Implementation of Choke Models in AD-GPRS

Nazari Moghaddam, Rasoul

Publication date
2017

Document Version
Final published version

Citation (APA)
Nazari Moghaddam, R. (2017). *Implementation of Choke Models in AD-GPRS*.

INTEGRATED SYSTEMS APPPROACH FOR PETROLEUM PRODUCTION

ISAPP Project: Coupled Well-Reservoir Models for Pressure Transient Analysis in Horizontal Wells

Report

Implementation of Choke Models in AD-GPRS

September 2017

Delft University of Technology (TU Delft), The Netherlands
Faculty of Civil Engineering and Geosciences
Department of Geoscience and Engineering

Author:
Rasoul Nazari Moghaddam
r.nazarimoghaddam@tudelft.nl
(Post-Doctoral researcher)

Reviewer:
Prof. dr. ir. Jan Dirk Jansen (TU Delft)
j.d.jansen@tudelft.nl
(Supervisor)

Abstract

This report concerns the addition of a new well control option 'Flow Through Surface Choke' (CHK) to AD-GPRS, the Automatic Differentiation General Purpose Research Simulator developed at Stanford University. The report provides background information on the modelling of fluid flow through surface chokes and gives details of the implementation in AD-GPRS.

Table of Contents

Abstract 2
Table of Contents 2
AD-GPRS 3
The Structure of AD-GPRS 3
The Multi-Segment Well in AD-GPRS 5
Automatically Differentiable Expression Templates Library 7
Flow Through Chokes 8
  Empirical models for critical flow 8
  Empirical models for sub-critical flow 9
Implementation of Choke Models in AD-GPRS 10
  Constraint Equations and Jacobian Matrix Structures 10
  Choke Option in the WCONPROD Keyword 14
Acknowledgments 15
References 15

AD-GPRS

The Automatic Differentiation General Purpose Research Simulator (AD-GPRS) is a simulator developed at Stanford University (SUPRI-B) for simulation of various processes in the field of petroleum engineering. For example, AD-GPRS can be used to simulate multiphase flow in reservoirs, wellbores and facilities, including various enhanced oil recovery (EOR) processes, CO_2 sequestration in saline aquifers and depleted oil reservoirs, shale gas/oil production etc.

In this simulator, there are no assumptions about the underlying grid structure, hence both structured and unstructured grids are supported for accurate representation of the complex structure and heterogeneity of reservoir. A standard well or a multi-segment well (MSWell) can be chosen while a drift-flux wellbore model can be defined to model pressure drop over the well. Fully implicit or sequentially implicit time-discretization schemes are available. Black oil and compositional formulations can be used in AD-GPRS. There are two discretization options: two-point and multi-point flux. As linear solver, direct Lapack solvers and several iterative linear solvers with many different pre-conditioners can be used.

AD-GPRS uses object–oriented design and programming with standard C++. It is a large and complex program which currently includes hundreds of files separated into many sub-directories. The main objective for the development of AD-GPRS was to make it a flexible and efficient reservoir simulation research laboratory with extensible modelling and solution capabilities. While this design is convenient for the developers to extend the simulator through incorporating new physics, complex processes, or new formulations and solution algorithms, it requires some effort for a new developer to become fully familiar with the structure and details of the entire program. In the following, the structure of AD-GPRS is briefly described.

The Structure of AD-GPRS

AD-GPRS employs automatic differentiation to construct the Jacobian allowing for an easy extension to new physics and constitutive relations. Basically, in AD-GPRS, the Jacobian matrices are calculated separately for the reservoir part and for the facilities part. Then, the equation selector is used to recast the Jacobian with desired variables and implicitness levels. After that, the Jacobian matrices are passed to the linear solver and pieced together. Finally, the solution goes back in the opposite direction. All of these procedures are controlled by SimMaster. A schematic view of SimMaster is shown in Figure 1.

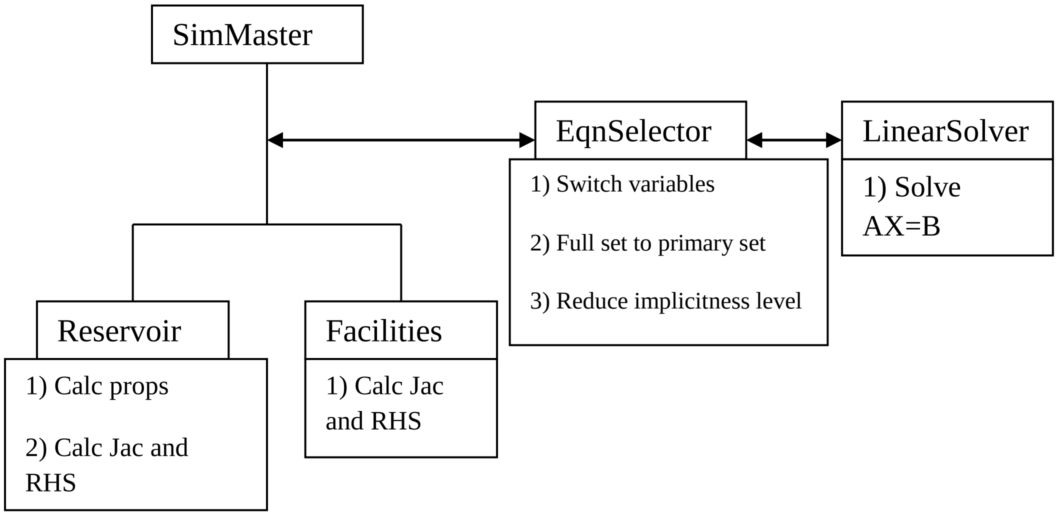

Figure 1. Structure of SimMaster in AD-GPRS

As mentioned, the Jacobian matrices are calculated separately for the reservoir part and for the facilities part. Figures 2 and 3 depict the system model for each of them. As shown in Figure 2, each reservoir includes a grid and a formulation, where the grid part generates the grid information and passes it to the formulation part. The formulation part calculates the grid block properties and builds the reservoir part of the Jacobian matrix and the right-hand side (RHS). In GPRS, the grid information is either internally generated (currently only for Cartesian grids), or read in from the output of an external gridding software package.

The system model for the well part has a similar structure (as shown in Figure 3). In this part, well information is generated from the well completion data and passed to the well control module, which calculates the well part of the Jacobian matrix and the RHS. Currently, six types of well controls are implemented: bottom hole pressure target (BHP), oil flow rate at standard conditions (ORAT), gas flow rate at standard conditions (GRAT), water flow rate at standard conditions (WRAT), liquid flow rate at standard condition (LRAT) and reservoir fluid volume rate (RESV) control. In this research, an important well control option, flow through surface choke (CHK) has been added to the simulator. In this report, it is aimed to explain this implementation, and provide the necessary background information required for the modelling of fluid flow through surface chokes.

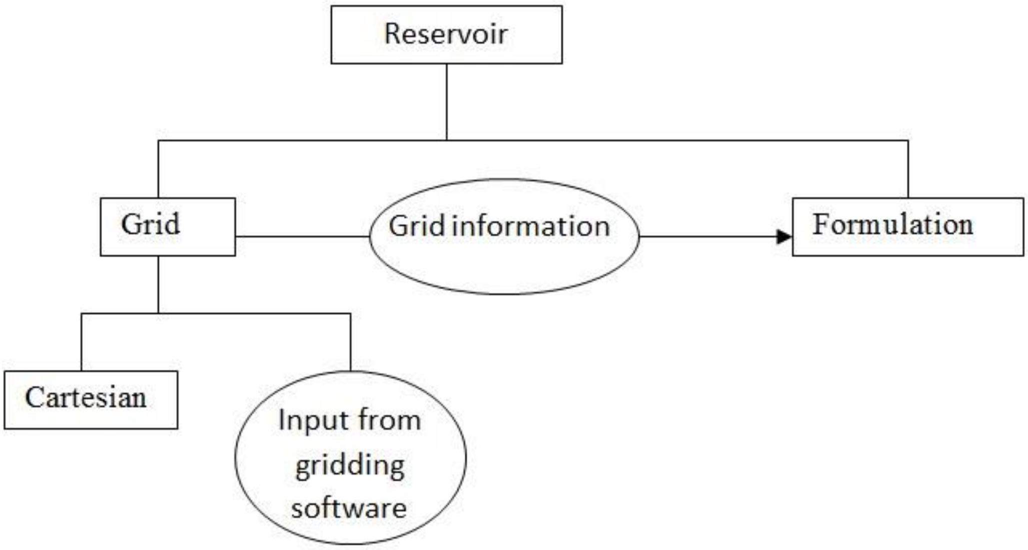

Figure 2. Structure of the reservoir model in AD-GPRS

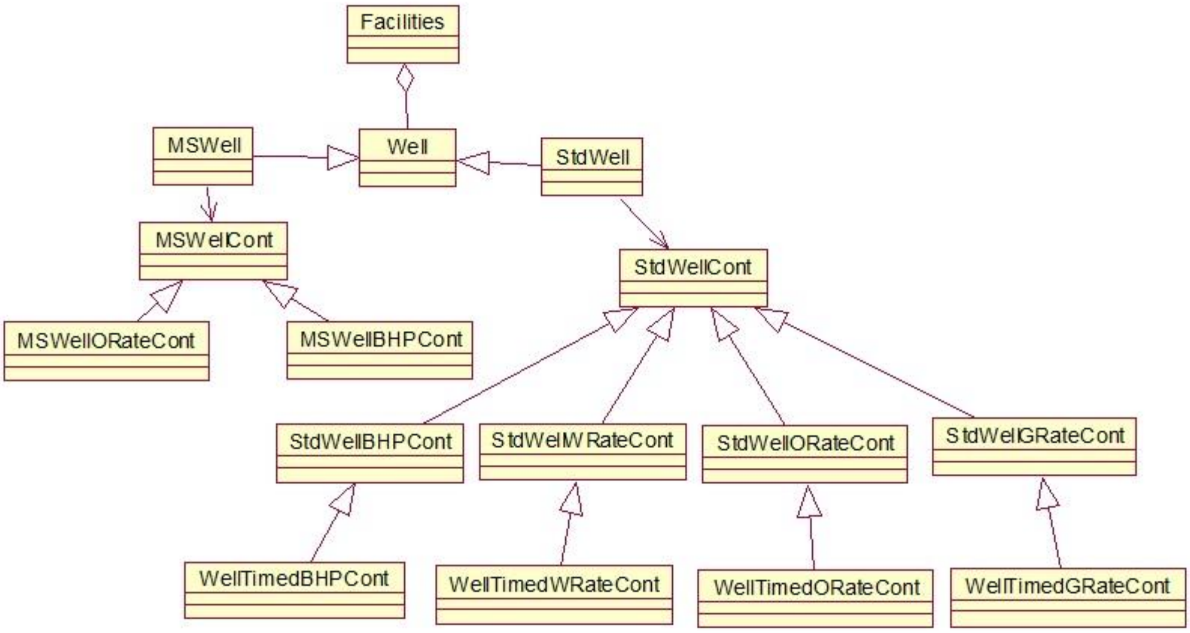

Figure 3. Structure of the well model in AD-GPRS

The Multi-Segment Well in AD-GPRS

Multiphase flow effects in wellbores and pipes can have a strong impact on the performance of reservoirs and surface facilities. In the case of horizontal or multilateral wells, for example, pressure losses in the well can lead to a loss of production at the toe or overproduction at the heel. The modelling of advanced (multilateral, horizontal, smart) wells requires more sophisticated well models than are required for conventional wells. Models for advanced wells must allow pressure, flow rate and fluid compositions to vary with position in the wellbore, and enable different fluid streams to co-mingle at branch junctions. This can be accomplished by discretizing the well into segments and solving the mass balance equations and pressure drop equation for each segment. In AD-GPRS, a Multi-Segment Well (MSWell) can have an arbitrary number of segments. In addition, an MSWell model may have many perforations along the wellbore. It is assumed that a segment can have at most one perforation. The segments are numbered from heel to toe. The toe-end of a perforated segment is always located in the centre of the perforated reservoir cell. In the black oil implementation, the system contains four primary variables in each segment at each time step which are free gas holdup (α_g), liquid holdup (α_w), mixture velocity (V_m), and pressure (P^{seg}). The pressure of a segment is defined at the toe-end of the segment. Since the toe-end of a perforated segment is aligned with the centre of a reservoir cell, the flux between the reservoir and the well can be calculated from the pressures of the perforated segment and the reservoir cell directly. It is noted that the gas and liquid phase fractions, α_g and α_w, are defined for the entire segment. The mixture velocity of a segment, V_m, is defined at the heel end of a segment. Figure 4 shows a schematic of a well segment in the MSWell model.

The governing equations for the system are the mass balance equation for each component and a pressure equation. These four equations are solved to determine the four primary unknowns. For the pressure equation, the components of the pressure drop can be selected to be either Hydrostatic + Friction + Acceleration (HFA), Hydrostatic + Friction (HF) or Hydrostatic only (H). In addition, there are two options for the computation of superficial phase flow rate. Both homogeneous flow (all phases flow with the same velocity) and the a drift-flux model (slip between phases is allowed) can be selected.

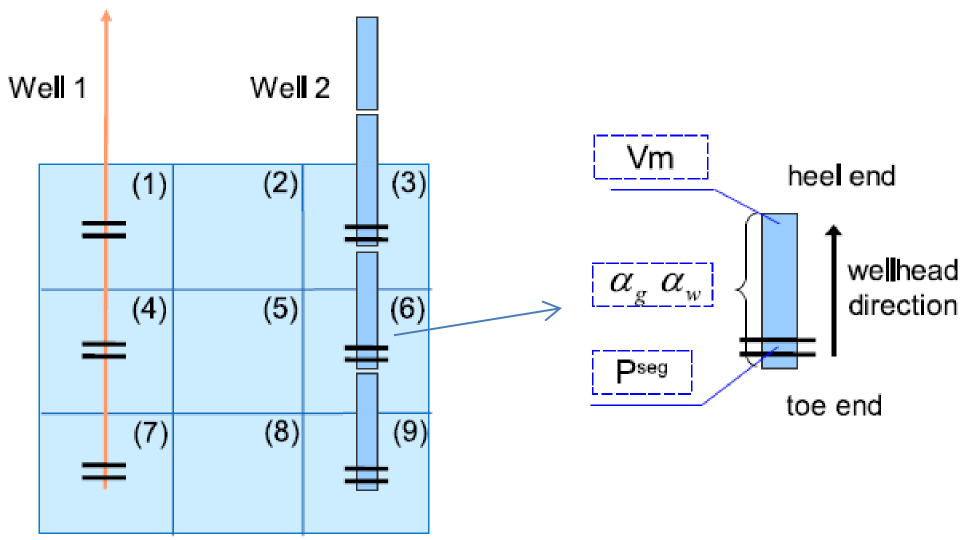

Figure 4. A schematic of a well segment in the MSWell model (Jiang, 2007)

As mentioned earlier, it is aimed to explain the choke implementation into AD-GPRS which has been performed as a part of this research. The choke option was implemented in MSWell in the well model (Figure 3). However, before continuing, it is required to gain some understanding of the "Automatically Differentiable Expression Library" which is a basic template required for the implementation of the choke formulations. In the following, the data structures in this library are briefly described.

Automatically Differentiable Expression Templates Library

The Automatically Differentiable Expression Templates Library (ADETL) is a large optimized generic library, which is composed of various data structures and algorithms for the purpose of automatic differentiation (AD). ADETL provides core infrastructure for AD-GPRS. With this library, only nonlinear residual equations need to be written and the associated Jacobian matrix can be automatically generated. In the next section, it is explained that the choke formulations have been implemented into the program in terms of the residual equations used for the generation of the Jacobian matrix.

The most important data type that is introduced by ADETL is the ADscalar, which contains a value member in the double type, and a gradient member in a customized data type (currently the default option is the block_sparse_vector with a fixed block size of 4). An ADscalar can be independent, dependent, or constant. When it is independent, the gradient member contains only one derivative of value 1 at a certain column (i.e., its independent position). If it is dependent, the gradient member stores the derivatives with respect to all related independent variables. The gradient member will be empty if the ADscalar is a constant. The ADvector, which is also a frequently-used data type in ADETL, represents a vector of ADscalars. It has an interface that is composed of those standard functions in the STL vector type and several additional ones specifically for the purpose of AD. A schematic of ADscalar and ADvector types are illustrated in Figure 5.

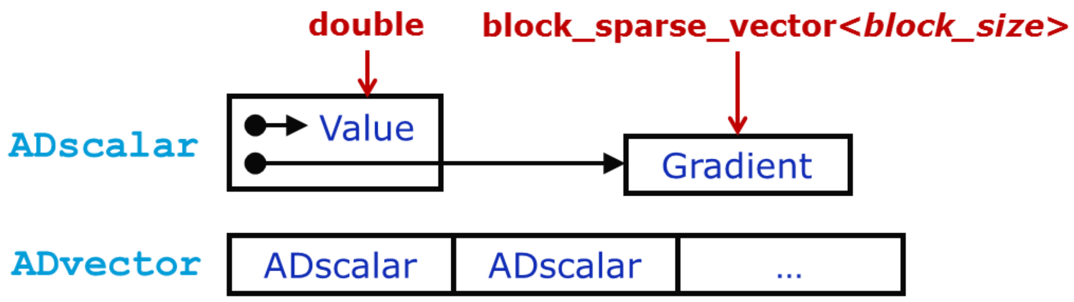

Figure 5. Schematic of the ADscalar and ADvector data types available in ADETL.

As mentioned, an ADscalar data type contains a value member and a gradient member or members in a customized data type. In the next section, it will be explained that the ADscalar data type should be used to define the variable parameters (such as gas and liquid flow rates) to keep the gradient members with respect to the other parameters. Constants used in choke models can be defined as simple double type value.

Flow Through Chokes

A wellhead choke controls the surface pressure and production rate from a well. Chokes are usually selected so that fluctuations in the line pressure downstream of the choke have no effect on the production rate. This requires that flow through the choke is at critical flow conditions, i.e. that the flow rate in the throat of the choke is so high that it reaches the sonic velocity of the gas-liquid mixture flowing through the choke. Under critical flow conditions, the flow rate is a function of the upstream (tubing head) pressure only. For this condition to occur, the downstream pressure must be approximately 0.6 or less of the tubing head pressure. Because in most real cases fluid flow at the top of a well is multiphase, models for multiphase flow through chokes have been considered here. In particular we will use an empirical model which is computationally much more efficient than more complex first-principles models.

Empirical models for critical flow

At critical conditions, there are a number of empirical correlations such as those of Gilbert, Ros, Baxendell and Achong. Each of these has been determined from a limited set of measurements for specific fluid properties and choke types and it is therefore not possible to make a general recommendation which of them is the preferred one (For more details see Jansen, 2017). All of them can be written in the form

$$P_1 = -Aq_{l,sc}\frac{\left(ER_{gl}\right)^B}{(Fd_{ch})^c} + D \tag{1}$$

where P_1 is the pressure upstream of the choke, $q_{l,sc}$ is the liquid flow rate through the choke, R_{gl} is the producing gas-liquid ratio, d_{ch} is the choke diameter and *A*, *B*, *C*, *D*, *E* and *F* are experimentally-determined constants which are different for different empirical models. These constant are given in Table 1 in two different units.

Table 1. Empirical coefficients for different choke models (Jansen, 2017).

SI Units						
Model	A	B	C	D	E	F
Gilbert	3.75E10	0.546	1.89	1.01E5	5.61	2.52E3
Ros	6.52E10	0.500	2.00	1.01E5	5.61	2.52E3
Baxendell	3.58E10	0.546	1.93	1.01E5	5.61	2.52E3
Achong	1.43E10	0.650	1.88	1.01E5	5.61	2.52E3
Field Units						
Model	A	B	C	D	E	F
Gilbert	10.0	0.546	1.89	14.7	1.00	1.00
Ros	17.4	0.500	2.00	14.7	1.00	1.00
Baxendell	9.56	0.546	1.93	14.7	1.00	1.00
Achong	3.82	0.650	1.88	14.7	1.00	1.00

Empirical models for sub-critical flow

For sub-critical flow, an extension of the above empirical models was used as proposed by Jansen (2017). From equation (1) it follows that the flow rate at the critical pressure ratio is given by:

$$q_{l,\ sc,crit} = \frac{(D - P_{1,crit})(Fd_{ch})^{c}}{A\left(ER_{gl}\right)^{B}} \tag{2}$$

The upstream pressure is related to downstream pressure as $P_{1,crit} \approx 1.7P_4$. Hence an appropriate expression that fulfils the following conditions can be used as an extension of the empirical model for sub-critical flow.

$$q_{l,\ sc} = 0 \quad : \quad P_1 = P_4 \tag{3a}$$

$$q_{l,\ sc} = 0 \quad : \quad \frac{dP_1}{dq_{l,\ sc}} = 0 \tag{3b}$$

$$q_{l,\ sc} = q_{l,sc,crit} \quad : \quad P_1 = P_{1,crit} \tag{3c}$$

$$q_{l,\ sc} = q_{l,sc,crit} \quad : \quad \frac{dP_1}{dq_{l,\ sc}} = P_{1,crit}{}' \tag{3d}$$

where $P_{1,crit}{}'$ can be obtained using Equation 1 as follow:

$$P_{1,crit}{}' = -A\frac{\left(ER_{gl}\right)^{B}}{(Fd_{ch})^{c}} \tag{4}$$

The following third-order polynomial can be an appropriate expression:

$$P_1 = c_0 + c_1 q_{l,sc} + c_2 {q_{l,sc}}^2 + c_3 {q_{l,sc}}^3 \tag{5}$$

provided that the four coefficients are derived from Equation 3a to 3d. It can be shown that these coefficients are (Jansen, 2017):

$$c_0 = P_4 \tag{6a}$$

$$c_1 = 0 \tag{6b}$$

$$c_2 = \frac{{P_{1,crit}}'}{2q_{l,sc,crit}} + \frac{3(P_{1,crit} - 0.5{P_{1,crit}}' q_{l,sc,crit} - P_4)}{{q_{l,sc,crit}}^2} \tag{6c}$$

$$c_3 = \frac{2(P_{1,crit} - 0.5{P_{1,crit}}' q_{l,sc,crit} - P_4)}{{q_{l,sc,crit}}^3} \tag{6d}$$

It is noted that there is no discontinuity between the results obtained from the critical and the sub-critical choke models at critical pressure. Hence, these models can be implemented in the simulator without the risk of sharp changes of calculated upstream pressures near critical values.

Implementation of Choke Models in AD-GPRS

The choke formulations have been implemented in AD-GPRS in terms of the residual equations for construction of the Jacobian matrix. In the following, it is briefly explained how the well constraint equation are involved in the Jacobian matrix.

Constraint Equations and Jacobian Matrix Structures

Wells in the field are subject to various control strategies. These controls are represented by constraint equations. For the MSWell model, the constraint equation takes the place of the "pressure equation" for the top segment. This can be shown clearly in a simple example of a two-phase reservoir model with 3 × 2 grid blocks illustrated in Figure 6.

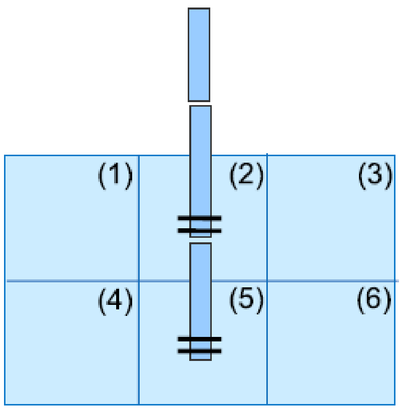

Figure 6. A simple example of a two-phase reservoir model with 3 × 2 grid blocks (Jiang, 2007)

In this model, for fully implicit oil-water simulation, there are two equations per cell. Therefore, the total number of the reservoir equations of this case is twelve. Since each segment has four equations, the three-segment well results in twelve well equations. The total number of the equations in the system is 24. A partitioned Jacobian matrix can be formed and Newton iteration can be repeated using:

$$\begin{pmatrix} J_{RR} & J_{RW} \\ J_{WR} & J_{WW} \end{pmatrix} \cdot \begin{pmatrix} \delta u_r \\ \delta u_w \end{pmatrix} = - \begin{pmatrix} R_r \\ R_w \end{pmatrix} \tag{7}$$

where J_{RW} is the sector of the Jacobian corresponding to the derivatives of the reservoir equations with respect to the MSWell variables, and J_{RR}, J_{WR}, and J_{WW} are defined in a similar manner. R_r and R_w are the residual vectors of the reservoir and well equations respectively. δu_r and δu_w are the corrections for the reservoir and well variables from the Newton iteration. Now assuming a well with pressure control, the constraint equation can be written as:

$$R_1{}^{seg} = P_1{}^{seg} - p_{target} \tag{8}$$

The above equation has only one non-zero derivatives:

$$\frac{\partial R_1{}^{seg}}{\partial P_1{}^{seg}} = 1 \tag{9}$$

The structure of the corresponding Jacobian matrix is shown in Figure 7. The dark blue cells represent non-zero elements. The first twelve rows of the matrix are derived from the reservoir equations, and the remaining twelve rows are from the well equations. The constraint equation is the first well equation, which corresponds to the 13th row of the matrix and is labelled with “ctrl”. The first twelve columns represent the reservoir variables and the remaining columns represent the well unknowns.

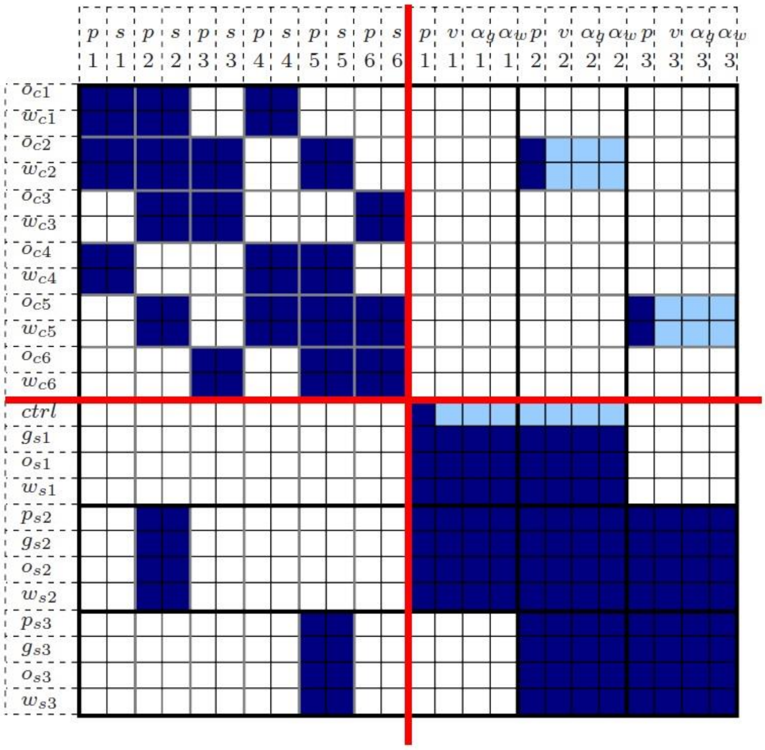

Figure 7. The structure of Jacobian matrix with pressure controlled well (Jiang, 2007)

A similar procedure can be followed when using the oil-rate target. For the oil-rate constraint equation, the mass rate of the oil component from all the perforations is equal to the specified rate:

$$R_1{}^{seg} = R_{ctrl} = \sum_{j=1}^{n_{perf}} \left(\sum_{p=1}^{n_p} \rho_p \, \lambda_p x_{0,p} \, WI(P_p{}^{res} - P^{seq})\right) - \rho_o q_o = 0 \qquad (10)$$

It is noted that R_{ctrl} depends on both formation and segment pressures at the perforations. It is a function of the sand face fluid saturation as well. Therefore, the derivatives of Equation 8 with respect to formation pressure $(\frac{\partial R_1{}^{seg}}{\partial P_j{}^{res}})$, segment pressure $(\frac{\partial R_1{}^{seg}}{\partial P_j{}^{seg}})$, and sand face fluid saturation $(\frac{\partial R_1{}^{seg}}{\partial S_j{}^{seg}})$, should be taken into account. As can be seen in Figure 8, these values are non-zero.

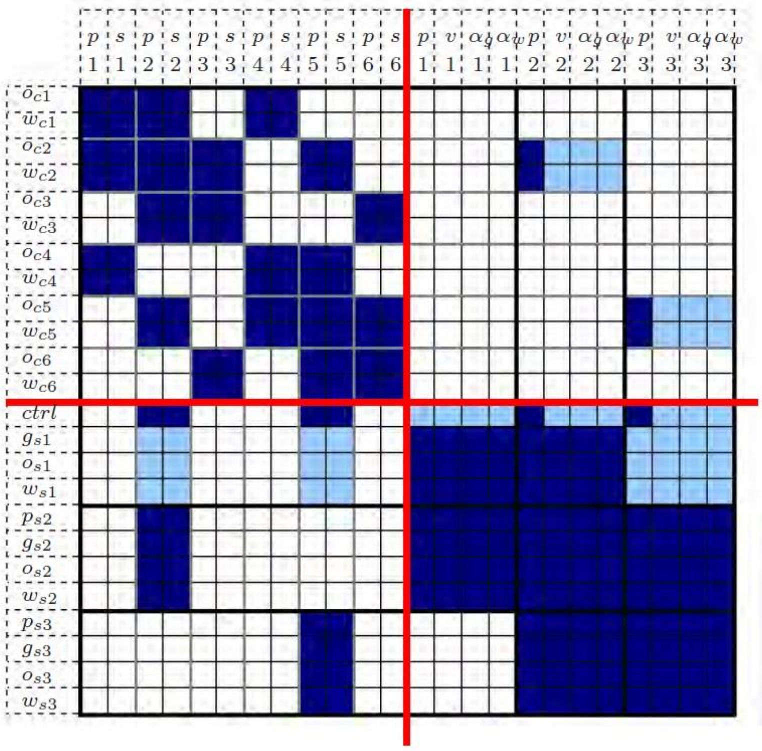

Figure 8. The structure of Jacobian matrix with oil-rate controlled well (Jiang, 2007)

From the above two examples, it is clear how the Jacobian matrix can be affected by the constraint equation. Now assume that the choke equation is replaced with p_{target} in Equation 8. In this case, the p_{target} should be replaced by Equation 1 for critical flow or Equation 2 for sub-critical flow. However, as the upstream pressure is not a constant anymore and related to other variables, the derivatives of the constraint equation with respect to other variables should be taken into accounts. This is the reason for using the ADscalar data type as it makes the automatic differentiation possible. Therefore, the constraint equation for critical flow can be defined as:

$$R_1{}^{seg} = P_1{}^{seg} - (-Aq_{l,1}{}^{seg} \frac{(ER_{gl}{}^{seg})^B}{(Fd_{ch})^c} + D) \tag{11}$$

or similarly:

$$R_1{}^{seg} = q_{l,1}{}^{seg} - (\frac{(D - P_1{}^{seg})(Fd_{ch})^c}{A\left(ER_{gl}{}^{seg}\right)^B}) \tag{12}$$

It is noted that $q_{l,1}{}^{seg}$, $R_{gl}{}^{seg}$ and $P_1{}^{seg}$ should have an ADscalar data type. Hence the derivatives of the constraint equation can be calculated automatically within AD-GPRS. For the sub-critical flow, the constraint equation is defined as

$$R_1{}^{seg} = P_1{}^{seg} - (c_0 + c_1 q_{l,1}{}^{seg} + c_2 q_{l,1}{}^{seg^2} + \ c_3 q_{l,1}{}^{seg^3}) \tag{11}$$

where c_0 to c_3 are defined from Equation 6. In this case, $P_{1,crit}{}'$, $q_{l,1}{}^{seg}$, $R_{gl}{}^{seg}$ and $P_1{}^{seg}$ should have the ADscalar data type.

It should be highlighted that, if the upstream pressure (obtained from the previous iteration) is equal to $1.7P_4$ or above, the flow is critical and the appropriate constraint equation (Equation 12) is used, otherwise Equation 11 is used as a constraint equation.

Choke Option in the WCONPROD Keyword

In AD-GPRS, most of the keywords are designed to be consistent with Eclipse keywords. The WCONPROD keyword specifies control data for production wells. Before choke implementation, the keyword should have been followed by nine parameters of well name, open/shut flag for the well, control mode, oil rate target or upper limit, water rate target or upper limit, gas rate target or upper limit, liquid rate target or upper limit, reservoir fluid volume rate target or upper limit and BHP target or lower limit. However, after choke implementation, three parameters of choke downstream pressure, choke size and choke model have been added to this keyword. Therefore, the required parameters for the WCONPROD keyword can be summarized as follow:

WCONPROD

1.Well name.

2. Open/shut for the well.

3. Control mode:

- ORAT: Controlled by oil rate target
- WRAT: Controlled by water rate target
- GRAT: Controlled by gas rate target
- LRAT: Controlled by liquid rate target
- RESV: Controlled by reservoir fluid volume rate target
- BHP: Controlled by BHP target
- CHK: Controlled by surface choke

DEFAULT: BHP

4. Oil rate target or upper limit, m^3/day

5. Water rate target or upper limit, m^3/day

6. Gas rate target or upper limit, m^3/day

7. Liquid rate target or upper limit, m^3/day

8. Reservoir fluid volume rate target or upper limit, m^3/day

9. BHP target or lower limit, bar.

10. Choke downstream pressure

11. Choke size, m

12. Choke model:

   1. Gilbert
   2. Ros

3. Baxendell
4. Achong

DEFAULT: Gilbert

As an example, the following code defines a Gilbert (default) choke model with a downstream choke pressure of 10 bars and a choke size of 4 cm.

```
WCONPROD
W1 OPEN CHK 5*   10  10   0.04   1*/
```

Acknowledgments

This research was carried out within the context of the Integrated Systems Approach to Petroleum Production (ISAPP) knowledge centre. ISAPP is a joint project of TNO, Delft University of Technology, ENI, Statoil and Petrobras. The authors acknowledge Schlumberger for providing multiple academic Eclipse licenses for this work. The authors also acknowledge Stefano Raniolo for assistance in setting up the simulation models.

Reference

Jansen, J.D., 2017: *Nodal analysis of oil and gas production systems.* SPE Textbook Series. SPE, Richardson. ISBN: 978-1-61399-564-8.

Other relevant documents

AD-GPRS, 2015: *AD-GPRS, Stanford University's Automatic Differentiation based General Purpose Research Simulator, Technical Documentation*. Department of Energy Resources Engineering, Stanford University.

AD-GPRS Manual, 2015: *AD-GPRS, Stanford University's Automatic Differentiation based General Purpose Research Simulator, Manual*. Department of Energy Resources Engineering, Stanford University.

Pan, H. and Cao, H., 2006: *Overview of General Purpose Research Simulator (GPRS)*. Department of Energy Resources Engineering, Stanford University.

Jiang, Y., 2007: *Techniques for modelling complex reservoirs and advanced wells*, PhD Thesis, Stanford University.

Shi, H., Jiang, Y., Batias, J., ?: *Implementation document of multi-segment well black oil model, GPRS Technical Report*. Department of Energy Resources Engineering, Stanford University.

Zhou, Y., ?: *Automatically differentiable expression templates library (ADETL). User Manual*. Department of Energy Resources Engineering, Stanford University.