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Road safety of passing maneuvers: a bivariate extreme value theory approach under non-stationary conditions

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1 Abstract

Observed accidents have been the main resource for road safety analysis over the past decades. Although such reliance seems quite straightforward, the rare nature of these events has made safety difficult to assess, especially for new and innovative traffic treatments. Surrogate measures of safety have allowed to step away from traditional safety performance functions and analyze safety performance without relying on accident records. In recent years, the use of extreme value theory (EV) models in combination with surrogate safety measures to estimate accident probabilities has gained popularity within the safety community.

In this paper we extend existing efforts on EV for accident probability estimation for two dependent surrogate measures. Using detailed trajectory data from a driving simulator, we model the joint probability of head-on and rear-end collisions in passing maneuvers. We apply the Block Maxima method and estimate several extremal univariate and bivariate models, including the logistic copula. In our estimation we account for driver specific characteristics and road infrastructure variables. We show that accounting for these factors improve the head-on and rear-end collision probabilities estimation. This work highlights the importance of considering driver and road heterogeneity in evaluating related safety events, of relevance to interventions both for in-vehicle and infrastructure-based solutions. Such features are essential to keep up with the expectations from surrogate safety measures for the integrated analysis of accident phenomena, which show to significantly improve from the best known stationary extreme value models.

Keywords

Road safety; Small probability estimation; Block Maxima; Multivariate EV distribution; Passing maneuvers; Non-stationary model

2 Introduction

2.1 Motivation

Prediction of accidents has been a major topic in traffic safety for the last couple of decades. Despite the huge efforts that researchers have put in developing accident prediction models (Bonneson, 2010), there is a great tendency in the last decades to develop new proactive methods for safety evaluation that are not based on accident records (Archer, 2004; Kraay & Van der Horst, 1985). Evaluating conflicts and risky situations between road users has been the main alternative and multiple methodologies can be found in the literature: the Swedish traffic conflict technique (Hydén, 1987), DOCTOR method (Kraay & Van der Horst, 1985), and the use of surrogate safety measures (Archer, 2004). The main challenge is the link between these measures and the number of accidents. Zheng, Ismail, and Meng (2014) indicate that the validity of surrogate safety measures is usually determined by its correlation with accident frequency which is usually assessed using regression analysis. However, regression analysis still incorporates accident counts which are known to suffer from underreporting and quality issues, and thus this approach is limited. Besides, it is difficult to insure the stability of the accident-to-surrogate ratio, and this relationship also hardly reflects the physical nature of accident occurrence (Zheng et al., 2014). Therefore, there is a need to develop an alternative approach to predict the number of accidents based on surrogate safety measures. Songchitruksa and Tarko (2006) proposed a new and more sophisticated approach based on the Extreme Value (EV) theory to estimate the frequency of accidents based on measured accident proximity.

2.2 Extreme Value (EV) Approach

The EV approach has three considerable advantages over the traffic conflict technique: (a) it abandons the assumption of fixed ratio converting the surrogate event frequency into accident frequency; (b) accident risk given the surrogate event is estimated based on the observed variability of accident proximity without using accident data; (c) the accident proximity measure precisely defines the surrogate event.

The implicit assumption of the EV theory is that the stochastic behavior of the process being modeled is sufficiently smooth to enable extrapolation to unobserved levels (De Haan & Ferreira, 2006). In the context of road safety, the more observable traffic conflict events are used to predict the less frequent accidents, which are often unobservable in a short time period (Zheng et al., 2014). The field of EV theory, pioneered by Fisher and Tippett (1928), is a commonly applied theory in many fields, such as in meteorology, hydrology, finance (Zheng et al., 2014) and very recently, road safety analysis Songchitruksa and Tarko (2006). Songchitruksa and Tarko (2006) used an EV approach to build up relationships between occurrences of right-angle accidents at urban intersections and frequency of traffic conflicts measured by using post-encroachment time. A major improvement of the latter study is that it links the probability of accident occurrence to the frequency of conflicts estimated from observed variability of accident proximity, using a probabilistic framework and without using accident records. The generic formulation of the application of EV to road safety analysis was then proposed by Tarko (2012) and it was only very recently applied to other accident types and data sets (Jonasson & Rootzén, 2014; Zheng et al., 2014; Åsljung et al., 2017; Orsini et al., 2018; Wang et al., 2018). The formulation relies usually, but not exclusively, on time-based surrogate measures and estimates the probability of accident occurrence using the EV fitted distribution of such measures (see Appendix 1 for formulation details). These studies have relied mostly on univariate EV models, and only very recent studies have explored the use of multi(bi)-variate approaches: Jonasson and Rootzén (2014) applied the bivariate block maxima approach to study near-crashes selection bias in a naturalistic driving setting, focusing on the relationship of different surrogate measures in rear-end collisions. In a series of recent studies published during the revision of this manuscript, Zheng et al. (2018. 2019a, 2019b) explored the use of bivariate EV to combine multiple surrogate safety measures obtained from the same pair of vehicles to predict the probability of specific crash events. In Zheng et al. (2018) post encroachment time (PET) and length proportion of merging (LPM) are used in a bivariate threshold excess model to estimate collision probability in freeway merging scenario. Later, in Zheng et al. (2019a) focuses on finding the best pair of severity-specific surrogate safety measures to estimate the probability of crash at signalized intersections and validates it with actual crash data. Finally, Zheng et al. (2019b) compares univariate and bivariate GEV and GP for left turning vehicles at intersections. This application series provide a concerted evidence of higher accuracy of bivariate against univariate EV predictions.

In this paper, we extend the EV -based road safety application state-of-the-art with the first non-stationary bivariate and copula-based EV models as a way to formalize the dependencies in events resulting from two interlinked phenomenon's and illustrate it for the context of passing maneuvers.

2.3 Risk of Passing Maneuvers

Passing maneuvers on two-lane roads (one lane per travel direction) carries several types of risks. The process of passing involves, synchronizing the vehicle's speed with that of the vehicle in front, estimating the available gap on the opposite direction and evaluating its suitability to successfully perform the passing maneuver, and finally return to the main driving lane while keeping a sufficient safe gap from the passed vehicle, as well as, from the vehicle on the opposite direction. The gap from the passed vehicle at the end of the passing maneuver is termed in this study as 'THW'. It reflects the time headway between the front of the passed vehicle and the rear of the passing vehicle – a measure for rear-end and side-collisions with the passed vehicle[1]. The gap from the vehicle on the opposite direction is termed in this study 'TTC' for time-to-collision between the passing and the opposite vehicle – a measure for head-on collisions. Both of these gaps are calculated at the end of the passing maneuver. In this study both measures will be used: the THW was calculated as the remaining distance between the passing and passed vehicle at the end of the passing maneuver divided by the driving speed of the passed vehicle, while the TTC was calculated as the remaining distance between the passing and opposing vehicle divided by the sum of their speeds.

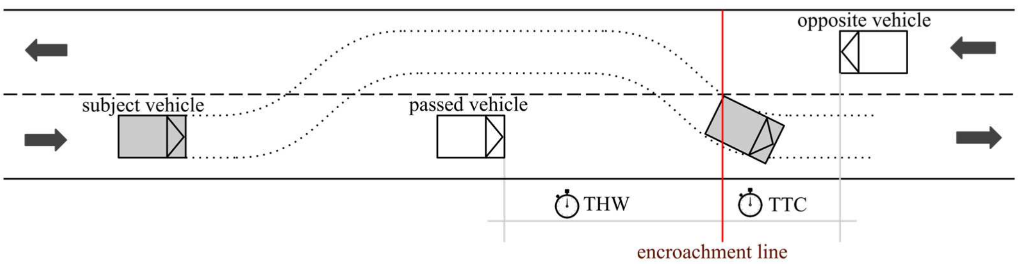

Figure 1 Schematic figure of maneuver and surrogate safety measures of interest

2.4 Drivers' Characteristics

Several studies have shown that there are significant differences in passing behaviors among different drivers. Farah (2011) using a driving simulator found that gender and age have a significant impact on the passing behavior. She found that male drivers pass more frequently than female drivers. They also maintain smaller following time gaps from the front vehicle before initiating a passing maneuver and accept shorter gaps in the opposite traffic for

[1] Note that the THW measure used in this paper is equal to the PET during a lane change maneuver between the merging subject vehicle and the following passed vehicle in the target destination lane. It is defined as the time difference between the end of the subject vehicle leaving the encroachment line and the front of the passed vehicle arriving at the encroachment line. Similarly to our case, the encroachment line is a virtual line perpendicular to the lane dividing marker and crossing the intersection point of the lane dividing marker and the lane change trajectory (Zheng et al., 2014).

passing. Younger drivers have significantly lower critical gaps and higher desired driving speeds compared to older drivers. They also keep smaller gaps from the front vehicle at the end of the passing maneuvers. These behaviors increase the risk of collisions. Llorca, Garcia, Moreno, and Perez-Zuriaga (2013) reached similar conclusions using an instrumented vehicle. The authors found that young male drivers have shown a more aggressive behavior when passing compared to other groups of drivers. Passing times were around 1s lower than other drivers, while average speed difference was 4 km/h higher. Farah, Polus, Bekhor, and Toledo (2007) tested the significance of including driving styles in the passing behavior model, and found that drivers who are characterized by an anxious driving style and/or patient and careful driving style have larger critical gaps. Vlahogianni and Golias (2012) emphasize that the behaviors of young male and female drivers during passing maneuvers are different and this is because of differences in the process of scanning and evaluating available opportunities for passing.

To summarize, the integration of drivers' characteristics and driving styles in accident prediction is valuable and have the potential to contribute to understanding accident causation. Previous EV models did not account for such factors.

3 Research Method

The aim of this study is to test two different methods to estimate accident probability in passing maneuvers. The first approach analyzes the risk of individual types of accidents during passing maneuvers, including: (1) head-on collisions using the proximity measure of the minimum TTC to the vehicle in the opposite direction; (2) rear-end collisions using the proximity measure of the minimum THW measured from the front of the passed vehicle to the rear-end of the passing vehicle. The second approach aims to analyze the joint risk of colliding with the opposite or passed vehicle during passing maneuvers using the two surrogate safety measures (THW and TTC).

3.1 Modeling Approach

There are two families of EV distributions which follow two different approaches to sample extreme events: (1) the Generalized Extreme Value (GEV) distribution which is used in the block maxima or minima (BM) approach, in which maxima over blocks of time (or space) are considered; (2) the Generalized Pareto (GP) distribution which is used in the peaks over threshold (POT) approach (Coles, Bawa, Trenner, & Dorazio, 2001), where all values above a certain threshold are used. In this paper we focus our attention on the application of the BM approach for estimating the risk of a single type of accident (head-on or rear-end collision), while for estimating the risk of both types of collisions jointly, the bivariate distribution with copula approach was considered. We opted for the BM in favor of the POT following, (a) our previous findings in terms of stability for the univariate case (Farah and Lima Azevedo, 2016), (b) weaker consistency in results from POT trials considered at an earlier stage of this research (c) BM favorable conditions in case of weak dependence and (d) that a detailed comparative study between BM and POT in a multivariate setting is out of scope for the present manuscript although much needed (Büncher and Zhou, 2018). Note that the univariate POT is well known and has been extensively discussed (De Haan & Ferreira 2006) and the estimation process to go to the bivariate POT is much similar to the case of going from the univariate to bivariate BM discussed below and also be found in recent applications (see for example Zheng & Sayed 2019).

3.2 The BM Approach

Mathematically, the standard GEV function is as follows (von Mises, 1936; Zheng et al., 2014):

$$G(x) = \exp\left(-\left[1+\xi\left(\frac{x-u}{\sigma}\right)\right]_{+}^{\frac{-1}{\xi}}\right) \quad \text{(Eq. 3)}$$

Following the Block Maxima approach, the observations are aggregated into fixed intervals over time or space, and then the extremes are extracted from each block by identifying the maxima in each single block. If $\{X_1, X_2, \ldots, X_n\}$ is a set of independently and identically distributed random observations with unknown distribution function $D(x) = Pr(X_i \leq x)$, the linearly normalized maximum $\mathrm{M_n} = \max\{\mathrm{X_1}, \mathrm{X_2}, \ldots, \mathrm{X_n}\}$ will converge to a GEV distribution when $n \to \infty$. Three parameters identify this distribution: the location parameter, $-\infty < u(\boldsymbol{z}) < \infty$*;* the scale parameter, $\sigma > 0$; and the shape parameter, $-\infty < \xi < \infty$. If the shape parameter, ξ, is positive, then this would yield the Frechet Cumulative Distribution Function (CDF) with a finite lower endpoint, $(u - \sigma/\xi)$, if ξ is negative, this will yield the (reversed) Weibull CDF with finite upper endpoint $(u + \sigma/|\xi|)$, and if $\xi = 0$ this yields the Gumbel CDF. In a non-stationary BM model several factors, $\boldsymbol{z}$, can be included in the location parameter to account for their impact on the probability of the extreme events, i.e. $u(\boldsymbol{z})$. More details on the GEV properties can be found in (Tarko, 2012). Details on the statistical properties of EV can be found in (Coles et al., 2001; Dombry & Ferreira, 2018), and on the theoretical background of its applicability for surrogate (road) safety analysis in (Tarko, 2012).

3.3 Parametric Bivariate EV distributions

In some applications, the study of accident probability using multivariate distributions is of interest. Traditionally, single surrogate safety measures are used to estimate a single type of events. However, it is expected that in some of the complex accident phenomena, multiple pre-accident events can play an important role in a potential accident.

Passing maneuvers are a typical case where both the opposite and passed vehicles are key stimulus during driver's decision making. The bivariate model aims to estimate a measure of risk that not only takes into account the possibility to collide with the opposite vehicle but also with the passed vehicle.

Given a bivariate random sample $(X_1, Y_1), \dots (X_n, Y_n)$, much of extreme value theory is concerned with the limiting behavior of a suitable normalization of the component wise maxima $(M_{1,n}, M_{2,n})$, where $M_{1,n} = \max(X_1, \dots, X_n)$ and $M_{2,n} = \max(Y_1, \dots, Y_n)$. More precisely, it is assumed that there exists a non-degenerate bivariate distribution function BG such that, as $n \to \infty$:

$$P\left\{\frac{M_{1,n} - b_{1,n}}{a_{1,n}} \leq x, \frac{M_{2,n} - b_{2,n}}{a_{2,n}} \leq y\right\} \to BG(x, y) \qquad \text{(Eq. 4)}$$

for sequences $a_{j,n} > 0, b_{j,n} \in \mathbb{R},\ j = 1,2$ (Capéraà & Fougères, 2000). To analyze separately the behavior of the marginals and the dependence structure of the distribution, it is convenient to write

$$BG(x, y) = C\{F(x), G(y)\} \qquad \text{(Eq. 5)}$$

in terms of univariate extreme value margins F and G, and a dependence function C (Capéraà & Fougères, 2000; De Haan & Ferreira 2006) defined for all $0 \leq w, v \leq 1$ by:

$$C(w, v) = P\{F(X) \leq w, G(Y) \leq v\} = \exp\left[\log(w\ v)\ A\left\{\frac{\log(w)}{\log(w\ v)}\right\}\right] \qquad \text{(Eq. 6)}$$

where $A(\cdot)$ is a convex function on $[0,1]$ such that $\max(t, 1 - t) \leq A(t) \leq 1$ for all $0 \leq t \leq 1$ (Pickands, 1981). Given this representation and except for the margins, the bivariate extreme value distribution BG for component wise maxima is characterized by a one-dimensional function $A(\cdot)$. Common used functions are the logistic, asymmetric logistic, negative logistic, asymmetric negative logistic, bilogistic and negative bilogistic. These extreme-extreme value specification for the dependence function along with common assumptions on the distribution for both marginals (e.g. Gumbel, Exponential, Fréchet or Weibull) allow for the formulation of well-defined parametric models. Information on the statistical properties of the estimation methods that have been developed in the context of bivariate EV can be found in (Capéraà & Fougères, 2000; Pickands, 1981). The few recent bivariate models found in the literature and described in Section 2 follow this approach (as, for example, the logistic case in Zheng & Sayed, 2019b). Extending this line of research, here we estimate a large set of stationary and non-stationary parametric EV models' using the maximum likelihood (ML) method in R (v3.0.3) using the `exTremes` and `evd` packages (Gilleland and Katz, 2011).

3.4 Copula-based Bivariate distributions

Here, we focus our attention on the bivariate distribution with a copula method as alternative approach due to the uncertain form of dependence (not necessary linear) between the two surrogate safety measures. While in the previous section, well defined parametric bivariate EV models rely on a clear predefined structure of $\mathrm{C}(\cdot)$, $\mathrm{F}(\cdot)$ ang $\mathrm{G}(\cdot)$, the Sklar's Theorem (1959) ensures that it is possible to estimate a multivariate distribution by separately estimating the marginal distributions and the copula separately. As in eq. 6, the copula $\mathrm{C}(\cdot)$ is a multivariate distribution whose margins $\mathrm{F}(\cdot)$ and $\mathrm{G}(\cdot)$ are all uniform over (0,1), and can be defined as:

$$\mathrm{C}(\mathrm{w},\mathrm{v}) = \mathrm{P}\{\mathrm{F}(\mathrm{X}) \leq \mathrm{w}, \mathrm{G}(\mathrm{Y}) \leq \mathrm{v}\} \qquad \text{(Eq. 7)}$$

The copula not only provides a structure for the dependence between the two variables but also reveals itself to be invariant under strictly monotone transformations. Using copulas allows to test and select different distributions for the marginals and for the copulas in a two-stage statistical procedure, thus relaxing some of the predefined assumptions needed for the parametric multivariate estimation case. In our particular overpassing event, here represented by the two interlinked phenomenona's (relative trajectories of subject-passed and subject-opposite vehicles), the dependence structure of the two surrogate safety measures is unclear and the assumption of strong extremal dependence may not be valid. For surrogate safety measures computed for the same phenomenon (as in the existing literature mentioned above) the dependence function may have a clearer dependence structure as the measures come from the same vehicle trajectory pairs. The two most frequently used copula families are elliptical copulas, extreme-value copulas, Archimedean and polynomial copulas. More details on these copula families can be found in Fang, Kotz, and Ng (1990), Nelsen (2007), and Genest and Rivest (1993). To assess if a given copula is well fitted to the data under analysis, a goodness-of-fit test is performed based on statistics such as the rank-based versions of the Cramer-von Mises or the Kolmogorov-Smirnov. An example of goodness-of-fit testing overview are given in Berg (2009). In this study, all copula models' were estimated in R (v3.0.3) using the package `VineCopula` (Brechmann & Schepsmeier, 2013) and `copula` (Yan, 2007).

3.5 Data Collection

The data for this study was obtained from a driving simulator experiment developed by Farah, Bekhor, and Polus (2009) for modelling drivers' passing behavior on two-lane rural highways. In this experiment the STISIM (Rosenthal, 1999) driving simulator was used. STISIM is a fixed-base interactive driving simulator, which has a 60° horizontal and 40° vertical display. The driving scene was projected onto a screen in front of the driver with a rate of 30 frames per second. A total of 16 simulator scenarios were designed in order to have a better understating of how different infrastructure and traffic related factors affect drivers' passing behavior. The 16 different scenarios are the result of an experimental design that included 4 factors in 2 levels, which are: the speed of the front vehicle (60 or 80 km/h), the speed of the opposite vehicle (65 or 85 km/h), the opposite lane traffic volume (200 or 400 veh/h), and the road curve radius (300-400 or 1500-2500 m). However, all the scenarios were composed of 7.5 km of two-lane rural highway section with no intersections, and good weather conditions. Each driver drove 4 scenarios out of the 16 scenarios which were selected following a partial confounding method that was adopted (Hicks & Turner). A more detailed information about this experiment can be found in (Farah, 2013; Farah et al., 2009).

A total of 100 drivers (64 males and 36 females) with at least 5 years of driving experience participated in the driving simulator experiment on a voluntary base. 67 drivers are with an age between 22 and 34 years old, 20 drivers with an age between 35 and 49 years old, and the remaining 12 with an age between 50 and 70 years old.

Prior to participating in the driving simulator experiment each driver filled a questionnaire composed of two parts: the first part included questions on the driver personal characteristics (including questions such as: gender, age, and driving experience), while the second part included the Multidimensional Driving Style Inventory (MDSI) developed by Taubman Ben-Ari et al. (Taubman-Ben-Ari, Mikulincer, & Gillath, 2004). The MDSI is a 6-point scale, which consists of 44 items that are used to characterize four factors that represent different driving styles: (1) *Reckless and careless driving style*, which refers to deliberate violations of safe driving norms, and the seeking of sensations and thrills while driving. It characterizes persons who drive at high speeds, race in cars, pass other cars in no-passing zones, and drive while intoxicated, probably endangering themselves and others; (2) *Anxious driving style*, which reflects feelings of alertness and tension as well as ineffective engagement in relaxing activities during driving; (3) *Angry and hostile driving style*, which refers to expressions of irritation, rage, and hostile attitudes and acts while driving, and reflects a tendency to act aggressively on the road, curse, blow horn, or "flash" to other drivers, and (4) *Patient and careful driving style*, which refers to planning ahead, attention, patience, politeness, and calmness while driving, as well as obedience to traffic rules. Factor scores were calculated for each respondent on each of these four driving styles.

4 Results and Analysis

The data set from the driving simulator experiment resulted in a total of 1287 completed passing maneuvers, 9 head-on collisions and 2 rear-end collisions. The detailed vehicle movement from the simulator was processed to obtain the two surrogate safety measures of interest at the end of passing maneuvers: the TTC with the opposing vehicles and the THW between the passed and passing vehicles.

4.1 Univariate Model

In the univariate model, a separate distribution was fitted to the minimum TTC and THW measurements resulting from the1287 passing maneuvers. In the GEV approach, each passing maneuver is represented by one block for which we take its minima for each of the surrogate safety measures considered. Note that accident observations were not used in the estimation procedure. GEV models pertain to continuous random variables that give zero mass to any real value, hence to zero. But accidents do happen and can be recorded with zero value with positive mass. The continuous random variables do not take such values into account and, thus, recorded accidents were not considered in the estimation dataset.

4.1.1 Head-on collisions

Aiming at estimating the probability of a head-on collision for a single passing maneuver, the minimum TTC was considered as a head-on accident surrogate measure. The data was then filtered to account only for values smaller than 1.5s (Hydén, 1987; Jonasson & Rootzén, 2014; Vogel, 2003), leading to a total of 463 observations. Knowing that 9 maneuvers ended with actual head-on collisions, the empirical probability of a head-on collision in a passing maneuver given that a critical TTC (i.e. TTC lower than 1.5 s), is 9/(463+9)=0.0191, with 95% binomial confidence interval (0.0089,0.0366). Note that different filtering conditions were also tested in estimation (see Appendix 2).

We start with an existing stationary BM model developed by Farah and Lima Azevedo (2016). The authors estimated that the parameters of the univariate GEV cumulative distribution function are $\hat{\mu} = -0.993\ (0.0212)$, $\hat{\sigma} = 0.0383\ (0.0163)$ and $\hat{\hat{\xi}} = -0.236\ (0.0500)$. Figure 1. presents the probability density function of the empirical and modeled negated TTC (*upper left*) and the simulated QQ plot (*upper right*). This model was then upgraded by the authors to a non-stationary BM model. They concluded that the covariates '*passinggap'*, '*tailgatetp*', '*speedfront*', '*curvature'* as defined below, related to the infrastructure and traffic, significantly contribute to the prediction of the probability of a head-on-collision during a passing maneuver. While the covariate '*speedpv*' was not found to be significant. Variables related to drivers' personal characteristic (gender, age, and driving style) were not tested. In this study, we will test whether drivers' personal characteristic significantly contribute to the model in addition to the traffic and road variables. The variables are defined as following:

- *passinggap*: is defined as the time gap between the opposing vehicle and the subject vehicle, at the time that the lead vehicle encounters the subject vehicle;
- *tailgatetp:* time gap between the subject vehicle and the front vehicle at the moment of start passing (s);
- *speedfront*: speed of the front vehicle at the moment of start passing (m/s);
- *curvature*: road curvature (1/m);
- *speedpv*: speed of the passing vehicle (m/s);
- *gender:* gender of the driver (1-male; 0-female)*;*
- *age:* categorical variable, with ranges 22-34; 35-49 and 50-70;
- *drivingstyle:* angry & hostile; anxious; reckless & careless; patient & careful (Taubman-Ben-Ari et al., 2004);

A driver was considered to have started the overtaking maneuver when the front left wheel crossed the centerline and to have completed the overtaking maneuver when the rear left wheel crossed the road centerline (Farah, 2016). A set of a non-stationary models considering different combinations of covariates were estimated. Table 1 presents the four best non-stationary models with a range of likelihood ratio p-value between 2.773×10^{-9} and 1.931×10^{-8} (Model #1 to #4). The estimated likelihood ratio tests are shown in Table 2. The previously estimated non-stationary model by Farah and Lima Azevedo (2016), presented as Model #0, is used as a benchmark for assessing the performance of the other models.

Table 1 Estimation results of the non-stationary BM approach for head-on collisions

Non-stationary model	#0	#1	#2	#3	#4
	Est. (Std. Error)	Est. (Std. Error)	Est. (Std. Error)	Est. (Std. Error)	Est. (Std. Error)
$\hat{\mu}_0$	-1.045 (0.137)	-0.983 (0.139)	-0.927 (0.145)	-0.953 (0.145)	-1.107 (0.139)
$\hat{\mu}_1$ (speedFront)	0.024 (0.006)	0.026 (0.006)	0.027 (0.006)	0.025 (0.006)	0.027 (0.006)
$\hat{\mu}_2$ (tailgatetp)	0.002 (0.002)	0.003 (0.002)	0.003 (0.002)	0.003 (0.002)	0.003 (0.002)
$\hat{\mu}_3$ (passinggap)	-0.022 (0.004)	-0.023 (0.004)	-0.023 (0.004)	-0.023 (0.004)	-0.023 (0.004)
$\hat{\mu}_4$ (curvature)	-33.653 (13.519)	-34.304 (13.419)	-34.068 (13.403)	-34.090 (13.488)	-34.139 (13.397)
$\hat{\mu}_5$ (Gender)	-	-0.097 (0.042)	-0.080 (0.043)	-	-
$\hat{\mu}_6$ (Angry&Hostile)	-	-	-0.021 (0016)	-0.029 (0.015)	-
$\hat{\mu}_7$ (F2234)	-	-	-	-	0.116 (0.044)
$\hat{\sigma}$	0.3639 (0.0145)	0.3616 (0.0143)	0.3607 (0.0142)	0.362 (0.014)	0.361 (0.014)
$\hat{\xi}$	-0.2196 (0.042)	-0.2176 (0.0413)	-0.216 (0.041)	-0.216 (0.041)	-0.217 (0.041)
Neg.LL	208.6541	206.0598	205.2183	206.8908	205.2379

Analyzing the results presented in Table 1, it is concluded that the inclusion of *Gender* (model #1) improves the accuracy of the model when compared to the non-stationary model (#0). The significance of this variable is given by the p-value of the likelihood ratio test, which is equal to 0.023 as presented in Table 2, with 95% confidence level.

Table 2 Likelihood Ratio Test (and p-value) for the non-stationary BM models for head-on collisions

Model	#0	#1	#2	#3	#4
#0	-				
#1	5.189 (0.023)	-			
#2	6.872 (0.032)	1.683 (0.194)	-		
#3	3.527 (0.060)	-1.662 (1.000)	3.345 (0.067)	-	
#4	6.832 (0.008)	1.644 (2.2E-16)	0.039 (0.843)	3.306 (2.2E-16)	-

The contribution of the variables representing driving styles (*Angry&Hostile*, *Anxious*, *Reckless&Careless* and *Patient&Careful*) was tested considering all the possible combinations of these variables besides the ones included in model #1. Comparisons between the different models were based on the likelihood ratio test. This procedure resulted in the inclusion of one driving style, *Angry&Hostile*, as presented in model #2. Analyzing the correlation between the different driving styles and the sociodemographic variables, a small but significant sample correlation of 0.29 was found between *Angry&Hostile* and *Gender*. For modelling purposes, and in order to test which variable among the two has a larger influence, the variable *Gender* was excluded from model #2, creating model #3. Comparing the results of models #1 to #3, it is concluded that the model that only includes *Gender* (model #1) has a better fit based on the p-value of the likelihood ratio test. Although *Angry&Hostile* could have higher explanatory power in other samples, a reason to prefer this model is the simplicity of collecting data on driver gender compared to drivers' driving styles, which requires the completion of the MDSI survey.

Aligned with the conclusions achieved by Farah (2011) and Llorca et al. (2013) regarding the impact of age, this variable (*age*) was found to improve the accuracy of the model when compared to the stationary model but turned out to have a non-significant contribution if gender is also included. After several attempts, we included the interaction variable between gender (female drivers, *Gender*) with age (range 22-34), *F2234*, and the final model (model #4) is shown in Table 1. This model considers a new variable that takes 1 if the driver is a female with age range between 22 and 34, and zero otherwise.

To estimate the probability of a head-on collision along with the conclusion about which model is the one with the better fit (models #1 and #4), two different approaches were considered. The first approach considers that the location parameter value is calculated using the covariates from the data, achieving the estimated probabilities of

0.0195 and 0.0198 for models 1 and 4, respectively, with 95% confidence level (0.0192; 0.0198) and (0.0195; 0.0201), respectively. These confidence intervals of estimation were computed assuming a normal distribution under regular parameters' conditions, a simulation experiment size of 1×10^6 and its simulated distribution quantiles. The second approach considers the estimation of the location parameters based on the estimation dataset, where normal distributions with means (standard deviations) of -0.989 (0.123) and -0.988 (0.125), for models #1 and #4, respectively were considered. The Kolmogorov-Smirnov test statistic of 0.0444 and 0.0479, respectively was achieved. This procedure simulates the values 0.0197 and 0.0202 for the probabilities of head-on collisions of models 1 and 4, respectively, with 95% confidence interval of (0.01939, 0.0199) and (0.0199, 0.0205). Comparing the probabilities of these two methods with the probability for a head-on collision assuming a near head-on collision in a passing maneuver of 0.0191, results in model #1 give a slightly better estimation compared to model #4 and the estimation performance is not significantly deteriorated.

According to the results of model #1 presented in Table 1, if the speed of the front vehicle (*speedfront*) increases, or if drivers start their passing maneuver from a larger gap from the front vehicle (*tailgatetp*), the negated TTC increases (corresponding to a decrease in the TTC). These are logical results since it is more difficult to end the passing maneuver if the front vehicle has a higher driving speed. Similarly, starting the passing maneuver from a larger gap from the front vehicle results in a longer time to finish the maneuver and consequently smaller TTC. If the passing gap (*passinggap*) that is accepted is larger, or the curvature of the road (*curvature*) is larger, the negated TTC is lower and the TTC is higher. This shows that drivers adapt their behavior in a passing maneuver if the road is too complex (i.e. sharp curves). Finally, male drivers have smaller TTC. This result is supported by previous studies (Farah et al., 2007; Vlahogianni & Golias, 2012), where it was found that male drivers usually drive faster, have shorter passing gaps, and conduct a higher number of passing maneuvers when compared to females.

The probability density function of the empirical and modeled standardized[2] maximum negated TTC and the simulated QQ-plot for the best non-stationary BM model (model #1) are shown in Figure 2. From these figures it can be concluded that the modeled GEV distribution has satisfactory fitting results to the empirical data.

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[2] For non-stationary models, it is common practice to transform the data to a density function that does not depend on the covariates, using the following function $Z_i = -\log\left(1 + \frac{\xi}{\sigma} \times (X_i - \mu_i)\right)^{-\frac{1}{\xi}}$, Gilleland and Katz (2011).

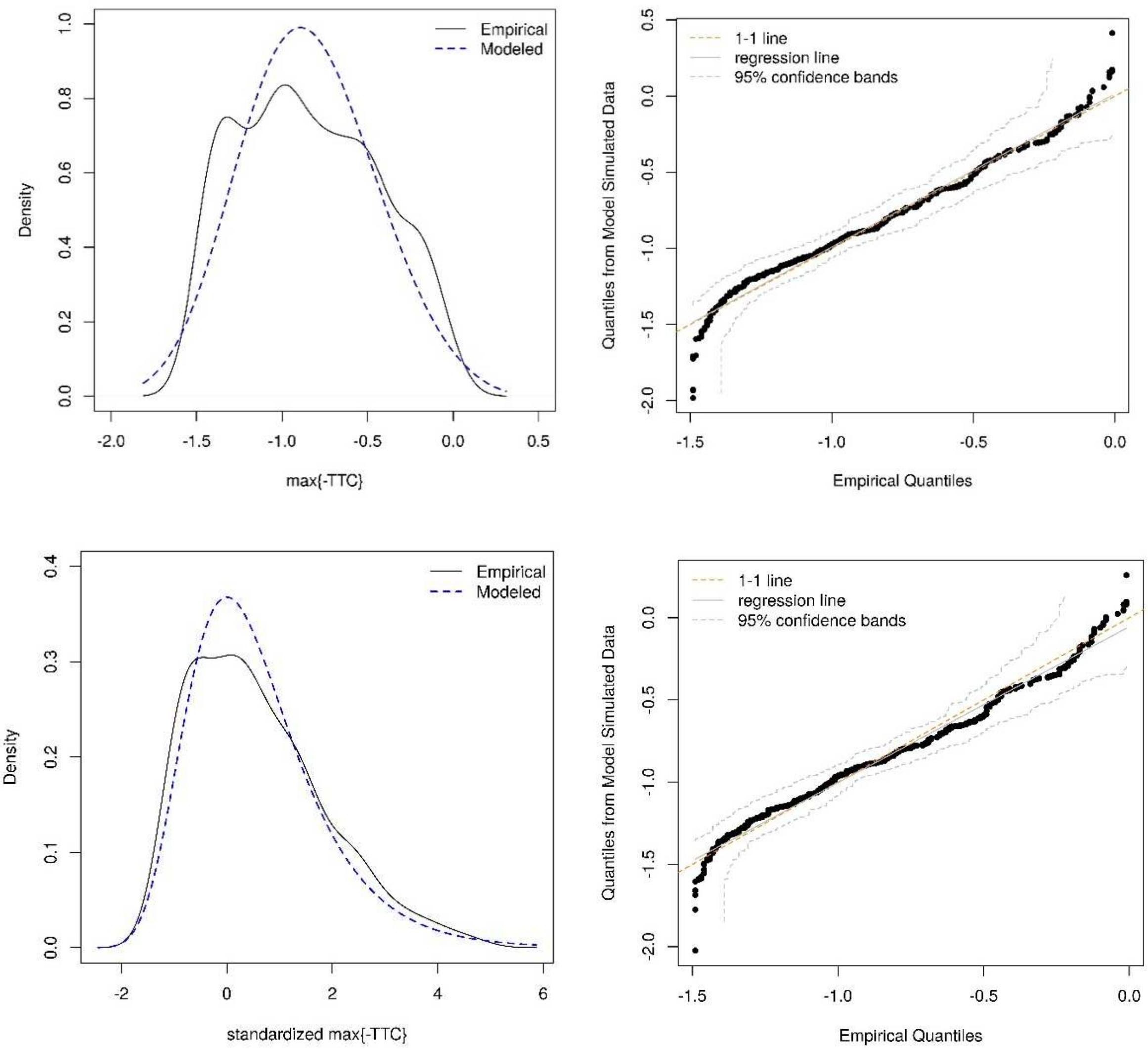

Figure 2 Probability Density plots (left) and simulated QQ-plots (right) for the stationary BM model (top) and the best non-stationary BM model (model #1, bottom) for head-on collisions.

4.1.2 Rear-end collisions

In order to estimate the probability of rear-end collisions, the headway between the passed vehicle and the passing vehicle at the end of the passing maneuver (THW) is used as accident surrogate. Similarly to the probability estimation process for head-on collisions, the minimum THW should be smaller than a limit to be useful as an accident surrogate. Based on the literature, this value varies between <0.6s (Vogel, 2003) and <2.0s (Evans & Wasielewski, 1982; Vogel, 2003). Considering these thresholds, several BM stationary models were developed and evaluated (see Appendix 2) and the value of 2.0s was ultimately adopted.

With a total of 492 observations with a THW smaller than 2.0s and knowing that 2 rear-end collisions occurred, the theoretical probability of a rear-end collision was calculated as 2/(492+2)= 0.00405, with a 95% binomial confidence interval (-0.00155, 0.00964).

The estimation of the stationary BM for the model of the negated values of the THW as carried out using the Gumbel distribution as the stable region for the shape parameter around the 2.0s THW filter resulted in $\hat{\xi} \approx 0$. Further, as explored in Appendix 2, we use the normalized dataset $\widetilde{X_i} = -X_i - \max\ (-\{X_j\}_{j=1}^{n})$, X_i being the THW for observation i, that was proven to provide a better prediction performance. Thus, we obtained the parameters $\hat{\mu} = -1.456\ (0.0121)$, $\hat{\sigma} = 0.256$ (0.0093) and $\hat{\xi} = 0$. The density function of the empirical and modeled negated THW and the simulated QQ plot are shown in Figure 3. Due to the small sample and high variance of the surrogate measure at stake (low THW) the non-normalized BM estimation resulted in positive shape parameter for low filtering conditions.

Using the fitted Gumbel distribution to the normalized data, the estimated probability of this stationary model is 0.00334 with 95% confidence interval (0.00317, 0.00340). This interval was computed assuming a normal distribution under regularity conditions of the parameters, simulating an experiment with a size of $1x10^6$ and its simulated distribution quantiles. This estimated probability is relatively close to the empirical probability of 0.00405. Notwithstanding, the passing maneuver may be affected by specific passing conditions, such as speeds of the vehicles surrounding the subject vehicle. Therefore, several linear combinations of covariates were tested according to a non-stationary BM model approach. This process was conducted in a similar way to the model developed to estimate the probability of a head-on-collision.

Taking this into account, we start with the non-stationary BM model #0, which includes the covariates related with the maneuver and the environment that were found to be significant during the estimation: passing vehicle speed (*speedpv*) and the passing gap time (*passinggap*). Testing this non-stationary model against the stationary one through the likelihood ratio test, a p-value of 0.0002 is achieved with a direct value of 17.508 (note, 2 degrees of freedom). We then estimate the probability of a rear-end collision for a single passing maneuver (see Appendix 2). This distribution with a mean of -1.454, a standard deviation of 0.0232 and a Kolmogorov-Smirnorv test statistic of 0.025, lead to a simulated rear-end collision probability of 0.00339 with 95% confidence interval (0.00328, 0.00351), resulting in a slightly better estimation than the stationary model. The probability density plot as well as the QQ-plot for model #0 are shown in Figure 3.

Table 3 Estimation results of the non-stationary BM approach for rear-end collisions

Non-stationary model	#0	#1	#2
	Est. (Std. Error)	Est. (Std. Error)	Est. (Std. Error)
$\hat{\mu}_0$	-1.42 (0.0626)	-1.37 (0.0653)	-1.42 (0.0694)
$\hat{\mu}_1$ (speedpv)	0.00587 (0.00193)	0.00686 (0.00198)	0.00673 (0.00197)
$\hat{\mu}_3$ (passinggap)	-0.00916 (0.00263)	-0.0101 (0.00267)	-0.0107 (0.00271)
$\hat{\mu}_5$ (Gender)	-	-0.0590 (0.0282)	-0.0753 (0.0296)
$\hat{\mu}_6$ (Angry&Hostile)	-	-	0.0185 (0.00992)
$\hat{\sigma}$	0.254 (0.00914)	0.253 (0.00910)	0.252 (0.00906)
$\hat{\xi}$	0	0	0
Neg.LL	104.09	101.98	100.26

Table 4 Likelihood Ratio Test (and p-value) for the non-stationary BM models for rear-end collisions

Model	#0	#1	#2
#0	-		
#1	4.23 (0.0398)	-	
#2	7.657 (0.0218)	3.43 (0.0640)	-

From all models including driver's characteristics, Table 3 presents the two best models for rear-end collisions, which actually relied in the same key co-variates as the head-on collision estimation results. While age related co-variates didn't bring any improvement in the estimation results, *Gender* and *Angry&Hostile* co-variates managed to improve it. Again, the significance of these variable is given by the p-value of the likelihood ratio tests presented in Table 4. We note again the correlation found between these two variables and as previously discussed, followed with model #1 in the rest of this paper.

Similarly to the head-on estimates, two different approaches were considered for probability estimation. The first approach considers that the location parameter value is calculated using the covariates from the data. The second approach considers the estimation of the location parameter distribution based on the estimation dataset. The estimated probabilities of 0.00333 and 0.00337, respectively, with 95% confidence level (0.00322; 0.00345) and (0.00326; 0.00349) were obtained. All confidence intervals of estimation were computed assuming a normal distribution under regular parameters' conditions, a simulation experiment size of 1×10^6 and its simulated distribution quantiles.

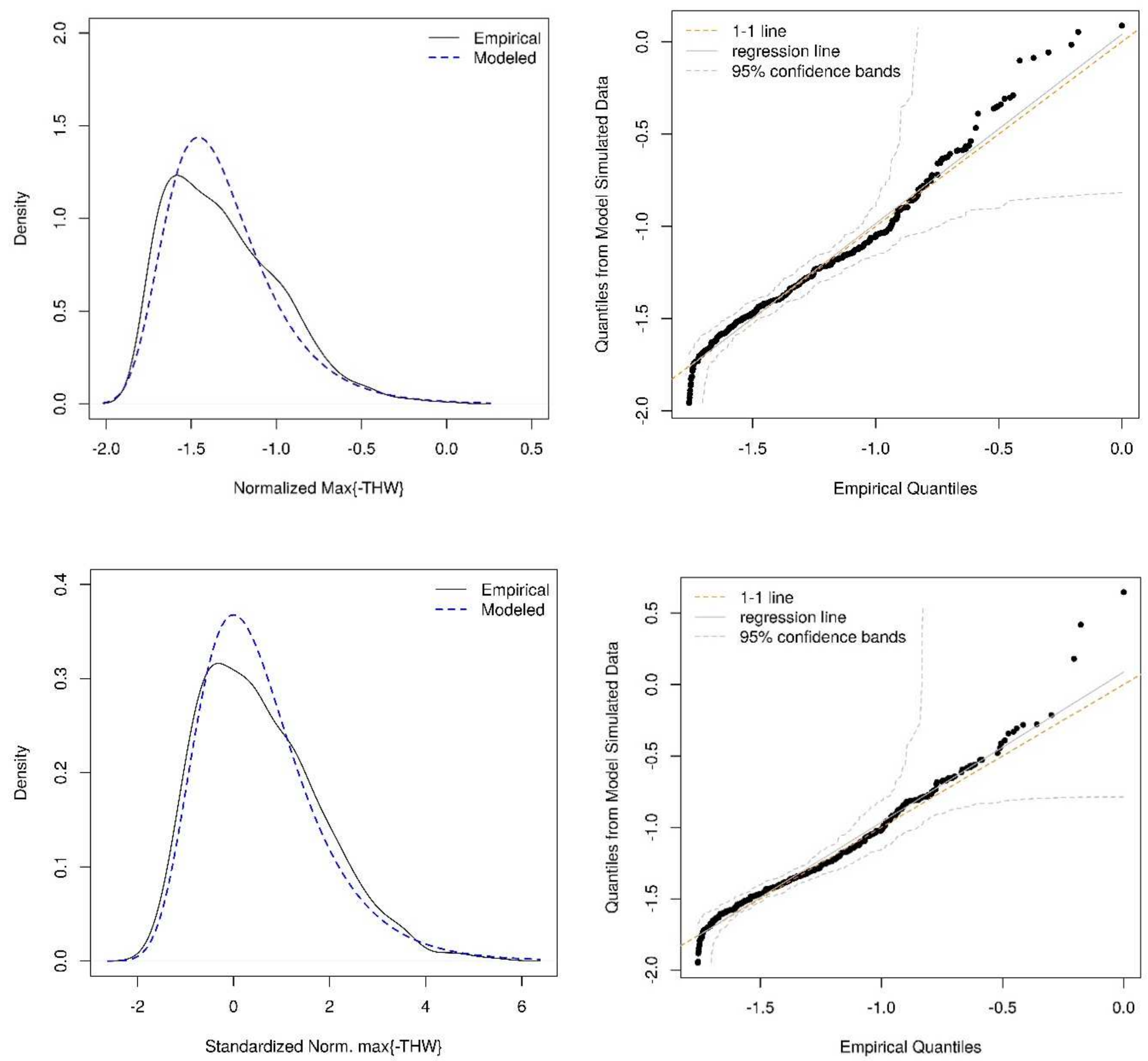

Figure 3 Probability Density plots (left) and simulated QQ-plots (right) for the stationary BM model (top) and the non-stationary BM model (model #1, bottom) for rear-end collisions.

In summary, the univariate fitting resulted in a non-stationary GEV with *speedFront, tailgatetp. passinggap, curvature* and *Gender* as scale-specific covariates for head-on collisions and a non-stationary Gumbel with *speedpv*, *passinggap* and *Gender* as scale-specific covariates for rear-end collisions as the best models. One can interpret that the scale is larger in TTC and the tails (shape) are much lighter (close to 0) for THW which shows a significant different behavior between the two. Interestingly, Zheng and Sayed (2019b) also reported similar findings for the univariate GEV of similar surrogate safety measures but in a very different context (crossing maneuvers). Ultimately and as shown in Annex 2, understanding the behavior of the fitted distributions and its theoretical suitability for each application at stake needs to be carefully handled both at the level of improving estimation or/and at the level of defining the safety index, as very little applications to road safety are yet available.

4.2 Bivariate Model

It is aimed to estimate a measure of risk that not only takes into account the possibility to collide with the opposite vehicle but also with the passed vehicle. Performing a passing maneuver requires a split of attention by the driver regarding its location relative to the surrounding vehicles, and in this case mainly the opposite and lead vehicle. As drivers attempt to keep larger gap from the passed vehicle, this directly means that the gap from the opposite vehicle will be smaller, assuming constant speeds of the opposite and passed vehicles. In other words, there might be a correlation between these two surrogate safety measures. In this paper, it is assumed that the dependence between the TTC and the THW is unknown. Furthermore, an integrated analysis is possible to be developed if, and only if, a relationship of dependence can be found between TTC and THW. When examining the correlation between these two variables with using the whole dataset, a Pearson-correlation value of 0.186 was found. This value shows the lack of linear correlation between the TTC and THW. However, this does not mean that TTC and THW are independent (Embrechts, McNeil, & Straumann, 2002). To further examine potential correlation, the Kendall's rank correlation τ was computed and found to be significantly greater than zero, indicating the existence of dependence between TTC and THW (τ-0.192, p-value$<2.2\text{x}10^{-16}$). This statement is corroborated by the independence test Global Cramer-von Mises, where a significant p-value close to zero (p-value=0.000499) gives evidence against the null hypothesis of independence.

4.2.1 Stationary Marginals

Due to Sklar's Theorem (Sklar, 1959) any multivariate distribution can be expressed in terms of its marginals and the copula (see Section 3.3). Hence, in a first stage we estimated the marginals separately, considering the stationary univariate BM distributions for each variable. By the probability integral transform the marginals turn to approximately uniform variables, used next to copula estimation that contains all the dependence information. Depending on the selected copula-parametric model, its dependence parameters are consequently estimated. This integration estimates the probability of an accident conditioned on TTC and THW being smaller than their filters (1.5s and 2.0s, respectively). Recall that these filters are the established limits for the variables to be useful as an accident surrogate. Since a marginal fitting is necessary prior to the dependence fitting, we think it is more coherent to use only the specific sample information related to accident surrogate. Thus, to perform the estimation for the remaining TTC and THW sample, other distributions for the margins should be tested (e.g., gamma distributions). This is mainly due to the reported non-extreme value distribution of safety measures beyond surrogate safety analysis conditions.

Using the R package `evd` (Stephenson, 2002), we explored different bivariate functions: logistic, asymmetric logistic, Husler–Reiss, negative logistic, asymmetric negative logistic and bilogistic. For further details on its formulation and implementation in R the reader is referred to the `evd` package documentation (Stephenson, 2002). Here we present the results for the distributions with the highest AIC and predictive power: logistic; asymmetry logistic and asymmetric negative logistic distributions. The bivariate logistic distribution function has underlying dependence parameter r (within the dependence function A in Eq. 6 above) where complete dependence is obtained in the limit as r approaches zero (and independence is obtained when $r = 1$). It is a special case of the asymmetric logistic model where two additional asymmetry parameters are considered within A. Here, independence is obtained when $r = 1$ or when the asymmetry parameters equal 0. Complete dependence is obtained in the limit when the asymmetry parameters are equal to 1 and r approaches zero. The asymmetric negative logistic distribution is equivalent to the negative logistic model. Independence is obtained in the limit as either r or the asymmetry parameters approaches zero. Complete dependence is obtained in the limit when the asymmetry parameters are equal to 1 and r tends to infinity (Capéraà & Fougères, 2000; De Haan & Ferreira 2006). The results are presented in Table 5. The estimated probability was obtained from the estimated joint cumulative distribution function and considering either and both of the surrogate measures equal or less than zero (see Appendix 1).

Table 5 Estimation results for the stationary parametric bivariate BM model for the Logistic, Asymmetric Logistic and Asymmetric Negative Logistic and the best Copula based model (Joe-Frank)

	Parameter	**Logistic**	**Asym. Log.**	**Asym. Neg. Log.**	**Joe-Frank Copula**
Marginals	$\hat{\mu}_{TTC}$	-0.886 (0.031)	-0.877 (0.031)	-0.883 (0.031)	-0.886 (0.031)
	$\hat{\sigma}_{TTC}$	0.431 (0.024)	0.445 (0.026)	0.431 (0.024)	0.432 (0.025)
	$\hat{\xi}_{TTC}$	-0.417 (0.060)	-0.424 (0.069)	-0.415 (0.062)	-0.432 (0.059)
	$\hat{\mu}_{THW}$	-1.417 (0.021)	-1.422 (0.021)	-1.421 (0.021)	-1.417 (0.019)
	$\hat{\sigma}_{THW}$	0.280 (0.016)	0.283 (0.016)	0.280 (0.016)	0.280 (0.014)
	$\hat{\xi}_{THW}$	-0.0008 (0.060)	-0.0059 (0.061)	-0.0017 (0.058)	-
Dependence	r	0.865 (0.039)	0.774 (0.100)	0.580 (0.154)	-
	$Asym_1$	-	0.497 (0.328)	0.379 (0.199)	-
	$Asym_2$	-	0.999 ($2x10^{-6}$)	0.999 ($2x10^{-6}$)	-
	θ	-	-	-	1.815 (1.101)
	δ	-	-	-	0.729 (0.384)
	AIC	419.197	422.948	421.043	418.4
	Estimated Probability	0.0141	0.0190	0.0149	0.0209

The empirical probability for comparison was calculated by knowing that a maximum of 5 (4 head-on and 1 rear-end collisions) out of the total 11 collisions had the other surrogate safety measure value below its filter (i.e., if head-on collision, then THW was below 2.0 s, and if rear-end collision, then the TTC was below 1.5 s), and that the sample of size is given by the number of collisions plus the 256 observations where both TTC and THW were below 1.5 and 2.0, respectively. Therefore, the empirical collision probability is 0.0191 (0.0025, 0.0358). Despite slightly higher AIC, the asymmetric negative logistic distribution was able to provide better probability estimates. Note again the obtained shape parameter for the THW marginal close to zero.

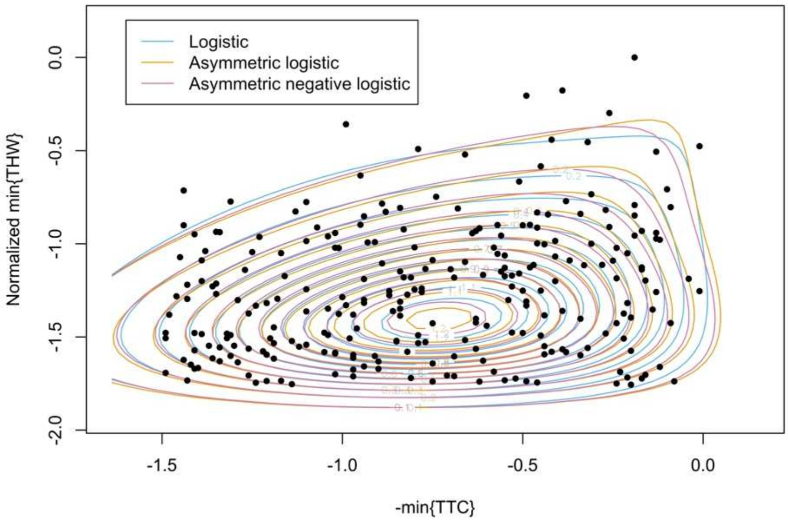

Figure 4 Probability density contours for stationary bivariate EV distributions and the observed data.

Further explore non-EV copula families to model the dependence between the negated values of TTC and of (normalized) THW using the range of families available in the R packages `VineCopula` and `copula` (namely, Gaussian, Student t, Clayton, Gumbel, Frank, Joe, Joe-Gumbel, Joe-Clayton, Joe-Frank and their rotations). To explore these families the marginals are estimated first (as per the previous section 4.1 of this paper), and the copula is here estimated subsequently. We first concluded that the copula with the best AIC is the Joe-Frank (Brechmann & Schepsmeier, 2013), with parameters 1.95 and 0.71. This result was also confirmed by performing the goodness-of-fit test based on Kendall's process (0.67 for both p-values of Cramer-von Mises statistic and Kolmogorov-Smirnov statistics). Simulating elements for the copula distribution analyzed in this exploratory approach, with a Joe-Frank copula and GEV (TTC) and Gumbel (THW) distributions for the margins, a maximum log likelihood of -202.2. Using this fitted distribution (see Table 5), the estimated probability of having an accident, conditioned that both surrogate measures are below their filter, was slightly overestimated at 0.0209 (0.0206, 0.0212). For head-on collisions the obtained probability is 0.0178 (0.0175, 0.0181) and for rear-end collisions 0.0033 (0.0032, 0.0034).

The probability density function of this bivariate model is displayed in Figure 5. The dependencies obtained for both the full maximum likelihood estimation under the bivariate EV distribution functions or the non-EV Copula-based estimation for the Joe-Frank copula revealed the suitability of our approach to estimate the joint probability of an accident based on the two surrogate measures (TTC and THW). Yet, as previously mentioned, further analysis should be performed with approaches such as other EV and non-EV copulas together with the inclusion of other distributions for the margins should be explored (Capéraà, Fougères, & Genest, 1997).

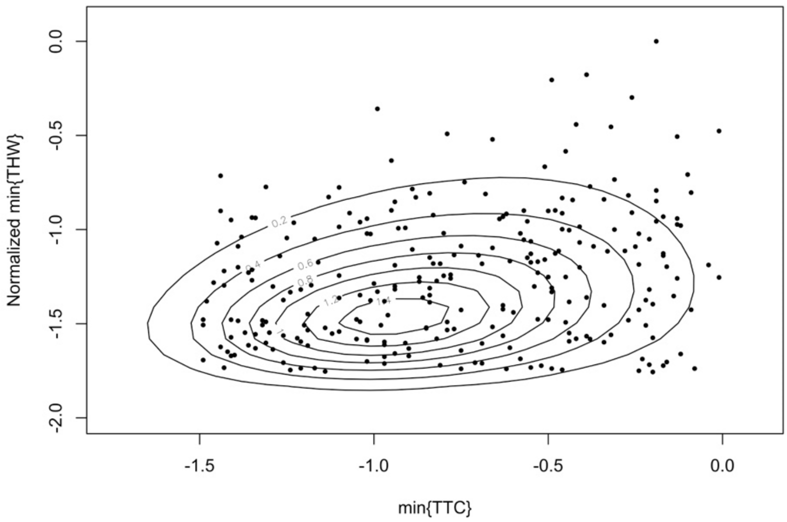

Figure 5 Probability density contour for the stationary Joe-Frank copula and the observed values.

4.2.2 Non-stationary Marginals

As in the univariate case, we now aim at studying the importance of driver characteristics in the estimation of collision probability under the bivariate approach. Such analysis allows us to shed light on these variables in the interactions between the driver, the opposing and the passed vehicle. We consider again the covariates selected in the best univariate BM models and we estimate the three bivariate EV distributions from section 4.2. A bivariate BM model was fitted to the joint distribution of max{-TTC} and (normalized as detailed in as in Appendix 2) max{-THW}, achieving the parameters shown in Table 6 and the contours shown in Figure 6. Both the dependence and the predictive power improve significantly, while keeping the signs and magnitude of the covariates consistent with the phenomenon at stake and the previous estimations. All confidence intervals for the estimated probabilities were again computed assuming a normal distribution under regular parameters' conditions and a simulation experiment size of 1×10^6.

Table 6 Estimation results for the non-stationary bivariate BM model for the Logistic, Asymmetric Logistic and Asymmetric Negative Logistic

Parameter	**Logistic**	**Asym. Log.**	**Asym. Neg. Log.**
$\hat{\mu}_0^{TTC}$	-1.004 (0.177)	-1.018 (0.177)	-0.958 (0.170)
$\hat{\mu}_{speedFront}^{TTC}$	0.028 (0.009)	0.028 (0.009)	0.025 (0.009)
$\hat{\mu}_{tailgatetp}^{TTC}$	0.004 (0.002)	0.0033 (0.002)	0.033 (0.002)
$\hat{\mu}_{passinggap}^{TTC}$	-0.022 (0.006)	-0.021 (0.005)	-0.021 (0.006)
$\hat{\mu}_{curvature}^{TTC}$	-31.193 (19.231)	-34.777 (19.44)	-34.790 (18.505)
$\hat{\mu}_{Gender}^{TTC}$	-0.076 (0.060)	-0.061 (0.061)	-0.063 (0.061)
$\hat{\sigma}^{TTC}$	0.400 (0.023)	0.398 (0.023)	0.397 (0.023)
$\hat{\xi}^{TTC}$	-0.366 (0.068)	-0.371 (0.071)	-0.381 (0.068)
$\hat{\mu}_0^{THW}$	-1.353 (0.115)	-1.358 (0.116)	-1.312 (0.114)
$\hat{\mu}_{speedPV}^{THW}$	0.014 (0.004)	0.014 (0.004)	0.013 (0.004)
$\hat{\mu}_{passinggap}^{THW}$	-0.016 (0.005)	-0.016 (0.005)	-0.017 (0.005)
$\hat{\mu}_{Gender}^{THW}$	-0.103 (0.045)	-0.104 (0.046)	-0.110 (0.045)
$\hat{\sigma}^{THW}$	0.281 (0.015)	0.283 (0.015)	0.277 (0.014)
$\hat{\xi}^{THW}$	-0.008 (0.049)	-0.083 (0.050)	-0.073 (0.049)
r	0.907 (0.037)	0.813 (0.099)	0.456 (0.195)
$Asym_1$	-	0.277 (0.202)	0.350 (0.336)
$Asym_2$	-	0.999 ($2x10^{-6}$)	0.999 ($2x10^{-6}$)
AIC	395.396	397.553	397.85
Estimated Probability	0.0180 (0.0178,0.0183)	0.0179 (0.0176,0.0181)	0.0153 (0.0149,0.0154)

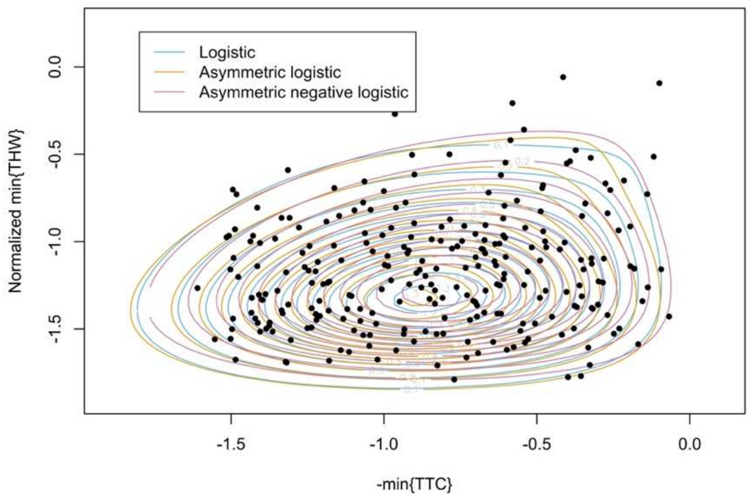

Figure 6 Probability density contour plot for the best non-stationary bivariate distributions and the observed data.

In Figure 7 we present the summary of the estimated probabilities for all best models presented above, and its empirical values.

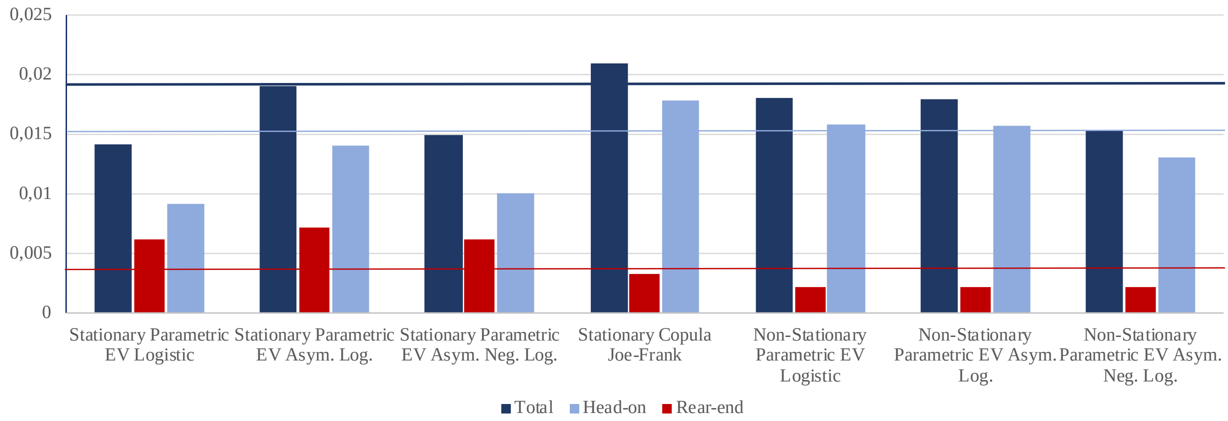

Figure 7 Probability estimates for all best bivariate models (empirical values as horizontal lines).

As a final comparison, and as in the previous sections, we look at the obtained AIC, the amplitude of the estimates' confidence interval reflecting higher precision, and the prediction power (estimated probabilities) of each model. The non-stationary parametric asymmetric logistic bivariate EV seems to perform better overall. More interestingly, and having in mind the considered surrogate safety measures, the models that perform best in terms of these three criteria are not the ones with the strongest dependence, especially noticing that the same model with covariates increases the degree of independence. Thus, we are consistently observing weak dependence between the two indicators. Also, adding the covariates allowed once again to improve the overall performance of all models.

5 Conclusions

This paper analyzed the individual and joint probabilities of head-on collisions and rear-end collisions through the Block Maxima approach using the Univariate and Bivariate distributions to model dependence between the two surrogate measures that capture those types of collisions during passing maneuvers.

We investigated the fitting of EV models allowing for any real extreme value index, to understand how informative these models may be in respect to the extreme value indices pertaining to surrogate measures of safety.

The univariate non-stationary estimation allowed to conclude that aspects linked to drivers' characteristics, namely the driving style/gender, have a significant impact on the prediction of collisions. Furthermore, we show how different measures may have different behaviors that need for careful EV theoretical and application-specific attention. In future application, understanding such behavior of the fitted distributions and its theoretical suitability for each application at stake needs to be handled both at the level of improving estimation or/and at the level of defining the safety index, as very little applications to road safety are yet available. Furthermore, with the increased availability of comprehensive naturalistic datasets, we hope that EV approaches will be able to provide a robust quantified link between driver's characteristics, detailed surrogate safety measures and the instant probability of different types of accidents.

The bivariate model approach integrated the two different surrogate measures, TTC and THW, in order to estimate the risk of colliding with the opposite or with the passed vehicle in a single passing maneuver. Although the linear correlation between the two surrogate measures has proved to be weak, the bivariate distribution estimation shown the existence of dependency between these two surrogate measures, relative improved the predictive power of the EV model and set the ground for further the exploration of multi-variate and copula EV approaches in complex road safety phenomena. However, the models that perform best in terms of fit and prediction power are not the ones with the strongest dependence, especially noticing that the same model with covariates increases the degree of independence. Also, adding the covariates allowed to improve the overall performance of all models.

In terms of future work, and regarding the rear-end collisions model estimation, we suggest that the fitting must concentrate on the more theoretically consistent Gumbel or Weibull distributions under the BM approach and GP distributions restricted to non-positive extreme value index under POT approach to improve precision on accident probability estimation especially under small samples. Further exploration of the wide range of Copula family models should also be explored in the field of road crash probability estimation based on surrogate safety measures. To sustain the preliminary conclusions that both TTC and THW are good surrogate safety measures for near-accidents, head-on collisions and/or rear-end collisions, further analysis should be developed in order to validate through simulated data and/or data from other experimental scenarios the conclusions drawn by these models. Finally, following the very recent univariate applications (Wang, Xu, Xia, Qian & Lu, 2019), the proposed multivariate probabilistic surrogate safety models should be integrated in traffic microscopic simulation frameworks and advanced vehicle control and driving assistance systems for promising safety assessment, where the estimation of safety for individual maneuvers would not need to rely on accident records nor on the limitations of simulation's and algorithm's premise of accident-free models.

References

Archer, J. (2004). Methods for the assessment and prediction of traffic safety at urban intersections and their application in micro-simulation modelling. *Royal Institute of Technology*.

Åsljung, D., Nilsson, J. & Fredriksson, J., (2017) Using extreme value theory for vehicle level safety validation and implications for autonomous vehicles. *IEEE Trans. Intell. Veh. 2* (4), 288–297

Berg, D. (2009). Copula goodness-of-fit testing: an overview and power comparison. *The European Journal of Finance, 15*(7-8), 675-701.

Bonneson, J. A. (2010). Highway safety manual. *Washington, DC: American Association of State Highway and Transportation Officials*.

Brechmann, E. C., & Schepsmeier, U. (2013). Modeling dependence with C-and D-vine copulas: The R-package CDVine. *Journal of Statistical Software, 52*(3), 1-27.

Bücher, A., & Zhou, C. (2018). A horse racing between the block maxima method and the peak-over-threshold approach. *arXiv*. arXiv:1807.00282.

Capéraà, P., & Fougères, A.-L. (2000). Estimation of a bivariate extreme value distribution. *Extremes, 3*(4), 311-329.

Capéraà, P., Fougères, A.-L., & Genest, C. (1997). A nonparametric estimation procedure for bivariate extreme value copulas. *Biometrika*, 567-577.

Coles, S., Bawa, J., Trenner, L., & Dorazio, P. (2001). *An introduction to statistical modeling of extreme values* (Vol. 208): Springer.

De Haan, L., & Ferreira, A. (2006). Extreme value theory: an introduction. Springer Science & Business Media.

Dombry, C., & Ferreira, A. (2017). Maximum likelihood estimators based on the block maxima method. arXiv preprint arXiv:1705.00465.Embrechts, P., McNeil, A., & Straumann, D. (2002). Correlation and dependence in risk management: properties and pitfalls. *Risk management: value at risk and beyond*, 176-223.

Evans, L., & Wasielewski, P. (1982). Do accident-involved drivers exhibit riskier everyday driving behavior? *Accident Analysis and Prevention, 14*(1), 57-64.

Fang, K.-T., Kotz, S., & Ng, K. W. (1990). *Symmetric multivariate and related distributions*: Chapman and Hall.

Farah, H. (2011). Age and gender differences in overtaking maneuvers on two-lane rural highways. *Transportation Research Record: Journal of the Transportation Research Board*(2248), 30-36.

Farah, H. (2013). Modeling drivers' passing duration and distance in a virtual environment. *IATSS research, 37*(1), 61-67.

Farah, H. (2016). When do drivers abort an overtaking maneuver on two-lane rural roads? *Transportation Research Record: Journal of the Transportation Research Board*(2602), 16-25.

Farah, H., & Lima Azevedo, C. (2016). Safety analysis of passing maneuvers using extreme value theory. *IATSS Research*.

Farah, H., Bekhor, S., & Polus, A. (2009). Risk evaluation by modeling of passing behavior on two-lane rural highways. *Accident Analysis & Prevention, 41*(4), 887-894.

Farah, H., Polus, A., Bekhor, S., & Toledo, T. (2007). Study of passing gap acceptance behavior using a driving simulator. *Advances in Transportation Studies an International Journal*, 9-16.

Fisher, R. A., & Tippett, L. H. C. (1928). *Limiting forms of the frequency distribution of the largest or smallest member of a sample.* Paper presented at the Mathematical Proceedings of the Cambridge Philosophical Society.

Genest, C., & Rivest, L.-P. (1993). Statistical inference procedures for bivariate Archimedean copulas. *Journal of the American statistical Association, 88*(423), 1034-1043.

Gilleland, E., & Katz, R. W. (2011). New software to analyze how extremes change over time. *Eos, Transactions American Geophysical Union, 92*(2), 13-14.

Hicks, C. R., & Turner, K. Fundamental concepts in the design of experiments. 1999. In: Oxford University Press, New York.

Hydén, C. (1987). The development of a method for traffic safety evaluation: The Swedish Traffic Conflicts Technique. *Bulletin Lund Institute of Technology, Department*(70).

Jonasson, J. K., & Rootzén, H. (2014). Internal validation of near-crashes in naturalistic driving studies: A continuous and multivariate approach. *Accident Analysis and Prevention, 62*, 102-109.

Kraay, J. H., & Van der Horst, A. (1985). The Trautenfels study: a diagnosis of road safety using the Dutch conflict observation technique DOCTOR.

Llorca, C., Garcia, A., Moreno, A. T., & Perez-Zuriaga, A. M. (2013). Influence of age, gender and delay on overtaking dynamics. *IET Intelligent Transport Systems, 7*(2), 174-183.

Nelsen, R. B. (2007). *An introduction to copulas*: Springer Science & Business Media.

Orsini, F., Gecchele, G., Gastaldi, M. & Rossi R. (2018). Collision prediction in roundabouts: a comparative study of extreme value theory approaches. *Transportmetrica A: Transport Science, DOI: 10.1080/23249935.2018.1515271*.

Pickands, J. (1981). *Multivariate extreme value distributions.* Paper presented at the Proceedings 43rd Session International Statistical Institute.

Rosenthal, T. (1999). STISIM drive user's manual. *Systems Technology Inc., Hawthorne, CA*.

Sklar, A. (1959). Fonctions de répartition à n dimensions et leurs marges. *Publ. Inst. Statist. Univ. Paris 8*, 229-231.

Songchitruksa, P., & Tarko, A. P. (2006). The extreme value theory approach to safety estimation. *Accident Analysis & Prevention, 38*(4), 811-822.

Stephenson, A. G. (2002). evd: Extreme Value Distributions. *R News*, 2(2):31-32, June 2002.

Tarko, A. P. (2012). Use of crash surrogates and exceedance statistics to estimate road safety. *Accident Analysis & Prevention, 45*, 230-240.

Taubman-Ben-Ari, O., Mikulincer, M., & Gillath, O. (2004). The multidimensional driving style inventory—scale construct and validation. *Accident Analysis and Prevention, 36*(3), 323-332.

Vlahogianni, E. I., & Golias, J. C. (2012). Bayesian modeling of the microscopic traffic characteristics of overtaking in two-lane highways. *Transportation research part F: traffic psychology and behaviour, 15*(3), 348-357.

Vogel, K. (2003). A comparison of headway and time to collision as safety indicators. *Accident Analysis and Prevention*, *35*(3), 427-433.

von Mises, R. (1936) La distribution de la plus grande de *n* valeurs. *Rev. Math. Union Interbalcanique 1*, 141--160. Reproduced in: Selected Papers of Richard von Mises, Amer. Math. Soc. 2, 271--294 (1964).

Wang, C., Xu, C., Xia, J., Qian Z. & Lu L. (2018) A combined use of microscopic traffic simulation and extreme value methods for traffic safety evaluation. *Transportation Research Part C: Emerging Technologies*,90, 281-291.

Yan, J. (2007) Enjoy the Joy of Copulas: With a Package copula. Journal of Statistical Software 21(4), 1–21.

Zheng, L., Ismail, K., & Meng, X. (2014). Freeway safety estimation using extreme value theory approaches: A comparative study. *Accident Analysis & Prevention*, *62*, 32-41.

Zheng, L., Ismail, K., Sayed T., & Fatema, T. (2018). Bivariate extreme value modeling for road safety estimation. *Accident Analysis and Prevention 120*, 83-91.

Zheng, L., Sayed T. & Essa, M. (2019a) Validating the bivariate extreme value modeling approach for road safety estimation with different traffic conflict indicators. *Accident Analysis & Prevention* 123, 314-323.

Zheng, L. & Sayed, T. (2019b) From univariate to bivariate extreme value models: Approaches to integrate traffic conflict indicators for crash estimation. *Transportation Research Part C: Emerging Technologies*, 103, 211-225.

6 Appendix 1: Probability of Collision

The maximum domain of attraction condition holds if, for $\{-X_1, \ldots, -X_n\}$ an i.i.d. sample of the surrogate measure of safety x,

$$\exists_{a_n>0}, b_n \in \mathbb{R}: \lim_{n\to\infty} P\left(\frac{\max_{i\le n} -X_i - b_n}{a_n} \le y\right) = e^{-\left(1+\frac{\xi(y-u)}{\sigma}\right)_+^{-\frac{1}{\xi}}} \quad , \xi \in \mathbb{R} \qquad \text{(Eq. 8)}$$

with $1 + \xi\,\frac{(y-u)}{\sigma}$ always positive. This formulation simplifies to a standard EV in the limit if a_n and b_n are chosen appropriately. Hence under the BM approach we define:

$$P_{BM}(collision) = \lim_{n\to\infty} P\left(\frac{\max_{i\le n} X_i - b_n}{a_n} \ge 0\right) = \lim_{n\to\infty} P\left(\max_{i\le n} X_i \ge b_n\right) \qquad \text{(Eq. 9)}$$

and estimate it by,

$$\hat{P}_{BM}(collision) = 1 - e^{-\left(1-\hat{\xi}\frac{\hat{u}}{\hat{\sigma}}\right)^{-1/\hat{\xi}}} \qquad \text{(Eq. 10)}$$

The above equation can also be interpreted as estimating the probability until achieving that “least real value” of the surrogate safety measure before collision, or approximately the probability of this measure being as close to zero up to the actual observed sample maximum.

7 Appendix 2: Summary of the Stationary Model Estimation and Filter Sensitivity Analysis

To validate the TTC and THW filter used in Section 4, a sensitivity analysis was performed. This analysis allows not only to analyze the stability of the estimation in the selection of the filter, but also to explore the range of shape parameter values to be expected in non-stationary estimation. It also allows to evaluate the prediction performance of a simplified stationary approach. We also computed the estimates for the normalized model with $\{a_n, b_n\} = \{1, -\min(X_i)\}\ with\ i \in 1, \ldots, n$ for the BM and study it’s performance compared to the non-normalized model. Such normalization can be of particular relevance under small samples and increased variability of the observed variables.

In Figure 8 we present the results for BM approach. The red curve in the bottom plot represents the empirical conditional collision probability from the used data. We first highlight the stable region for the estimate of the shape parameter for TTCs between 0.6 and 1.5s (head-on collision). The normalized model does not provide significant prediction improvement nor different estimates (as the dark blue and light blue curves for parameter estimates are overlapping), likely due to a large number of surrogate observations. From the analysis values between 1.2 and 1.5 s are likely to provide both good statistical properties and prediction performances. The value of 1.5s was thus selected for further analysis due to its smaller variance.

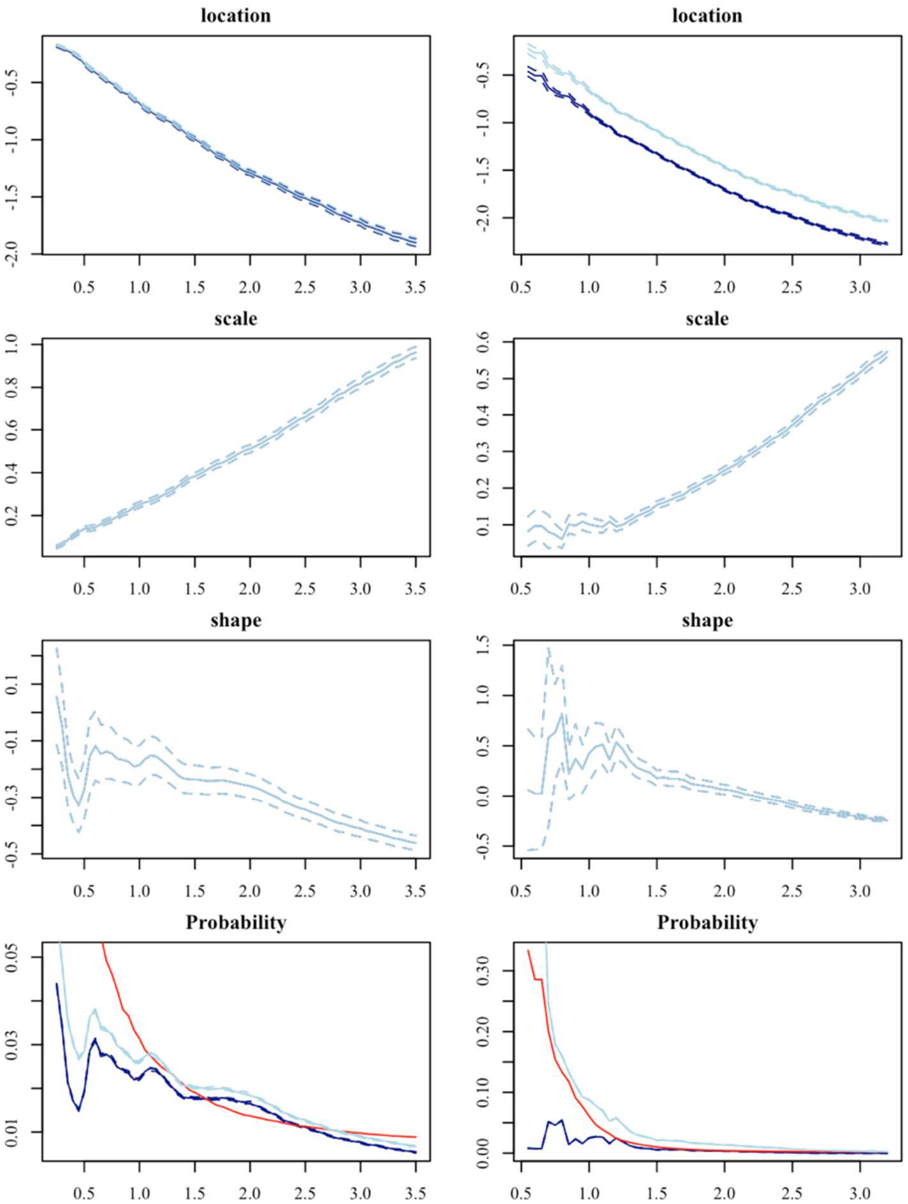

Figure 8 ML estimates of the univariate stationary BM location, scale and shape parameters and the resulting probability of collision for the TTC (left) and the THW (right) for the original (dark blue) and normalized (light blue) model for different filtering criteria. In red is the empirical conditional collision probability.

A first stable region for the estimate of the shape parameter for THWs happens between 1.0 and 1.3s (rear-end collision). However, within this region the shape parameter has positives values (between 0.35 and 0.55) therefore conflicting with the theory, since a positive shape parameter implies an infinite right-endpoint for the underlying distribution of THW (as explained in Section 3.2). These results are mainly due to the small sample (<100 observations) and the high variability of THW within this region. Thus, we further explore the stable region between 2.0 and 2.4s where the shape parameter has values close to zero (between 0.06 and -0.05). The Gumbel distribution (GEV with $\xi = 0$) was tested within this region and used for further analysis. Finally, due to limitations of the THW dataset mentioned above, it is worth noting the increased prediction performance of the normalized model.