the main results of this paper, where the SDOC of the partially coherent LG_{0l} beam in the focal plane has been measured using the experimental setup in Fig. 2. We have shown that coherence singularities of the partially coherent vortex beam can be easily found by moving the perturbation point. For different locations of the perturbation, one finds different distributions of coherence singularities and simultaneously extracts the information of topological charge. More importantly, the experimental results where the SDOC is reconstructed with respect to an off-axis reference point are more robust to noise, and simultaneously contain more intuitive and richer information. Not only the magnitude, but also the sign of the topological charge can be directly obtained from only one phase pattern.

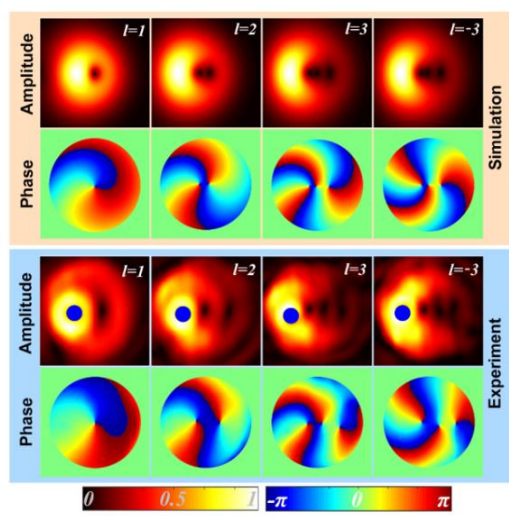

FIG. 4. Theoretical simulations and experimental results of the amplitude and phase distributions of the spectral degree of coherence with an off-axis reference point for the focused partially coherent LG_{0l} beams with different topological charge.

In conclusion, we have demonstrated that the complex-valued SDOC of a partially coherent beam can be determined using two perturbations and solving a simple linear system of equations, which makes it possible to study coherence singularities through the measured amplitude and phase patterns in real-time. Most importantly, our results suggest a straightforward route for determining the real and imaginary part of the SDOC efficiently, without operating for a long time, without extensive modification of existing setups, and which can be used for all kinds of correlation structures. Furthermore, the method presented here does not depend on the wavelength, thus allowing this method to be applied to UV, mid-IR sources, microwaves, and even X-rays. The results presented here will not only aid the determination of SDOC, but one can also apply this method to other topics, such as information transmission and imaging using partially coherent vortex beams.

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