

## Probabilistic prediction of the failure mode of the Ruytenschildt Bridge

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Title: Probabilistic prediction of the failure mode of the Ruytenschildt Bridge

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Abstract: In the Netherlands, the shear capacity of a large number of existing reinforced concrete solid slab bridges is subject to discussion, as initial assessments indicated that their capacity was insufficient. In certain cases, the deterministic value of the moment capacity is larger than the deterministic value of the shear capacity. However, when the variability of the material properties, and of the capacity models themselves are factored in, a probability of a certain failure mode can be calculated. Here, a method is introduced to calculate the chance that a cross-section fails in shear before it fails in bending. The method that is derived here is applied to the Ruytenschildt Bridge. This case study is a reinforced concrete solid slab bridges that was tested to failure in two spans during the summer of 2014. The relative probability of failure in shear of the bridge was determined. The predictions indicated a smaller probability of a shear failure than of a bending moment failure. In the first tested span, failure was not reached, but indications of flexural distress were observed. In the second span, a flexural failure was achieved, in line with the probabilistic predictions. The presented method can be used in the assessment of existing bridges to determine which failure mode is most probable, taking into account the variability of materials and capacity models.

**Comments:**

The reviewer believes that there are still two vital problems in this manuscript:

1) Using only two tests (and one of them did not achieve failure) to prove the proposed method is not sufficient. And if a method is not verified, it could not be used in any way.

The authors responded: “The authors do not think comparing the method to laboratory tests would give interesting results, because typical shear tests are overdesigned in bending to make sure that the element fails in shear. Similarly, experiments on beams in bending will be designed so that the shear span is large enough to avoid a shear failure, or beams for flexural tests can be reinforced with stirrups to avoid a shear failure”.

However, I think that laboratory tests would still give some benefits to the verification of the proposed method. For example, although typical beam or slab shear tests are overdesigned in bending to make sure that the members fail in shear, the ratios of the bending capacity to shear capacity of the test members are different for each test, which means that the predicted probability of shear failure would be different. Furthermore, it could be inferred that if the ratio of the bending capacity to shear capacity is getting smaller, the probability of shear failure is also getting smaller. If the method proposed by the authors could predict this tendency, it could be stated that the method was verified reasonably. Actually, it is the core value of the probabilistic method to obtain a specific failure probability regardless of how a member is designed.

I've verified the method with four representative laboratory tests: S1T1, S5T4, S8T1 and S9T1:

	Case	Failure mode
S1T1	Reference case	WB
S5T4	a = 400 mm	WB + B
S8T1	High strength concrete	WB
S9T1	High strength concrete + a = 400 mm	WB + P

I set up a MathCad sheet to determine the Unity Check in bending moment when the Unity Check in shear equals 1, see Annex for S1T1 as an example.

The results are as follows for the UC in moment:

S1T1	0.353
S5T4	0.377

S8T1	0.563
S9T1	0.444

The ratio of  $f_y/f_u$  is now based on the measured properties of the steel from the experiment:  $542\text{MPa}/658\text{MPa} = 0,824$ .

Then, the expression from Eq. 13 from the paper becomes, for S1T1 for example:

$$0.8 \frac{f_{ck}^{1/3}}{f_{c,mean}^{1/3}} \frac{1}{\left(\frac{Test}{Predicted}\right)_V} - 0.824 \times 0.353 \frac{1}{\left(\frac{Test}{Predicted}\right)_M} \leq 0$$

Slabs S1 and S5 are normal strength concrete, properties are based on all samples of the B35 concrete used for the experiments [1]:

	$\lambda$	$\varepsilon$
$f_{c,meas}$	3.76 MPa	0.08 MPa
$f_{ck}$	3.56 MPa	0.10 MPa

Slabs S8 and S9 are high strength concrete, properties are based on all samples of the B65 concrete used for the experiments [2]:

	$\lambda$	$\varepsilon$
$f_{c,meas}$	4.293 MPa	0.047 MPa
$f_{ck}$	4.177 MPa	0.053 MPa

The results of the monte carlo simulations ( $10^5$  simulations) are then:

	Probability of bending failure (%)	Probability of shear failure (%)
S1T1	0,64	99,36
S5T4	1,82	98,19
S8T1	52,90	47,10
S9T1	9,14	90,86

We can draw a few comments from these results:

- all slabs failed in shear, as they were designed as shear experiments. For the experiments that are analysed with the proposed method, we see that for S1T1, S5T4 and S9T1, the method clearly shows that a bending moment failure is unlikely. For S8T1, the method indicates that the chance is more fifty-fifty, with slightly more chance of a bending moment failure. The fact that this test still failed in shear does not contradict the results of the method (47,10% chance is still a significant chance).
- we can observe a relationship between the Unity Check for bending when  $UC_{shear} = 1$  and the calculated probabilities of a bending failure: lower values for  $UC_{bending}$  lead to lower probabilities of a bending failure relative to shear failure, as expected.
- S9T1 also had some punching damage – this method did not take punching into account.
- The verification with the slab shear experiments shows that the method is valid.

I've added the following new paragraph to the manuscript:

### **6. Verification with selected slab shear experiments**

To verify the proposed method for future use, 4 experiments from the slab shear experiments [14-17] are selected, see Table 1. Slabs with normal and high strength concrete are selected, and two positions of the concentrated load are studied. The value of the Unity Check for bending is then determined for the load that causes a Unity Check = 1 for shear. These values are given in Table 1. Then, Eq. (13) can be applied. For the considered slab shear experiments, S500 steel was used with a yield strength of 542 MPa and an ultimate strength of 658 MPa. The ratio of  $f_y/f_u$  is then 0.824. As a result, Eq. (13) becomes as follows for S1T1:

$$0.8 \frac{f_{ck}^{1/3}}{f_{c,mean}^{1/3}} \frac{1}{\left(\frac{Test}{Predicted}\right)_V} - 0.824 \times 0.353 \frac{1}{\left(\frac{Test}{Predicted}\right)_M} \leq 0 \quad (1)$$

The properties of  $f_{ck}$  and  $f_{c,mean}$  are based on a large number of material samples that were taken alongside the slab testing program, so that the statistical properties could be derived for the Dutch concrete classes B35 (normal strength concrete) and B65 (high strength concrete) used in the experiments. The sample test results follow a lognormal distribution, and the

properties are given in Table 2.  $\left(\frac{Test}{Predicted}\right)_V$  and  $\left(\frac{Test}{Predicted}\right)_M$  follow the lognormal

distribution with properties given in **Error! Reference source not found.**

With all random variables defined, the Monte Carlo simulations can be used to determine the probability of failure in bending moment relatively compared to the probability of failure in shear. The results are given in Table 1. S1T1, S5T4 and S9T1 are clearly predicted to fail in shear, as happened in the experiment. S8T1 has about a fifty-fifty chance to fail in shear or moment, and failed in shear in the experiment. As such, the prediction method for the failure mode gives good results. Moreover, it can be seen that for higher Unity Checks for bending moment for the load that results in a Unity Check = 1 for shear, the chance of failure in bending moment becomes relatively higher as well. The verification procedure has thus shown that the proposed method is valid.

**Table 1 – Verification with slab shear experiments [14-17]**

Test nr	$a_{pos}$ (mm)	class	FM	$UC_{moment}$	$p_{bf}$ (%)	$p_{sf}$ (%)
S1T1	600	B35	WB	0.353	0,64	99,36
S5T4	400	B35	WB + B	0.377	1,82	98,19
S8T1	600	B65	WB	0.563	52,90	47,10
S9T1	400	B65	WB + P	0.444	9,14	90,86

**Table 2 - Overview of input values for lognormal distributions of B35 and B65**

**concrete**

	Random variable	$\lambda$	$\epsilon$
B35	$f_{c,mean}$ (MPa)	3.76	0.08
	$f_{ck}$ (MPa)	3.56	0.10
B65	$f_{c,mean}$ (MPa)	4.29	0.05
	$f_{ck}$ (MPa)	4.18	0.05

And the following paragraph is added to the conclusions:

*The proposed method for estimating a failure mode was verified with four of the slab shear experiments. It was found that the shear failures were correctly predicted.*

**2) The Unity Check expressed as a ratio of predicted to tested capacities is not convincing.**

**The authors responded: “Instead of writing the Unity Check as the load effect / capacity, here the Unity Check is written as predicted load / failure load. In other words, what this method does, is (for example, for shear) taking the maximum shear stress that could occur in a cross-section, provided that it is designed properly (because the code equations take load effect  $\leq$  capacity) and comparing it to the “real” average capacity of the cross-section. The “real” capacity is the code equation with average material properties, and multiplied with a correction factor based on experimental results, the ratio test/predicted. As such, on average the “real” capacity as determined here will equal the tested capacity.”**

**However, I still cannot agree with the authors, here is my consideration:**

**According to the original definition by the authors, the probability of failure in bending  $p_f$  can be calculated by the equation (1), where  $V_d$ ,  $M_d$ ,  $V_c$ , and  $M_c$  is the shear demand (load effect), moment demand (load effect), shear capacity (real),**

and moment capacity (real), respectively.

$$p_f = P(\text{moment} < \text{shear}) = P(\text{UC}_{\text{shear}} < \text{UC}_{\text{moment}}) = P\left(\frac{V_d}{V_c} < \frac{M_d}{M_c}\right) \quad (1)$$

The authors proposed another formulation trying to substitute the above one, which can be expressed as the equation (2), where  $V_c$  and  $M_c$  is the shear capacity (predicted) and moment capacity (predicted).

$$p'_f = P\left(\frac{V_c}{V_c} < \frac{M_c}{M_c}\right) \quad (2)$$

Assuming

$$V_c = n_v V_d \quad (3)$$

$$M_c = n_m M_d \quad (4)$$

Where  $n_v$  and  $n_m$  is the over-designed coefficient for shear and moment, respectively. And  $n_v$  and  $n_m$  are larger than 1.0 if the structural member is “properly designed”.

The authors concluded that the  $p_f$  is equal to  $p'_f$ ; however, it is true only if  $n_v$  is equal to  $n_m$ . Unfortunately,  $n_v$  is not necessarily equal to  $n_m$ . Making  $n_v$  equal to  $n_m$  is not one of the considerations when we design a structure. Actually,  $n_v$  is sometimes designed to be larger than  $n_m$  considering shear failure is brittle and could cause more serious results.

If we follow the code (for example the Eurocode), and we design a new structure, then we have our load and resistance factors in such a way that every failure mode has the same probability of failure.

If, however, we say that we will overdesign something in shear because it is a brittle failure mode, then we do not follow the original safety philosophy anymore. This approach seems to be more popular in North America, or in seismic regions, where your design will select and properly design the plastic hinges. In that case, it becomes harder to equalize calculations with respect to the chosen reliability indices, and engineering judgement takes a larger role.

The method that we have followed is in line with the first approach we outlined above.

#### **Cited references**

1. Lantsoght, E.O.L., et al., *Proposal for the extension of the Eurocode shear formula for one-way*

*slabs under concentrated loads*. Engineering Structures, 2015. **95**: p. 16-24.

2. Lantsoght, E.O.L., *Shear in Reinforced Concrete Slabs under Concentrated Loads Close to Supports*. 2013, Delft University of Technology; PhD Thesis. p. 340.

Bepaling dwarskrachtcapaciteit van plaatbrug onder mobiele belasting, eigengewicht en permanente belasting

## Geometrie

$d_1 := 265\text{mm}$	nuttige hoogte: dekhoogte - 45mm // dekhoogte - dekking - halve staafdia
$b_r := 1250\text{mm}$	randafstand (minimum 30cm, maximum 140cm)
$l_{\text{span}} := 3.6\text{m}$	overspanning dek
$b := 2.5\text{m}$	breedte dek
$h := 300\text{mm}$	constructiehoogte dek
$b_{\text{sup}} := 100\text{mm}$	breedte van de oplegging, tenzij de werkelijke grootte gekend is
$b_{\text{load}} := 200\text{mm}$	breedte van de wiellast
$a := 600\text{mm}$	$z := a + 300\text{mm}$

## Materiaaleigenschappen

$$f_{c\_cube} := 35.8\text{MPa}$$

$$f_{ck} := 0.82 \cdot f_{c\_cube} - 8\text{MPa} = 21.356\text{MPa}$$

$$f_{yk} := 500\text{MPa}$$

$$A_s := 6597\text{mm}^2$$

$$\rho_1 := \frac{A_s}{b \cdot d_1} = 9.958 \times 10^{-3}$$

waarde aanpassen tot UC = 1

$$P_u := 536\text{kN}$$

$$F_{\text{pres}} := 163\text{kN}$$

$$q_{\text{self}} := 18.75 \frac{\text{kN}}{\text{m}}$$

## Momentcapaciteit

$$a_{\text{bending}} := \frac{A_s \cdot f_{yk}}{0.85 \cdot f_{ck} \cdot b} = 0.073\text{m}$$

$$M_{\text{Rd}} := A_s \cdot f_{yk} \cdot \left( d_1 - \frac{a_{\text{bending}}}{2} \right) = 754.229 \cdot \text{kN} \cdot \text{m}$$

## Dwarskrachtcapaciteit

$$v_{\text{Rd}} := 0.12 \cdot \left[ 1 + \left( \frac{200\text{mm}}{d_1} \right)^{\frac{1}{2}} \right] \cdot (\rho_1 \cdot 100 \cdot f_{ck} \cdot 1\text{MPa} \cdot 1\text{MPa})^{\frac{1}{3}} = 0.621 \cdot \text{MPa}$$

$$v_{\text{Rd\_mag}} := 1.545 \cdot v_{\text{Rd}} = 0.96 \cdot \text{MPa}$$

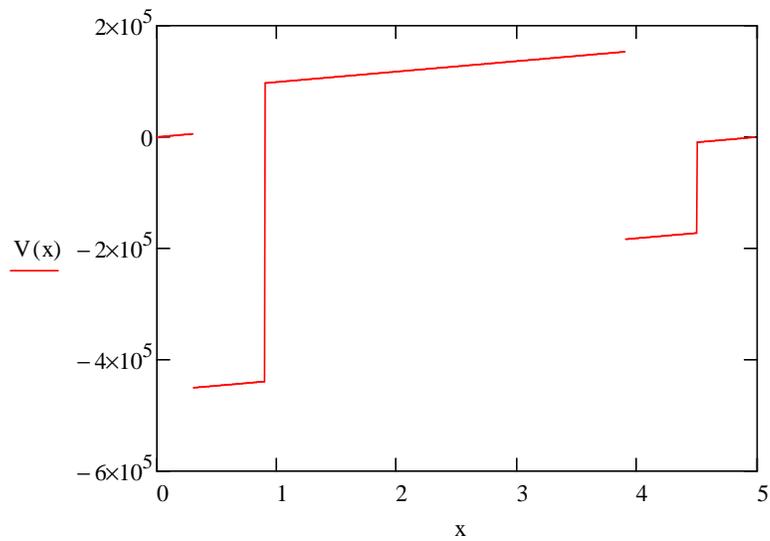
## Belasting voor UC = 1

$$R_{\text{CS}} := \frac{P_u \cdot (z - 0.3\text{m}) + F_{\text{pres}} \cdot 4.2\text{m} + q_{\text{self}} \cdot 5\text{m} \cdot 2.2\text{m}}{3.6\text{m}} = 336.792 \cdot \text{kN}$$

$$R_{\text{SS}} := q_{\text{self}} \cdot 5\text{m} + F_{\text{pres}} + P_u - R_{\text{CS}} = 455.958 \cdot \text{kN}$$

Bepaling dwarskrachtcapaciteit van plaatbrug onder mobiele belasting, eigengewicht en permanente belasting

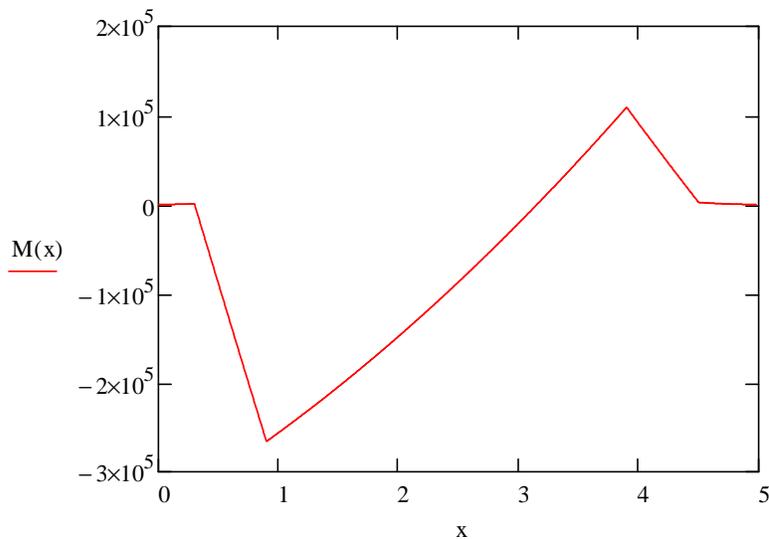
$$V(x) := \begin{cases} (q_{\text{self}} \cdot x) & \text{if } 0\text{m} \leq x < 0.3\text{m} \\ (q_{\text{self}} \cdot x - R_{\text{SS}}) & \text{if } 0.3\text{m} \leq x < z \\ (q_{\text{self}} \cdot x - R_{\text{SS}} + P_u) & \text{if } z \leq x < 3.9\text{m} \\ (q_{\text{self}} \cdot x - R_{\text{SS}} + P_u - R_{\text{CS}}) & \text{if } 3.9\text{m} \leq x < 4.5\text{m} \\ (q_{\text{self}} \cdot x - R_{\text{SS}} + P_u - R_{\text{CS}} + F_{\text{pres}}) & \text{if } 4.5\text{m} \leq x < 5\text{m} \end{cases}$$



$$V_{\text{sup}} := -V(0.3\text{m}) = 450.333 \text{ kN}$$

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$$M(x) := \begin{cases} \left( q_{\text{self}} \cdot \frac{x^2}{2} \right) & \text{if } 0\text{m} \leq x < 0.3\text{m} \\ \left[ q_{\text{self}} \cdot \frac{x^2}{2} - R_{\text{SS}} \cdot (x - 0.3\text{m}) \right] & \text{if } 0.3\text{m} \leq x < z \\ \left[ q_{\text{self}} \cdot \frac{x^2}{2} - R_{\text{SS}} \cdot (x - 0.3\text{m}) + P_{\text{u}} \cdot (x - z) \right] & \text{if } z \leq x < 3.9\text{m} \\ \left[ q_{\text{self}} \cdot \frac{x^2}{2} - R_{\text{SS}} \cdot (x - 0.3\text{m}) + P_{\text{u}} \cdot (x - z) - R_{\text{CS}} \cdot (x - 3.9\text{m}) \right] & \text{if } 3.9\text{m} \leq x < 4.5\text{m} \\ \left[ q_{\text{self}} \cdot \frac{x^2}{2} - R_{\text{SS}} \cdot (x - 0.3\text{m}) + P_{\text{u}} \cdot (x - z) - R_{\text{CS}} \cdot (x - 3.9\text{m}) + F_{\text{pres}} \cdot (x - 4.5\text{m}) \right] & \text{if } 4.5\text{m} \leq x < 5\text{m} \end{cases}$$



$$M_{\text{sup}} := M(0.3\text{m}) = 0.844 \cdot \text{kN} \cdot \text{m}$$

$$M_{\text{max}} := -M(z) = 265.981 \cdot \text{kN} \cdot \text{m}$$

$$a_v := a - \frac{b_{\text{load}}}{2} - \frac{b_{\text{sup}}}{2} = 450 \cdot \text{mm}$$

$$\beta := \max \left( 0.5, \frac{a_v}{2d_1} \right) = 0.849$$

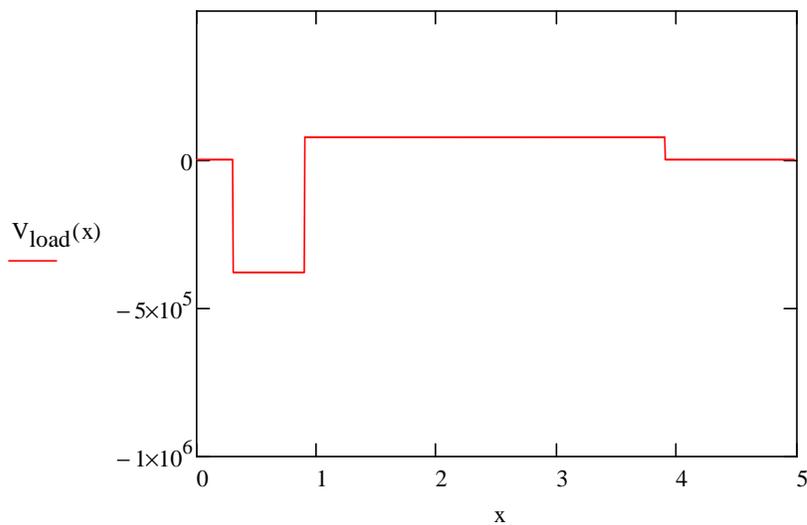
Bepaling dwarskrachtcapaciteit van plaatbrug onder mobiele belasting, eigengewicht en permanente belasting

$$P_{EC} := \beta \cdot P_u = 455.094 \cdot \text{kN}$$

$$R_{CS\_load} := \frac{P_{EC} \cdot (z - 0.3\text{m})}{3.6\text{m}} = 75.849 \cdot \text{kN}$$

$$R_{SS\_load} := P_{EC} - R_{CS\_load} = 379.245 \cdot \text{kN}$$

$$V_{load}(x) := \begin{cases} (0) & \text{if } 0\text{m} \leq x < 0.3\text{m} \\ (-R_{SS\_load}) & \text{if } 0.3\text{m} \leq x < z \\ (-R_{SS\_load} + P_{EC}) & \text{if } z \leq x < 3.9\text{m} \\ (0 - R_{SS\_load} + P_{EC} - R_{CS\_load}) & \text{if } 3.9\text{m} \leq x < 4.5\text{m} \\ (0 - R_{SS\_load} + P_{EC} - R_{CS\_load}) & \text{if } 4.5\text{m} \leq x < 5\text{m} \end{cases}$$



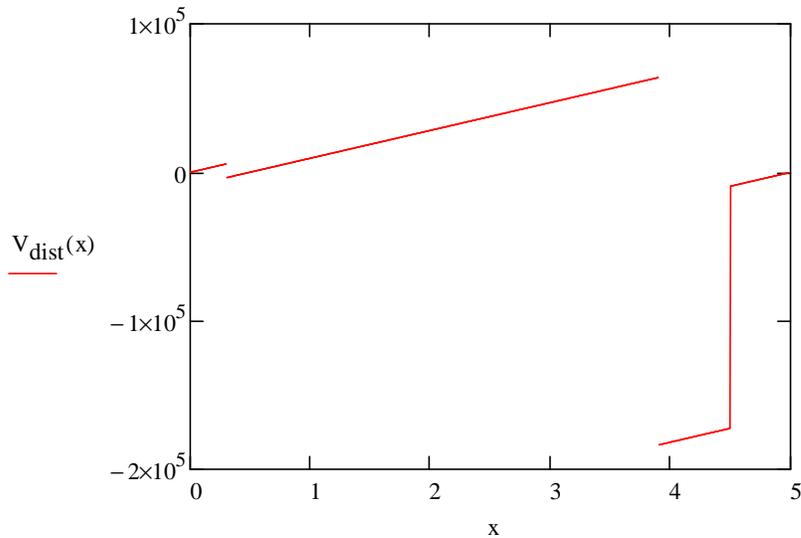
$$V_{sup\_load} := -V_{load}(0.3\text{m}) = 379.245 \cdot \text{kN}$$

$$R_{CS\_dist} := \frac{F_{pres} \cdot 4.2\text{m} + q_{self} \cdot 5\text{m} \cdot 2.2\text{m}}{3.6\text{m}} = 247.458 \cdot \text{kN}$$

$$R_{SS\_dist} := q_{self} \cdot 5\text{m} + F_{pres} - R_{CS\_dist} = 9.292 \cdot \text{kN}$$

Bepaling dwarskrachtcapaciteit van plaatbrug onder mobiele belasting, eigengewicht en permanente belasting

$$V_{\text{dist}}(x) := \begin{cases} (q_{\text{self}} \cdot x) & \text{if } 0\text{m} \leq x < 0.3\text{m} \\ (q_{\text{self}} \cdot x - R_{\text{SS\_dist}}) & \text{if } 0.3\text{m} \leq x < z \\ (q_{\text{self}} \cdot x - R_{\text{SS\_dist}}) & \text{if } z \leq x < 3.9\text{m} \\ (q_{\text{self}} \cdot x - R_{\text{SS\_dist}} - R_{\text{CS\_dist}}) & \text{if } 3.9\text{m} \leq x < 4.5\text{m} \\ (q_{\text{self}} \cdot x - R_{\text{SS\_dist}} - R_{\text{CS\_dist}} + F_{\text{pres}}) & \text{if } 4.5\text{m} \leq x < 5\text{m} \end{cases}$$



$$V_{\text{sup\_dist}} := -V_{\text{dist}}(0.3\text{m}) = 3.667 \cdot \text{kN}$$

$$V_{\text{sup\_load}} + V_{\text{sup\_dist}} = 382.912 \cdot \text{kN}$$

$$b_{\text{eff}} := 2 \cdot (a_v + b_{\text{load}}) + b_{\text{load}} = 1.5 \text{ m}$$

$$v_{\text{Ed}} := \frac{V_{\text{sup\_dist}}}{b \cdot d_1} + \frac{V_{\text{sup\_load}}}{b_{\text{eff}} \cdot d_1} = 0.96 \cdot \text{MPa}$$

$$UC_{\text{shear}} := \frac{v_{\text{Ed}}}{v_{\text{Rd\_mag}}} = 1$$

$$UC_{\text{moment}} := \frac{-M(z)}{M_{\text{Rd}}} = 0.353$$

Bepaling dwarskrachtcapaciteit van plaatbrug onder mobiele belasting, eigengewicht en permanente belasting

Bepaling dwarskrachtcapaciteit van plaatbrug onder mobiele belasting, eigengewicht en permanente belasting

1

- This paper analysis a test to failure in flexure on an existing reinforced concrete slab bridge in two spans.
- A deterministic analysis predicted a possible shear capacity in the experiment.
- The prediction of the failure mode changes when the variability of materials and capacity models is taken into account.
- Monte Carlo simulations were used to find the most likely failure mode.
- The conclusion of the simulations was that the probability of failure in shear is smaller than in flexure.

## **Abstract**

In the Netherlands, the shear capacity of a large number of existing reinforced concrete solid slab bridges is subject to discussion, as initial assessments indicated that their capacity was insufficient. In certain cases, the deterministic value of the moment capacity is larger than the deterministic value of the shear capacity. However, when the variability of the material properties, and of the capacity models themselves are factored in, a probability of a certain failure mode can be calculated. Here, a method is introduced to calculate the chance that a cross-section fails in shear before it fails in bending.

The method that is derived here is applied to the Ruytenschildt Bridge. This case study is a reinforced concrete solid slab bridges that was tested to failure in two spans during the summer of 2014. The relative probability of failure in shear of the bridge was determined. The predictions indicated a smaller probability of a shear failure than of a bending moment failure. In the first tested span, failure was not reached, but indications of flexural distress were observed. In the second span, a flexural failure was achieved, in line with the probabilistic predictions. The presented method can be used in the assessment of existing bridges to determine which failure mode is most probable, taking into account the variability of materials and capacity models.

1 **Probabilistic prediction of the failure mode of the Ruytenschildt Bridge**

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10 Netherlands

## 1 **Abstract**

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17

## 18 **Keywords**

19 Assessment; Bending moment capacity; Deterministic capacity; Failure modes; Field test;  
20 Probabilistic capacity; Reliability analysis; Shear capacity; Slab bridge.

21

## 1 **1. Introduction**

### 2 ***1.1. Existing slab bridges in The Netherlands***

3 In The Netherlands, reinforced concrete solid slab bridges were a popular structural system,  
4 especially during the 1950s, 1960s and 1970s for relatively short spans. Their popularity  
5 coincided with the post-war expansion of the Dutch road network. These bridges were designed  
6 according to the load models and resistance models of that era. Nowadays, the live load models  
7 are heavier, and some resistance models, such as the shear model, allow smaller capacities. The  
8 result is that for a large number of existing reinforced concrete slab bridges, an assessment  
9 according to the design codes shows insufficient capacity for shear [1]. Of 2000 slab bridges,  
10 600 are subject to discussion with regard to their shear capacity. This result does not mean that  
11 these bridges can fail in shear at any time, and are unsafe for the traveling public, but it means  
12 that more suitable methods to assess existing reinforced concrete slab bridges need to be  
13 developed.

### 14 ***1.2. Assessment by Levels of Approximation***

15 An assessment of the Dutch reinforced concrete slab bridges according to the design codes often  
16 shows insufficient shear capacity [1]. However, it must be kept in mind that a number of load-  
17 bearing mechanisms that are activated in reality are not taken into account in the simplified,  
18 empirical code equations. For example, for the shear capacity of NEN-EN 1992-1-1:2005 [2] is  
19 derived based on beam shear experiments [3]. For slabs under concentrated loads, such as the  
20 wheel prints of a live load model, transverse load redistribution increases the shear capacity [4].  
21 As such, more refined methods can be derived for the shear assessment of reinforced concrete  
22 slab bridges. The *fib* Model Code [5] recommends the use of Levels of Approximation: higher  
23 Levels of Approximation will require more computational time and effort, but are expected to

1 give a result closer to the tested capacity of a member. The *fib* Model Code uses an approach  
2 based on Levels of Approximation for the shear capacity and the punching shear capacity: the  
3 lowest Level of Approximation can be used for a preliminary design, and the higher levels for  
4 optimization.

5 Currently, in the Netherlands [6], the following four Levels of Approximation are used  
6 for the shear assessment of reinforced concrete solid slab bridges, further called Levels of  
7 Assessment:

- 8 1. Level of Assessment 1: the Quick Scan method [7], a spreadsheet-based method  
9 that takes recommendations [8] derived from experiments into account. The shear  
10 stress is determined based on a method similar to a hand calculation and the shear  
11 capacity according to NEN-EN 1992-1-1:2005.
- 12 2. Level of Assessment 2: the shear stress distribution is determined based on a  
13 linear finite element model. The peak over a distance of  $4d$  is determined as the  
14 governing shear stress [9], and compared to the shear capacity according to NEN-  
15 EN 1992-1-1:2005.
- 16 3. Level of Assessment 3: using non-linear finite element models to determine the  
17 behavior of the structure under the assigned live load model [10]. Probabilistic  
18 methods are also part of this Level of Assessment.
- 19 4. Level of Assessment 4: Proof loading of structures [11, 12].

20 In this paper, a probabilistic approach is used to determine the probability of failure in bending  
21 as compared to the probability of failure in shear. While this procedure is not a standard part of  
22 the presented four Levels of Assessment, it can be used to interpret the assessment results taking

1 into account the variability of the material properties and the capacity models. Such a method is  
2 also useful when preparing a proof load experiment, Level of Assessment 4, to analyze which  
3 failure mode can be expected [13].

#### 4 ***1.3. Slab shear experiments***

5 Over the past few years, a large series of experiments on reinforced concrete slabs subjected to  
6 concentrated loads has been carried out [14-17]. The slabs had dimensions of 5000 mm × 2500  
7 mm × 300 mm. The span length was 3600 mm. The longitudinal reinforcement ratio was 1%.  
8 The size of the slabs was half scale of existing reinforced concrete slab bridges, to study the  
9 effect of the larger concentrated live loads prescribed by NEN EN 1991-2:2003 [18]. A single  
10 concentrated load (size of the loading plate was varied: 200 mm × 200 mm or 300 mm × 300  
11 mm) was applied close to the support, since loads close to the support result in the highest  
12 sectional shear forces, which in assessment would be the critical loading case. The concentrated  
13 load was applied in the middle of the slab width as well as close to the free edge. In the series of  
14 experiments, the following parameters were studied: size of the loading plate, amount of  
15 transverse reinforcement, concrete compressive strength, position of the concentrated load (along  
16 the span length and along the width), effect of existing cracks, line support versus bearings,  
17 ribbed reinforcement bars versus plain reinforcement bars, and the effect of combining the  
18 concentrated load with a line load applied further in the span (at 1200 mm from the center of the  
19 support). The main conclusion of this research was that slabs in shear behave differently than  
20 beams. The explanation for this conclusion is that slabs are three-dimensional elements, which,  
21 when subjected to concentrated loads, have a larger capacity because of their ability to  
22 redistribute stresses in the transverse direction [4].

## 1 **2. Literature review on probabilistic methods**

2 For the shear assessment of reinforced concrete solid slab bridges, the probability of failure of a  
3 cross-section in shear is determined by taking into account the prescribed probability density  
4 functions of the applied loads, the resistance models and the material models. Recommendations  
5 and guidelines for the probability density functions that can be used are given by the  
6 Probabilistic Model Code, as developed by the Joint Committee on Structural Safety [19, 20].  
7 Slightly different values for the resistance models can be found in a 1998 paper on bridge load  
8 and resistance models [21]. For reinforced concrete solid slab bridges subjected to concentrated  
9 wheel loads, the probability density function of the resistance model is different from the case of  
10 a beam failing in shear, because of transverse load redistribution [22]. It was found that the  
11 resistance model can be conservatively approximated based on a lognormal distribution, which  
12 will be used for the analysis here.

13 In the Netherlands, for existing structures, a set of codes are being developed, which  
14 allow lower reliability levels, as shown in Table 1. Here, the reliability levels of NEN-EN  
15 1990:2003 [23] and NEN 8700:2011 [24], also adopted by the Dutch Guidelines for the  
16 Assessment of Existing Bridges (“RBK”) [25], are summarized. The load factors are described in  
17 NEN 8700:2011 [24], the loads are described in NEN 8701:2011 [26], and in the future NEN  
18 8702 will contain the capacity models for concrete structures. The numbering of the NEN 8700-  
19 series of codes follows the structure of the Eurocodes. All considered values are for  
20 Consequences Class 3 (highway bridges). The values are for bridges designed and built under the  
21 regulations for construction of 2003 or earlier. For newer structures, higher reliability levels are  
22 required. The presented principles have been applied to reinforced concrete slab bridges [27].

1 Reliability-based methods for shear have gained more importance over the last few years.  
2 A more advanced probabilistic analysis (full probabilistic nonlinear analysis) of the shear and  
3 bending moment capacity of beams was developed based and showed that for beams subjected to  
4 a combination of shear and bending, the required reliability level of NEN-EN 1992-1-1:2005 is  
5 not achieved [28]. It also must be noted that the models presented by the JCSS are valid for  
6 concrete beams with steel reinforcement; the derivation for concrete beams with fibre-reinforced  
7 polymer bars is also available in the literature [29].

8 For the assessment of bridges, reliability-based systems also have been developed. These  
9 methods have led to an improvement of the current bridge rating practices [30-32]. A systems-  
10 level safety evaluation combined with nonlinear finite element analysis was developed in  
11 Switzerland [33]. In the United States, the shear capacity of existing reinforced concrete bridges  
12 was also studied based on a probabilistic analysis [34]. Moreover, the 2011 Virginia earthquake  
13 caused concern about the capacity of existing bridges under seismic events, which have also  
14 been studied based on a reliability analysis [35]. For Germany, procedures on how to combine  
15 non-destructive testing results with probabilistic analysis methods are available [36].

16 An interesting observation from the literature review is the safety philosophy for the  
17 reliability-based assessment of existing bridges. For design, the source of uncertainty in  
18 structural resistance is classified into three categories: material (mechanical and chemical  
19 properties), fabrication (geometrical properties), and analysis (approximate method of analysis)  
20 [37]. For assessment, the philosophy that is followed in the Netherlands is that only the material  
21 and analysis uncertainties remain. The geometric properties are not random variables anymore,  
22 since the structure has been built and the uncertainty on the dimensions are thus taken away [38].

1 However, in North America [32], a different safety philosophy is followed, and the uncertainty  
2 on the geometric properties is fully modelled.

### 3 **3. Description of case study bridge**

#### 4 **3.1. Structural system**

5 A full description and discussion of the load test and test to failure of the Ruytenschildt Bridge is  
6 discussed in a companion paper [39] to this work. Here, only the main properties of the structure  
7 will be described that are necessary for the probabilistic calculations in this paper.

8 The Ruytenschildt Bridge has five spans of each 9 m and is a reinforced concrete solid  
9 slab integral bridge, with an 18-degree skew angle. Staged demolition was used so that the tested  
10 structure had a width of 7.365 m. The slab thickness was 550 mm. The reinforced layout is  
11 shown in Figure 1.

12 The first experiment was carried out in span 1, at a face-to-face distance between the first  
13 axle of the load tandem and the support of 1250 mm ( $2.5d$ , the critical position for shear failure).  
14 The distance from the free edge and the side face of the wheel print was 800 mm. The second  
15 experiment was carried out in span 2, with the face of the front axle of the load tandem at 1250  
16 mm from the face of the support. The distance from the free edge and the side face of the wheel  
17 print was 600 mm. The load tandem was the tandem from Eurocode live load model 1 [18], with  
18 two axles 1.2 m apart. Each axle has two wheel prints of 400 mm  $\times$  400 mm, spaced 2 m apart.

#### 19 **3.2. Material Properties**

20 The material properties were determined through destructive tests. The compressive strength of  
21 concrete cores was on average  $f_{cm} = 63$  MPa, which gives a cylinder compressive strength  $f_{cm,cyl}$   
22 of 52 MPa. The conversion from core compressive strength, which is similar to the cube

1 compressive strength, to cylinder compressive strength is done with a factor 0.82, as used in the  
2 Netherlands for the assessment of existing bridge [40]. The measured standard deviation on the  
3 drilled cores was 11.55 MPa, so that the characteristic value of the concrete compressive strength  
4 becomes  $f_{ck} = 33$  MPa. The reinforcement is QR24 steel with a yield strength of  $f_y = 282$  MPa  
5 and a tensile strength of  $f_t = 360$  MPa, measured on samples from a similar bridge [41].

## 6 **4. Probability of shear failure in experiment**

### 7 ***4.1. Introduction***

8 As the deterministic values indicated a possibility of failure in shear before flexure in the second  
9 span, a probabilistic analysis was carried out to identify the probability of failure in shear as  
10 compared to the probability of failure in flexure. This type of analysis can also be carried out as  
11 part of an assessment, to evaluate the probability of a failure in shear, which is a brittle type of  
12 failure. Given that the first assessments according to the governing codes [42] indicated that the  
13 shear capacity of the existing reinforced concrete solid slab bridges is critical, in the Netherlands,  
14 the transportation officials want to have a solid estimate of the probability of a brittle shear  
15 failure in existing bridges. Part of developing this estimate was geared towards studying the  
16 shear capacity of reinforced concrete solid slabs under concentrated wheel loads close to  
17 supports through experiments [14-17]. These experimental results are also used for the  
18 probabilistic analysis presented hereafter.

### 19 ***4.2. Limit state function***

20 The probability of a shear failure as compared to the probability of a flexural failure is  
21 determined with a Monte Carlo simulation. The question of the probability of failure in flexure

1 before shear (expression used for this derivation) is translated into a limit state function. The  
 2 chance of failure,  $p_f$ , is sought for the following situation:

$$3 \quad p_f = P(\text{moment} < \text{shear}) \quad (1)$$

4 The relative capacities for shear and moment will be expressed based on a Unity Check. The  
 5 Unity Check, as typically used for assessment in The Netherlands, is the ratio of load effect over  
 6 resistance. For example, the Unity Check for shear,  $UC_{shear}$  is the ratio of the shear stress caused  
 7 by the applied load over the shear capacity as described by the governing code. As such, the limit  
 8 state function is rewritten as:

$$9 \quad p_f = P(UC_{shear} < UC_{moment}) \quad (2)$$

10 In a next step, the Unity Checks for shear and moment are expressed. The Unity Check  
 11 for shear can be expressed as the ratio of the shear stress caused by the applied loads to the shear  
 12 capacity. The loading side of the equation is not a function of random variables, since in the test  
 13 to failure a deterministic value of the load is applied (i.e, the load at which failure is achieved).  
 14 Therefore, the Unity Check can be expressed as a function of the ratio of predicted to tested  
 15 capacities, as used to analyse code equations and as proposed by Yura et al. [43, 44]. For shear,  
 16 the capacity that replaces the loading side of the equation is the characteristic value according to  
 17 NEN-EN 1992-1-1:2005 [2], based on the code assumption that a factored load effect is always  
 18 smaller than or equal to the design capacity. The resistance side of the equation is the expression  
 19 for the mean value of the shear capacity, multiplied by the ratio of tested to predicted values  
 20 observed from slab shear experiments. As such, the resistance side of the equation on average is  
 21 equal to the “real” capacity of a slab tested in shear. The expression for the Unity Check is thus:

$$UC_{shear} = \frac{v_{Ed}}{v_{Rd,c}} = \frac{\frac{C_{Rd,c}}{\gamma_c} k (100\rho_l f_{ck})^{1/3}}{\left(\frac{Test}{Predicted}\right)_V C_{Rd,c,test} k (100\rho_l f_{c,mean})^{1/3}} \quad (3)$$

2 with:

3  $\left(\frac{Test}{Predicted}\right)_V$  the ratio of experimental sectional shears to predicted values according to NEN-

4 EN 1992-1-1:2005 [2], based on half-scale experiments of slab bridges [45]

5  $C_{Rd,c,test}$  0.15: average value [3]

6  $f_{c,mean}$  average concrete cylinder compressive strength

7  $f_{ck}$  characteristic concrete cylinder compressive strength

8  $C_{Rd,c}$  0.18 as used in NEN-EN 1992-1-1:2005 [2]

9  $\gamma_c$  1.5 as used in NEN-EN 1992-1-1:2005 [2]

10 Equation (3) can be simplified as follows:

$$UC_{shear} = \frac{\frac{C_{Rd,c}}{\gamma_c} (f_{ck})^{1/3}}{\left(\frac{Test}{Predicted}\right)_V C_{Rd,c,test} (f_{c,mean})^{1/3}} = \frac{0.8(f_{ck})^{1/3}}{\left(\frac{Test}{Predicted}\right)_V (f_{c,mean})^{1/3}} \quad (4)$$

12 The Unity Check for the moment capacity is also expressed based on the ratio of the predicted to

13 the observed moment capacity. The predicted moment capacity is the moment capacity that

14 would be derived based on the characteristic yield strength of the steel. The observed moment

15 capacity is based on the tensile capacity of the steel and takes the tested to predicted ratio as

16 recommended by the JCSS probabilistic model code [19] into account. This value is derived for

17 beams, and data for slabs under concentrated loads failing in bending are not available to fine-

1 tune this parameter. With these considerations, the Unity Check for the bending moment  
 2 capacity becomes:

$$3 \quad UC_{moment} = \frac{A_s f_{yk} \left( d - \frac{a}{2} \right)}{\left( \frac{Test}{Predicted} \right)_M A_s f_t \left( d - \frac{a}{2} \right)} \quad (5)$$

4 with

5  $A_s$  the area of steel

6  $f_{yk}$  the characteristic yield strength of the steel

7  $d$  the effective depth of the cross-section

8  $a$  the height of the compressive stress block

9  $\left( \frac{Test}{Predicted} \right)_M$  the experimental to predicted capacity of beams in bending

10  $f_t$  the average tensile strength of the steel

11 With a characteristic yield strength of the steel of 240 MPa and an average tensile strength of  
 12 360 MPa the ratio of  $f_{yk}/f_t = 240\text{MPa}/360\text{MPa} = 0.667$ . The expression for the Unity Check of the  
 13 bending moment from Equation (5) can thus be simplified as:

$$14 \quad UC_{moment} = \frac{f_y}{\left( \frac{Test}{Predicted} \right)_M f_u} = \left( \frac{Test}{Predicted} \right)_M 0.667 \quad (6)$$

15 In the first span, the value of the Unity Check for moment can be calculated deterministically as  
 16 a magnification factor for the loading for which the Unity Check for shear equals one. The Unity  
 17 Check for shear equals one for a load on the tandem of 1972 kN. This value is determined by  
 18 taking into account all load and resistance factors. The load spreading method  $b_{para}$  is used to  
 19 determine the effective width. The load factors then are  $\gamma_{traffic} = 1.3$  for the live loads and  $\gamma_{permanent}$

1 = 1.15 for the dead load and the load caused by the wearing surface. The material factor for  
 2 concrete  $\gamma_c = 1.5$  is used. These load factors correspond with the reconstruction level of the RBK  
 3 (Guideline for the Assessment of Existing Bridges) [25]. This load of 1972 kN causes a  
 4 maximum moment in the cross-section of  $M_{UC, shear} = 3363$  kNm, as determined with a beam  
 5 model.

6 The value of the design moment capacity can be determined as:

$$7 \quad M_{Rd} = A_s f_{yk} \left( d - \frac{a}{2} \right) \quad (7)$$

8 Using a rectangular stress block, the value of the height of the stress block is:

$$9 \quad a = \frac{A_s f_{yk}}{0.85 f_{ck} b} \quad (8)$$

10 In the first span, the available reinforcement is  $A_s = 3866$  mm<sup>2</sup> per 1 m of width, which gives (for  
 11  $f_{yk} = 240$  MPa and  $f_{ck} = 33$  MPa) a height of the concrete stress block of  $a = 33.08$  mm, so that  
 12 the design moment capacity becomes:

$$13 \quad M_{Rd} = 7.365m \times 3866 \frac{mm^2}{m} \times 240MPa \left( 499mm - \frac{33.08mm}{2} \right) = 3297kNm \quad (9)$$

14 The ratio of the moment when the design value of the Unity Check for shear equals one to the  
 15 calculated design moment capacity is thus for the first span:

$$16 \quad \frac{M_{UC, shear}}{M_{Rd}} = \frac{3363kNm}{3297kNm} = 1.020 \quad (10)$$

17 A similar analysis can be made for the second span. For this case, a total load on the  
 18 tandem of 2308 kN causes a design Unity Check of 1. This load corresponds with a span moment  
 19 of 2462 kNm. The design capacity of this cross-section is 3119 kNm. The design values thus

1 predict that the cross-section, when loaded with the proof loading tandem at the critical position,  
 2 could fail in shear. The magnification factor is now:

$$3 \quad \frac{M_{UC, shear}}{M_{Rd}} = \frac{2462kNm}{3119kNm} = 0.789 \quad (11)$$

4 The limit state function from Equation (2) is expressed as:

$$5 \quad p_f = P(UC_{shear} - UC_{moment} < 0) \quad (12)$$

6 The expression between brackets is now filled in with the results from Equations (4), (6) and the  
 7 magnifications factors from Eq. (10) and (11) for spans 1 and 2 respectively. The limit state  
 8 function for span 1 thus becomes:

$$9 \quad 0.8 \frac{f_{ck}^{1/3}}{f_{c, mean}^{1/3}} \frac{1}{\left( \frac{Test}{Predicted} \right)_V} - 0.667 \times 1.020 \frac{1}{\left( \frac{Test}{Predicted} \right)_M} \leq 0 \quad (13)$$

10 Similarly, for span 2 the limit state function becomes:

$$11 \quad 0.8 \frac{f_{ck}^{1/3}}{f_{c, mean}^{1/3}} \frac{1}{\left( \frac{Test}{Predicted} \right)_V} - 0.667 \times 0.789 \frac{1}{\left( \frac{Test}{Predicted} \right)_M} \leq 0 \quad (14)$$

### 12 **4.3. Random variables and distributions**

13 In the identified limit state functions Eqs. (13) and (14), the following random variables  
 14 can be identified:

- 15 •  $f_{ck}$ , which follows a lognormal distribution according to the JCSS [19]
- 16 •  $f_{c, mean}$ , which follows a lognormal distribution according to the JCSS [19]
- 17 •  $\left( \frac{Test}{Predicted} \right)_V$  which follows a lognormal distribution determined on slab shear  
 18 experiments [22]

- $\left(\frac{Test}{Predicted}\right)_M$  which follows a lognormal distribution according to JCSS [19]

1  
2 The concrete compressive strength was determined based on a large number of cores drilled from  
3 the bridge. A correction factor of 1.575 was used for the poor surface treatment of the cores [46]  
4 and a factor of 0.82 was used to convert core strengths into cylinder strengths [40]. The factor for  
5 the poor surface treatment is necessary, since capping was not used on the samples. Afterwards,  
6 a series of cores with and without capping was studied to see the reduction in measured capacity  
7 when the surface treatment is not optimal. The conversion factor from cores to cylinder  
8 compressive strengths is based on the idea that the core strength is about the same as the cube  
9 compressive strength. The cylinder compressive strength is 0.80 to 0.85 times the cube  
10 compressive strength [47]. In the Netherlands, 0.82 is used as conversion factor. Figure 2 shows  
11 the resulting histogram of the concrete compressive strengths. It can be seen that the assumption  
12 of a lognormal distribution corresponds to the measured probability density function and  
13 cumulative distribution function.

14 As an input for a lognormal distribution, the mean of the natural logarithm  $\lambda$  and its  
15 associated standard deviation  $\varepsilon$  need to be determined. For the mean concrete cylinder  
16 compressive strength,  $f_{c,mean}$ , an average value of 52 MPa and a standard deviation of 11.55 MPa  
17 are used, which gives a coefficient of variation of 22%. For the characteristic value of the  
18 concrete cylinder compressive strength  $f_{ck}$ , an average value of 33 MPa is found. Assuming that  
19 the coefficient of variation is the same as for the mean values, a standard deviation of 7.33 MPa  
20 is found. These values have to be converted to input for the lognormal distribution,  $\lambda$  and  $\varepsilon$ . The  
21 following formulas are used for the conversion:

$$\mu = e^{\left(\lambda + \frac{\varepsilon^2}{2}\right)} \quad (15)$$

$$\sigma = \sqrt{\left(e^{\varepsilon^2} - 1\right)e^{2\lambda + \varepsilon^2}} \quad (16)$$

3 A MathCad sheet is used to make the conversion. The resulting input values for the lognormal

4 distributions of  $f_{ck}$  and  $f_{c,mean}$  are given in Table 2. The values of  $\left(\frac{Test}{Predicted}\right)_v$  from the slab

5 shear experiments is taken from [22] and given in Table 2. The values of  $\left(\frac{Test}{Predicted}\right)_v$  and

6  $\left(\frac{Test}{Predicted}\right)_M$  from the JCSS Probabilistic Model Code [19] for reinforced concrete beams are

7 also given in Table 2. The JCSS is based on average and coefficient of variation. Here, the same

8 MathCad sheet is used to convert these results to  $\lambda$  and  $\varepsilon$  with Equations (15) and (16).

#### 9 **4.4. Monte Carlo simulations and results**

10 The limit state functions from Equations (13) and (14) for spans 1 and 2, with the random

11 variables and their properties as given in Table 2 are solved with a crude Monte Carlo

12 simulation. A total of  $10^5$  drawings were used for the simulations. The equations are solved both

13 by using the ratio  $\left(\frac{Test}{Predicted}\right)_v$  recommended for beams from the JCSS probabilistic model

14 code [19] and based on the slab shear experiments [22], for spans 1 and 2. In the Monte Carlo

15 simulations, the probability density functions of  $f_{c,mean}$  and  $f_{ck}$  are constructed with the input from

16 Table 2. The resulting function for  $f_{c,mean}$  is shown in Figure 3 and for  $f_{ck}$  in Figure 4. Similarly,

17 the probability density functions of  $\left(\frac{Test}{Predicted}\right)_v$  for beams and slabs, and  $\left(\frac{Test}{Predicted}\right)_M$  are

1 constructed based on the parameters from Table 2. The resulting function for  $\left(\frac{Test}{Predicted}\right)_V$  for  
 2 beams is shown in Figure 5, for slabs in Figure 6, and the function for  $\left(\frac{Test}{Predicted}\right)_M$  in Figure 7.  
 3 The distribution of the limit state function for span 1, using  $\left(\frac{Test}{Predicted}\right)_V$  for beams is shown in  
 4 Figure 8 and using  $\left(\frac{Test}{Predicted}\right)_V$  based on slab shear experiments in Figure 9. For span 2, the  
 5 results using  $\left(\frac{Test}{Predicted}\right)_V$  from the JCSS probabilistic model code are given in Figure 10 and  
 6 with  $\left(\frac{Test}{Predicted}\right)_V$  based on the slab shear experiments in Figure 11. The probability of failure  
 7 in bending moment is then calculated by counting the occurrences of the limit state function  
 8 below 0. Dividing this value by the number of drawings,  $10^5$ , gives the probability of failure in  
 9 bending moment. The probability of shear failure is then calculated as 100% minus the  
 10 probability in failure in bending moment. The results for the four analysed cases are given in  
 11 Table 3.

12 The first observation one can make from Table 3, is that in span 1 a bending moment  
 13 failure is expected, while the situation in span 2 might not be that clear. The conclusions with the  
 14 deterministic analysis of the Unity Checks for design, showing that the Unity Check in bending  
 15 is 1.020 when the Unity Check in shear is 1, was that bending moment and shear failure could  
 16 possibly occur. However, taking into account the variability of the materials and, most  
 17 importantly, the variability on the resistance models, shows, in Table 3, that a bending moment

1 failure is much more likely than a shear failure. Taking into account the data from the slab shear  
2 tests, this conclusion is even more reinforced, showing a very low probability of a shear failure.

3 For the second span, the deterministic analysis of the Unity Checks for design showed  
4 that the Unity Check in bending is 0.789 when the Unity Check in shear is 1. A failure in shear  
5 would be expected from the deterministic analysis. This observation is reflected in Table 3, when  
6 the distribution on the shear model from the JCSS probabilistic model code is taken into account.  
7 When the results of the slab shear experiments are added, the probability of failure in shear  
8 becomes very low. This observation shows as well that using experimental results to improve  
9 probabilistic analyses can have a large impact on the outcome of a probabilistic analysis.

10 Comparing the deterministic analysis and the probabilistic analysis, it can be seen that the  
11 deterministic analysis not always immediately catches the failure mode that a more advanced  
12 analysis identifies. As such, the probabilistic analysis gives more information about the failure  
13 mode that can be expected in an experiment, taking into account the variability on the material  
14 parameters and the capacity models. The variability on the moment capacity, as shown in Figure  
15 7, is much lower than the variability on the shear capacity, as shown in Figure 5 and Figure 6.  
16 Therefore, a probabilistic analysis can give more insight in the behaviour that can be expected  
17 during a load test.

## 18 **5. Comparison between prediction and test results**

19 The maximum load during the test on span 1 was 3049 kN, but failure was not achieved  
20 as the maximum load was determined by the maximum available counter weight. Flexural  
21 cracking was observed. This flexural distress is in correspondence with the predicted failure  
22 mode according to the probabilistic analysis. The deterministic analysis showed that both shear

1 and flexural failure could occur. As no failure could be achieved in this span, a final conclusion  
2 on the predictions cannot be given.

3 For testing on span 2, additional loading was ordered. The maximum load was 3991 kN.  
4 Flexural failure was achieved, in combination with a settlement of the pier at support 2. Here, the  
5 benefit of the probabilistic analysis is clear. Whereas the deterministic results showed that shear  
6 failure would occur before flexural failure, the first probabilistic analysis showed a reasonable  
7 probability of failure in bending moment, albeit smaller than the probability of failure in shear.  
8 Adding the results from the slab shear experiments then showed that the probability of failure in  
9 bending moment is significantly larger than the probability of failure in shear. This prediction  
10 corresponded to the test result on span 2 of the Ruytenschildt bridge.

11 The presented method is based on the Eurocodes. Similar expressions for the limit state  
12 function could be derived for other codes, such as AASHTO LRFR [48]. The expression for  
13  $UC_{moment}$  would remain unchanged. The expression for  $UC_{shear}$  would change, but would still be a  
14 function of the material and model uncertainties. The ratio of the Unity Checks to build into the  
15 expression for the limit state function would also have to be altered.

## 16 **6. Verification with selected slab shear experiments**

17 To verify the proposed method for future use, 4 experiments from the slab shear  
18 experiments [14-17] are selected, see Table 4. Slabs with normal and high strength concrete are  
19 selected, and two positions of the concentrated load are studied. The value of the Unity Check  
20 for bending is then determined for the load that causes a Unity Check = 1 for shear. These values  
21 are given in Table 4. Then, Eq. (13) can be applied. For the considered slab shear experiments,

1 S500 steel was used with a yield strength of 542 MPa and an ultimate strength of 658 MPa. The  
 2 ratio of  $f_y/f_u$  is then 0.824. As a result, Eq. (13) becomes as follows for S1T1:

$$3 \quad 0.8 \frac{f_{ck}^{1/3}}{f_{c,mean}^{1/3}} \frac{1}{\left(\frac{Test}{Predicted}\right)_V} - 0.824 \times 0.353 \frac{1}{\left(\frac{Test}{Predicted}\right)_M} \leq 0 \quad (17)$$

4 The properties of  $f_{ck}$  and  $f_{c,mean}$  are based on a large number of material samples that were  
 5 taken alongside the slab testing program, so that the statistical properties could be derived for the  
 6 Dutch concrete classes B35 (normal strength concrete) and B65 (high strength concrete) used in  
 7 the experiments. The sample test results follow a lognormal distribution, and the properties are  
 8 given in Table 5.  $\left(\frac{Test}{Predicted}\right)_V$  and  $\left(\frac{Test}{Predicted}\right)_M$  follow the lognormal distribution with  
 9 properties given in Table 2.

10 With all random variables defined, the Monte Carlo simulations can be used to determine  
 11 the probability of failure in bending moment relatively compared to the probability of failure in  
 12 shear. The results are given in Table 4. S1T1, S5T4 and S9T1 are clearly predicted to fail in  
 13 shear, as happened in the experiment. S8T1 has about a fifty-fifty chance to fail in shear or  
 14 moment, and failed in shear in the experiment. As such, the prediction method for the failure  
 15 mode gives good results. Moreover, it can be seen that for higher Unity Checks for bending  
 16 moment for the load that results in a Unity Check = 1 for shear, the chance of failure in bending  
 17 moment becomes relatively higher as well. The verification procedure has thus shown that the  
 18 proposed method is valid.

## 1 **7. Summary and conclusions**

2           The shear capacity of existing reinforced concrete solid slab bridges is the subject of  
3 discussion in the Netherlands, as the recently implemented Eurocodes prescribe higher live loads  
4 and lower shear capacities. To study the behaviour of this type of bridges, the Ruytenschildt  
5 bridge was tested to failure in two spans in the summer of 2014. In the first span, not enough  
6 load was available to achieve failure, but flexural distress was observed. In the second span, a  
7 flexural failure combined with a significant settlement of the pier occurred. The experiments  
8 were carried out at the critical position for shear, and indicated that shear was not the governing  
9 failure mode in the field for this bridge. An assessment based on the Eurocodes for design is thus  
10 found to be very conservative for existing reinforced concrete slab bridges.

11           For the shear assessment of reinforced concrete slab bridges, currently different Levels of  
12 Assessment are used. Reliability analysis can be used as an additional assessment tool, and load  
13 testing can be used at Level of Assessment 4. The presented study is not part of a traditional  
14 shear assessment. It has proven to be a suitable tool for the preparation of load tests.

15           Reliability methods for shear from the literature show the importance of taking into  
16 account the variability of material parameters and capacity models. These concepts were  
17 investigated by means of Monte Carlo simulations. In these simulations, the probability of failure  
18 in bending moment as compared to the probability of failure in shear was studied. The limit state  
19 function was expressed based on the Unity Checks of predicted to experimental capacities. The  
20 studied random variables were the concrete compressive strength and the calibration factors of  
21 the capacity models. All random variables follow a lognormal distribution.

22           The results of the Monte Carlo simulations indicate that in span 1, a flexural failure is  
23 expected. The deterministic analysis showed that both shear or flexural failure could occur. In

1 span 2, the deterministic analysis showed that a shear failure would occur. The first Monte Carlo  
 2 simulation, with the calibration factors on the capacity models from the JCSS probabilistic model  
 3 code, showed a larger probability of failure in shear than in bending, but with a considerable  
 4 probability of failure in bending moment. Adding the calibration factor on the design model  
 5 based on experiments on slabs failing in shear indicated that the probability of failure in bending  
 6 moment is considerably higher than in shear. These results can be attributed to the larger  
 7 variability on the calibration factors for shear models than for bending moment capacity models.

8 The proposed method for estimating a failure mode was verified with four of the slab  
 9 shear experiments. It was found that the shear failures were correctly predicted.

10 To conclude, this paper has presented a method for estimating the probability of failure in  
 11 a certain failure mode that can be used to assess structures and prepare load tests. This analysis  
 12 showed the need for taking the variability of material models, and, most importantly, capacity  
 13 models into account.

#### 14 **Acknowledgements**

15 The authors wish to express their gratitude and sincere appreciation to the Province of Friesland  
 16 and the Dutch Ministry of Infrastructure and the Environment (Rijkswaterstaat) for financing this  
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 18 Hemert and Y. Yang, of the contractor de Boer en de Groot and of Mammoet, responsible for  
 19 applying the load, are also gratefully acknowledged.

#### 20 **List of notation**

21  $a$  the height of the compressive stress block  
 22  $a_{pos}$  the center-to-center distance between the load and the support

1	$b$	the member width
2	class	the considered concrete class
3	$d$	the effective depth of the cross-section
4	$f_{ck}$	the characteristic concrete cylinder compressive strength
5	$f_{cm}$	the compressive strength of concrete cores
6	$f_{c,mean}$	average concrete cylinder compressive strength
7	$f_{cm,cyl}$	the cylinder concrete compressive strength
8	$f_t$	the tensile strength
9	$f_y$	the yield strength
10	$f_{yk}$	the characteristic yield strength of the steel
11	$p_f$	the probability of failure
12	$p_{bf}$	the relative probability of a bending failure
13	$p_{sf}$	the relative probability of a shear failure
14	$A_s$	the area of steel
15	B	beam shear failure mode with a clear shear crack on the side face of the specimen
16	$C_{Rd,c,test}$	0.15: average value [3]
17	$C_{Rd,c}$	0.18 as used in NEN-EN 1992-1-1:2005 [2]
18	FM	failure mode
19	$M_{Rd}$	the design value of the moment capacity
20	$M_{UC,shear}$	the moment that corresponds with the load that causes a design Unity Check for
21		shear of 1
22	$P$	function of a chance
23	P	punching failure mode, with a partially developed cone

1	$\left( \frac{Test}{Predicted} \right)_M$	the experimental to predicted capacity of beams in bending
2	$\left( \frac{Test}{Predicted} \right)_V$	the ratio of experimental sectional shears to predicted values according to NEN-
3		EN 1992-1-1:2005 [2], based on half-scale experiments of slab bridges [45]
4	$UC_{moment}$	the Unity Check for moment capacity
5	$UC_{shear}$	the Unity Check for shear capacity
6	WB	wide beam shear failure mode: the bottom face of the slab shows inclined cracks,
7		indicating shear distress
8	$\beta$	the reliability index
9	$\gamma_c$	1.5 as used in NEN-EN 1992-1-1:2005 [2]
10	$\varepsilon$	the standard deviation of a lognormal distribution
11	$\lambda$	the mean of the natural logarithm
12	$\mu$	the mean value
13	$\sigma$	the standard deviation

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1

1 **List of tables and figures**2 **List of Tables**3 **Table 1 – Different reliability levels for assessment of existing structures according to NEN**4 **8700:2011, used in the Guideline for existing bridges (RBK).**

Reliability level	$\beta$	Reference period
ULS Eurocode	3.8	50 years
RBK Design	4.3	100 years
RBK Reconstruction	3.6	30 years
RBK Usage	3.3	30 years
RBK Disapproval	3.1	15 years
SLS Eurocode	1.5	50 years

5

6 **Table 2 - Overview of input values for lognormal distributions of random variables**

Random variable	$\lambda$	$\varepsilon$
$f_{c,mean}$	3.927	0.219
$f_{ck}$	3.472	0.219
$\left(\frac{Test}{Predicted}\right)_V$ , slab shear experiments	0.697	0.130
$\left(\frac{Test}{Predicted}\right)_V$ , JCSS	0.306	0.246
$\left(\frac{Test}{Predicted}\right)_M$	0.177	0.104

7

1 **Table 3 – Results of Monte Carlo simulations**

	Span 1		Span 2	
	beam results	slab results	beam results	slab results
probability of bending failure (%)	66.49	99.50	31.36	90.14
probability of shear failure (%)	33.61	0.50	68.64	9.86

2

3 **Table 4 – Verification with slab shear experiments [14-17]**

Test nr	$a_{pos}$ (mm)	class	FM	$UC_{moment}$	$p_{bf}$ (%)	$p_{sf}$ (%)
S1T1	600	B35	WB	0.353	0,64	99,36
S5T4	400	B35	WB + B	0.377	1,82	98,19
S8T1	600	B65	WB	0.563	52,90	47,10
S9T1	400	B65	WB + P	0.444	9,14	90,86

4

5 **Table 5 - Overview of input values for lognormal distributions of B35 and B65 concrete**

	Random variable	$\lambda$	$\varepsilon$
B35	$f_{c,mean}$ (MPa)	3.76	0.08
	$f_{ck}$ (MPa)	3.56	0.10
B65	$f_{c,mean}$ (MPa)	4.29	0.05
	$f_{ck}$ (MPa)	4.18	0.05

6

7

1 **List of Figures**

2 **Figure 1** - Reinforcement layout of the Ruytenschildt Bridge (symmetric structure, one half is  
3 shown): (a) plan view; (b) side view. Units: mm.

4 **Figure 2** - Distribution of the concrete cylinder compressive strength.

5 **Figure 3** – Probability density function of  $f_{c,mean}$

6 **Figure 4** – Probability density function of  $f_{ck}$

7 **Figure 5** – Probability density function of  $\left(\frac{Test}{Predicted}\right)_V$  for beams

8 **Figure 6** – Probability density function of  $\left(\frac{Test}{Predicted}\right)_V$  for slabs

9 **Figure 7** – Probability density function of  $\left(\frac{Test}{Predicted}\right)_M$

10 **Figure 8** – Probability density function of limit state function for span 1, with  $\left(\frac{Test}{Predicted}\right)_V$  for  
11 beams

12 **Figure 9** – Probability density function of limit state function for span 1, with  $\left(\frac{Test}{Predicted}\right)_V$  for  
13 slabs

14 **Figure 10** – Probability density function of limit state function for span 2, with  $\left(\frac{Test}{Predicted}\right)_V$   
15 for beams

16 **Figure 11** – Probability density function of limit state function for span 2, with  $\left(\frac{Test}{Predicted}\right)_V$   
17 for slabs

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Figure 1  
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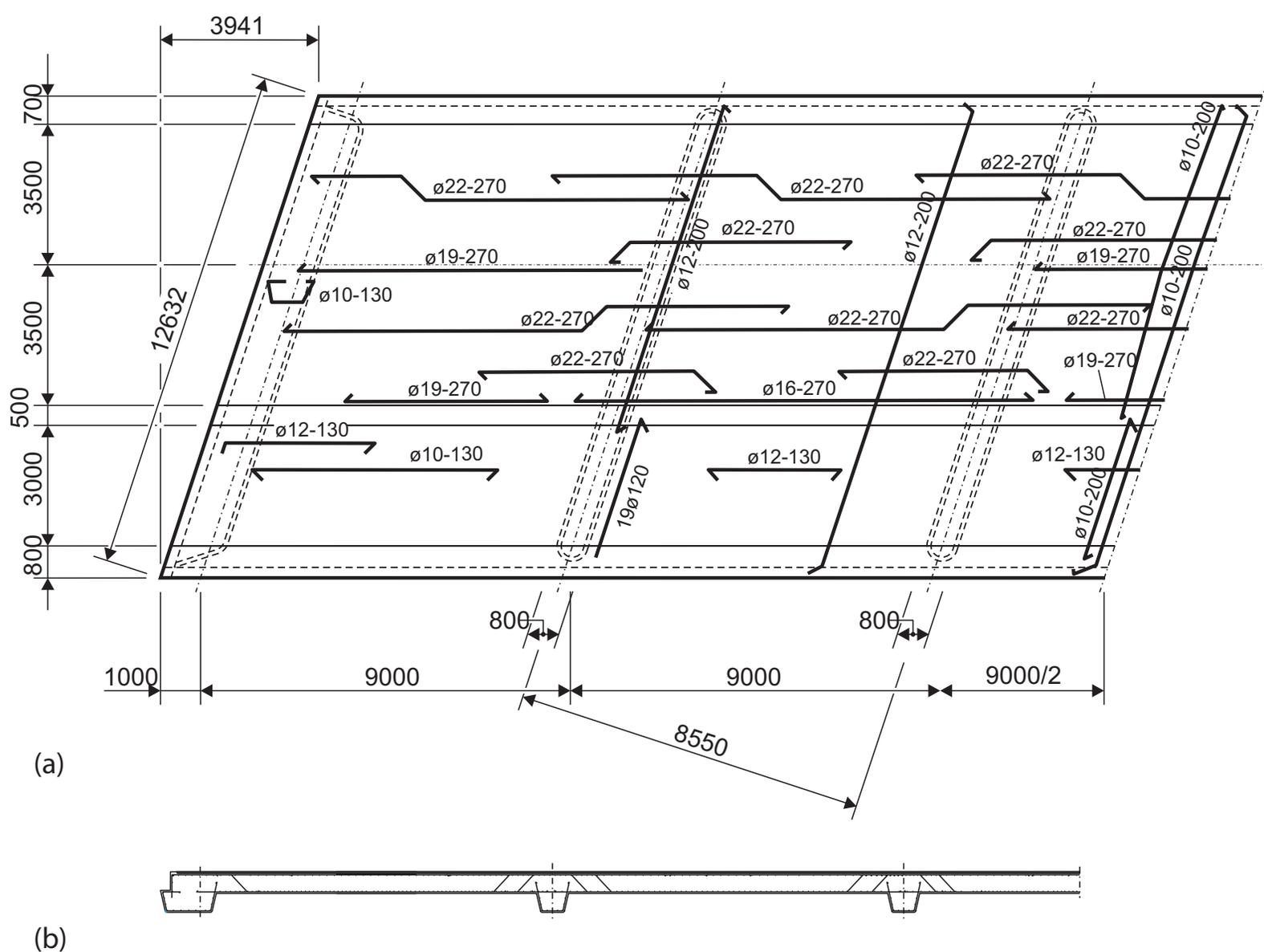


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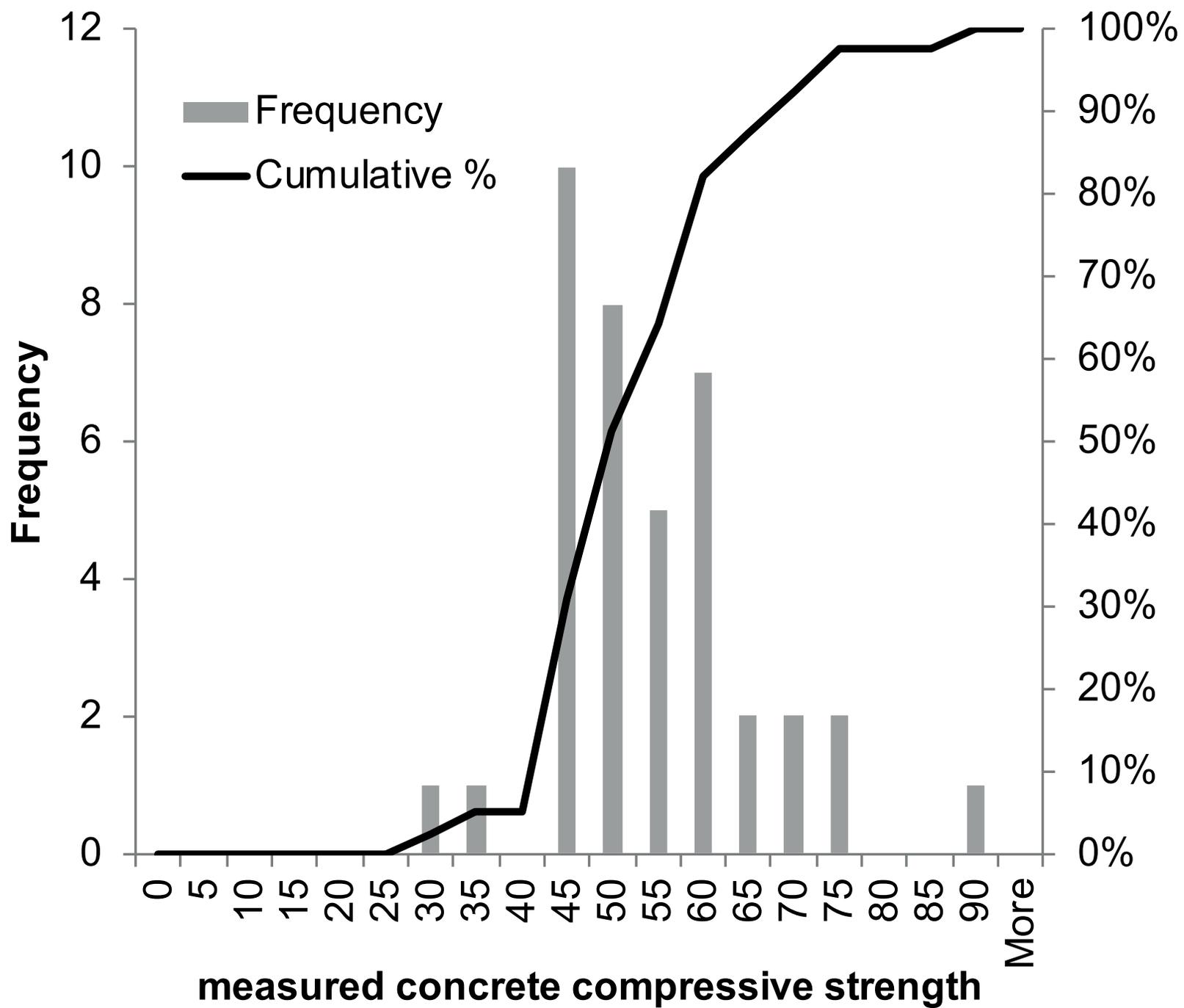


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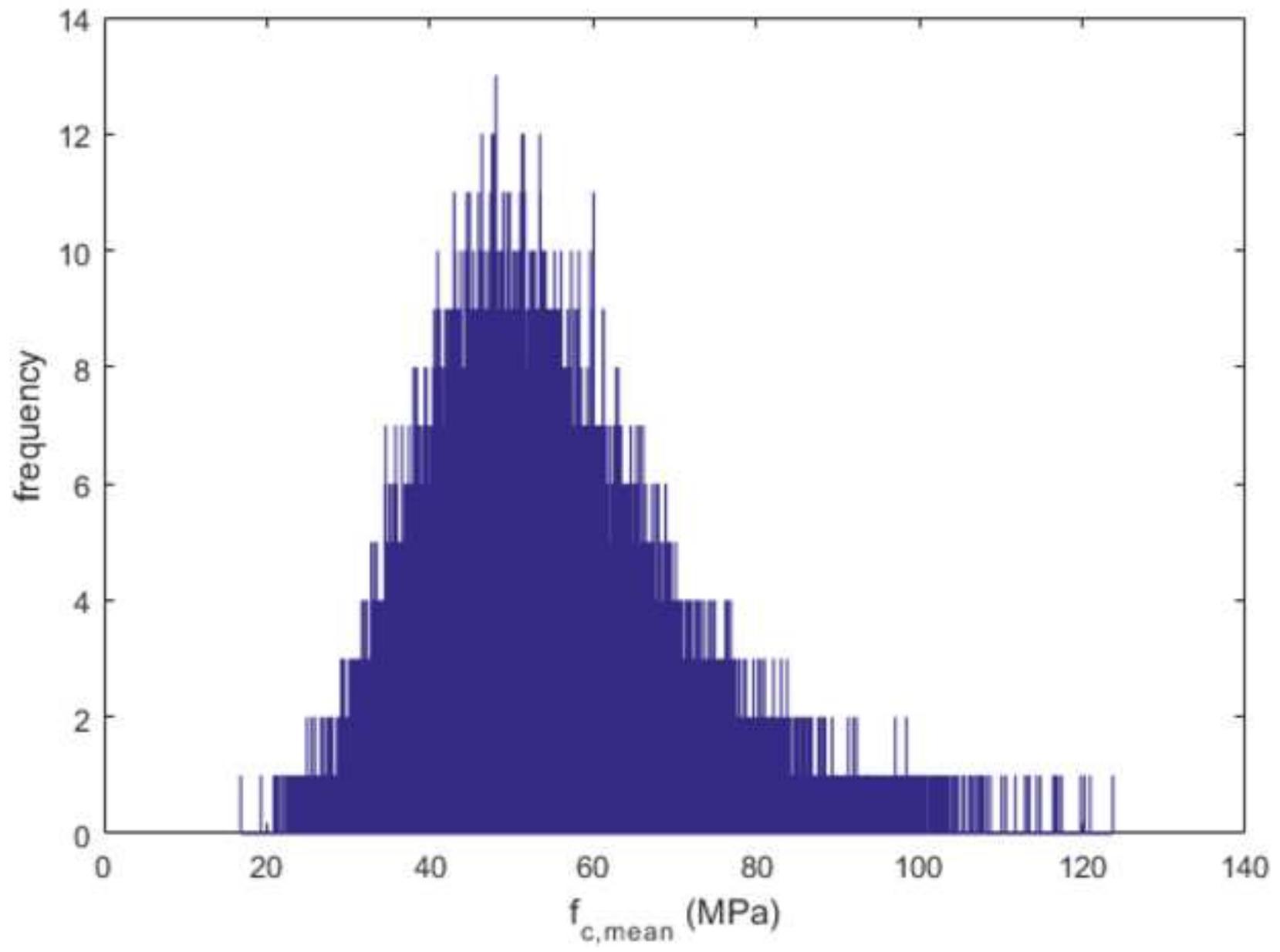


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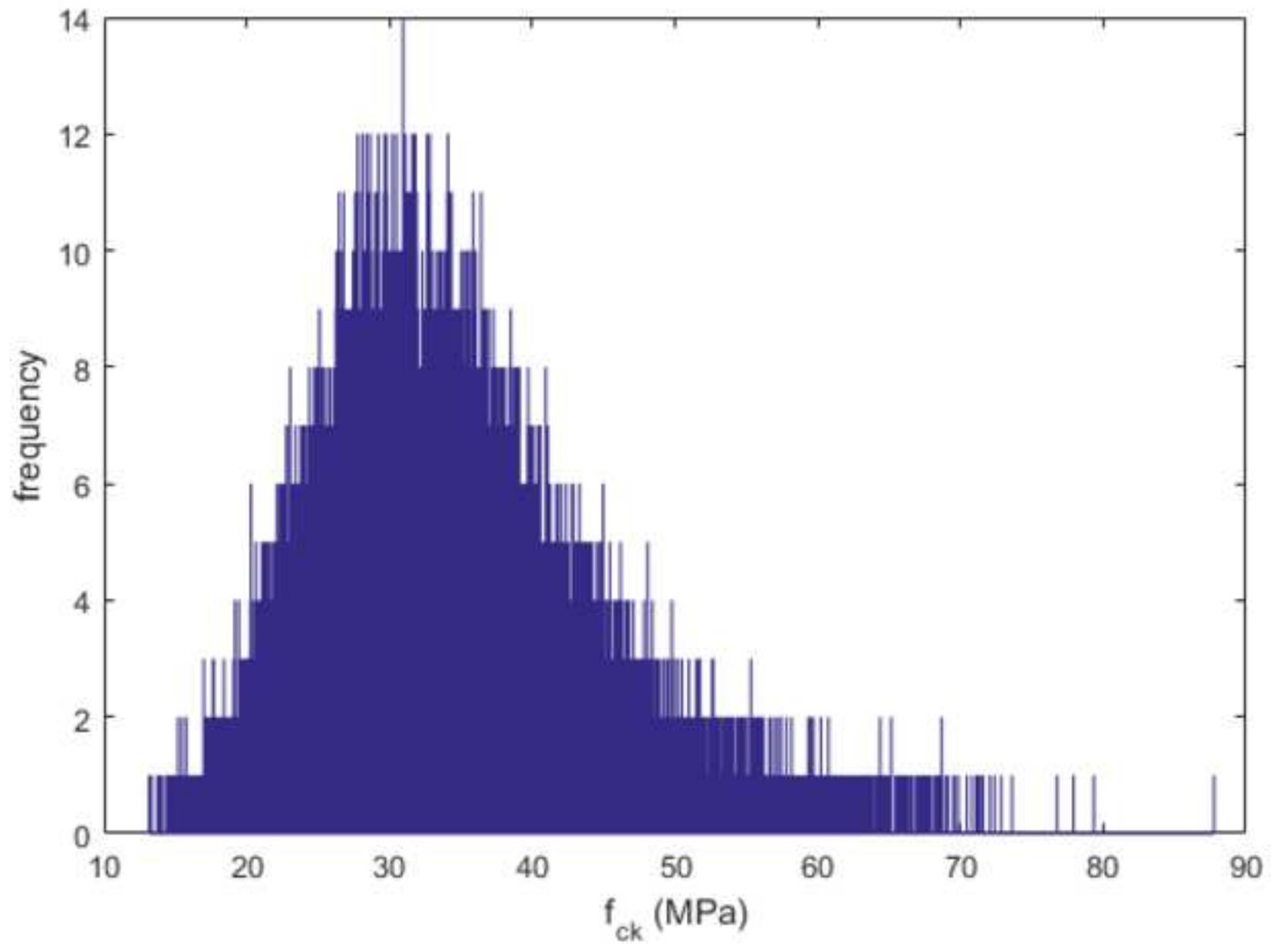


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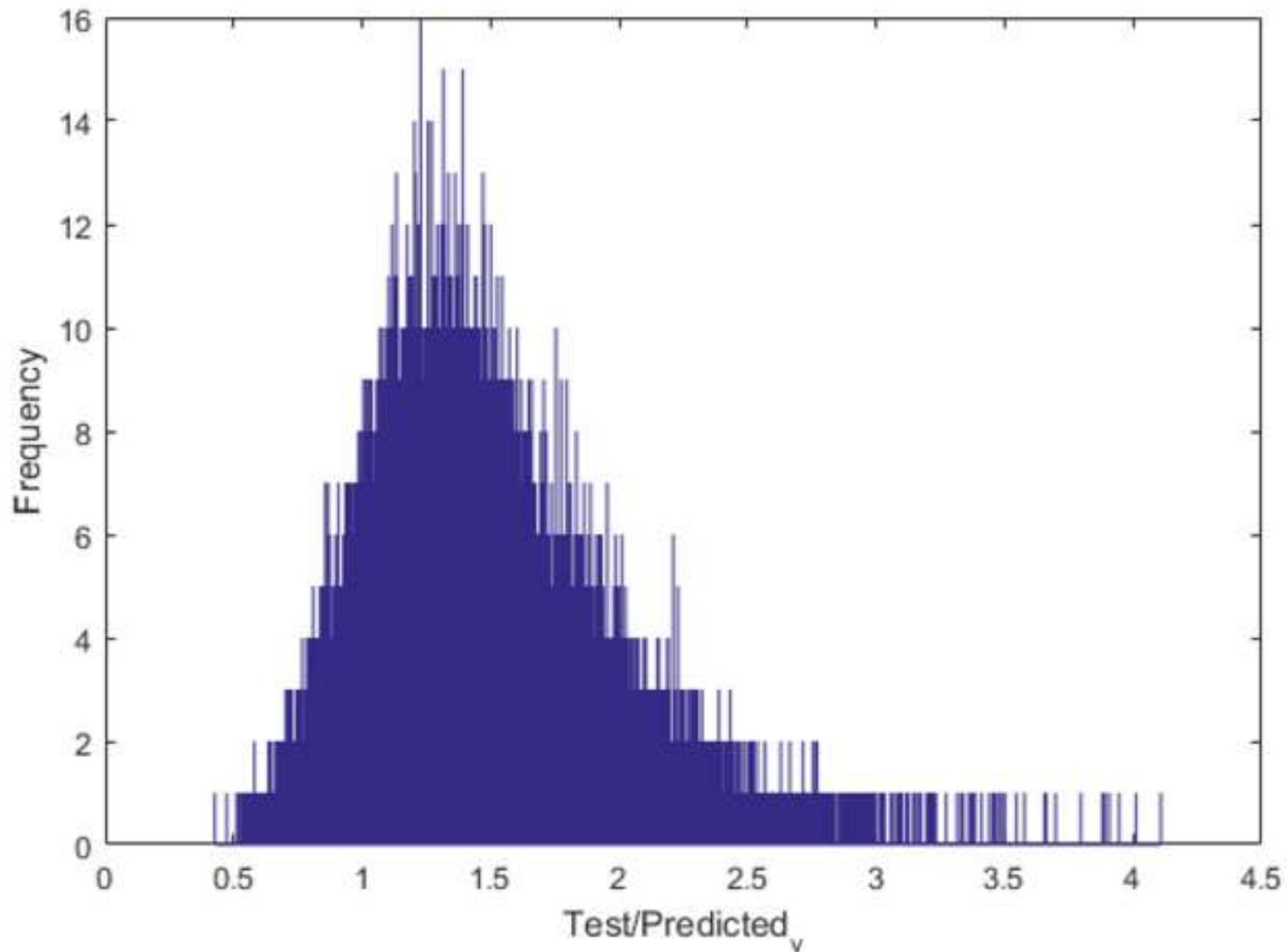


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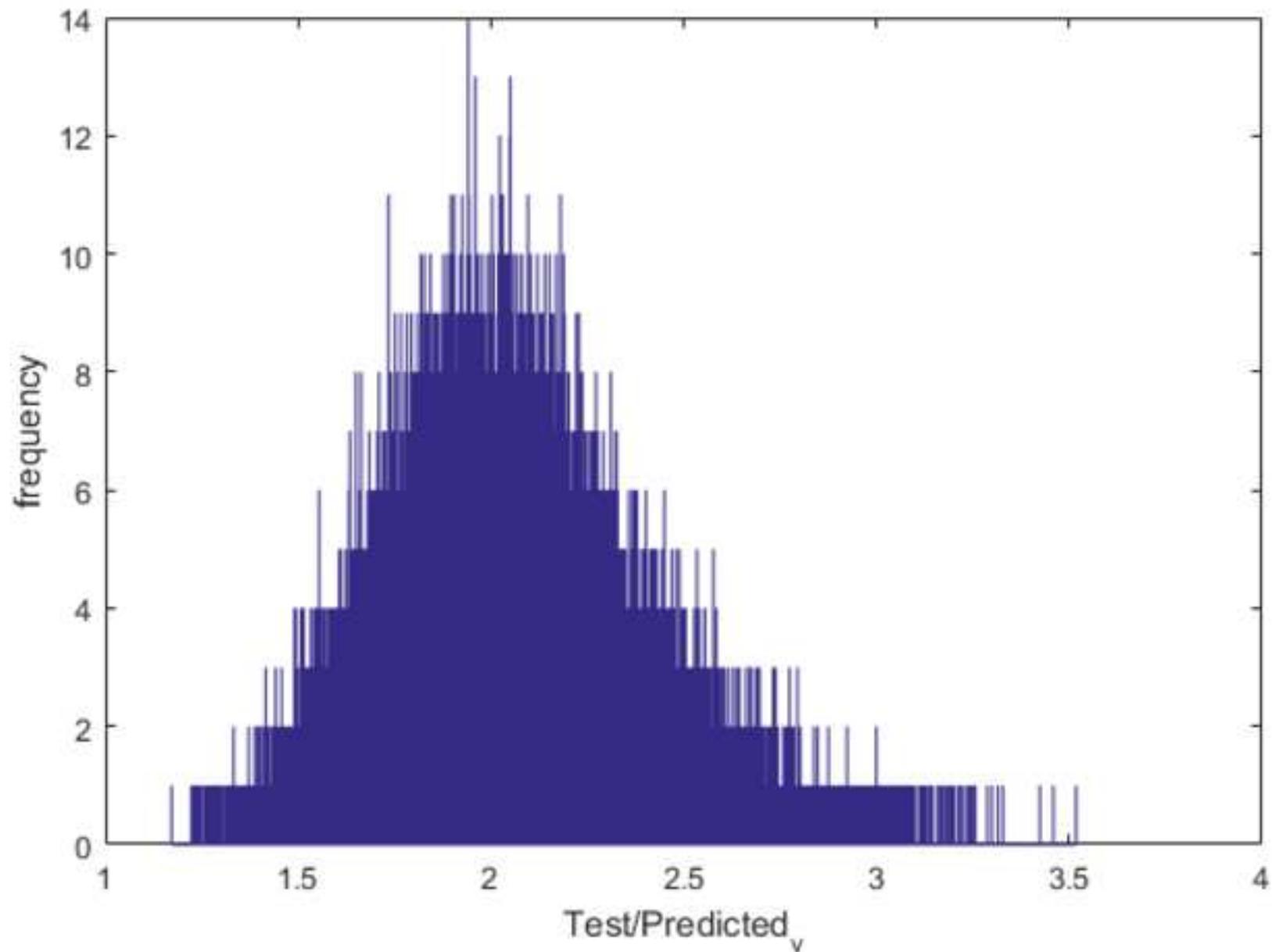


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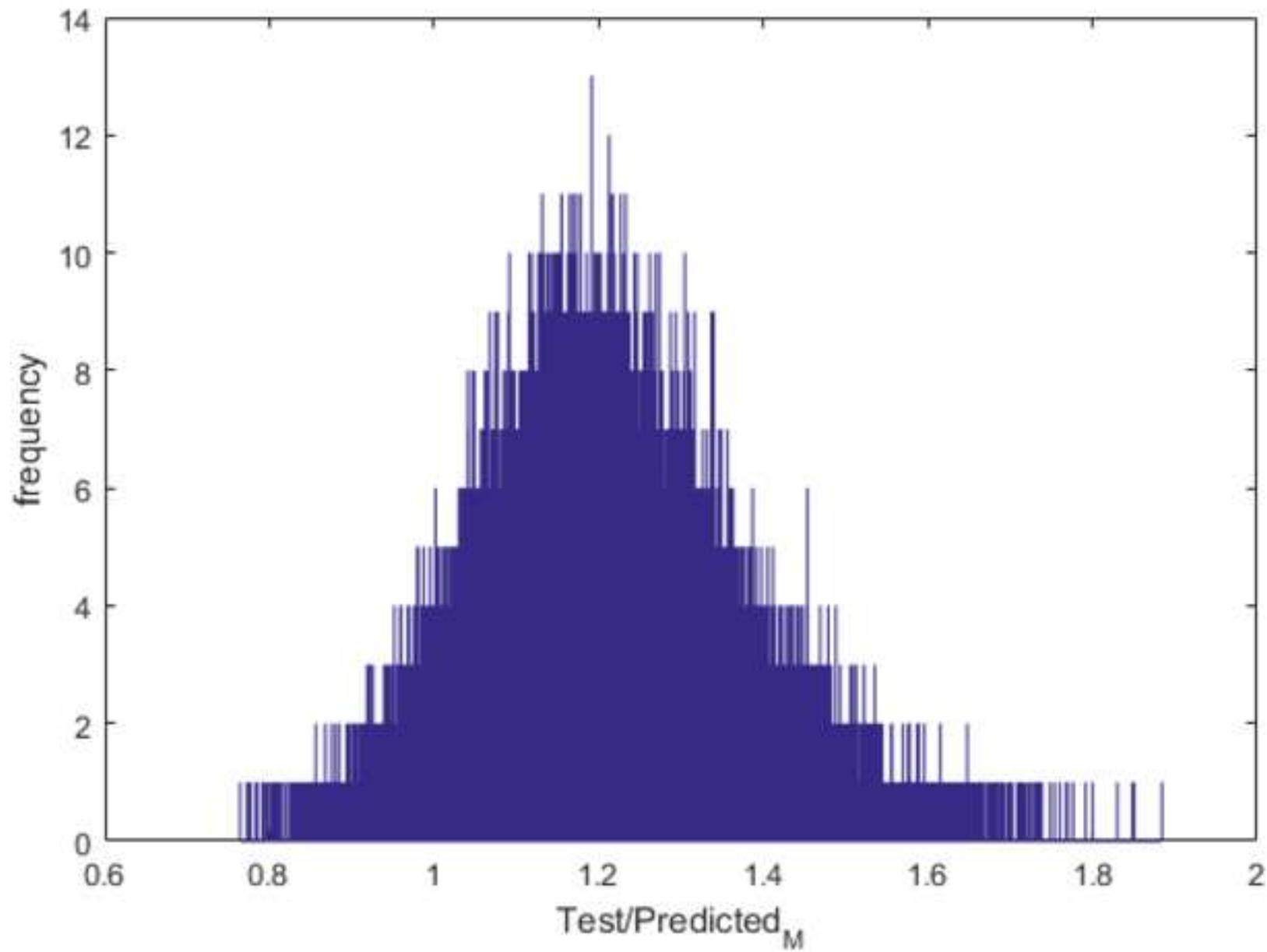


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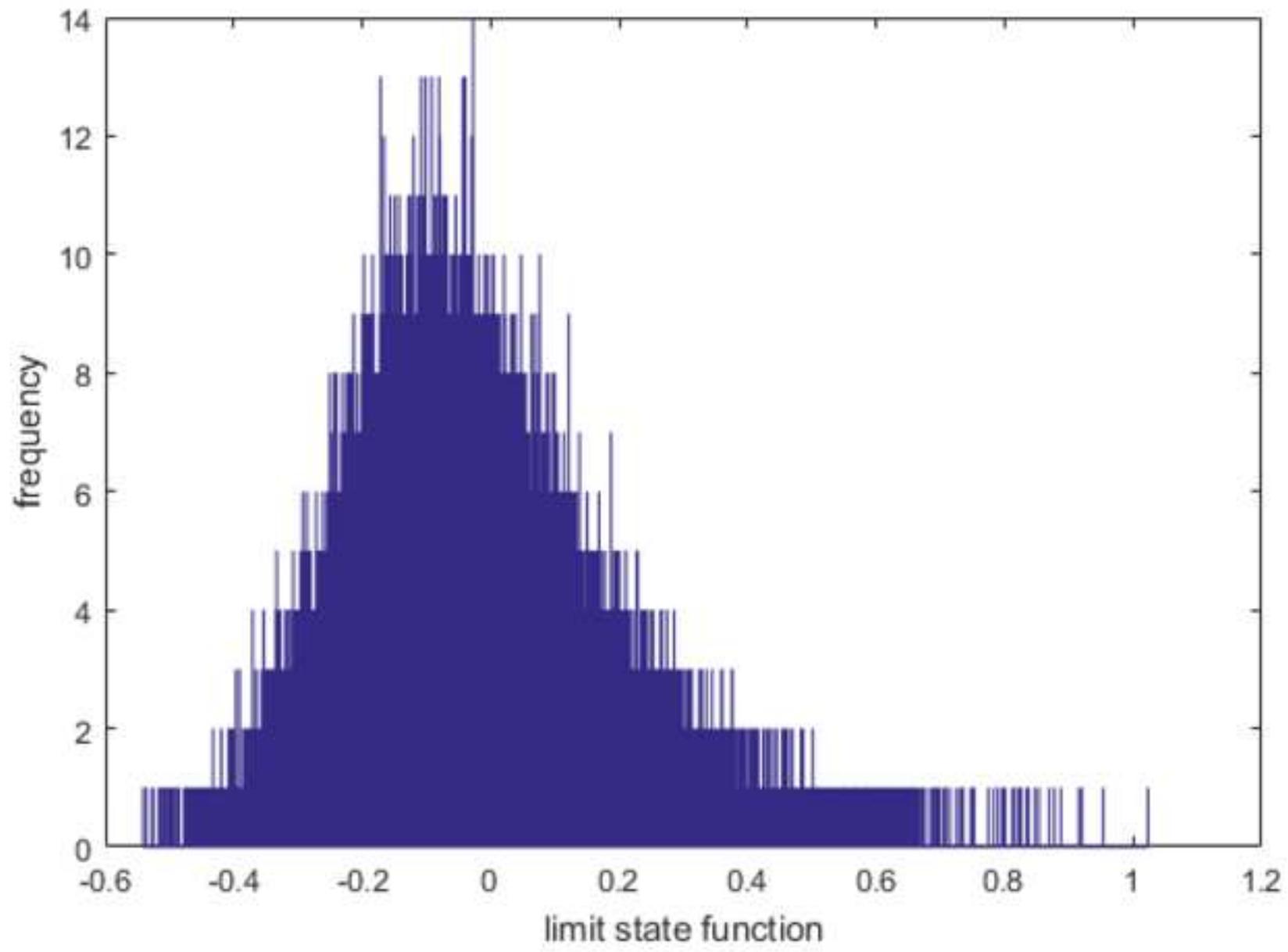


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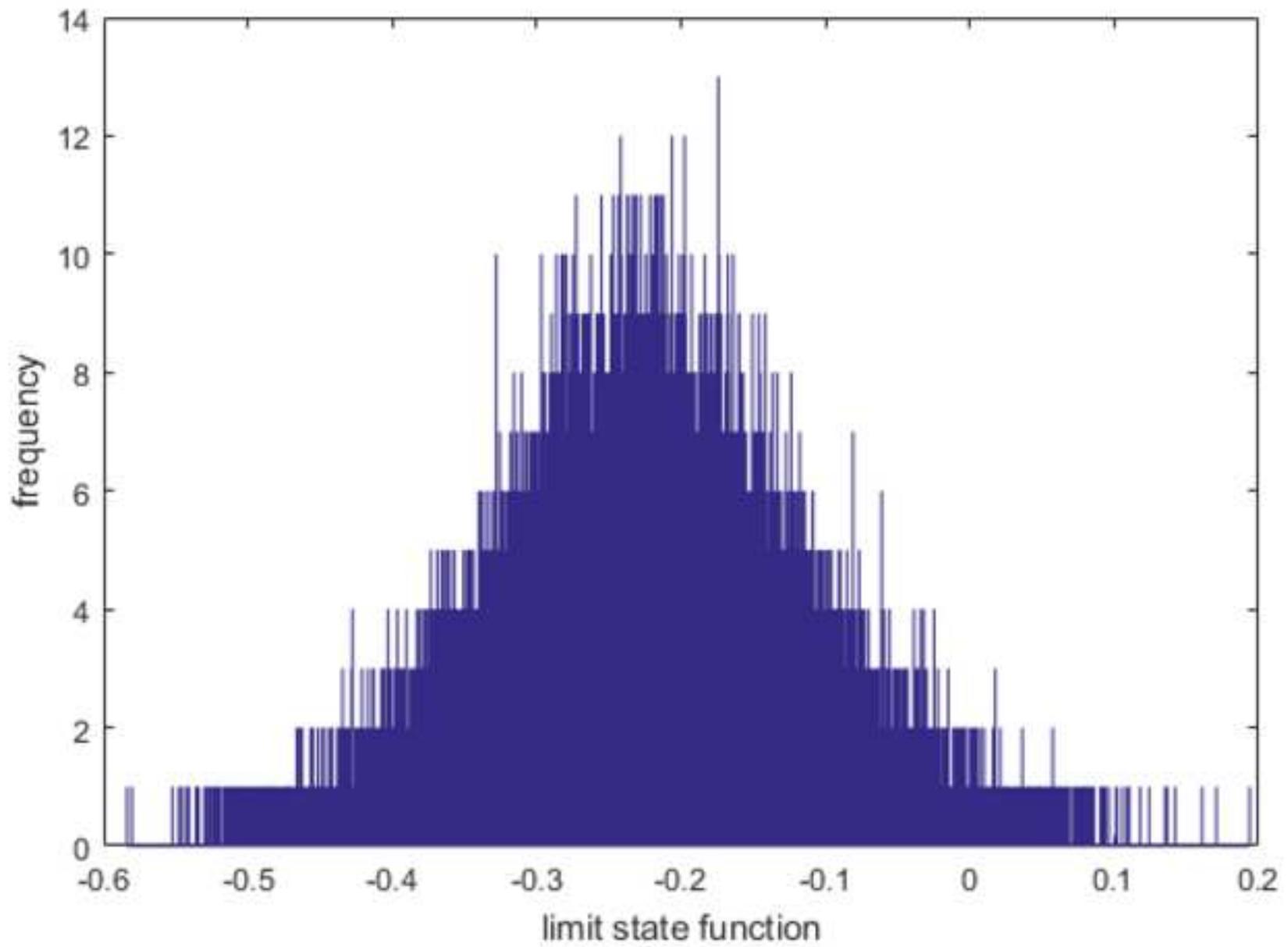


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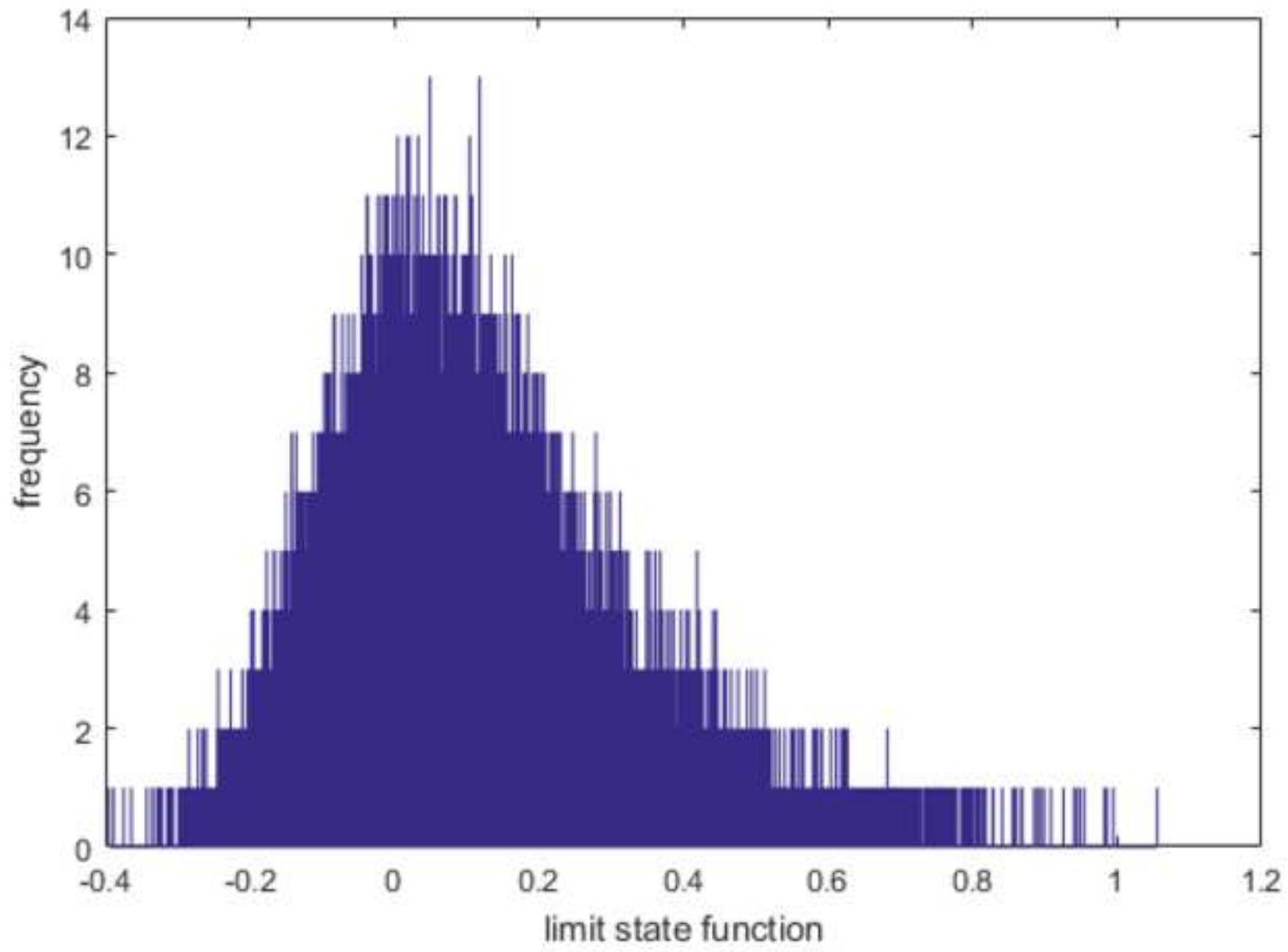
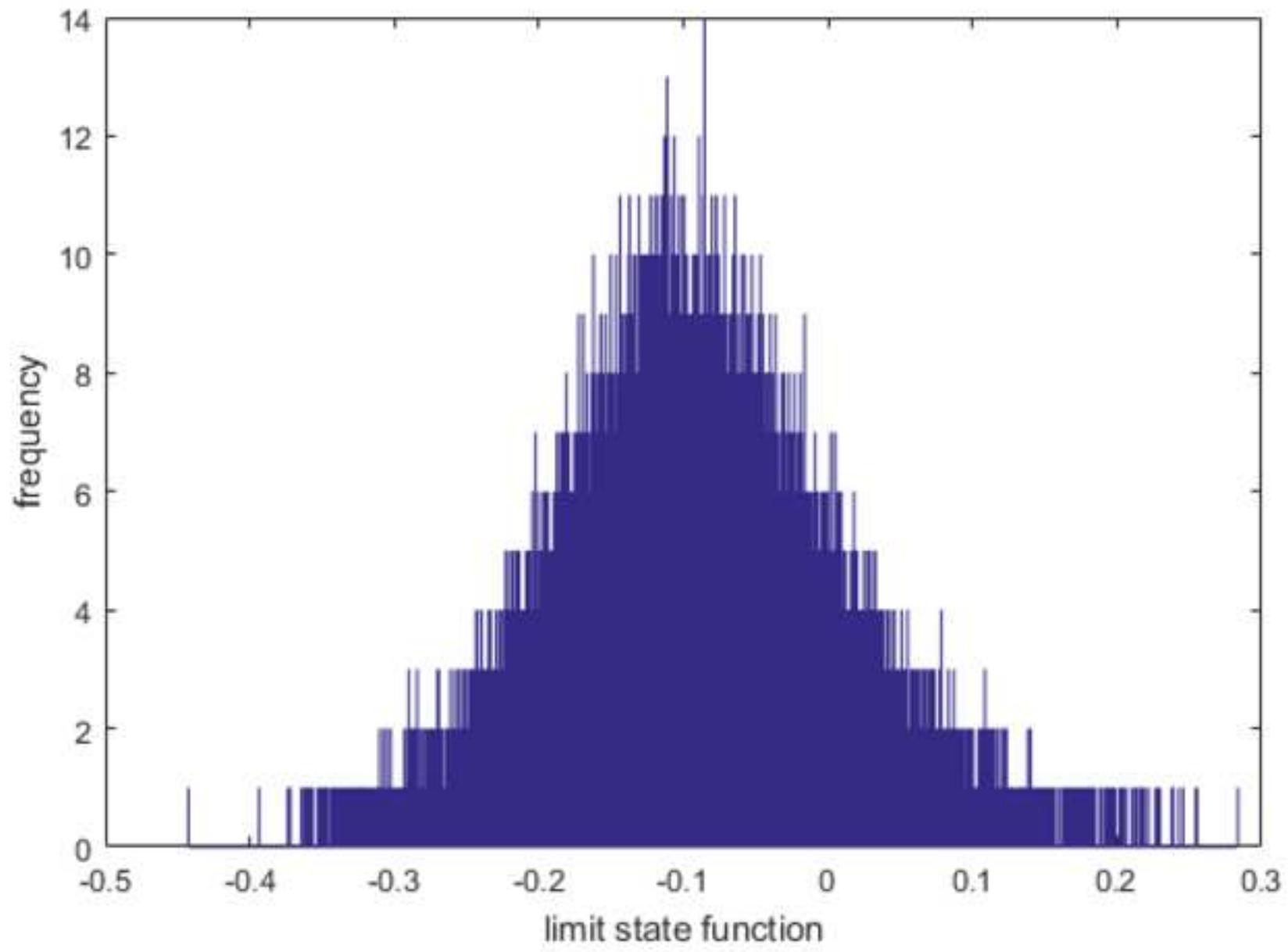


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