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Optimizing the joint collision avoidance operations of multiple ships from an overall perspective

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Abstract

Ship collision is the main type of maritime accidents, which causes great losses on human lives and economy, and brings negative impacts to the maritime environment. In crowded waters such as the sea area near a seaport, multiple ships encountering situations happen frequently. While several methods have been proposed for solving multiple ships collision avoidance problem, most research focuses on the safety guarantee and time-availability of anti-collision decisions, and less attention is paid on improving the efficiency of collision avoidance maneuvers. This paper proposes a rolling horizon optimization approach for multiple ships from a global optimal perspective, with the aim to minimize the time costs and course angle alterations of the anti-collision operations. A ship maneuverability model is used to make predictions and calculations of inter-ship collision risks, upon which an overall optimization problem is formulated. Continuous time is divided into a set of discrete time slots, and an iterative optimization procedure is carried out to determine the optimal course angles for the ships at each time slot. A PID heading controller is designed to implement the optimal course angles. When collision risks among ships no longer exist, the optimization procedure terminates. To evaluate the performance of the proposed approach, simulation experiments regarding 7-ships and 12-ships encounter situations are carried out.

Keywords: collision avoidance, decision support, ship navigation, maritime transportation

1. Introduction

Ship collision is the main type of maritime accidents, which causes great losses on human lives and economy, and brings negative impacts to the maritime environment. When two ships encounter one another, they are supposed to obey the 1972 International Regulations for Preventing Collision at Sea (COLREGs), proposed by the International Maritime Organization (IMO). The COLREGs describe potential collision scenarios between encountering ships and provide a set of guidelines for safe maneuvering at sea. Over the last two decades, advanced assistant systems such as GPS (Global Positioning System), ARPA (Automatic Radar Plotting Aid), AIS (Automatic Identification System), and ECDIS (Electronic Chart Display and Information System), have been developed and installed on ships.

The collision avoidance decision making of a ship depends on many factors, such as ship speed, course, relative position, and ship maneuverability. Several ship collision avoidance decision making methods have been proposed (Ahn et al., 2012; Goerlandt et al., 2015; Johansen et al., 2016; Chai et al., 2017; Wang et al., 2017; Szlapczynski et al., 2018; Huang et al., 2019), and relevant literature reviews can be found in (Tam et al., 2009; Johansen et al., 2016). Meanwhile, a number of safe path planning methods have been proposed to help ships find collision-free paths based on Genetic Algorithm (Tam and Bucknall, 2010), Fuzzy Logic (Perera et al., 2011), Branch and Bound (Mohamed-Seghir, 2012), A* Algorithm (Naeem et al., 2012), Ant Colony Optimization (Escario et al., 2012; Lazarowska, 2014), Cooperative Path Planning (Tam and Bucknall, 2013), Neural Network (Simsir et al., 2014), Fast Marching Method (Liu and Bucknall, 2015), Multi-criteria Optimization (Lazarowska, 2017a).

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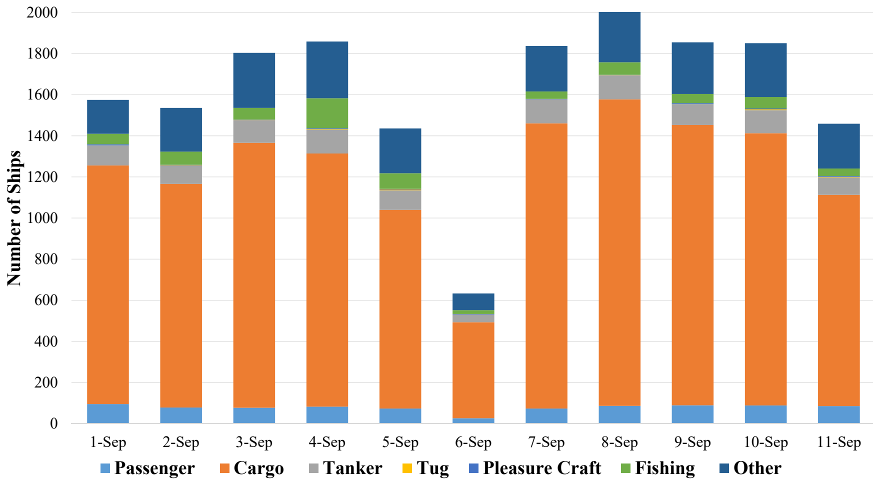


Figure 1: Ship arrivals by type from September 1 to September 11, 2019 in Yangshan Port in Shanghai, P. R. China (MarineTraffic, 2019).

Most research focuses on one-to-one encounters, in which a ship labels itself as own ship and the encountering ship as a target ship. The own ship only considers one target ship at one time. However, when there are more ships around the own ship, the collision avoidance maneuver with regard to one ship may create a collision situation with another ship. In practice, multiple ships encountering situations happen frequently in crowded waters such as the sea area near a seaport. Figure 1 gives an example to show the dense ship arrivals per day at Yangshan Port in Shanghai, P. R. China. Dense ship traffic creates complicated encounters among ships, which could turn into collision accidents.

Regarding multi-ship encounter situations, a decision support system for ship collision avoidance is proposed for Istanbul Strait in (Perera et al., 2015), which makes use of ships' exchanged data to train artificial neural networks so as to make predictions on ship trajectories. Kim et al. (2015) use Distributed Local Search Algorithm (DLSA) and a Distributed Tabu Search Algorithm (DTSA) to find optimal courses for involved ships. The priority of ships in course alteration is given to the ship that could reduce collision risks most significantly. Then Kim et al. (2017) extend this approach by introducing a Distributed Stochastic Search Algorithm (DSSA), which allows each ship to change its intention in a stochastic manner immediately after receiving all of the intentions from the other ships. A multi-target ARPA (Automatic Radar Plotting Aid) system is proposed in (Ożoga and Montewka, 2018) to provide ship navigators with information on safe navigation operations on all possible courses when the ship encounters with other ships. The safe ship courses are determined based on the levels of collision risks the other ships bring based on the time to their closest points of approach (TCPA) and distances at the closest points of approach (DCPA). Li et al. (2019) proposed a distributed constraint optimization approach for assisting ships in making decisions on rudder angles and the rudder deflection time to avoid collisions with the other ships. Simulation experiments are carried out to evaluate the communication and computation costs incurred in the distributed coordination process.

On safe path planning in many-to-many encountering situations, Szlapczynski (2011, 2013b,a) proposed a series of trajectory planning methods based on evolutionary algorithms. Tam and Bucknall (2013) developed a deterministic collision avoidance path planning algorithm to provide collision-free paths for all involved ships, assuming that all encountering ships are in a cooperative mode. Hornauer et al. (2015) proposed a partly-cooperative decentralized trajectory optimization algorithm, in which the movement for non-cooperative ships computed by a Bayesian model using the data from AIS. The estimated positions for non-cooperative ship is predicted by historic probabilistic models. In (Szlapczynska, 2015), an auto-negotiation system for ships collision avoidance is proposed, in which the ships can communicate with each other and negotiate on their maneuvers via a negotiation procedure. A decision support system in (Lazarowska, 2017b) is proposed, in which a trajectories database is constructed, upon which a method called the Trajectory Base Algorithm is used to find optimal ship paths. Ni et al. (2018) adopted a hybrid genetic algorithm to find safe ship trajectories, in which types of ships encountering situations and the obligation of collision avoidance are determined according to COLREGs, and then considered as restricted conditions in population initialization.

While a number of methods have been proposed for solving many-to-many ship collision avoidance problem, most research focuses on the safety guarantee and time-availability of the generated anti-collision decisions, and that less attention is paid on the efficiency of collision avoidance maneuver of ships. Therefore,this paper considers the collision avoidance problem between multiple ships from a global optimal perspective, with the aim to find safe and efficient anti-collision decisions with smaller course alterations and time costs for all involved ships.

This paper proposes an optimization approach to assist ships in finding optimal course angles to avoid collisions with other ships. Firstly, a ship maneuverability model is used to make predictions and calculations of inter-ship collision risks, upon which an overall optimization problem is formulated, with the aim to minimize the time costs and course angle alterations. Different types of ships' priorities in changing their courses are constructed via different optimization objectives. An iterative optimization procedure is carried out to determine the optimal course angles for the ships at each discrete time slot. A PID heading controller is designed to implement the anti-collision decisions. When the collision risks among ships have been eliminated, they will go back to original courses. In order to evaluate the performance of the proposed approach on course deviations and time costs, experiments are carried out.

The main contribution of this paper are threefold:

- A rolling horizon optimization approach is proposed, in which continuous time is divided into a set of discrete time slots, and an optimization procedure is carried at every time slot. Meanwhile, splitting the overall collision avoidance optimization into a series of smaller optimization problems could also reduce the computational complexity and speed up the optimization process, and that re-optimization can be carried out whenever necessary.

- The efficiency of collision avoidance maneuver is considered by transforming the time costs and heading angle deviations for collision avoidance maneuvers into the optimization objective that aims to minimize the sum of times that the ships spend in heading angle alterations.

- Ship dynamics and the inter-related characteristic of the collision avoidance decision making among multiple ships, as well as the optimal solution's compliancy with COLREGs are considered in the model formulations. Under certain circumstance when multiple ships encounter, it may be difficult for COLREGs to describe all possible conditions in the form of rules due to the complexity of encountering situations. This paper deals with such situations by defining different types of ship priorities in changing their courses in the optimization objectives.

This remainder of the paper is organized as follows. Section 2 gives the definitions of relevant parameters and variables. Section 3 introduces the optimization procedure of the proposed approach. Detailed regarding model formulations and optimization steps are given in Section 4. Experimental results are presented in Section 5. Conclusions and future work are given in Section 6.

2. Definitions of parameters and decision variables

This section introduces the definitions of important parameters for the collision avoidance algorithm. Figure 2 gives an example of a 3-ships encounter situation. Ships i, j and k are approaching one another with heading angles φ_i, φ_j and φ_k, and velocities V_i, V_j and V_k. Ship i, it will collide with ships j and k in the future, as the distances at its closest point of approach DCPA_{ij} and DCPA_{ik} are smaller than the minimum safety distance D^{safe}. If ship i changes its heading angle φ_i to φ_i^*, the relative velocity V_{ij}^{R} becomes $V_{ij}^{\mathrm{R}*}$, and the velocity V_{ik}^{R} becomes $V_{ik}^{\mathrm{R}*}$. With the altered heading angle φ_i^*, collisions between ship i and the other two ships could be avoided. Therefore, it is important to find out the most suitable heading angles for the ships.

Definition 1 (Avoidance direction): Let α_{ij} denote the difference between relative heading angle $\varphi_{ij}^{\mathrm{R}}$ and relative bearing angle $\varphi_{ij}^{\mathrm{S}}$ of ships i and j: $\alpha_{ij} = \varphi_{ij}^{\mathrm{R}} - \varphi_{ij}^{\mathrm{S}}$.

If $\alpha_{ij} < \beta_{ij}$, the distance between ships i and j at their closest point will be smaller than the safe distance D^{safe}. Therefore, for ship i, in order to avoid collision with ship j, it is necessary to ensure that $\alpha_{ij} \geq \beta_{ij}$.

Definition 2 (Completion of collision avoidance): As can be seen from Figure 2, if the projection of $\overrightarrow{R_{ij}}$ on relative velocity $\overrightarrow{V_{ij}^{R}}$ is opposite to the direction of $\overrightarrow{V_{ij}^{R}}$, ships i and j no longer have a collision risk. In other words, the condition of completion of collision avoidance between ships i and j is $\alpha_{ij} \geq \pi/2$.

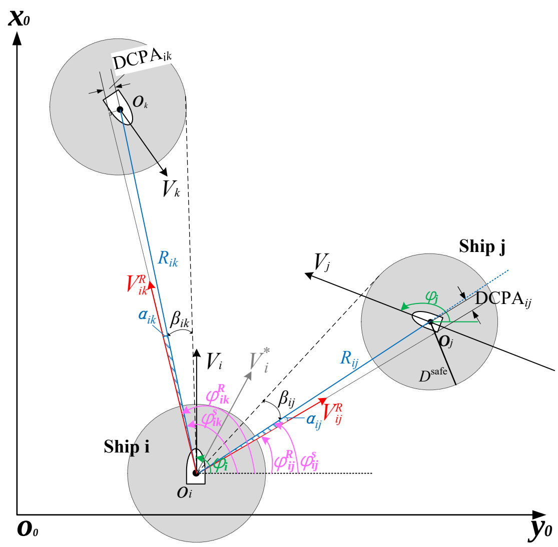


Figure 2: An example of a 3-ships encountering situation.

Table 1 summarizes the parameters that are used in this paper. Considering typical ship maneuvering, this paper assumes that the rudder angle ranges from +20° on the port side to +20° on the starboard side (±20°), and formulate the rudder angle variable domain as $D_i = \{-20^\circ, -10^\circ, -5^\circ, 0^\circ, 5^\circ, 10^\circ, 20^\circ\}$. In order to consider COLREGs, parameters S_i^{port} and $S_i^{\text{starboard}}$ are used to distinguish port side and starboard side ships.

Table 1: List of parameters that should be defined before optimization procedure starts.

Symbols	Definitions
M	a set of encountering ships
D^{safe}	the minimum safety distance that any two ships should keep to avoid collision
$T^{\text{prediction}}$	the prediction time horizon of ship states
$T^{\text{encounter}}$	the deadline for ships to take collision avoidance operations
D_i	the domain of variable rudder angles for ship i
$S_i^{\text{port}}/S_i^{\text{starboard}}$	the set of ships that are on the port/starboard side of ship i, respectively

Table 2 gives the list of decision variables that represent ship states at time t during each time slot p ($t \in p$). Rudder angle δ_i, rudder maneuvering time $T_i^{\text{maneuvering}}$, and heading angle φ_i are the main decision variables for each ship $i \in M$. Variables $TCPA_{ij}, DCPA_{ij}, \alpha_{ij}, \beta_{ij}$ and $T_{ij}^{\text{maneuvering}}$ are the variables that reflect the collision risks between any two ships $i, j \in M$.

3. Optimization procedure of the proposed approach

Figure 3 and Algorithm 1 describe the optimization procedure of the proposed approach. Initially, the set of ships with potential collision risks need to be identified. Figure 4 presents the time horizon regarding the optimization procedure. It is assumed that ship set M consists of ships that will encounter each other within a time of $T^{\text{encounter}}$, i.e., the times to their closest points of approach are smaller or equal to $T^{\text{encounter}}$. Each ship $i \in M$, based on its initial coordinates (x_i, y_i), heading angles φ_i, and speed V_i, calculates its closest point of approach with the any other ship $j \in M$. If $\text{DCPA}_{ij} < D^{\text{safe}}$, collision risk exists between ships i and j. Continuous time are discretized into a series of discrete time slots $1, 2, 3 \ldots, p, \ldots, P$. Each ship makes decisions regarding the optimal heading angle alteration in each time slot p.

In each time slot p, given ships current states and a set of candidate rudder angles, and using ship maneuverability models, the future states of ships can be predicted with a prediction horizon $T^{\text{prediction}}$. Then the collision risks

Table 2: List of decision variables.

Symbols	Definitions
$V_i(t)$	velocity of ship i at time t
$(x_i(t), y_i(t))$	coordinates of ship i on x axis and y axis at time t
$\varphi_i(t)$	heading angle of ship i at time t
$T_{ij}^{\text{maneuvering}}(t)$	shortest rudder maneuvering time for ship i to avoid collisions with ship j
δ_i	rudder angle that ship i takes
$R_{ij}(t)$	relative distance between ships i and j at time t
$V_{ij}^{\text{R}}(t)$	relative velocity between ships i and j at time t
$\varphi_{ij}^{\text{R}}(t)$	relative heading angle between ships i and j at time t
$\varphi_{ij}^{\text{S}}(t)$	relative bearing between ships i and j at time t
$\alpha_{ij}(t)$	the angle between ships i and j' relative heading angle $\varphi_{ij}^{\text{R}}(t)$ and the angle between their displacements $\varphi_{ij}^{\text{S}}(t)$
$\beta_{ij}(t)$	the angles of tangent of safety distance D^{safe} with respect to the vector $R_{ij}(t)$ between ships i and j at time t
$\text{DCPA}_{ij}(t)$	distance between ships i and j at their closest point of approach (CPA) at time t
$\text{TCPA}_{ij}(t)$	time to the closest point of approach (CPA) between ships i and j at time t

parameters between any two ships, when they take different rudder angles are determined. Based on these parameters, safety constraints are constructed. The optimization objective is to minimize the heading angle deviations and time costs for collision avoidance maneuvering of all ships, also considering the compliancy with COLREGs. Then the multiple ships anti-collision decision optimization problem are formulated and solved.

After the solutions are obtained, the optimal heading angles that the ships should select are determined. PID-based heading control is adopted to implement the selected heading angle for each ship, and each ship will keep the selected heading angle during time slot p. At the end of p, ships' states will be updated as the initial states at the beginning of next time slot $p+1$, and the same procedure starts over again until all potential collision risks are eliminated. When the ships have reached their closest of point of approach with each other, the decision optimization process can be terminated.

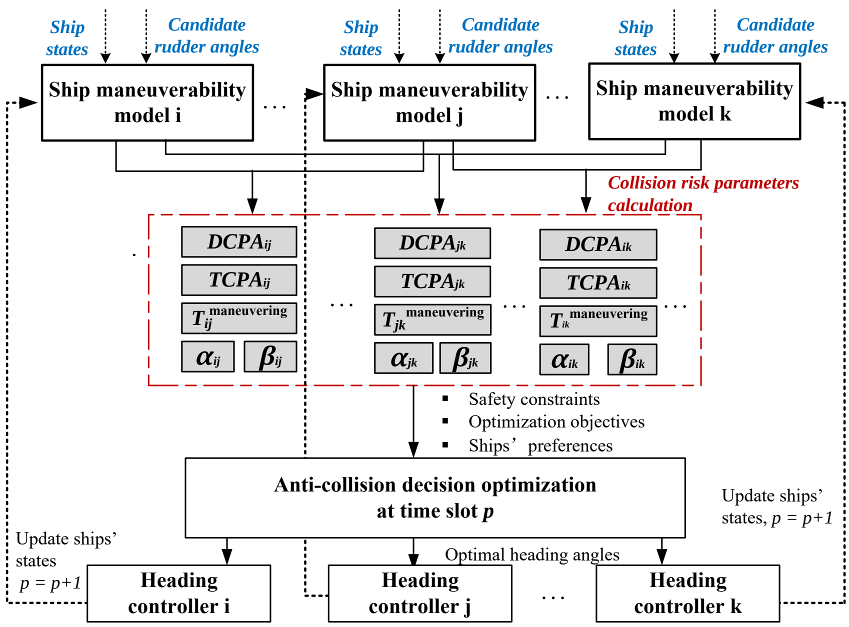


Figure 3: The structure of proposed approach.

Algorithm 1 The steps of collision avoidance decision optimization procedure among multiple ships.

1. Initialization

- Initialize parameters $T^{\text{encounter}}$, D^{safe}, $T^{\text{prediction}}$, the length of a time slot p, and variable domain D_i for each ship i
- Calculate port side S_i^{port} and starboard side $S_i^{\text{starboard}}$ ships of each ship i
- Acquisition of each ship i's current state: coordinates (x_i, y_i), speed V_i, rudder angle δ_i, heading angld φ_i

2. Prediction of ship states based on maneuverability model

Given different candidate rudder angles from $\delta_i \in D_i$ and current ship states as inputs, calculate ship coordinates, speed, and heading angles over prediction time horizon $T^{\text{prediction}}$.

3. Calculation of inter-ship collision risk parameters

Based on ships' states in Step 2, calculate relevant collision risk parameters between any two ships i and j: α_{ij}, β_{ij}, $TCPA_{ij}$, $DCPA_{ij}$, $T_{ij}^{\text{maneuvering}}$.

4. Iterative process: formulate multiple ships anti-collision decision optimization problem at time slot p

Construct the optimization objective and safety constraints:

(a) DCPA between any two ships should be larger than D^{safe};

(b) $\alpha_{ij}(p) \geq \beta_{ij}(p)$.

5. Iterative process: solve multiple ships anti-collision decision optimization problem at time slot p

Solve the optimization problem at time slot p, to get the solution $\delta_i^*(p)$, and thereby deduce optimal heading angle φ_i^* according to predicted ship states in Step 2.

6. Iterative process: heading control for ships to execute collision decisions at time slot p

Design heading control law to make sure that all ships change their courses $\varphi \rightarrow \varphi^*$ with a time length of $|p|$, update ships' states at the end of time slot p.

7. Termination criteria

According to the update ships' states, check if the ships have passed their CPAs:

- Use the designed heading control law in Step 6 to calculate the simulated ships' states when $\varphi \rightarrow \varphi_0$ (φ_0: original courses)
- Calculate α_{ij}, β_{ij} and $DCPA_{ij}$ based on the simulated ships' states
- Test for each ship i if Conditions (1) $\alpha_{ij} \geq \beta_{ij}, \forall j \in M$; (2) $DCPA_{ij} \geq D_{ij}^{\text{safe}}, \forall j \in M$ are satisfied:
  - **If** all ships satisfy (1)(2): optimization procedure terminates, and all ships switch back to their original courses;
  - **Otherwise**: go to Step 2 with update ship states at the end of time slot p.

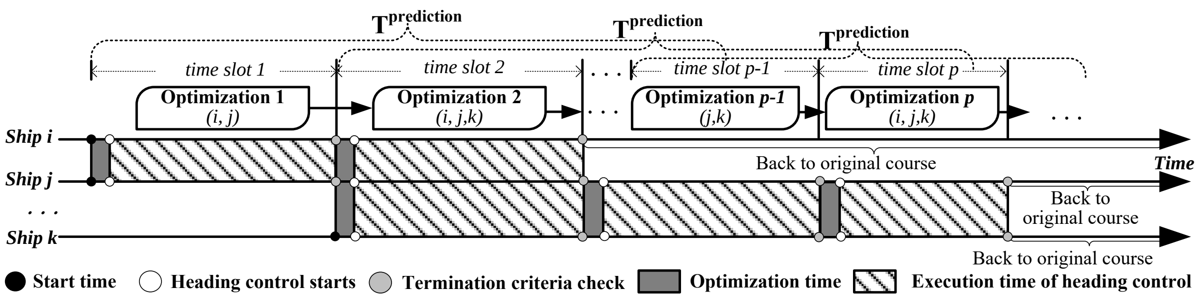


Figure 4: Time horizon.

4. Model formulations and optimization steps

This section gives details regarding the collision avoidance decision optimization process, which consists of 7 steps: 1) relevant parameters and decision variables are initialized; 2) predictions of ship states are made according to maneuverability model; 3) five inter-ship collision risk parameters are calculated based on predicted ship states; 4) the collision avoidance decision optimization problem at time slot p is formulated; 5) solve the optimization problem and suggested optimal heading angles are given to multiple ships as solutions; 6) PID-based heading control is used to maneuver ships to desired states; 7) check if the termination criteria is satisfied.

4.1. Step 1: Initialization with parameters and defined decision variables

Firstly, each ship i's current states including its coordinates (x_i, y_i), speed V_i, rudder angle δ_i, heading angle φ_i should be acquired. To determine when the optimization procedure should start, parameter $T^{\text{encounter}}$ is introduced. When the TCPA between any two ships $TCPA_{ij} \leq T^{\text{encounter}}$, calculate the DCPAs between them, if the $DCPA_{ij} < D^{\text{safe}}$, the ships need to take collision avoidance operation. In order to determine ships' priorities in changing courses, the port side S_i^{port} and starboard side $S_i^{\text{starboard}}$ ships of each ship $i \in M$ need to be identified. In addition, the length of a time slot p, ship state prediction time $T^{\text{prediction}}$, variable domain D_i for each ship i are also determined in this step.

4.2. Step 2: Prediction of ship states based on maneuverability model

In order to make predictions of ships' states within a prediction time of $T^{\text{prediction}}$, for each ship $i \in M$, given different rudder angles δ_i from domain D_i, its trajectories, speed change and heading angle alteration are determined, according to ship maneuvering model expressed as follows:

$$\begin{cases} (m+m_x)\dot{u} - (m+m_y)vr - x_G m r^2 = X_H + X_P + X_R \\ (m+m_y)\dot{v} + (m+m_x)ur + x_G m\dot{r} = Y_H + Y_P + Y_R \\ (I_z + x_G^2 m + J_z)\dot{r} + x_G m(\dot{v} + ur) = N_H + N_P + N_R \end{cases} \tag{1}$$

where, subscripts H, P, R represent the hull, the propeller and the rudder; m, m_x and m_y are ship mass, added mass in x-direction, and added mass in y-direction; I_z and J_z are moment of inertia and added moment of inertia around the z-axis, u and v are ship longitudinal and lateral speed, r is ship yaw rate around midship, and the dot notation of u, v and r represents the derivative of each parameter. For more details regarding the model, we refer the readers to (Liu et al., 2017).

Given different rudder angles, the hydrodynamic force X_R due to rudder acting on midship in $x-$ direction is determined, thereby the forward speed u and acceleration $\dot{u}$ in $x-$axis, as well as sway speed v and acceleration $\dot{v}$ in $y-$axis are also determined. Based on the ship motion variables (u, v) and $(\dot{u}, \dot{v})$, coordinates $(x(t), y(t))$ of the ship on x-axis and y-axis at time t can be calculated. A series of ship coordinates over time constitutes the ship's trajectory.

4.3. Step 3: Calculation of inter-ship collision risk parameters

Based on the predicted ship states, relevant collision risk parameters $TCPA_{ij}, DCPA_{ij}, \alpha_{ij}, \beta_{ij}$ and $T_{ij}^{\text{maneuvering}}$ are calculated.

Figure 5 gives an illustrative example of the collision risk parameters. Parameters $TCPA_{ij}$ and $DCPA_{ij}$ are chosen because they are commonly used parameters in literature (Xu and Wang, 2014). Parameter $DCPA_{ij}$ implies whether collision risks exist between ships i and j, and parameter $TCPA_{ij}$ reflects when the ships will meet and could go back to their original courses. Parameters α_{ij} and β_{ij} are required to evaluate whether the ships have passed their CPAs and whether their collision risks no longer exist. If $\alpha_{ij} \geq \beta_{ij}$, collisions between ships i and j can be avoided. Parameter $T_{ij}^{\text{maneuvering}}$ represents the minimum rudder steering time required for ship i to avoid collisions with ship j. This parameter is chosen to reflect the heading angle alterations: longer rudder maneuvering time implies larger heading angle alterations. Parameter $T_{ij}^{\text{maneuvering}}(\delta_i, \delta_j, t)$ is calculated as the earliest time t when $DCPA_{ij}(\delta_i, \delta_j, t) \geq D^{\text{safe}}$.

In addition, to avoid situations in which the close quarters situation happen long before the closest point of approach is actually reached, the relative distance between them should always be larger than the minimum safe distance D^{safe} during time $t \in [0, T_{ij}^{\text{maneuvering}}]$, otherwise $T_{ij}^{\text{maneuvering}} = +\infty$. In other words, if any two ships i and j

have already reached their closest point before the time when their $DCPA_{ij} \geq D^{\text{safe}}$, the chosen rudder angles pair (δ_i^*, δ_j^*) are infeasible for them.

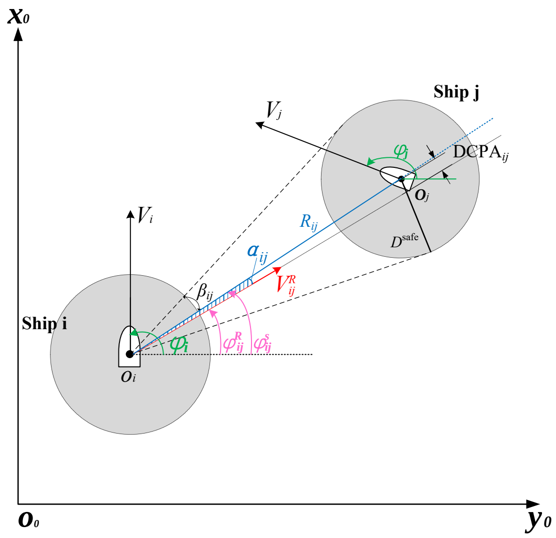


Figure 5: A two ships encounter case as an illustrative example of collision risk parameters.

When any two ships $i, j \in M$ select a rudder angle $\delta_i \in D_i$ and $\delta_j \in D_j$, the collision risk parameters are calculated as follows:

$$R_{ij}(\delta_i,\delta_j,t) = \sqrt{(x_i(\delta_i,t)-x_j(\delta_j,t))^2+(y_i(\delta_i,t)-y_j(\delta_j,t))^2} \qquad \forall i,j \in M, \forall \delta_i \in D_i, \forall \delta_j \in D_j \tag{2}$$

$$\beta^*_{ij}(\delta_i,\delta_j,t) = \arcsin\left(\frac{D^{\text{safe}}}{R_{ij}(\delta_i,\delta_j,t)}\right) \qquad \forall i,j \in M, \forall \delta_i \in D_i, \forall \delta_j \in D_j \tag{3}$$

$$\theta^{\text{R}}_{ij}(\delta_i,\delta_j,t) = \arctan\left(\frac{x_j(\delta_j,t)-x_i(\delta_i,t)}{y_j(\delta_j,t)-y_i(\delta_i,t)}\right) \qquad \forall i,j \in M, \forall \delta_i \in D_i, \forall \delta_j \in D_j \tag{4}$$

$$\varphi^{\text{S}}_{ij}(\delta_i,\delta_j,t) = \begin{cases} -(\pi - |\theta^{\text{R}}_{ij}(\delta_i,\delta_j,t)|), & x_j(\delta_j,t) < x_i(\delta_i,t), y_j(\delta_j,t) < y_i(\delta_i,t), \forall i,j \in M, \forall \delta_i \in D_i, \forall \delta_j \in D_j \\ |\theta^{\text{R}}_{ij}(\delta_i,\delta_j,t)|, & x_j(\delta_j,t) \geq x_i(\delta_i,t), y_j(\delta_j,t) > y_i(\delta_i,t), \forall i,j \in M, \forall \delta_i \in D_i, \forall \delta_j \in D_j \\ \pi - |\theta^{\text{R}}_{ij}(\delta_i,\delta_j,t)|, & x_j(\delta_j,t) > x_i(\delta_i,t), y_j(\delta_j,t) < y_i(\delta_i,t), \forall i,j \in M, \forall \delta_i \in D_i, \forall \delta_j \in D_j \\ -|\theta^{\text{R}}_{ij}(\delta_i,\delta_j,t)|, & x_j(\delta_j,t) < x_i(\delta_i,t), y_j(\delta_j,t) > y_i(\delta_i,t). \forall i,j \in M, \forall \delta_i \in D_i, \forall \delta_j \in D_j \end{cases} \tag{5}$$

$$V^x_{ij}(\delta_i,\delta_j,t) = V_j(\delta_j,t)\cos\varphi_j(\delta_j,t) - V_i(\delta_i,t)\cos\varphi_i(\delta_i,t) \qquad \forall i,j \in M, \forall \delta_i \in D_i, \forall \delta_j \in D_j \tag{6}$$

$$V^y_{ij}(\delta_i,\delta_j,t) = V_j(\delta_j,t)\sin\varphi_j(\delta_j,t) - V_i(\delta_i,t)\sin\varphi_i(\delta_i,t) \qquad \forall i,j \in M, \forall \delta_i \in D_i, \forall \delta_j \in D_j \tag{7}$$

$$\chi^{\text{R}}_{ij}(\delta_i,\delta_j,t) = \arctan\left(\frac{V^x_{ij}(\delta_i,\delta_j,t)}{V^y_{ij}(\delta_i,\delta_j,t)}\right) \qquad \forall i,j \in M, \forall \delta_i \in D_i, \forall \delta_j \in D_j \tag{8}$$

$$\varphi^{\text{R}}_{ij}(\delta_i,\delta_j,t) = \begin{cases} |\chi^{\text{R}}_{ij}(\delta_i,\delta_j,t)|, & V^x_{ij}(\delta_i,\delta_j,t) \leq 0, V^y_{ij}(\delta_i,\delta_j,t) < 0, \forall i,j \in M, \forall \delta_i \in D_i, \forall \delta_j \in D_j \\ \pi - |\chi^{\text{R}}_{ij}(\delta_i,\delta_j,t)|, & V^x_{ij}(\delta_i,\delta_j,t) < 0, V^y_{ij}(\delta_i,\delta_j,t) > 0, \forall i,j \in M, \forall \delta_i \in D_i, \forall \delta_j \in D_j \\ -(\pi - |\chi^{\text{R}}_{ij}(\delta_i,\delta_j,t)|), & V^x_{ij}(\delta_i,\delta_j,t) > 0, V^y_{ij}(\delta_i,\delta_j,t) > 0, \forall i,j \in M, \forall \delta_i \in D_i, \forall \delta_j \in D_j \\ -|\chi^{\text{R}}_{ij}(\delta_i,\delta_j,t) & V^x_{ij}(\delta_i,\delta_j,t) > 0, V^y_{ij}(\delta_i,\delta_j,t) < 0, \forall i,j \in M, \forall \delta_i \in D_i, \forall \delta_j \in D_j \end{cases} \tag{9}$$

$$\alpha^*_{ij}(\delta_i,\delta_j,t) = \varphi^{\text{R}}_{ij}(\delta_i,\delta_j,t) - \varphi^{\text{S}}_{ij}(t) \qquad \forall i,j \in M, \forall \delta_i \in D_i, \forall \delta_j \in D_j \tag{10}$$

$$V^{\text{R}}_{ij}(\delta_i,\delta_j,t) = \sqrt{(V^x_{ij}(\delta_i,\delta_j,t))^2+(V^y_{ij}(\delta_i,\delta_j,t))^2} \qquad \forall i,j \in M, \forall \delta_i \in D_i, \forall \delta_j \in D_j \tag{11}$$

$$DCPA^*_{ij}(\delta_i,\delta_j,t) = R_{ij}(\delta_i,\delta_j,t)\sin\alpha^*_{ij}(\delta_i,\delta_j,t) \qquad \forall i,j \in M, \forall\delta_i \in D_i, \forall\delta_j \in D_j \tag{12}$$

$$TCPA^*_{ij}(\delta_i,\delta_j,t) = \frac{R_{ij}(\delta_i,\delta_j,t)\cos\alpha^*_{ij}(\delta_i,\delta_j,t)}{V^{\mathrm{R}}_{ij}(\delta_i,\delta_j,t)} \qquad \forall i,j \in M, \forall\delta_i \in D_i, \forall\delta_j \in D_j \tag{13}$$

4.4. Step 4: Formulating collision avoidance optimization problem at time slot p

At time slot p, based on the risk parameters calculated in Step 3, the multiple ships collision avoidance decision optimization problem is formulated.

4.4.1. Optimization objectives

We propose three types of optimization objectives Obj_1, Obj_2 and Obj_3 to minimize the sum of times that ships spend in heading angle alterations, with different weights w_i^1, w_i^2 and w_i^3 assigned to ships:

$$Obj_1 : \sum_{i\in V} w_i^1 \left(\max_{j\in V}(T_{ij}^{\text{maneuvering}}) \right)$$

The aim of Obj_1 is to minimize the sum of rudder maneuvering times of all involved ships, where $\max_{j\in V}(T_{ij}^{\text{maneuvering}})$ equals the longest rudder steering time that ship i spends in avoiding collision with the other ships, and w_i is the weight given to each ship $i \in M$. As shorter rudder steering time implies smaller changes of heading angle, the aim of Obj_1 is to ensure that all the ships could avoid collisions with the other ships with smaller heading angle alterations.

According to Rule 15 (crossing situation) of COLREGs, a ship that has the other ships on its starboard side shall keep out of the way. However, in many-to-many ship encountering situations, it is difficult to define the starboard side ship, as one ship may be on the starboard side of one ship while being on the port side of another ship. In addition, under certain circumstances, it is hard to find feasible solutions if the starboard side ships do not change their courses at all. We assume that the ship with more ships on its port side have higher priority to make smaller changes on its heading angles, and that the ship with more ships on its starboard side needs to make bigger changes on its heading angles to keep out of the way. Therefore, to determine the weight w_i^1 assigned to each ship $i \in M$ in Obj_1, the ships are ranked according to the number of ships on each ship's port side in descending order, the ships with more ships on its port side will be given higher weights.

$$Obj_2 : \sum_{i\in V} w_i^2 \left(\max_{j\in V}(T^{\text{maneuvering}}) \right)$$

In objective Obj_2, to determine the weight w_i^2 assigned to each ship $i \in M$, the ships are ranked according to the number of ships it has risks with in ascending order. If the ships with more risky ships make larger changes, it may bring larger improvements from a global perspective. This is because the collision avoidance decisions of these ships may affect more ships. As higher weights may lead to smaller heading angle alterations, therefore, the ships with less risky ships will be given higher weights in Obj_2.

$$Obj_3 : \sum_{i\in V} w_i^3 \left(\max_{j\in V}(T_{ij}^{\text{maneuvering}})) \right)$$

In objective Obj_3, it is assumed that all ships are in equal positions, and that they all need to change courses to avoid collisions whenever necessary. This implies that $w_1^3 = w_2^3 = \cdots = w_{|M|}^3 = 1$.

4.4.2. Constraints

The parameters calculated via Equations (2)-(13) are used to construct Constraints (14)-(27). It is noted that $M = +\infty$. Auxiliary variable $\varepsilon_{im} \in [0,1]$ is introduced in Constraints (14) and (15), and $\varepsilon_{im} = 1$ if ship i chooses rudder angle $\delta_m \in D_i$, otherwise $\varepsilon_{im} = 0$. Constraints (16) and (17) represent that if ships i and j select rudder angles δ_m and δ_n, respectively, then $\alpha_{ij}(p) = \alpha^*_{ij}$. Similarly, Constraints (18)-(25) are used to determine the values for variables $\beta_{ij}, TCPA_{ij}, DCPA_{ij}$ and $T_{ij}^{\text{maneuvering}}$ when ships i and j select different rudder angles. Constraint (26) ensures that the ships keep enough distances with each other and avoid collisions. Constraint (27) ensures that any two ships i and j have passed their CPA and that collision risks have been eliminated.

$$\delta_i \leq \delta_m \varepsilon_{im} + M(1-\varepsilon_{im}) \qquad \forall i \in M, \forall \delta_m \in D_i \quad (14)$$
$$\delta_i \geq \delta_m \varepsilon_{im} - M(1-\varepsilon_{im}) \qquad \forall i \in M, \forall \delta_m \in D_i \quad (15)$$
$$\alpha_{ij}(p) \geq \alpha^*_{ij}(\delta_m, \delta_n, p) - M(1-\varepsilon_{im}) - M(1-\varepsilon_{jn}) \qquad \forall i,j \in M, \forall p \in P, \forall \delta_m \in D_i, \forall \delta_n \in D_j \quad (16)$$
$$\alpha_{ij}(p) \leq \alpha^*_{ij}(\delta_m, \delta_n, p) + M(1-\varepsilon_{im}) + M(1-\varepsilon_{jn}) \qquad \forall i,j \in M, \forall p \in P, \forall \delta_m \in D_i, \forall \delta_n \in D_j \quad (17)$$
$$\beta_{ij}(p) \geq \beta^*_{ij}(\delta_m, \delta_n, p) - M(1-\varepsilon_{im}) - M(1-\varepsilon_{jn}) \qquad \forall i,j \in M, \forall p \in P, \forall \delta_m \in D_i, \forall \delta_n \in D_j \quad (18)$$
$$\beta_{ij}(p) \leq \beta^*_{ij}(\delta_m, \delta_n, p) + M(1-\varepsilon_{im}) + M(1-\varepsilon_{jn}) \qquad \forall i,j \in M, \forall p \in P, \forall \delta_m \in D_i, \forall \delta_n \in D_j \quad (19)$$
$$TCPA_{ij}(p) \geq TCPA^*_{ij}(\delta_m, \delta_n, p) - M(1-\varepsilon_{im}) - M(1-\varepsilon_{jn}) \qquad \forall i,j \in M, \forall p \in P, \forall \delta_m \in D_i, \forall \delta_n \in D_j \quad (20)$$
$$TCPA_{ij}(p) \leq TCPA^*_{ij}(\delta_m, \delta_n, p) + M(1-\varepsilon_{im}) + M(1-\varepsilon_{jn}) \qquad \forall i,j \in M, \forall p \in P, \forall \delta_m \in D_i, \forall \delta_n \in D_j \quad (21)$$
$$DCPA_{ij}(p) \geq DCPA^*_{ij}(\delta_m, \delta_n, p) - M(1-\varepsilon_{im}) - M(1-\varepsilon_{jn}) \qquad \forall i,j \in M, \forall p \in P, \forall \delta_m \in D_i, \forall \delta_n \in D_j \quad (22)$$
$$DCPA_{ij}(p) \leq DCPA^*_{ij}(\delta_m, \delta_n, p) + M(1-\varepsilon_{im}) + M(1-\varepsilon_{jn}) \qquad \forall i,j \in M, \forall p \in P, \forall \delta_m \in D_i, \forall \delta_n \in D_j \quad (23)$$
$$T^{\text{maneuvering}}_{ij}(p) \geq T^{\text{maneuvering}*}_{ij}(\delta_m, \delta_n, p) - M(1-\varepsilon_{im}) - M(1-\varepsilon_{jn}) \quad \forall i,j \in M, \forall p \in P, \forall \delta_m \in D_i, \forall \delta_n \in D_j \quad (24)$$
$$T^{\text{maneuvering}}_{ij}(p) \leq T^{\text{maneuvering}*}_{ij}(\delta_m, \delta_n, p) + M(1-\varepsilon_{im}) + M(1-\varepsilon_{jn}) \quad \forall i,j \in M, \forall p \in P, \forall \delta_m \in D_i, \forall \delta_n \in D_j \quad (25)$$
$$\alpha_{ij}(p) \geq \beta_{ij}(p) \qquad \forall i,j \in M, \forall p \in P \quad (26)$$
$$DCPA_{ij}(p) \geq D^{\text{safe}} \qquad \forall i,j \in M, \forall p \in P \quad (27)$$

To consider the compliancy with COLREGs, this paper takes head-on and crossing situations into account. Rule 15 has been considered via optimization objective Obj_1. In Rule 14 (head-on situation), as the two encountering ships should change their courses to starboard side, condition $\delta_i \geq 0$ is required as an additional constraint.

4.5. Step 5: Solving the optimization problem and determine optimal heading angles

The aim of Step 5 is to find the optimal rudder angles $\delta^*(p)$. The formulated optimization problem in Step 4 can be easily solved with a commercial or open-source solver. When constructing the solution for collision avoidance decision instance, the values of rudder angle δ^*_i for each ship $i \in M$ need to be retrieved. After the optimal rudder angle δ^*_i is obtained for each ship, the rudder deflection time $T^{\text{maneuvering}}_{ij}$ for each ship $i \in M$ is also determined. Based on the ship states prediction in Step 2, with δ_i and $\max_{j \in V} T^{\text{maneuvering}}_{ij}$, the associated optimal heading angle φ^*_i can be found. The optimal heading angles will be given as inputs for ship heading control in the next step.

4.6. Step 6: PID control mechanism of ship maneuvering motion

For ship $i \in M$, the rudder deflection angle δ_i is controlled by PID control law based on the error of the heading angle, in which $e_i = \varphi^*_i - \varphi_i$, and that φ^*_i is the desired course of ship $i \in M$ obtained from Step 5, and φ_i is its current course, which can be expressed as follows:

$$\delta_i = K_p e + K_d \dot{e} + K_i \int e dt,$$

where K_p, K_d and K_i are designed parameters.

The PID heading control will be carried out for a duration of $|p|$ time, at the end of time slot p, the ship states will be updated as the initial states at the beginning of time slot $p+1$.

4.7. Step 7: Termination criteria

Based on updated ship states at the end of time slot p, it is important to check if the ships could go back to their original heading angles. Firstly, use the heading control law in Step 6 to maneuver the ships to their original heading angles and a set of simulated ship trajectories is generated. Then calculate parameters α_{ij}, β_{ij} and $DCPA_{ij}$, and check for each ship i if (1) $\alpha_{ij} \geq \pi/2$ and (2) $DCPA_{ij} \geq D^{\text{safe}}$ are simultaneously satisfied:

Table 3: Spatial relations among the ships in the 7-ships case.

	Port side ships	**Starboard ships**	**Ships with risks**
Ship 1	Ships 2, 3, 4, 7	Ships 5, 6	Ship 3
Ship 2	Ships 6, 7	Ships 1, 3, 4, 5	Ships 4, 5, 6
Ship 3	Ships 2, 6, 7	Ships 1, 4, 5	Ships 1, 5
Ship 4	Ships 2, 3, 7	Ships 1, 5, 6	Ship 2
Ship 5	Ships 1, 2, 3, 4	Ships 6, 7	Ships 2, 3, 6, 7
Ship 6	Ships 1, 2, 3, 4, 5	Ship 7	Ships 2, 5
Ship 7	Ships 1, 5, 6	Ships 2, 3, 4	Ship 5

- Case 1: if conditions (1) and (2) are satisfied for all ships, then all ships can switch back to their original courses with the proposed heading control law and the optimization procedure terminates.
- Case 2: if conditions (1) and (2) are both satisfied for several ships, then the optimization procedure starts from Step 2 again with the update ship states at the end of Step 6, and the ships that satisfy the two conditions do no need to change their courses in the optimization model at time slot $p+1$.
- Case 3: if conditions (1) and (2) are satisfied for none of the ships, then the optimization procedure starts over again from Step 2 with the update ship states at the end of Step 6.

5. Experimental results

Simulation experiments are carried out to evaluate the proposed method. Simulated ship trajectories, heading angle changes over time, TCPA and DCPA changes over time in a 7-ships encounter situation and a 12-ships encounter situation are given. Then, the performance of different experiments are compared with respect to heading angle deviations, the finish time of collision avoidance of the last ship in the ship set, as well as the finish time of collision avoidance operation per ship.

5.1. Experimental setup

Our experiments are performed on an Intel Core i7-7500 CPU with 8GB RAM running Windows 10 and are implemented in MATLAB, in which CPLEX 12.8.1 is used as the optimization solver. We use the KVLCC2 tanker as a sample ship and adopt the ship parameters from (SIMMAN 2008 committee, 2008). When the TCPA between any two ships is smaller than 6 minutes, i.e, $T^{\text{encounter}} = 360s$, the optimization procedure is initiated. The minimum safety distance that each two ships should keep is set as $D^{\text{safe}} = 100m$. The prediction time $T^{\text{prediction}} = 900s$ and the length of each time slot $p = 60s$.

5.2. 7-ships encounter situation

Figure 6 presents a case in which 7 ships are encountering one another in a “crossing” situation. Figure 6a shows the ships’ simulated trajectories when they keep their original courses unchanged. Table 3 gives the port side and starboard ships of each involved ship, and the ships that it has collision risks with. Therefore, the weight assigned to each ship in the optimization objective function as constructed as follows:

- Constructing weights w^1 in Obj_1: rank the ships according to the number of ships on its starboard side in ascending order: Ship 6 (1 ship)< Ship 1, 5 (2 ships) < Ship 3, 4, 7 (3 ships) < Ship 2 (4 ships), therefore $w_6 = 4, w_1 = w_5 = 3, w_3 = w_4 = w_7 = 2, w_2 = 1$;
- Constructing weights w^2 in Obj_2: rank the ships according to the number of ships it has risks with in ascending order: Ship 1, 4, 7 (1 ship)<Ship 3,6 (2 ships)<Ship 2 (3 ships)<Ship 5 (4 ships), therefore $w_1 = w_4 = w_7 = 4, w_3 = w_6 = 3, w_2 = 2, w_5 = 1$;
- Constructing weights w^3 in Obj_3: all ships are in equal status and need to change courses, therefore $w_1 = w_2 = w_3 = w_4 = w_5 = w_6 = w_7 = 1$.

Figures 6b, 6c and 6d show the simulated ship trajectories with different optimization objectives. All the ships could successfully avoid collisions with the other ships. Table 4 summarizes the deviations of heading angles of each ship from its original heading angles. As can be seen from the simulated results with Obj_3 Ships 1, 4 and 6 do not change their heading angles, while Ships 2, 5 and 7 make relatively larger changes on their heading angles. Among the simulation results of the three optimization objectives, Obj_3 generate solutions with smallest heading angle deviations. This implies that with Obj_3 as the optimization objective in this case, the involved ships can avoid collisions with smaller heading angle deviations.

Figures 7a, 7b and 7c show the heading angle changes over time with different optimization objectives. It can be seen that the ships in Figure 7b could switch back to their original courses at an earlier time comparing with Figures 7a and 7c. This implies that with optimization objective Obj_2, the involved ships could finish the collision avoidance operation with a shorter time.

Table 4: Largest heading angle deviations of each ship compared with original heading angles (/degrees) in 7-ships case.

	Ship 1	**Ship 2**	**Ship 3**	**Ship 4**	**Ship 5**	**Ship 6**	**Ship 7**	**Avg.**
Obj_1	12	60	20	0	40	25	88	34.85
Obj_2	0	25	20	0	45	10	28	17.85
Obj_3	0	25	20	0	37	0	40	17.14

Figure 8 shows the changes of TCPAs and DCPAs over time between ships with different optimization objective. Firstly, it can be seen from Figures 8b, 8d and 8f that all ships keep enough distances from one another as $DCPA \geq D^{\text{safe}}$. The ships in Figure 8f keep relative larger distances comparing with the ships in Figures 8b and 8d.

The ships in Figure 8a finish collision avoidance operation with a sequence of Ship 3, Ship 5, Ship 6, Ship 7, Ship 2, Ship 1, Ship 4. The ships in Figure 8c finish collision avoidance operation with a sequence of Ship 5, Ship 3, Ship 6, Ship 7, Ship 2, Ship 1, Ship 4. The ships in Figure 8e finish collision avoidance operation with a sequence of Ship 3, Ship 6, Ship 5, Ship 7, Ship 2, Ship 1, Ship 4. In comparison with Figures 8a and 8e, the ships in Figure 8c finish the collision avoidance operation with a shorter time.

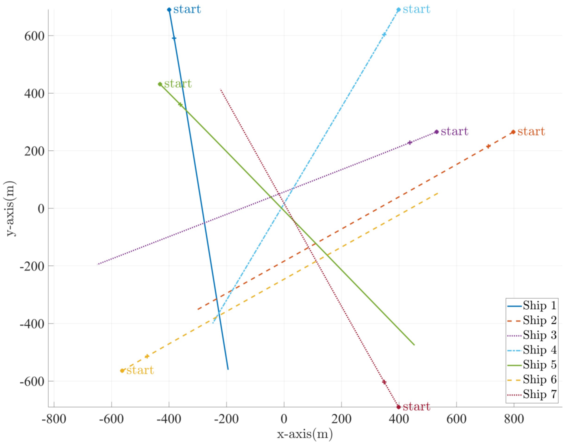
5.3. 12-ships encounter situation

Figure 9 presents a case in which 12 ships are encountering one another, considering both "head-on" and "crossing" situations simultaneously. Figure 9a shows the ships' simulated trajectories when they keep their original courses unchanged. Similar to the 7-ships case, the weight assigned to each ship in the optimization objective functions are constructed as follows:

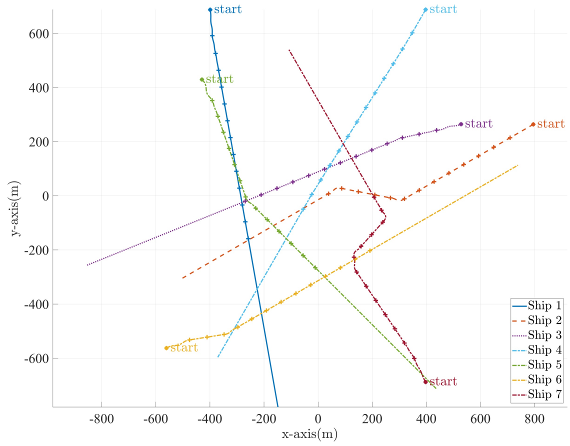
- Constructing weights w^1 in Obj_1: rank the ships according to the number of ships on its starboard side in ascending order: $w_5 = w_6 = w_{10} = 8, w_1 = w_8 = w_9 = 4, w_3 = w_4 = w_7 = 2, w_{11} = w_2 = w_{12} = 1$;
- Constructing weights w^2 in Obj_2: rank the ships according to the number of ships it has risks with in ascending order: $w_2 = w_{11} = w_4 = 1, w_5 = w_6 = w_8 = 2, w_9 = w_{10} = w_3 = 4, w_7 = w_1 = w_{12} = 8$;
- Constructing weights w^3 in Obj_3: all ships are in equal status and need to change courses, therefore $w_1 = w_2 = w_3 = w_4 = w_5 = w_6 = w_7 = w_8 = w_9 = w_{10} = w_{11} = w_{12} = 1$.

The simulated ship trajectories with different optimization objectives are given in Figures 6b, 6c and 6d. All the ships could successfully avoid collisions with the other ships. As can be seen from Figure 9a, Ships 5 and 7, Ships 8 and 9, as well as Ships 2 and 6 are in "head-on" situations. According to Rule 14 of COLREGs, they shall change their courses to starboard so that each shall pass on the port side of the other. This is obeyed by the proposed approach, as shown in Figures 9b, 9c and 9d.

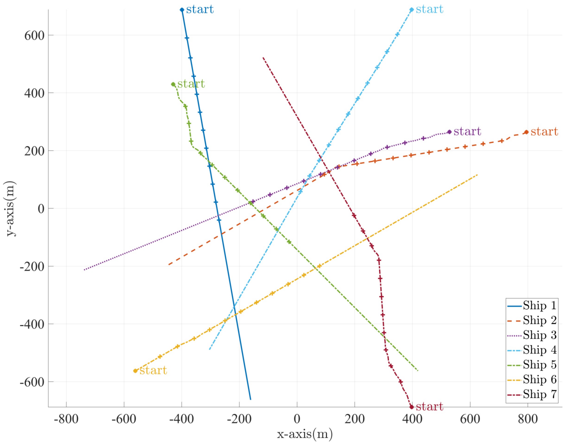
The ships' heading angles over time are given in Figure 10. Table 5 summarizes the largest heading angle deviation of each ship during their collision avoidance maneuvers. With optimization objective Obj_1, the ships can avoid collisions with each other with smaller heading angle deviations compared with objectives Obj_2 and Obj_3, as shown in Figure 10 and Table 5. Figures 11 and 12 present the DCPAs and TCPAs between ships. In general, the DCPAs between ships with optimization objective Obj_3 are larger than the ones with Obj_1 and Obj_2.



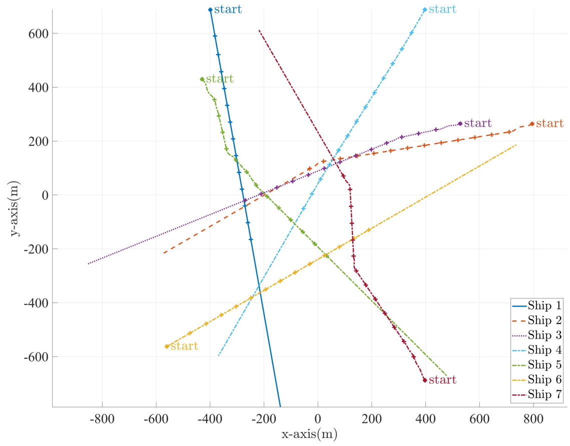
(a) Simulated trajectories with original courses unchanged in 7-ships case.



(b) Simulated ship trajectories with Obj_1 in 7-ships case.



(c) Simulated ship trajectories with Obj_2 in 7-ships case.

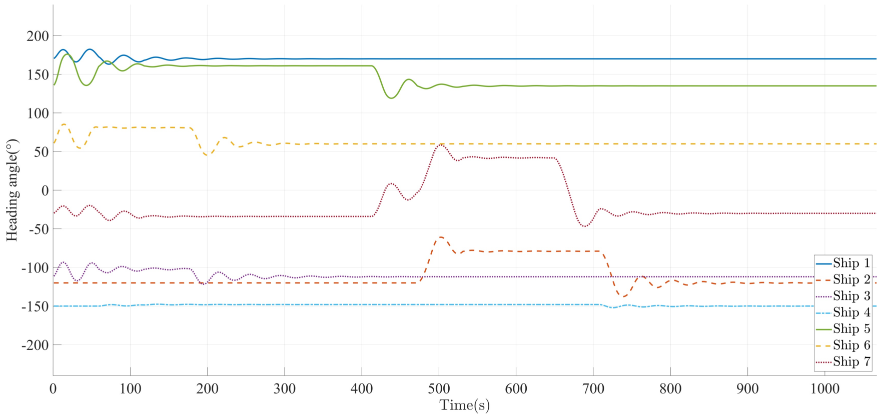


(d) Simulated ship trajectories with Obj_3 in 7-ships case.

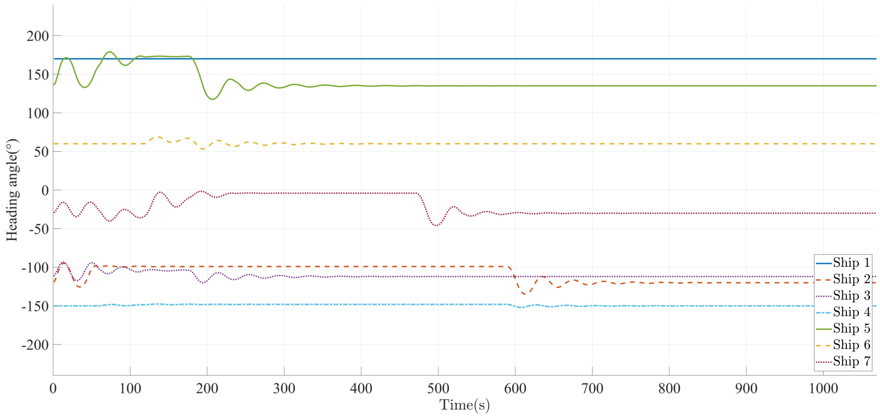
Figure 6: Simulated ship trajectories with different optimization objectives in 7-ships case.

Table 5: Largest heading angle deviations of each ship in all experiments compared with its original heading angles (/degrees) in 12-ships case.

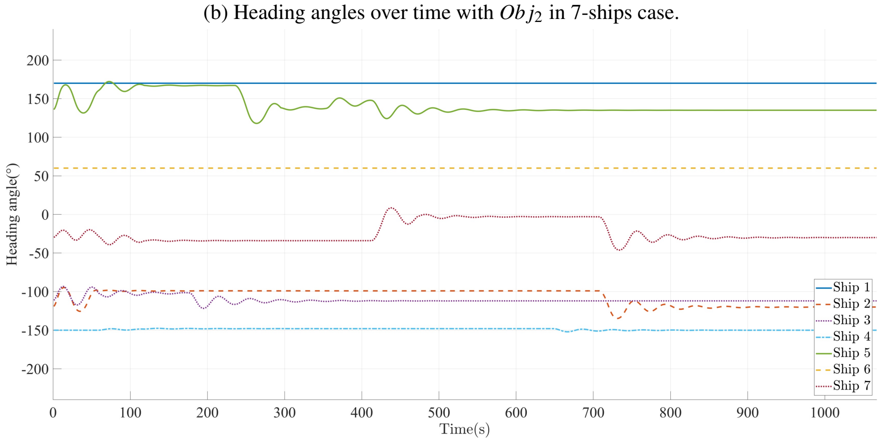
	Ship 1	**Ship 2**	**Ship 3**	**Ship 4**	**Ship 5**	**Ship 6**	**Ship 7**	**Ship 8**	**Ship 9**	**Ship 10**	**Ship 11**	**Ship 12**	**Avg.**
Obj_1	0	37	36	25	40	8	25	8	19	56	71	0	27.08
Obj_2	0	46	36	37	47	60	35	12	35	53	37	0	33.16
Obj_3	0	36	41	33	25	14	25	14	50	30	65	0	27.83



(a) Heading angles over time with Obj_1 in 7-ships case.

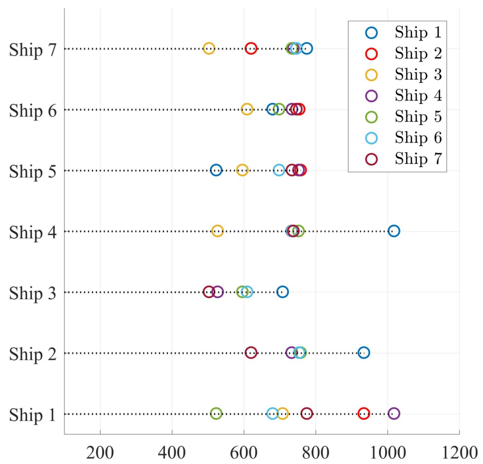


(b) Heading angles over time with Obj_2 in 7-ships case.

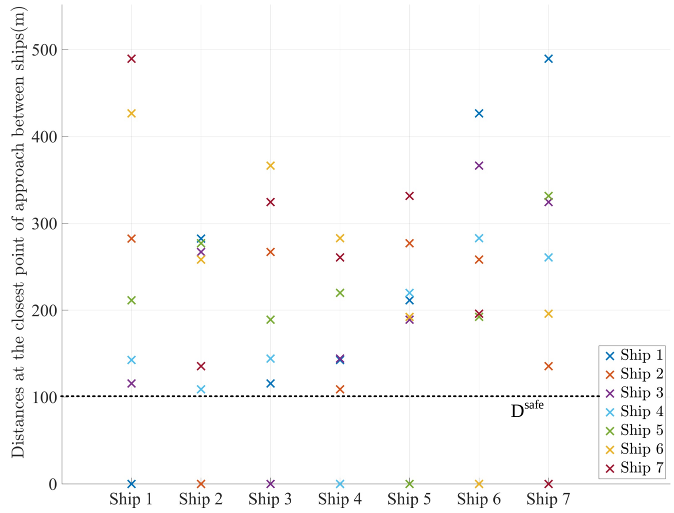


(c) Heading angles over time with Obj_3 in 7-ships case.

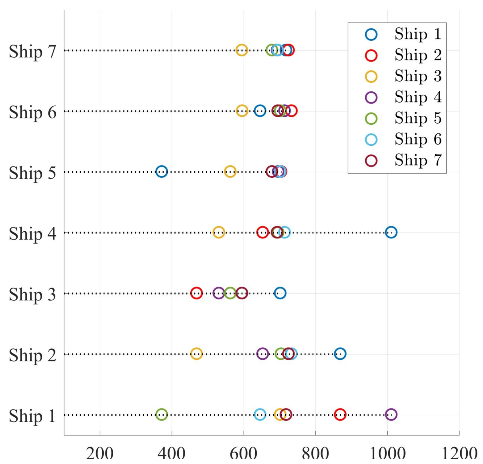
Figure 7: Simulated heading angles over time with different optimization objectives in 7-ships case.



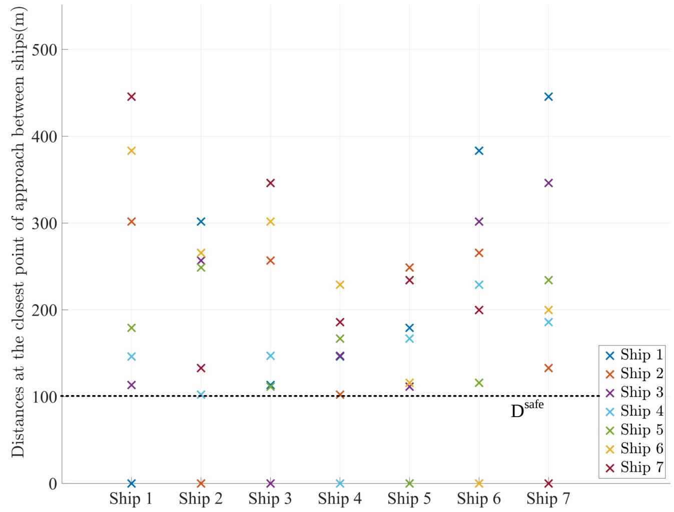
(a) TCPA between any two ships with Obj_1.



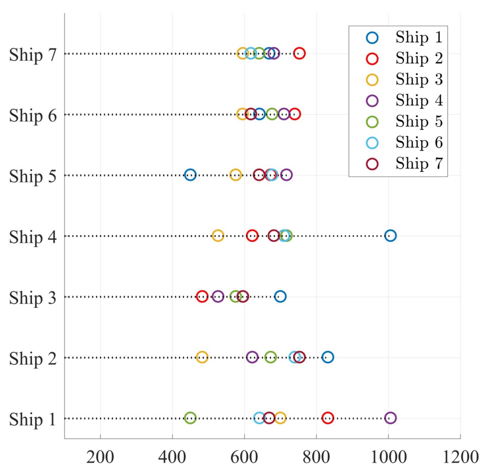
(b) DCPA between any two ships with Obj_1.



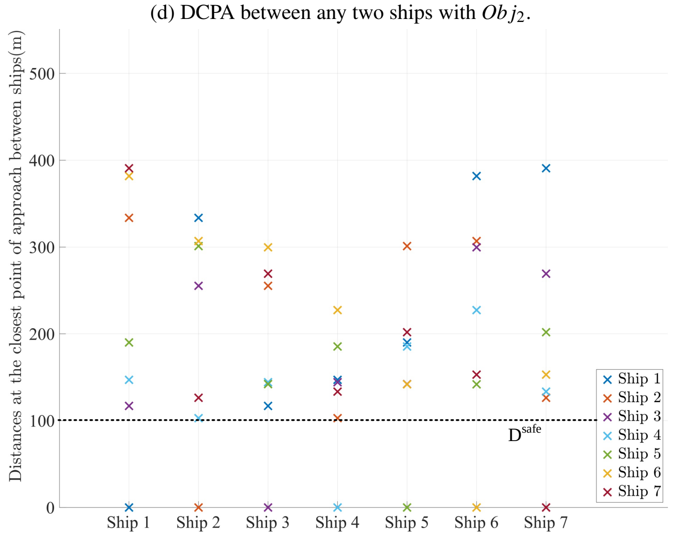
(c) TCPA between any two ships with Obj_2.



(d) DCPA between any two ships with Obj_2.

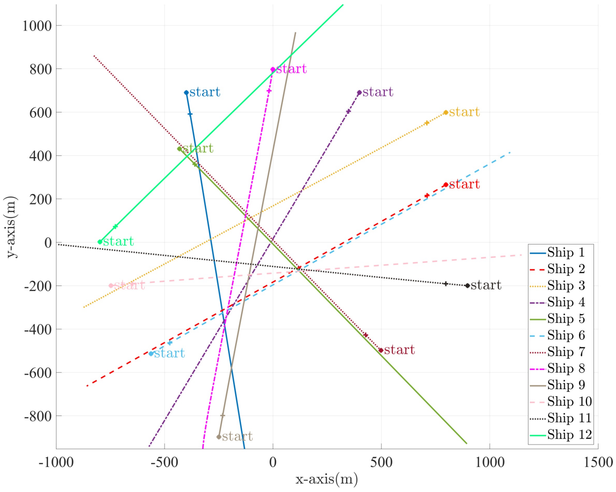


(e) TCPA between any two ships with Obj_3.

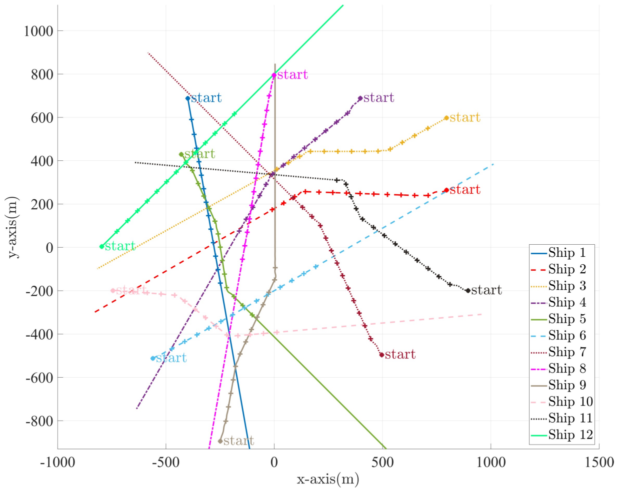


(f) DCPA between any two ships with Obj_3.

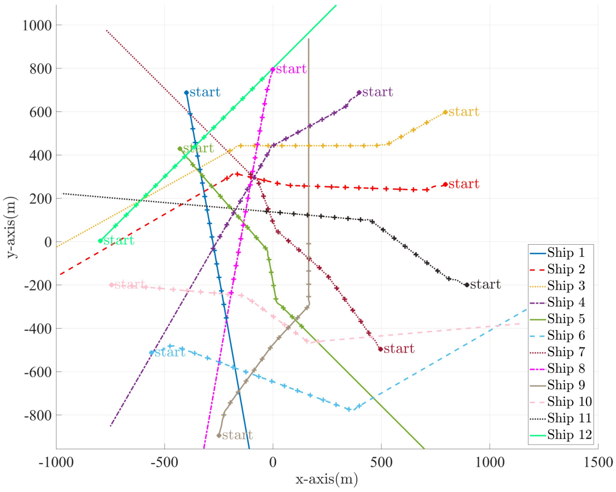
Figure 8: TCPAs and DCPAs between any two ships with different optimization objectives in 7-ships case.



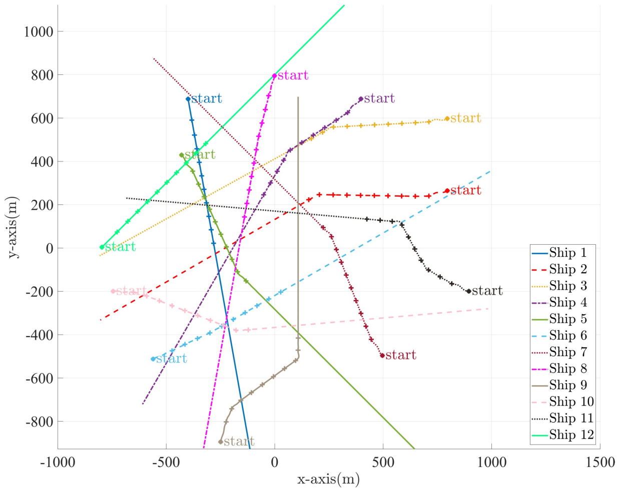
(a) Simulated ship trajectories with original courses.



(b) Simulated ship trajectories with Obj_1.

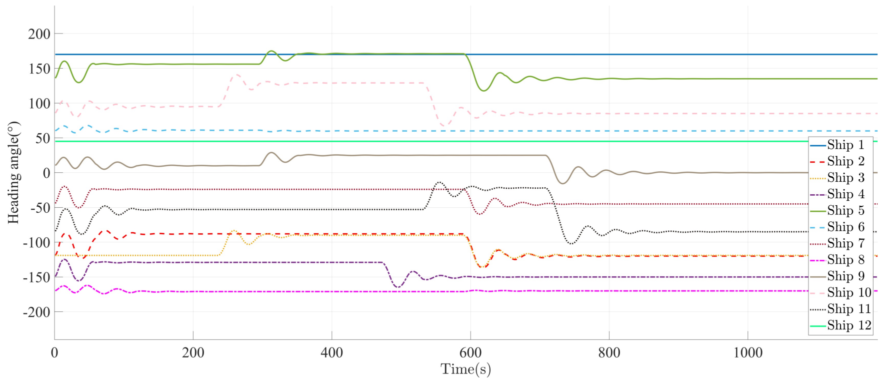


(c) Simulated ship trajectories with Obj_2.

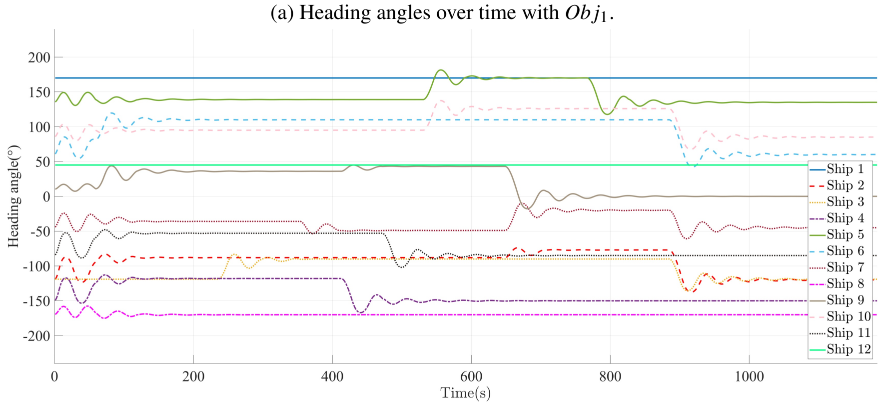


(d) Simulated ship trajectories with Obj_3.

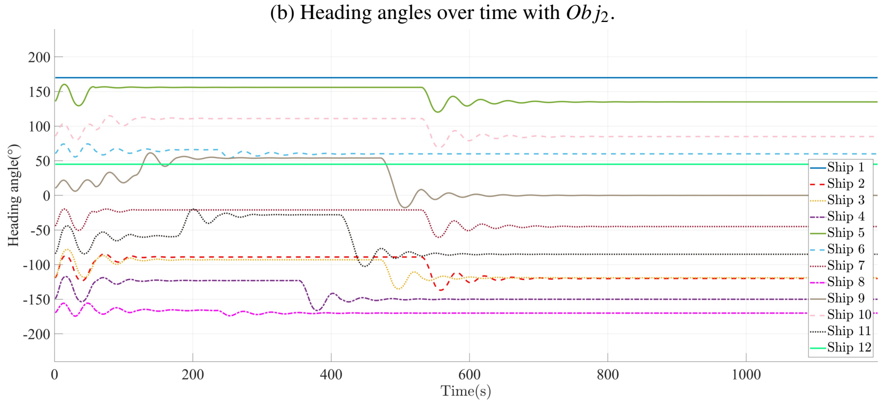
Figure 9: Simulated ship trajectories with different optimization objectives in 12-ships case.



(a) Heading angles over time with Obj_1.

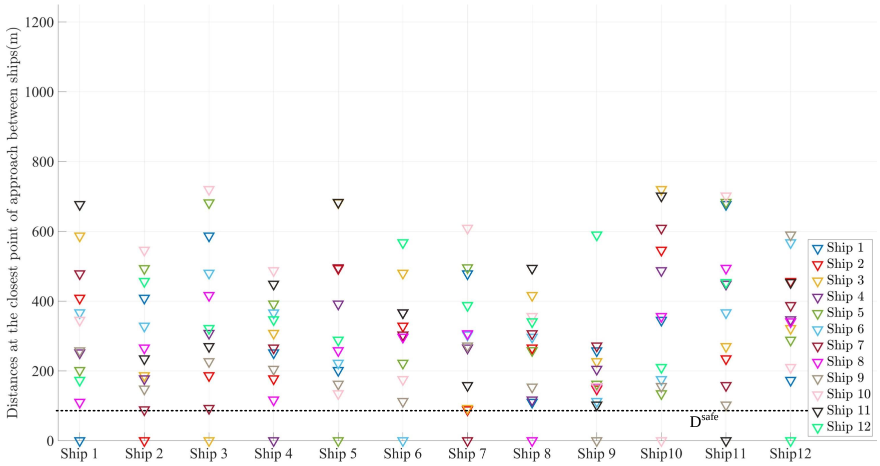


(b) Heading angles over time with Obj_2.

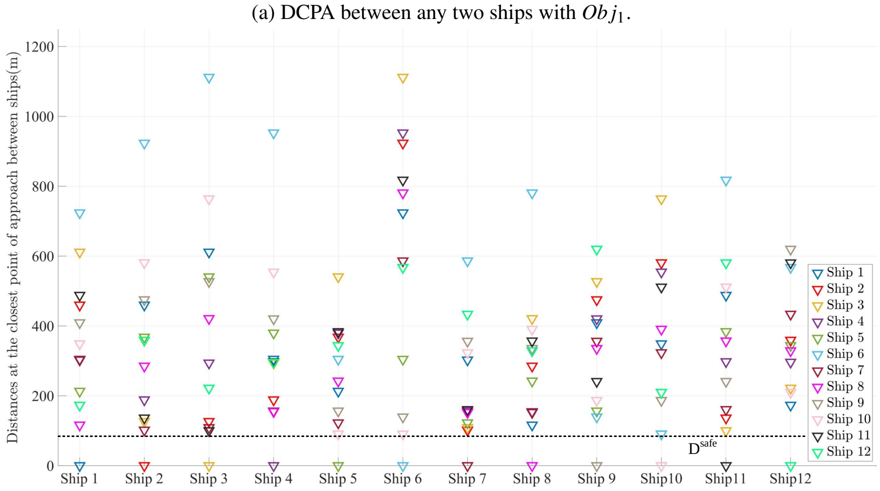


(c) Heading angles over time with Obj_3.

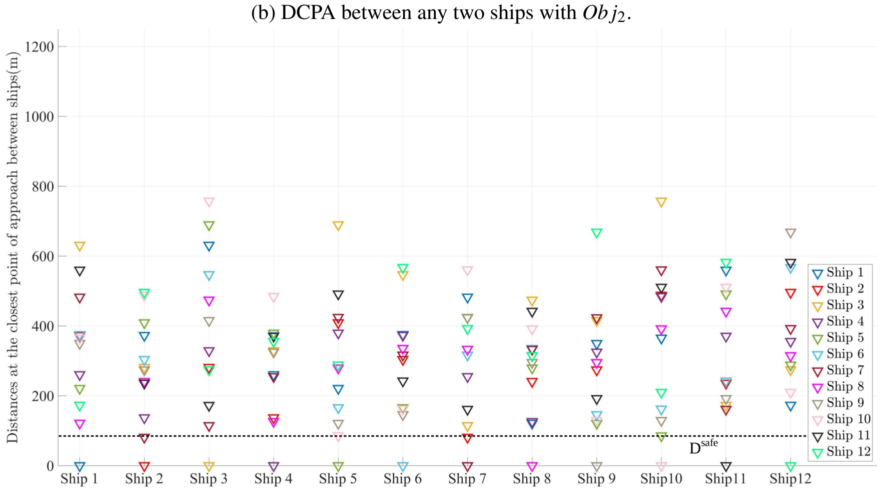
Figure 10: Simulated heading angle changes over time with different optimization objectives in 12-ships case.



(a) DCPA between any two ships with Obj_1.

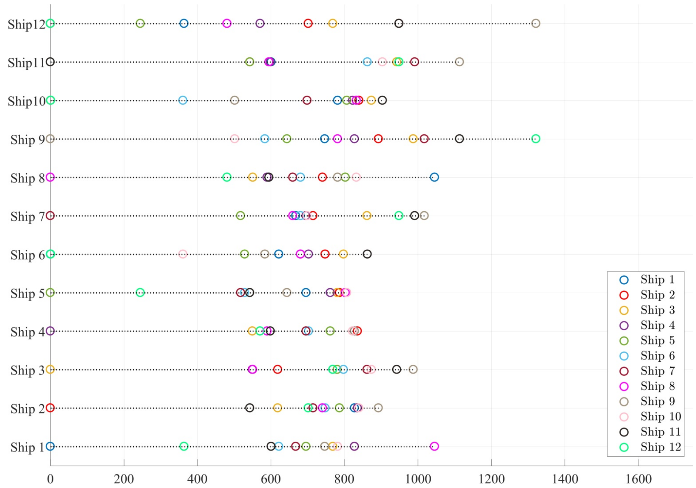


(b) DCPA between any two ships with Obj_2.

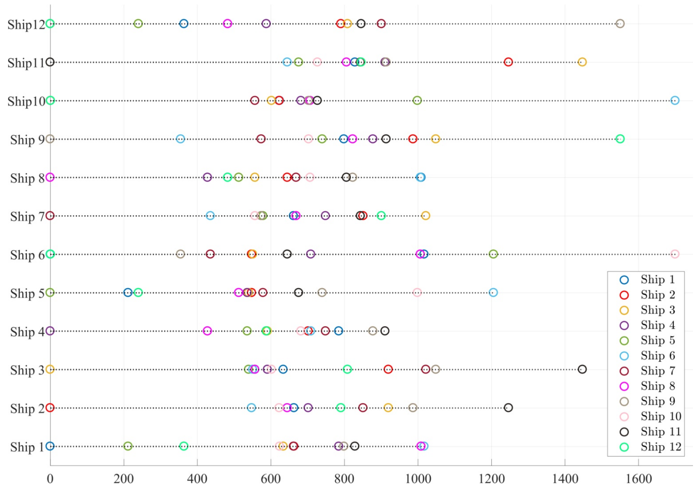


(c) DCPA between any two ships with Obj_3.

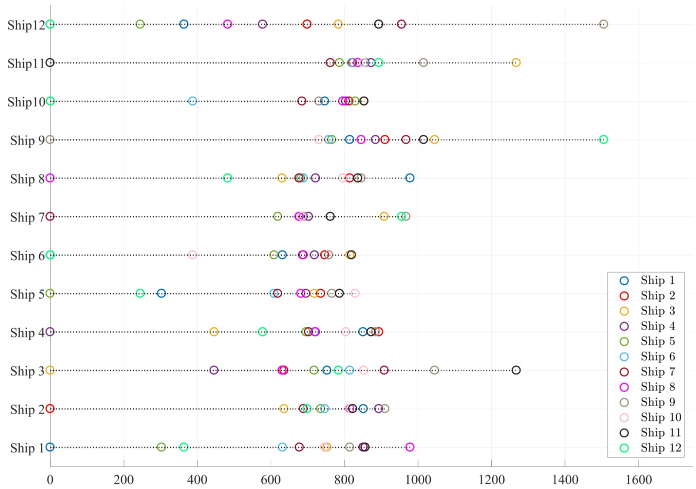
Figure 11: DCPAs between any two ships with different optimization objectives in 12-ships case.



(a) TCPA between any two ships with Obj_1.



(b) TCPA between any two ships with Obj_2.



(c) TCPA between any two ships with Obj_3.

Figure 12: TCPAs between any two ships with different optimization objectives in 12-ships case.

5.4. *Comparison of performances with different objectives*

In order to evaluate the efficiency of collision avoidance operations with different optimization objectives, we set up 10 scenarios in which 7 homogeneous ships are encountering one another, with different courses and coordinates. Table 13 summarizes the performance with respect to three types of performance indicators:

1. Heading angle deviation[1](%): the maximum, minimum and average ratio of the ship's heading deviations in each case with different objective value to the best-obtained results in that case.
2. Finish time of the last ship[2](%) : the maximum, minimum and average ratio of the time when the last ship involved in the set of ships passes the last CPA with the other ships in each case with different objective value to the best-obtained results in that case. In other words, it is the finish time of the collision avoidance operation of the last ship in the multiple ship system.
3. Finish time per ship[3](%): the maximum, minimum and average ratio of the set of ships' average ship's heading deviations in each case with different objective value to the best-obtained results in that case.

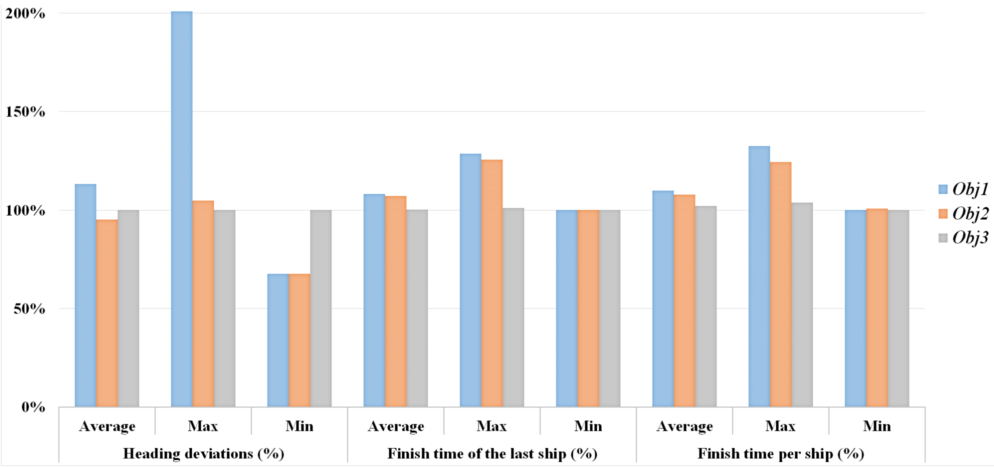


Figure 13: Comparison of performances with different objectives.

As can be seen from Figure 13, the ships could on average avoid collisions with smaller heading angle alterations with Obj_2, which means Obj_2 performs best in finding optimal heading angles with smaller deviations. Regarding the overall performance for multiple ships as a system on the finish time of collision avoidance operation, Obj_3 performs best, as the last ship in the system could go back to its original courses earlier, according to the average, maximum and minimum values. Moreover, Obj_3 also performs best in the individual performance per ship, as each ship could finish spend less time in collision avoidance operation. Therefore, it can be concluded from Figure 13 that with ship priories w^2, if the ship that has collision risks with more ships changes courses more substantially, the set of ships could finish collision avoidance earlier. Meanwhile, if all ships are in equal position and are given the same priority in changing heading angles from a global perspective, they could all finish the collision avoidance operation with a shorter time.

6. Conclusions and future work

This paper proposed an iterative optimization strategy for assisting ships in making optimal decisions on heading angle alterations to avoid collisions, when multiple ships encounter with one another. The proposed method

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[1]ratio of heading angle deviations with Obj_i= $\frac{(\sum_{m\in M}\Delta\varphi_m)\text{ with objective } Obj_i}{\text{Best value of }(\sum_{m\in M}\Delta\varphi_m)\text{ from }\{Obj_1,Obj_2,Obj_3\}}$

[2]ratio of finish time of the last ship with Obj_i= $\frac{(\max_{m\in M}(\max_{n\in M} TCPA_{mn}))\text{ with objective } Obj_i}{\text{Best value of }(\max_{m\in M}(\max_{n\in M} TCPA_{mn}))\text{ from }\{Obj_1,Obj_2,Obj_3\}}$

[3]ratio of finish time per ship with Obj_i= $\frac{(\sum_{m\in M}(\max_{n\in M} TCPA_{mn}))\text{ with objective } Obj_i}{\text{Best value of }(\sum_{m\in M}(\max_{n\in M} TCPA_{mn}))\text{ from }\{Obj_1,Obj_2,Obj_3\}}$

considers both the ship dynamics and the inter-related characteristic of the collision avoidance decision making among multiple ships. Meanwhile, a rolling horizon optimization perspective is taken, so that optimization can be carried out in a flexible way based on updated information. Moreover, the time costs and heading angle deviations for collision avoidance maneuvers are both considered via setting different optimization objectives. The compliancy with COLREGs is also considered in the model formulations. Simulation results show that the proposed method can provide ships with the optimal heading angles they could choose with different objectives, and finish collision avoidance operations efficiently. This contribute to increasing the safety and reliability of a ship's automated navigation, reduce the psychological and physical burden of ship operators, and reduce the occurrence of ship collisions.

There are several directions for future research. Firstly, this paper assumes that all ships are cooperative and will execute the optimal decisions. However, in reality, there may be uncooperative ships or dynamic obstacles that cannot be involved in the optimization model. Therefore, it is important to extend our optimization strategy to deal with such situations. Secondly, this paper adopts TCPA/DCPA as the main collision risk indicators, for future work it would be interesting to also consider other risk indicators and compare their performances. Thirdly, experiments are carried out based on simulated data in this paper, for validation with real-world instances, more experiments with real data are required. Last but not least, extensive experiments are also required for identification of the values to the weights assigned to each ship, in order to reach the best performance.

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