Rail network robustness

The role of rapid development and a polycentric structure in withstanding random and targeted attacks

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METROPOLITAN RAIL NETWORK ROBUSTNESS: INVESTIGATING THE ROLE OF RAPID DEVELOPMENT AND A POLYCENTRIC STRUCTURE

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Abstract

In large-scale urban agglomerations, heavy rail in the form of metro and commuter train serves as the backbone of the metropolitan public transport network (MPTN). Transport systems, including MPTN, are subject to recurrent disruptions that may result in severe consequences for network performance and society at large. The objective of this paper is to compare the robustness of selected public transport networks which exhibit different properties to both random and targeted attacks and gain insights on the role of network structure and development pattern. We adopt a complex network theory approach, inveistgating network performance under alternative sequential disruption scenarios corresponding to the successive closure of either stations or track segments. Targeted attacks are simulated by the removal of the most central network elements. Network performance is measured both in terms of the capacity of the network to function in terms of connectivity as well as the impedance induced that remain connected. Two exemplary networks are selected – the polycentric Randstad region and the largest and rapidly developing Shanghai metro network. The results indicate that both networks are highly vulnerable to targeted attacks with the Shanghai network exhibiting greater robustness in all scenarios. The connections between the urban cores are most critical in the multi-centric Randstad network while the Shanghai network is most susceptible to disruptions in the edge of its core, risking cutting off its long branches. Our findings provide more nuanced insights on the relations between network structure and robustness.

1 INTRODUCTION

Urban population increases rapidly worldwide as people are seeking better quality of life and job opportunities and governments stimulate agglomeration effects through investments in infrastructure. Faced with this situation, local authorities aim to improve mobility by densifying and expanding the respective public transport system. In large-scale urban agglomerations, heavy rail in the form of metro and commuter train serves as the primary high-level metropolitan public transport network (MPTN). Transport systems, including MPTN, are subject to recurrent disruptions that may result in severe consequences for network performance and the metropolitan metabolism as a whole. The robustness of critical infrastructure such as mass MPTNs is high on the planning and policy agenda (*1*). Notwithstanding, there is lack of knowledge on how network structure and design philosophy impacts its robustness. This is the topic of this study which adopts a complex network theory approach.

MPTN can be represented as a graph where stations correspond to nodes and track segments to links. Lin and Ban (*2*) provide a review of the literature devoted to transport network topology and measures adopted from complex network perspective since the early 2000s. A large number of studies that examined MPTN worldwide concluded that they exhibit scale-free and small-world properties. The former implies that regardless of the size of the network any node can be reached by a few steps while the latter implies that the degree of a graph follows a power-law distribution with the number of connections decreasing exponentially – i.e. few stations with many connections and many stations with few connections. The combination of these two properties is the blueprint of a hub and spoke network that branches out as you move away from the central core which Roth et al. (*3*) found to characterize the world's largest subway systems. This can be expected to happen if network evolution, the process in which new stations and track segments are added to the network, follows a preferential attachment mechanism where new stations are more likely to be connected to other stations which are already well-connected (i.e. what has become known as the rich getting richer phenomenon).

The structure of the public transport network is intertwined with the urban and regional development and the underlying policy making process. Metropolitan areas are often characterized as monocentric areas (as implied by the etymology of the word metropolitan) with radial networks. Notwithstanding, MPTN vary in the extent to which they are indeed concentrated around a single centre with secondary centres being the norm rather than the exception. The extent to which an urban agglomeration exhibits monocentric or polycentric land-use and travel patterns depends, among other things, on the underlying MPTN and the planning policy. The latter is also reflected in the development of MPTN which is an important driver in realizing such plans. Examples of the latter are the light rail projects in Paris and the on-going metro extension in Stockholm (*4*), both of which devised to stimulate a more polycentric development. In several cases such as in the Randstad in the Netherlands and the Ruhr area in Germany, an agglomeration of cities which is dominated by any one city, results with a polycentric structure that is then reflected by a more distributed MPTN.

All of the abovementioned networks have evolved over many decades and their structure and developments reflects the evolution in planning policies and realities and are the outcome of the decisions of a large number of planners with none of them envisioning the current state of the network. In contrast, MPTN in large cities in China have developed swiftly with unprecedented investments. These rapid developments are the result of a masterplan that is overseen by the transport authority. For illustration, Shanghai, Guangzhou and Nanjing, which are the first, fourth and sixth longest metro networks in the world in terms of track-km, have seen the first line inaugurated in 1993, 1997 and 2005, respectively. Metro systems in three more Chinese cities opened as recently as 2004-2005 and their network already exceeds in length than the Paris network. This strikingly different planning trajectory implies less path-dependence and reflects potentially a more cohesive top-down approach. It remains unknown what are the consequences of these differences for network robustness.

The complex network theory approach enables the robustness analysis of MPTN and the relation between the MPTN topology and its robustness. Derrible and Kennedy (*5, 6*) defined metrics that they believe are indicative of network robustness. They postulated using the number of cyclic paths available in the network since it approximates the possibility to use alternative routes under disruption. However, the relevance of this indicator in explaining network performance in the event of disruptions through experiments was not established. According to their criterion, the MPTN of Tokyo and Seoul are particularly robust followed by among others, Paris and London. Zhang et al. (*7*) studied 17 generic network structures and concluded that redundancy is a key determinant of network capability to withstand disasters. While some insights can be gained from analyzing a taxonomy of networks structures, real-world MPTN are complex and are comprised of a large number of diverse building blocks (e.g. hub-and-spoke, grid, ring, diamond etc.) which cannot be represented as direct extrapolation of their fundamental elements. Rodriguez-Nunez and Garcia-Palomares (*8*) and Jenelius and Cats (*9*) modelled the MPTN service network concluded that lines that offer many transfer opportunities to other lines such as ring and cross-radial lines are especially important in adding cycles and thus contributing to network robustness.

A systematic analysis of network robustness requires performing a full-scan of the impacts of failures on network performance. In the field of MPTN, the studies of von Ferber et al. (*10*) and Zhang et al. (*11*) pave the way by examining the impact of a sequential failure of network elements – either nodes (stations) or links (track segments). In both studies, three strategies for determining the failure sequence were investigated – random, in descending order of node degree and in descending order of betweenness centrality. The former reflects random failures while the latter two are designed to examine MPTN ability to withstand targeted attacks. von Ferber et al. (*10*) applied the analysis to the MPTNs of London and Paris. The results indicate that Paris is significantly more robust than London in the event of node degree removal strategy while the networks perform very similar under the other two strategies. However, the analysis was performed for the entire public transport network, treating metro and bus lines as if they were indistinguishable. The results of the analysis in (*11*) for the network of Shanghai from 2010 cannot unfortunately be directly compared due to differences in implementation and reporting.

The objective of this paper is to compare the robustness of selected public transport networks which exhibit different properties to both random and targeted attacks and gain insights on the role of network structure and development pattern. Two exemplary networks are selected – the polycentric Randstad region and the rapidly developing Shanghai network.

This paper is structured as follows. In the following section we present our method and detail the network representation, failure scenarios and the measures used for quantifying network robustness. In section 3 we present our two case study heavy rail networks of Shanghai and Randstad. Results are reported and analyzed in the section 4. The conclusion and network design implications are then discussed in section 5.

2 METHOD

2.1 Network representation and centrality indicators

The MPTN infrastructure is represented as a graph by representing each station as a node and introducing a link between each pair of stations that are directly connected by a track segment. This follows the so-called L-space representation (*12*). The physical MPTN is thus represented as an undirected graph $G(N, E)$ where the set N denotes rail stations and the link set $E \subseteq N \times N$ represents direct connections between stations. Each link may be operated by one or several public transport lines. A is the adjanceny matrix of the graph G, where each entry in the matrix, a_{ij}, equals one if there is a link connection stations $i, j \in N$ and is otherwise zero.

Even though the graph is unlabeled, network topology result with considerable differences among nodes and links in terms of their network centrality. Two measures of network centrality are used in this study, degree and betweenness and both are defined for both nodes and links. Degree is an indicator of local connectivity while betweenness measures global connectivity. Node degree is the number of direct neighbors, hence directly connected stations, and is defined as

$$k_i = \sum_{j \in N} a_{ij} \quad \forall\, i \in N \tag{1}$$

where k_i is the degree of node i. Following the definition proposed in (*10*), link degree is defined as the sum of degrees of the nodes it connects minus its own contribution to their degrees, hence

$$k_e = k_{e^-} + k_{e^+} - 2 \quad \forall\, e \in E \tag{2}$$

Where e^- and e^+ denote the upstream and downstream nodes of link e, respectively.

Unlike degree centrality, betweenness centrality measures the role of the node or link in the network as a whole, not only in relation to its direct neighbors. Betweenness centrality is defined here in relative terms, as the share of shortest paths connecting origin-destination pairs in the network that traverse through a certain node or link. The formula for node betweenness centrality is

$$b_i = \sum_{j \in N; j \neq i} \sum_{k \in N, k \neq j} \frac{\sigma_{jk}(i)}{\sigma_{jk}} \tag{3}$$

where σ_{jk} is the number of shortest paths between nodes $j, k \in N$ and $\sigma_{jk}(i)$ is the number of these paths that go through node $i \in N$. And for link betweenness centrality

$$b_e = \sum_{j \in N} \sum_{k \in N, k \neq j} \frac{\sigma_{jk}(e)}{\sigma_{jk}} \tag{4}$$

where $\sigma_{jk}(e)$ is the number of shortest paths between nodes $j, k \in N$ that go through link $e \in E$.

The abovementioned indicators are used in determining the sequence in which disruption scenarios are simulated as described in the following sub-section.

2.2 Network element failure scenarios

Network elements – either nodes or links – can be subject to both random and malicious failures. Both stations and track segments may temporarily be closed due to construction or maintenance works, as well as unplanned closures due to technical or mechanical failures such as vehicle breakdowns and switch or signal failure. Other causes of unplanned disruptions are accidents, suicide attempts and terror-related threats and attacks. The latter typically affect stations.

The approach taken in this study, inspired by (*10*), is to simulate a sequence of link or node failures. Each scenario involves the sequential removals of either nodes or links. In the pursue of the most devastating effect, the perpetrators of a targeted attack are assumed to target network elements that are most heavily loaded and that will result with the most adverse conditions. Degree centrality and betweenness centrality are therefore used as a possible attack strategy. Note that the removal of a node or link results with a new (reduced) network which affects the centrality indicators for the remaining nodes and links. The metrics are therefore recalculated and updated after each removal step in order to ensure that the most central network element is selected to be removed in the successive step.

Random attacks involve the random removal of links or nodes, each one picked in random from the remaining set of links or nodes. Crucially, the order in which links or nodes are removed may have great consequences for network performance due to both the specific elements selected as well as path dependency in the removal sequence. Neither the analysis in (*10*) nor (*11*) report how the random removal strategy was precisely performed. In order to attain meaningful results, a number of simulation needs to be performed. At the same time, the number of possible sequences is prohibitive even for fairly small networks. We assure statistically robust results by running a number of random removal sequences and then determining how many such replications are needed to attain a statistically significant result within a high (i.e. 95%) level of confidence.

In summary, six failure scenarios are analyzed; the combination of three successive removal strategies – random, degree and betweenness, each of which applied to either nodes or links. The following sub-sections describes the network robustness indicators calculated to assess the impacts of the failure scenarios.

2.3 Network performance and robustness indicators

System robustness is defined in the context of this study as system's ability to maintain its functionality under disruptions. In the context of MPTN, the core function of the system is to enable users to travel efficiently between different parts of the network. Various measures can be used to quantify network performance. Network robustness is then assessed by comparing the network performance under disruption to the original undisrupted performance.

We use two indicators to describe network performance, pertaining to the ability to travel and the detour inflicted by the disrupted situation. The former is assessed by identifying the largest sub-network that remains intact and measuring its size in relation to the original (complete) network size, or mathematically

$$S(r^n) = \frac{|N_r|}{|N|} \text{ or } S(r^e) = \frac{|E_r|}{|E|} \tag{5}$$

where $S(\cdot)$ is the relative size of the largest connected sub-network in removal step r^n or r^e of removing nodes or links, respectively. N_r (or E_r) is the set of nodes (or links) that remained in step r. The larger the sub-network is, the more likely it is that one can travel between a given origin-destination pair.

Even if it is still possible to travel using the MPTN between a given origin and destination, the removal of nodes or links may require performing detours and hence induce additional impedance. The second network performance indicator is designed to assess this effect. The average shortest path length is commonly used as a network efficiency indicator since it captures network transition capability. In order to facilitate the comparison of different networks as given the objective of this study, the normalized average shortest path, $\hat{l}$, is calculated using the following equation:

$$\hat{l} = \frac{2}{|N|(|N|-1)} \sum_{i \in N} \sum_{j \in N, i \neq j} l_{ij} \tag{6}$$

where $\langle l \rangle$ is the mean shortest path length and l_{ij} is the length of a shortest path between nodes i and j (here, the number of stations that need to be traversed).

Similar to the relative size metric, network performance under a given disruption state can be then assessed in terms of its deterioration compared to the original state, denoted as $\tilde{l}$, as follows:

$$\tilde{l}(r^n) = \frac{\hat{l}(r^n)}{\max_{i,j} l_{ij}(0)} \quad \text{or} \quad \tilde{l}(r^e) = \frac{\hat{l}(r^e)}{\max_{i,j} l_{ij}(0)} \tag{7}$$

Where $\hat{l}(\cdot)$ is the normalized average shortest path in a certain removal step r^n or r^e with the undisrtuped initial phase denoted by zero. In the course of removal steps, the network may not remain intact anymore. The original case here pertains to the longest rather than the average shortest path. A problem with the $\hat{l}$ metric arises when a pair of nodes is totally disconnected as this will result in infinite value of the mean shortest path. In this study, the infinite value is replaced by the 'diameter' of the network, i.e. the maximal shortest path length, in the initial undisrupted network. Hence in case there is no path available between an OD pair, $l_{ij}(\cdot) = \max_{i,j} l_{ij}(0)$.

The two network performance indicators can be calculated for the network resulting from each removal step. This allows analyzing how performance losses evolve over the course of the sequence of removals and what was coined by Cats et al. (*13*) the degrading rapidity. This is especially useful for identifying how long does the network remain robust to disruption, whether there is a transition point and the overall trend. It is however also desirable to quantify the overall robustness by considering the accumulated effect of disruption of different scenarios on different MPTN. To this end, following (*13*), the integral over $S(\cdot)$ in relation to the sequence of increasingly disrupted networks (from $|N_r| = \{|N|, |N| - 1, \dots, 0\}$, can be calculated. The larger the value the more robust the network is

$$A = 100 \cdot \int_0^1 S(\cdot) \tag{8}$$

$A = [0,1]$, where 0 is the most vulnerable and 1 is the most robust.

The following pseudocode describes the implementation of node removal strategies with the link removal strategies following a similar procedure.

Algorithm 1: **Node removal strategies**

```
Input: Rail station list N, edge list E, strategy
Output: Network robustness measures
Er ← E, Nr ← N
while |Nr| == 0 do
    if strategy == degree
        calculate node degree k of G(Nr, Er)
```

```
        [k_r, order] ← DescendingSort(k)
        N_r ← N_r[order]
        i ← N_r[1]
    if strategy == betweenness
        calculate node betweenness b of G(N_r, E_r)
        [b_r, order] ← DescendingSort(b)
        N_r ← N_r[order]
        i ← N_r[1]
    if strategy == random
        generate random number 1<m<|N_r|
        i ← N_r[m]
    E_r ← E_r – {a_ij, a_ki}        //Remove edges from the graph with node i as incoming or outgoing nodes
    N_r ← N_r – {i}                 //Remove node i from the graph
    calculate network robustness measures of updated G(N_r, E_r)
```

The following section describes the networks to which this method is applied.

3 APPLICATION

The 2018 heavy rail networks of the Randstad area in the Netherlands and Shanghai, China are the two case study networks. These two large networks are selected because they are exemplary of a polycentric development and a rapid top-down development, respectively.

The Randstad area is a megapolis, an urban agglomeration in the west of the Netherlands which extends into four provinces: North Holland, Flevoland, South Holland and Utrecht. The total population amounts to 8 million and includes the four largest cities of the Netherlands: Amsterdam, Rotterdam, The Hague and Utrecht. The MPTN consists of the inter-city and regional rail connections characterized by high frequencies and relatively short inter-station distances as well as metro services which are centered around Amsterdam and Rotterdam, including a metro-light rail line connecting the cities of Rotterdam and The Hague.

Shanghai is a fast growing city and is a global business hub with the metropolitan area encompassing more than 24 million inhabitants. Shanghai is characterized by a monocentric development pattern. The MPTN consists of the rapidly developing metro network which is now the most extensive network in terms of track length in the world with 200km of metro tracks currently under construction.

Table 1 summarizes key topological indicators for the two networks. Note that the Shanghai network analyzed by (*11*) consisted of 240 stations and 264 links. Even though the Shanghai network includes 30% more stations than the Randstad, the average number of intermediate stations that needs to be traversed when travelling between two stations is almost the same and in the worst combination possible (diameter) even worse off in the Randstad. This suggests that the Shanghai network is more efficient in terms of offering short travel alternatives between various OD pairs.

Table 1: Summary table of network topological indicators

Network	**Number of nodes, $\|N\|$**	**Number of links, $\|E\|$**	**Average shortest path, $\bar{l}$**	**Diameter, $\max_{i,j} l(i,j)$**
Randstad 2018	254	283	29.86	46
Shanghai 2018	329	377	31.19	41

4 RESULTS

An analysis of the characteristics and spatial distribution of centrality indicators in the two networks is first presented in section 4.1. Thereafter, the results of the disruption simulation and robustness analysis detailed in Section 2 for the 2018 networks of Randstad and Shanghai are presented in section 4.2.

4.1 Network centrality indicators

The simulated disruption scenarios are based on the dynamic update of degree and betweenness centrality rankings. Furthermore, network robustness is expected to depend on network properties and the availability of alternative routes, i.e. redundancy. It is therefore relevant to examine how the two networks – Shanghai and Randstad – differ in terms of the the spatial variation in link and node centrality indicators as this can explain their performance under disruptive situations. Figures 1 and 2 display the degree (left) and betweenness (right) centrality indicators for the Shanghai (top) and Randstad (bottom) networks, for node and links, respectively.

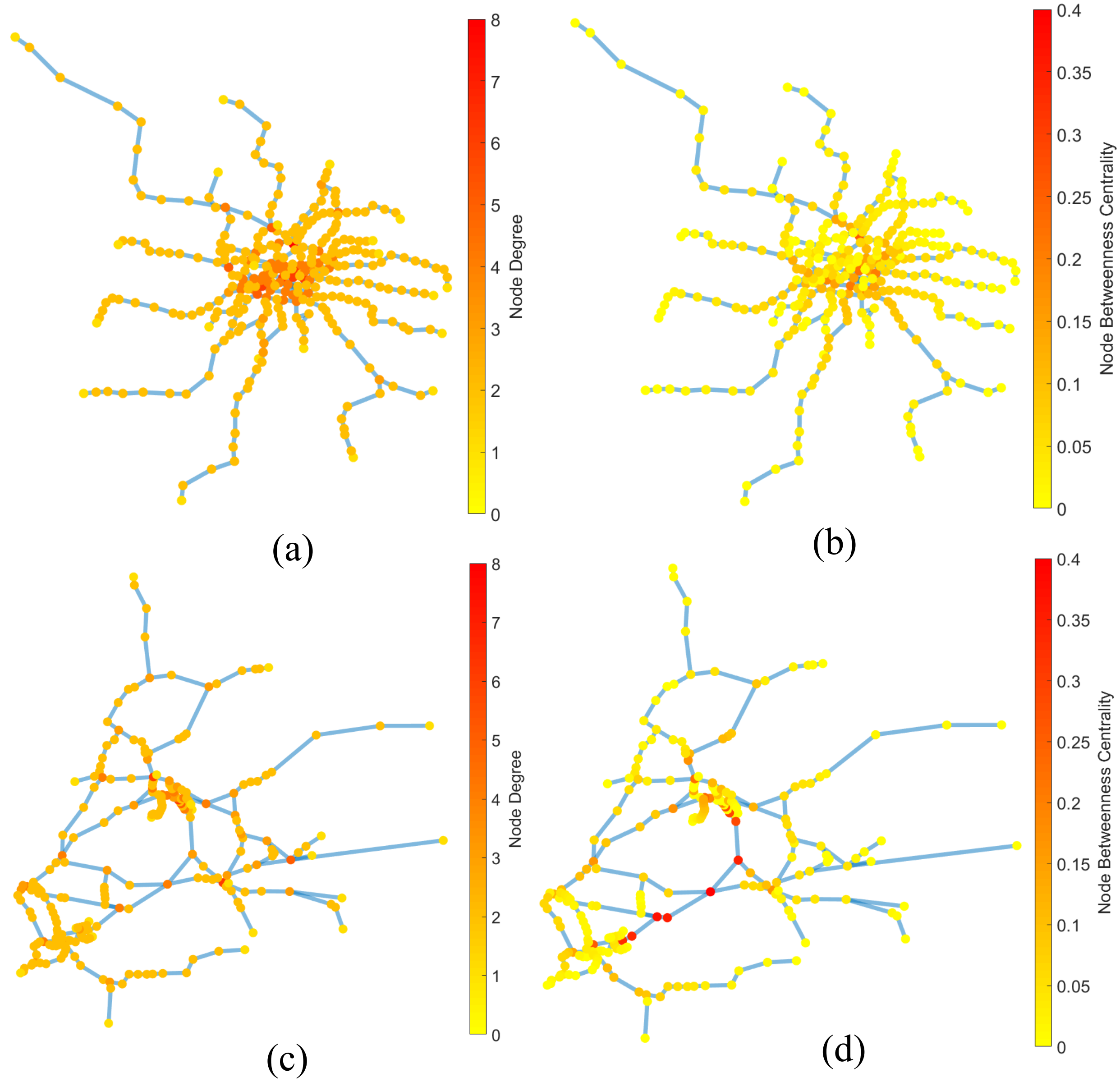

Figure 1: Node centrality indicators of the case study networks – (a) Shanghai, degree; (b) Shanghai, betweenness; (c) Randstad, degree; (d) Randstad, betweenness.

The two networks have a profoundly different structure as can be observed in figures 1 and 2. The Shanghai network has a typical radial structure, yet containing a finely meshed core with a large number of inner circuits. The Randstad network in contrast has a less pronounced center and can be characterized as a grid network with some additional cycles around its main cities. This differences in network structure are clearly reflected in the distribution of node degree (Fig 1, left), where the highest node degrees are all concentrated in the geographical core of the

Shanghai network, unlike their scattering in the Randstad case, corresponding to key rail intersections, often located between rather than within dense urban areas. The same pattern can be seen in relation to link degree (Fig 2, left).

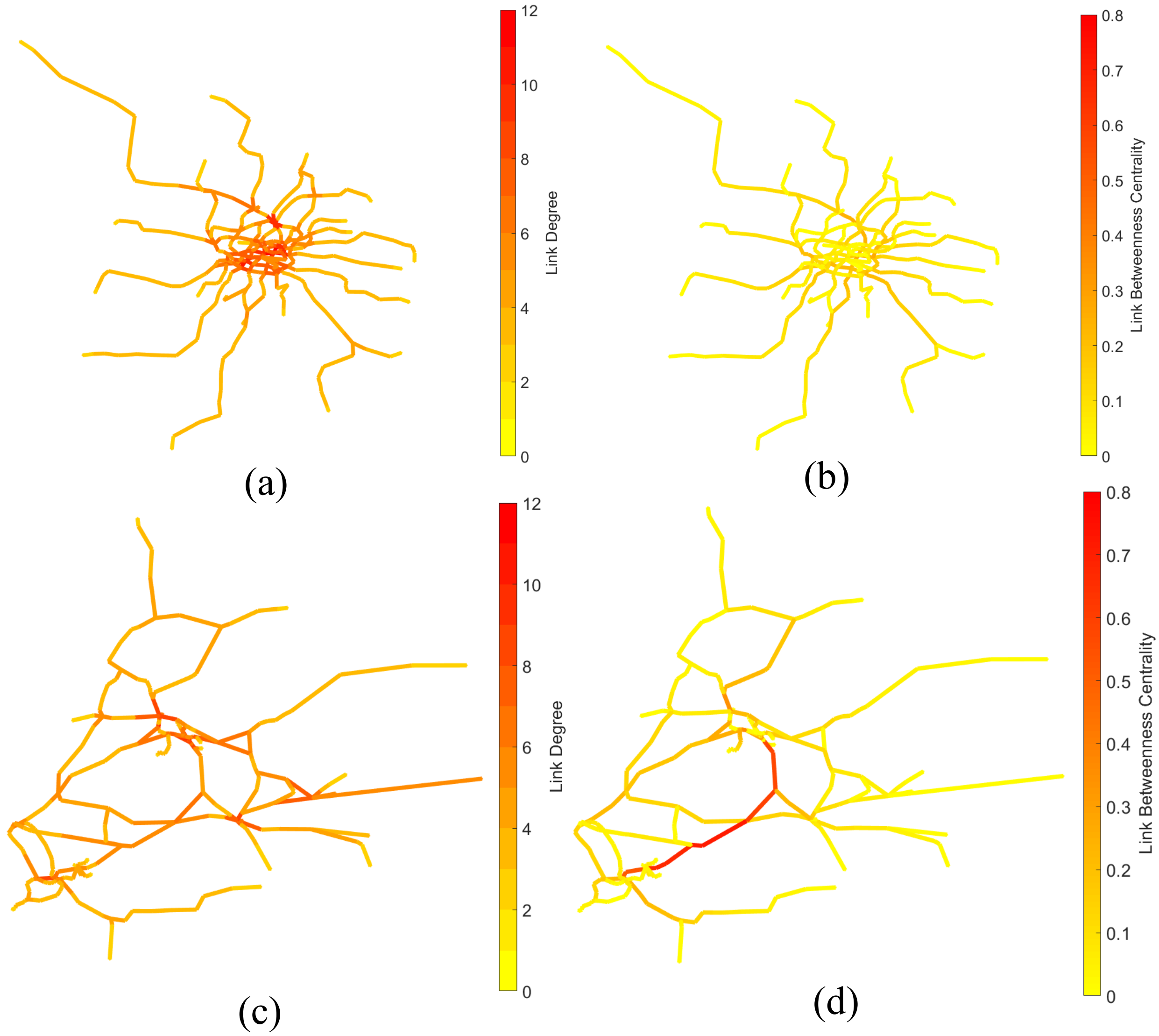

Figure 2: Link centrality indicators of the case study networks – (a) Shanghai, degree; (b) Shanghai, betweenness; (c) Randstad, degree; (d) Randstad, betweenness.

Nodes with high betweenness centrality (Fig 1, right) are mostly located in the southeastern part of the Shanghai core, reflecting the center of gravity of the network as the shortest paths between most OD pairs traverse this part of the network. Conversely, nodes with high betweenness centrality in the Randstad network are situated along the main northeast-southwest axis which connects Amsterdam and Rotterdam, the two largest cities in this polycentric agglomeration. The same axis is clearly visible also when examining the link betweenness centrality (Fig 2, right). It is evident that link betweenness centrality is much more evenly distributed in the case of Shanghai (note that the metric is displayed in relative terms, all values summing up to 1 for each network).

The variability of the centrality indicators is further investigated by plotting and comparing the distributions of both centrality indicators for both links and nodes for the two networks. While the node degree distributions are very similar, the Shanghai network exhibits higher link degree values. This indicates that nodes with a high degree value are more likely to be connected to other nodes with a high node degree in the Shanghai case than in the Randstad case, creating ‘cliques’ of high degree nodes, creating a set of highly connected nodes in the core.

Both node and link betweenness centrality are more skewed in the Randstad network than in the Shanghai network (Fig 3, right). Few nodes and links in the Randstad network are highly central in terms of them constituting part of the shortest path for a large share of the OD pairs, while the vast majority of nodes and links are on the shortest path of only few OD pairs. Betweenness centrality in the Shanghai network is much more evenly distributed, potentially making the network less dependent on any specific station or track segment.

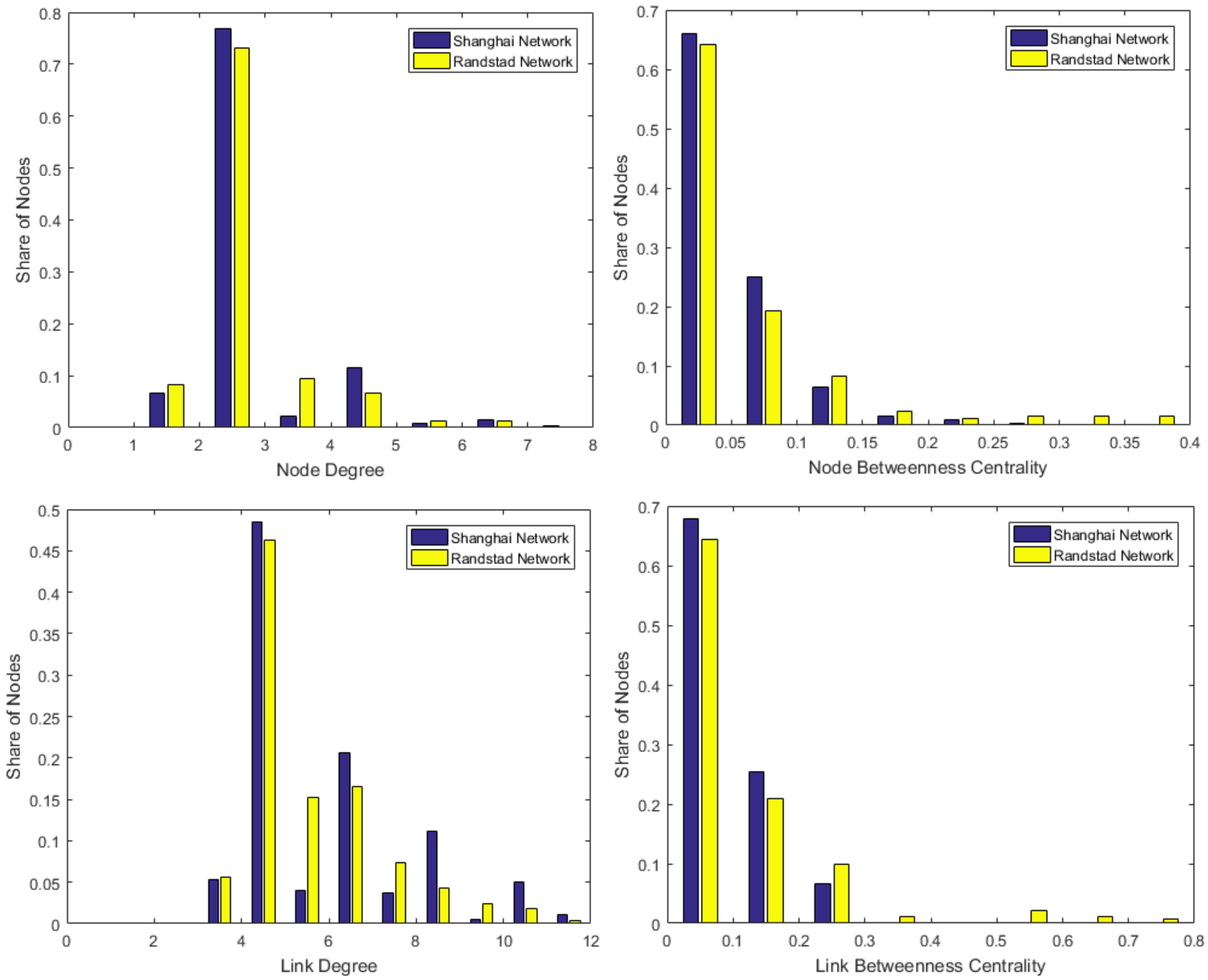

Figure 3: Comparison of node and link degree and betweenness centrality distributions for Shanghai and Randstad

4.2 Network robustness comparison

The results of the node and link removal strategies are summarized in Figure 4. Each curve corresponds to the sequence of either link or node removal strategy by one of the removal sequences – based on degree (k), betweenness (b) or random attacks (RA) – for either Randstad (R) or Shanghai (S). In all cases, the curve starts from the value which corresponds to the initial undisrupted state of the network. In the case of the relative decrease in network performance in terms of the size of the largest connected component, S, the intial value is one. The share of this component is monotonically decreasing with a larger share of the nodes or links removed. Conversely, when measuring changes in the average shortest path the initial value is zero and the value increases monotonically as a one progresses in the sequential removal of network elements.

The relative size of the largest component (Fig 4, left) decreases quickly and abruptly for both networks when removing nodes or links by order of importance, especially when removing based on betweenness. In Shanghai network, S starts to drop when about 8% of the links are removed while for Randstad it drops earlier at 6.5%. In contrast, a much slower and more gradual deterioration occurs when removing nodes or links randomly. After removing about 40% of the nodes or 50% of the links, the sub-networks are extremely fragmented with no sub-network consisting of more than 1% (3 nodes or links) of the original graph. Conversely, this does not happen until the removal of 80% of the nodes or links when those are removed at random. Both networks are more robust to link removal than to node removal. This could be expected since the removal of a node involves the removal of all links connected to this node, hence resulting with a faster breakdown. Interestingly, random removal can be more harmful than targeted removal when few links are removed from the network (bottom, left). The pace of network deterioration when removing links based on betweenness is twice as fast when removal is based on degree. Hence, global connectivity is more important than local connectivity in determining the most critical links.

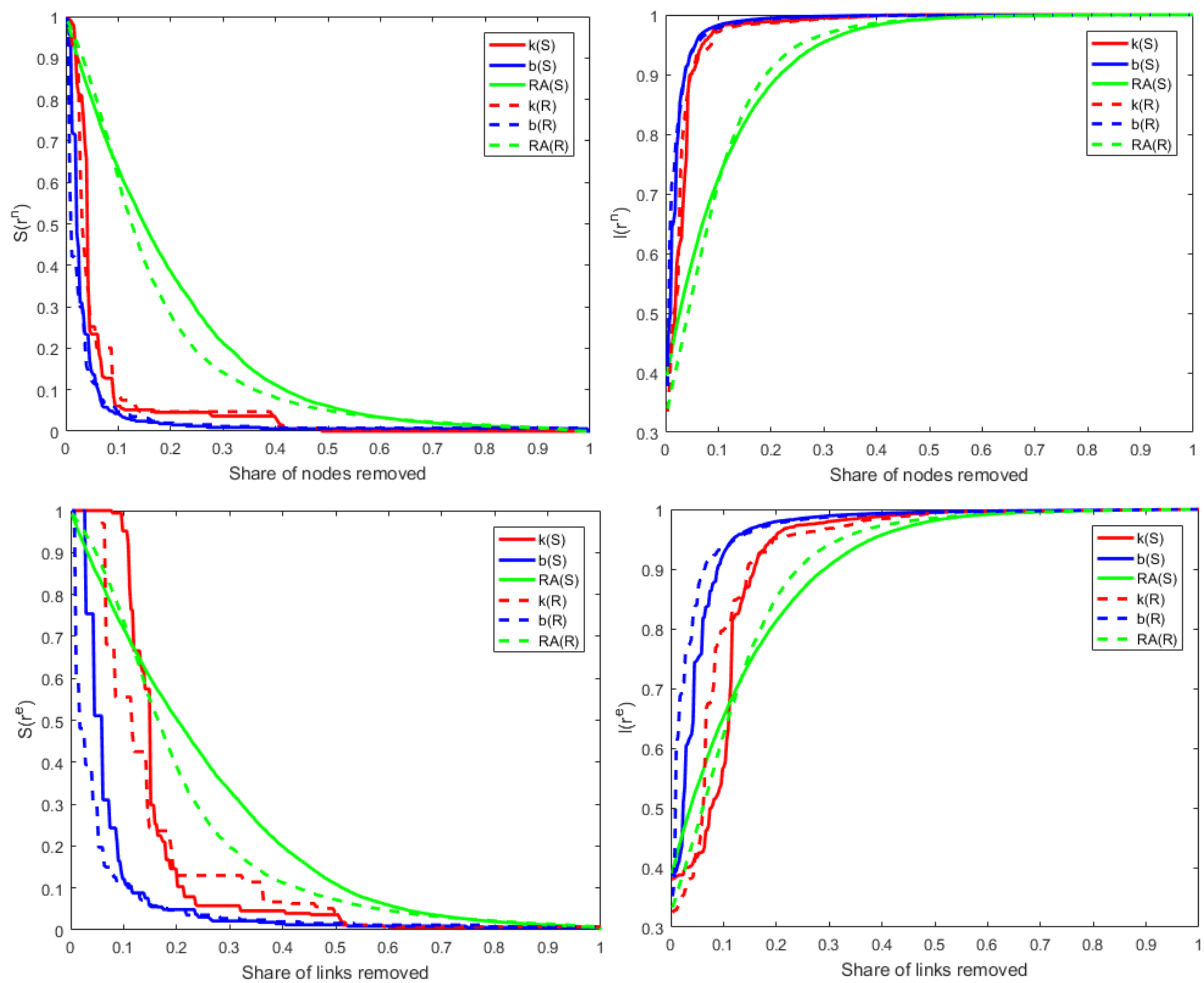

Figure 4: Shanghai (S) and Randstad (R) performance under alternative node (top) and link (bottom) removal strategies (k – node centrality; b – betweenness centrality; RA –random) in terms of Relative size, S, (left) and Shortest path (right).

In the case of node removal (top left), the patterns for the two networks is very similar. The performance of the Shanghai network exhibits greater robustness than the Randstad network in the link removal scenarios, with larger S values under most shares of links removed. For example, after the removal of 10% of the links with the highest degree centrality, the Shanghai network is still connected, allowing travelling between any OD pair, while the largest connected sub-network in the Randstad lost more than 40% of the original network elements. However, after the removal of 15%, both networks are equally (dis)connected with $S \approx 15\%$.

Similar trends are observed for the normalized average shortest path length metric, $\tilde{l}$ (Fig 4, right). Again, both networks are much more vulnerable to targeted attacks than to random attacks, with attacks targeting the network elements with the highest betweenness centrality values being the most devastating. No significant differences between the two networks are observed when removing nodes, while the Shanghai network prevails again as superior to the Randstad network in terms of its robustness to link removal scenarios. In targeted removal scenarios, after only 5% of the nodes are removed, $\tilde{l}$ already approaches 0.9. Hence, even though both networks have not broken down yet ($S = 1$), severe detours are needed, resulting in significantly longer paths. After removing 10% of the nodes or 20% of the links based on either degree or betweenness centrality, the network break down as indicated by the S metric is so severe, that the disconnected OD pairs dominate the shortest path calculations resulting with $\tilde{l}$ values approaching 1. This happens only after the removal of 40% of the nodes or 50% of the links when those are removed at random.

The analysis of the curves plotted in Figure 4 shed light on the degrading rapidity of the two networks. In addition, an aggregate metric of network robustness with respect to the largest connected component (Eq. 8), A, is calculated for all scenarios. A is the integral of the curves plotted in Figure 4, left. Hence, an extremely vulnerable network which breaks apart instantly after the first node or link has been removed will have $A = 0$. In contrast, the hypothetical case of an extremely robust network that remains intact until the very last brick is removed will yield $A = 1$.

In addition to the Randstad and Shanghai networks, we also calculate it for Paris and London based on the findings reported in (*10*). We remind the reader that the latter considered the entire public transport network, resulting in much larger graphs. The results show that the Randstad is consistently less robust than Shanghai. Both networks are however much more vulnerable than London and especially Paris, albeit the latter two include also public transport modes other than metro. The most pronounced differences are observed for link betweenness centrality removal. While the Randstad and Shanghai are most vulnerable to betweenness removal scenarios, London and Paris are more or equally vulnerable to degree-based removals. This is further discussed in the next section.

Table 1 : Aggregate metric of network robustness per scenario and network

Network	**Link removal**			**Nodes removal**		
	link degree (k)	*link betweenness centrality (b)*	*random attacks (RA)*	*node degree (k)*	*node betweenness centrality (b)*	*random attacks (RA)*
Shanghai	16.22	7.24	24.33	5.64	3.42	19.11
Randstad	14.95	5.06	20.76	5.54	2.97	16.82
London	20.95	27.20	27.45	5.45	8.71	29.31
Paris	47.12	55.93	56.04	10.77	10.67	37.93

5 DISCUSSION AND CONCLUSION

In this paper, a network vulnerability analysis is performed for both Shanghai and Randstad heavy rail network. The two network possesses different structures as the former one is a recent and rapidly developing network serving a monocentric metropolitan area while the latter is an exemplary of a polycentric urban agglomeration area that has developed over many decades by a large number of planning authorities. Both link-based and node-based sequential removal are applied to test the network and the performance of the degraded network is measured in terms of the relative size of the largest component (S), the relative mean shortest path ($\tilde{l}$) and an aggregate robustness measure (A). The first two measures are calculated for each phase in the iterative disruption strategy while the latter one is calculated after the whole sequences of attacks. Random failures as well as targeted attacks based on degree and betweenness centrality are examined.

The results are summarized in stylized schematic graphs in Figure 5, including the results for the Paris and London public transport networks based on the findings reported in (*10*). A clear pattern can be observed – The Paris network outperforms all other networks under all scenarios, exhibiting the most robust performance. In general, the remaining networks can be ranked in the following descending order in terms of their robustness: London, Shanghai and Randstad. While London is generally less vulnerable than Shanghai and Randstad in the event of targeted attacks, it exercises a more abrupt pattern in case of degree-based targeted attacks or random attacks, resulting with a more fragmented network than both Shanghai and Randstad beyond a certain share of the links has been removed. This results with comparable overall robustness for the three networks in the case of random link removal (Table 2).

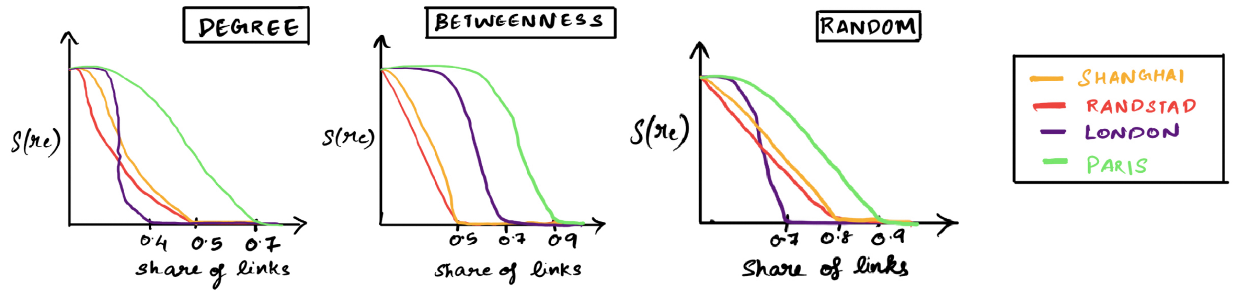

Figure 5: Schematic diagrams comparing the relative size deterioration for London, Paris, Shanghai and Randstad per link removal strategy.

The results of this study shed new light on some of the common conventions held in the network robustness discourse. The polycentric network of the Randstad was found the least robust, inferior to the monocentric networks of London, Paris and Shanghai. Unlike what might be expected, the network polycentricism does not yield in the case of the Randstad with a more distributed hub and spoke structure that is known to contribute to network robustness but rather results in few key stations and tracks that lie outside of the urban cores in a fork-like structure. The Shanghai network size provides opportunities for redundancy, the core of the network is finely meshed and the rapid and recent development – all within three decades – are all expected to be beneficial for network robustness. A closer inspection reveals however that the Shanghai network includes relatively more and longer branches which are vulnerable to targeted disruptions, especially in comparison to Paris which has many circuits due to short distances between stations and many peripheral hub connections. It is well-known that the connectivity of radial networks is highly vulnerable to disruption, isolating one branch from the remaining network. These results should be revisited by evaluating the redundancy offered by alternative public transport modes in the Randstad and Shanghai cases.

This study has several implications for network planning. First, the robustness of polycentric regions highly depends on the availability of routing alternatives between the most populated areas. Due to lower densities between the urban cores, networks serving polycentric areas may rely on a limited number of connecting corridors. This however has severe consequences for network robustness as their malfunction will result with an immediate breakdown and loss of a significant share of network functionality. Second, a finely meshed network core cannot substitute the availability of connections between outer hubs, in the absence of which long branches can become disconnected without any viable alternative.

Our findings suggest that the relation between network structure and its robustness is non-trivial. While in general more decentralized networks are more robust to targeted attacks, a polycentric urban agglomeration does not necessarily yield a more distributed network as measured in terms of centrality indicators. Furthermore, there is also no indication that a top-down planning style necessarily results with a more robust design. It may even be the case that a bottom-up and lengthy development may result with more fractal-like geometry due to underlying network evolutionary principles. Further research is needed in order to examine the generality of these results, related processes and their consequences for network design as well as the incorporation of demand and supply information.

AUTHORS' CONTRIBUTION:

The authors confirm contribution to the paper as follows: study conception and design: O. Cats; data collection: P. Krishnakumari; analysis and interpretation of results: K. Tundulyasaree, P. Krishnakumari, O. Cats; draft manuscript preparation: O. Cats. All authors reviewed the results and approved the final version of the manuscript.

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