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DOI

[10.1016/j.jrtpm.2019.100163](https://doi.org/10.1016/j.jrtpm.2019.100163)

Publication date

2020

Document Version

Final published version

Published in

Journal of Rail Transport Planning and Management

Citation (APA)

Scheepmaker, G. M., Willeboordse, H. Y., Hoogenraad, J. H., Luijt, R. S., & Goverde, R. M. P. (2020). Comparing train driving strategies on multiple key performance indicators. *Journal of Rail Transport Planning and Management*, 13, 1-26. Article 100163. <https://doi.org/10.1016/j.jrtpm.2019.100163>

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Journal of Rail Transport Planning & Management

journal homepage: <http://www.elsevier.com/locate/jrtpm>

Comparing train driving strategies on multiple key performance indicators

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ARTICLE INFO

Keywords:

Key performance indicators
Train driving strategies
Optimal train control

ABSTRACT

The driving strategy of train drivers has a large impact on the energy consumption. In recent studies the focus was on calculating the optimal eco-driving strategy, and measuring the exact amount of energy used during train runs. However, energy consumption is not the only key performance indicator that affects the operational performance of train or freight operating companies. In this study we define a set of key performance indicators, relevant to train operation, that are specific, measurable, assignable, realistic and time-related, and that are influenced by the driving strategy of the driver. These key performance indicators are *safety*, *timeliness*, *energy consumption*, *workload of the driver*, *the environment*, *cost of maintenance* and *brand image*. We chose four driving strategies that are most used in daily practice or most studied in the literature to assess these key performance indicators. Per key performance indicator we defined evaluation criteria to measure the impact of a driving strategy. We then defined key characteristics (e.g. track length, gradients), and conditions (e.g. speed restrictions, and load factor) based on which we defined test scenarios for three different train types. We then used the *Radau Pseudospectral Method* for solving the various optimal train control problems to compute the effect of the driving strategies on most of the key performance indicators. Our findings show amongst others, that a maximal coasting strategy causes the least environmental pollution, and in most scenarios its energy consumption coincided with the optimal energy-efficient train control strategy or it had an energy efficiency close to the optimal one. Furthermore, we found that on other key performance indicators there are differences between the driving strategies (e.g. in cost of maintenance), which should be considered when choosing a preferred driving strategy. Our results enable train and freight operating companies to make an informed decision when choosing a preferred driving strategy for their drivers, or when choosing a Driver Advisory System that supports this preferred driving strategy.

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<https://doi.org/10.1016/j.jrtpm.2019.100163>

Received 1 February 2019; Received in revised form 8 October 2019; Accepted 15 November 2019

Available online 29 November 2019

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1. Introduction

Energy consumption is one of the focus points of modern day train operation. In the last decade, almost every Train Operating Company (TOC), and Freight Operating Company (FOC) has taken measures to diminish its carbon footprint, and to save energy. These measures are meant to make rail transport more eco-friendly and cost effective (Luijt et al., 2017). The driving behaviour of the driver has a large impact on energy consumption during train trips. TOCs and FOCs therefore promote energy-efficient driving, and educate their drivers in one or more eco-driving strategies. However, there are more *Key Performance Indicators (KPIs)* than energy consumption that determine the performance of train operation. In this study we aimed at determining a set of SMART (i.e. Specific, Measurable, Assignable, Realistic and Time-related (Doran, 1981)) KPIs relevant to train operation, that are influenced by the driving strategy of the driver. Those KPIs are safety, timeliness, energy consumption, workload of the driver, the environment, cost of maintenance, and the brand image of a TOC.

We chose four of the most used train driving strategies to study the effect of the driving strategy on our set of KPIs. The first two are the main eco-driving strategies which are applied by drivers in everyday practice: the *Maximal Coasting (MC)* strategy, which applies maximum speed until coasting, and the *Reduced Maximum Speed (RMS)* strategy, which applies an optimal cruising speed and no coasting. Drivers apply any variation of these two strategies, either or not supported by a Driver Advisory System (DAS). We also included the *Energy-Efficient Train Control (EETC)* strategy, which calculates the optimal eco-driving strategy per train trip, and combines coasting and cruising. This strategy is described in multiple studies (Albrecht et al., 2016a, b; Howlett and Pudney, 1995; Howlett, 2000; Khmelnsky, 2000; Liu and Golovitcher, 2003; Lu et al., 2013). A review of energy-efficient train control can be found in Scheepmaker et al. (2017). The fourth driving strategy is the *Minimum Time Train Control (MTTC)* strategy. In this strategy the driver minimizes the total running time by driving as fast as possible. While this is not an eco-driving strategy, it is often used to make up for lost time in case of a delay. This driving strategy is mostly used in the literature as a reference scenario (see Albrecht et al. (2016a, b) and Scheepmaker and Goverde (2015, 2016)).

The main contributions of this paper are:

- The definition of a toolbox of KPIs, that can be used to determine the most appropriate driving strategy for a specific TOC or FOC, or even for a specific part of the track.
- A broader view on the effect of train driving strategies, by studying the effect on a set of KPIs relevant to train operation.
- A comparison of eco-driving strategies not only to MTTC, but also to each other.
- Elaborate multiple case studies based on several characteristics (e.g. track length, gradients) and conditions (e.g. speed restrictions, load factor) on three types of rolling stock (i.e. Intercity/long distance, Sprinter/regional and freight).

Although, we are aware that there are many DASs available (see Panou et al. (2013)), based on one or more driving strategies, we refrained from using a DAS in this study. First of all, because the purpose of this study was to establish a toolbox of KPIs to measure the performance of train operation and not to compare the effectiveness of DASs, although it can be used for this. Besides that, in practice it has been shown that significant energy savings can be accomplished by implementing an eco-driving strategy without the use of a DAS (see Luijt et al. (2017)) and lastly, because we wanted to make an objective comparison between the driving strategies on the KPI workload. Although we admit that it is difficult to apply a driving strategy such as EETC without a DAS.

We applied a mathematical model for different driving strategies based on the optimal control theory. Most of our test scenarios were based on the actual situation at the largest Dutch TOC, the Netherlands Railways (NS or in Dutch: *Nederlandse Spoorwegen*).

In this paper we first describe the KPIs studied, together with the evaluation criteria used to measure these KPIs (Section 2). In Section 3 the driving strategies, and the main model used in this paper are described. This section also includes an explanation on how we solved the optimal train control problem. In Section 4 we present our test scenarios, with the key characteristics and conditions on which they are based. We then present our results in Section 5. Our interpretation of these results, and the impact of the results will be discussed in Section 6. We will conclude this paper by highlighting the main conclusions based on the findings in this study (see Section 7).

Table 1
KPIs and evaluation criteria.

KPI	Evaluation Criteria
Safety	Distance remaining (between first and final braking regime) Time remaining (between first and final braking regime)
Timeliness	Deviations from planned time of arrival
Energy consumption	Total amount of energy
Workload	Total interference score
Environment	Average pass-by noise Wear of braking blocks
Cost of maintenance	Wear of braking blocks
Brand image	Risk of unplanned stops

2. Key performance indicators and evaluation criteria

In this section we describe the set of SMART KPIs, that are relevant to train operation, and that can be influenced by the driving strategy. This set consists of seven KPIs, of which the first three are the most important for train operation. They are presented in descending order of importance (Clements et al., 2011) in this section: safety (see Section 2.1), timeliness (see Section 2.2), energy consumption (see Section 2.3), workload of the driver (see Section 2.4), cost of maintenance (see Section 2.5), the environment (see Section 2.6), and the brand image of a TOC (see Section 2.7). Although the KPIs are selected based on SMART criteria, the T (time-related) of SMART does not apply so much in the process of finding a SMART set of KPIs for train operation.

Per KPI we describe the evaluation criteria and the methods used (except for the main method which is described in Section 3) to measure these KPIs in this section. We evaluated the KPI passenger and guard comfort during this study as well. However, we found that this KPI does not depend on the driving strategy, because the maximal acceleration and deceleration are the same for all driving strategies in our test scenarios, and the maximum speed hardly differs. Therefore, this KPI is not mentioned further in this paper. Table 1 gives an overview of the different KPIs and evaluation criteria used in our paper.

2.1. Safety

We defined safety as the absence of the risk of causing a railway accident (Era, 2016). In this paper we focused on Signals Passed At Danger (SPADs), one of the most common events preceding a railway accident. Note that we only measured this risk for conventional Automatic Train Protection (ATP) systems such as the Dutch NS '54/ATB system. We assumed that under ETCS L2 with continuous braking curve supervision, this risk does not exist. In agreement with NS safety experts we evaluated the KPI safety by choosing the distance and time between the end of the first braking regime (after an unplanned Signal Passed At Warning (SPAW)), and the beginning of the following braking regime (needed to come to a full stop in front of the red signal) as our evaluation criteria. After the SPAW the driver needs to decelerate to a speed of 40 km/h with a minimal ATP enforced braking deceleration of 0.43 m/s². For the Signal At Danger (SAD), a minimum braking deceleration of 0.5 m/s² is required. For most of the test scenarios in this paper, we assumed that the arrival of the test train was not hindered by a previous train at the arrival platform. The KPI safety is not of influence in these scenarios, since there is no unplanned SPAW. In order to study the effect on the KPI safety we introduced a so called platform clearing scenario (see scenario 6.2 in Table 4), in which the test train under NS '54/ATB is hindered by a previous train, that has not left the arrival platform. This results in an unplanned SPAW for the test train. The results of this scenario are described in Section 5.7.

2.2. Timeliness

Timeliness is defined as the ability to depart and arrive exactly to the second of the planned time. Note that we did not use the KPI punctuality, which is defined as arrival or departure within a specified number of minutes of the designated timetable time (see Olsson and Haugland (2004)). At NS for instance, trains are considered to be punctual if their departure or arrival time is within 3 min (180 s) of the planned time. Per train trip we determined the deviation from the designated timetable time in seconds.

2.3. Energy consumption

Energy consumption is defined as the use of the least possible amount of energy to complete a train run. We measured this by determining the total amount of energy (in kWh) per train trip i.e. the amount of traction energy used (including electrical losses), minus the amount of regenerative energy gained by braking. The use of, for instance, auxiliary power was excluded from our study, because this is not related to, or influenced by the driving strategy. Since not all the rolling stock of NS is equipped with regenerative brakes we added one test scenario with mechanical braking. We compute the total energy consumption E [kWh] at the catenary by using

$$E = (P_t + P_c)t, \quad (1)$$

with the powers computed by

$$P_t = Fv/\eta \quad (2)$$

$$P_c = (P_t/V)^2 R, \quad (3)$$

where P_t [kW] is the traction power of the train, P_c [kW] is the power over the catenary, t [s] is the time, F [N] is the traction force, v [m/s] is the speed, η [-] is the traction efficiency, V [V] is the voltage, and R [Ω] is the resistance. We consider the Dutch 1.5 kV DC catenary network with an average resistance over catenary and tracks of 0.1136 Ω . For stability of the calculations, we fixed the voltage at the pantograph on 1.5 kV, and added the dissipated power. In this way, the maximal tractive force remains unchanged, and iterations in the calculations were avoided.

2.4. Workload

In RSSB (2008, p.125) the workload of the train driver is defined as follows: "Workload refers to the effort demanded from people

by the tasks they have to do”, and on the same page the following statement is made: ”Workload is a problem for safety-critical operations if it’s too low - and an even bigger one if it’s too high.” We evaluated this KPI by measuring the Total Interference Score (TIS) per driving strategy (see Section 2.4.1), using the multiple resource model by Wickens (Horrey and Wickens, 2004; Wickens, 2002). We then compared the TISs of different driving strategies to each other. Note that we concentrated on differences in driving strategies, and therefore excluded tasks that are common to all driving strategies such as monitoring signs. In order to make a valid comparison between driving strategies we assumed that the driver does not use any form of electronic DAS. We are aware that without the use of such a device, a high level of memorization of routes and circumstances is needed to comply with both timeliness and energy consumption. Furthermore, we assumed that the driver in our study is an experienced driver with sufficient route knowledge.

2.4.1. Workload analysis

In order to determine the workload we first defined the tasks of the driver, and the order in which these tasks are carried out. For this purpose we used a hierarchical task analysis (based on RSSB (2008)). The initial results of this analysis were based on the expert judgment of the authors, and later verified by two expert drivers.

First we defined the goal per scenario. For the standard (golden) run, i.e. the reference scenario, the goal is: ”Driving a train from A to B” (from the moment the driver prepares for driving, until the moment the train comes to a full stop at the next station). Secondly, we defined the starting points of the driving regimes as subgoals, since these are the points at which the driver has to perform one or more tasks. We then made a breakdown of these subgoals into tasks, and determined the order of the tasks, the situation or time the task would be carried out, and the precondition(s) of the task per driving strategy. Note that the workload analysis was only performed on test scenarios besides the reference scenario, which are of influence on workload (e.g. not on scenarios concerning load or mechanical braking see test scenarios in Section 5).

We used the multiple resource model by Wickens (see Wickens (2002) and Horrey and Wickens (2004)) to determine the total task interference per driving strategy. This model is suitable for the analysis of work environments with a high workload where multiple tasks are performed. It is therefore appropriate for the analysis of train driving, although a train driver can also experience periods of under load. The multiple task model assumes that tasks can claim one or more resources. These resources are limited in capacity. The model is especially suitable to calculate the degree of interference in cases where there are concurrent tasks with a demand on the same resources. The multiple resource model (see Fig. 1) consists of four dimensions each with two different levels:

- The information processing stage consists of a perception/cognition level and a responding level.
- Perceptual modalities are either visual or auditory.
- Processing codes can be spatial (relating to environment) or verbal (spoken).
- The visual channel (the fourth dimension which is embedded in the visual modality resource) can be focal (e.g. reading a message) or ambient (peripheral vision e.g. monitoring signs and signals).

For the test scenarios relevant to the KPI workload we determined the TIS per task and driving strategy in the following three steps (see Table 2):

1. Determine demand scalar. Resources demanded by each task are scored according to the demand the task makes on the resource. The resources are scored as follows:
 - 0: the task is not dependent on this particular resource.

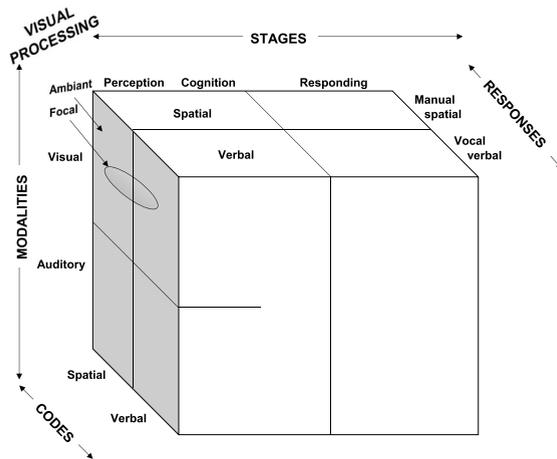


Fig. 1. The Multiple Resource Model adapted from Wickens (2002, p.163).

Table 2

Total task interference for reference scenario (golden run), short = short distance variant (SPR); long = long distance variant (IC/FT); SPR = Sprinter; IC = Intercity; FT = Freight Train; Vf = Visual focal; Va = Visual ambient; As = Auditory Spatial; Av = Auditory Verbal; Cs = Cognitive Spatial; Cv = Cognitive Verbal; Rs = Response Spatial; Rv = Response Verbal; CC = Conflict Component; TIS = Total Interference Score. *Note that tasks 1.2 and 1.3 are concurrent tasks. The TIS for these combined tasks is $3 \times 3 \times 0.8 = 7.2$. This value is only presented for task 1.3.; MA = (Maximum) acceleration; CR = Cruising; CO = Coasting; MB = (Maximum) Braking (see Section 3.1).

Nr	Subgoals/tasks	Total interference score					Perceptual				Cognitive		Response		CC
		MTTC	RMS	MC short	MC long	EETC	Vf	Va	As	Av	Cs	Cv	Rs	Rv	
	Reference scenario - Golden run														
1	MA - Accelerate to (maximum) speed target														
1.1	Decide on MTTC (based on assessment of departure time) or on an eco-driving strategy	3.0	3.0	3.0	3.0	3.0	2.0				1.0				
1.2	Determine when to start coasting	0.0	0.0	0.0	3.0	*					3.0			0.8	
1.3	Determine the MA speed target	0.0	3.0	3.0	0.0	7.2					3.0			0.8	
1.4	Accelerate to MA speed target	1.0	1.0	1.0	1.0	1.0							1.0		
2	CR - Starting a cruising regime														
2.1	Start cruising	2.0	2.0	0.0	2.0	2.0	1.0						1.0		
2.2	Monitor that the cruising speed is not overrun	2.0	2.0	0.0	2.0	2.0		1.0					1.0		
3	CO - Starting a coasting regime														
3.1	Start coasting (turn off traction)	0.0	0.0	2.0	2.0	2.0	1.0						1.0		
4	MB- starting (maximum) braking														
4.1	Determine when to start braking	1.0	1.0	1.0	1.0	1.0	1.0								
4.2	Turn off traction (before braking)	2.0	2.0	0.0	0.0	0.0	1.0						1.0		
4.3	Start decelerating to a speed limit which enables the driver to come to a full stop at the signal	2.0	2.0	2.0	2.0	2.0	1.0						1.0		
4.4	Start braking and come to a full stop in front of the signal	2.0	2.0	2.0	2.0	2.0	1.0						1.0		
	TIS per driving strategy	15.0	18.0	14.0	18.0	22.2									

- 1: there is some dependency on this resource.
- 2: the task demands this resource twice, or this is a more complex task.
- 3: complex task with a high demand on this particular resource.

The resource scores of each task form a demand vector. For instance, task 1.1 demands two types of visual perception. Firstly, perception of the departure time on the timetable and secondly perception of the actual time. The score for the visual focal level therefore is 2. On the information processing dimension the driver needs to choose a driving strategy based on his assessment of time. The cognitive verbal level is 1. The demand vector for this task is presented as 2–1.

2. Determine total demand score. The total demand score is then derived by adding up the scores from the demand vector. In case there is no concurrence of this particular task with another task the total demand score is equal to the TIS for this particular task.
3. Determine the TIS. In cases where two tasks use the same resources simultaneously, a resource-conflict score is derived from a conflict matrix (see Horrey and Wickens (2004)). Then the TIS is calculated as follows: total demand score task 1 \times total demand score task 2 \times conflict component. For instance, tasks 1.2 and 1.3 are concurrent tasks for the driving strategy EETC. The TIS for these combined tasks is $3 \times 3 \times 0.8 = 7.2$ for EETC.

The total workload per driving strategy in a certain scenario is determined by adding up the TISs of the tasks per driving strategy. Table 2 gives an elaboration of the TIS for the reference scenario (golden run). The rows in this table represent the tasks in consecutive order. The columns under ‘Total interference score’ show the interference score per driving strategy and per task. These scores are derived by adding up the demand scalars per task, which are presented under the columns from ‘Perceptual’ to ‘Response’. If a task does not apply for a certain driving strategy the interference score = 0. The interference score for concurrent tasks are determined by multiplying the demand scalars and then multiplying this result with a conflict component (see Wickens (2002) and Horrey and Wickens (2004)). The TIS per driving strategy (see last row of Table 2) is then derived by adding up the interference scores per task for a driving strategy.

2.5. Cost of maintenance

This KPI is linked to the amount of wear and tear on the braking blocks of the rolling stock. We would have liked to include wear on tracks. However, to our knowledge there was no suitable algorithm available in the literature on the basis of which we could determine this. Since wear on braking blocks is directly proportional to the absorbed energy in the mechanical brakes (this can be concluded from Vernersson and Lundén (2014)), we normalized it by setting wear of the MTTC reference scenario per rolling stock to 100% and compared results of other scenarios to it.

2.6. Environment

In this study we focused on two aspects of environmental pollution, namely the average amount of pass-by noise created during driving, and the amount of brake grindings produced during braking (which also depends on the material used in the brakes). In this study the evaluation criterion of the latter is equal to the wear of the braking blocks. We will refer to this as wear throughout the rest of the paper. We calculated the average pass-by noise L_p [dB] per train trip by measuring it on every 100 m of the track and then averaging it in dB over the full track length using the following equation (see [Ministerie van Infrastructuur en Milieu \(2012\)](#)):

$$L_p(v) = \frac{1}{n} \sum_{x=1}^n \log_{10} \left(\frac{v(x_i)}{v_{\text{MTTC}}(x_i)} \right), \tag{4}$$

with $v(x_i)$ the speed of the given driving style in km/h at location x_i [km] for the n measurements and $v_{\text{MTTC}}(x_i)$ the speed of the MTTC profile in km/h at the same location. The value of b [dB] is prescribed to be 16.1 dB for passenger trains and 19.6 dB for freight trains ([Ministerie van Infrastructuur en Milieu, 2012](#)).

We chose this way of averaging over the produced noise energy, because it corresponds best to the way other noise reduction measures are determined (e.g. calculation of surface roughness of rail). In our calculations the average pass-by noise of MTTC in each scenario is set to 0.0 dB, and the results of other driving strategies are compared to it. The influence of train speed on vibrations of the surroundings was excluded from this study, because we learned that this could vary per location, and that in some cases higher speeds result in less vibration (see [Van Leeuwen \(2017\)](#)). In the scope of this paper, location-independent averaging is used to assess the usefulness of this KPI.

2.7. Brand image

Brand image is defined as the view of target customers (i.e. passengers) on the service of a TOC. Note that we limited this KPI to TOCs. We assumed that for passengers the brand image would be influenced negatively by unplanned stops. Therefore, we assessed the likelihood of an unplanned stop for each of the driving strategies at the entry signal.

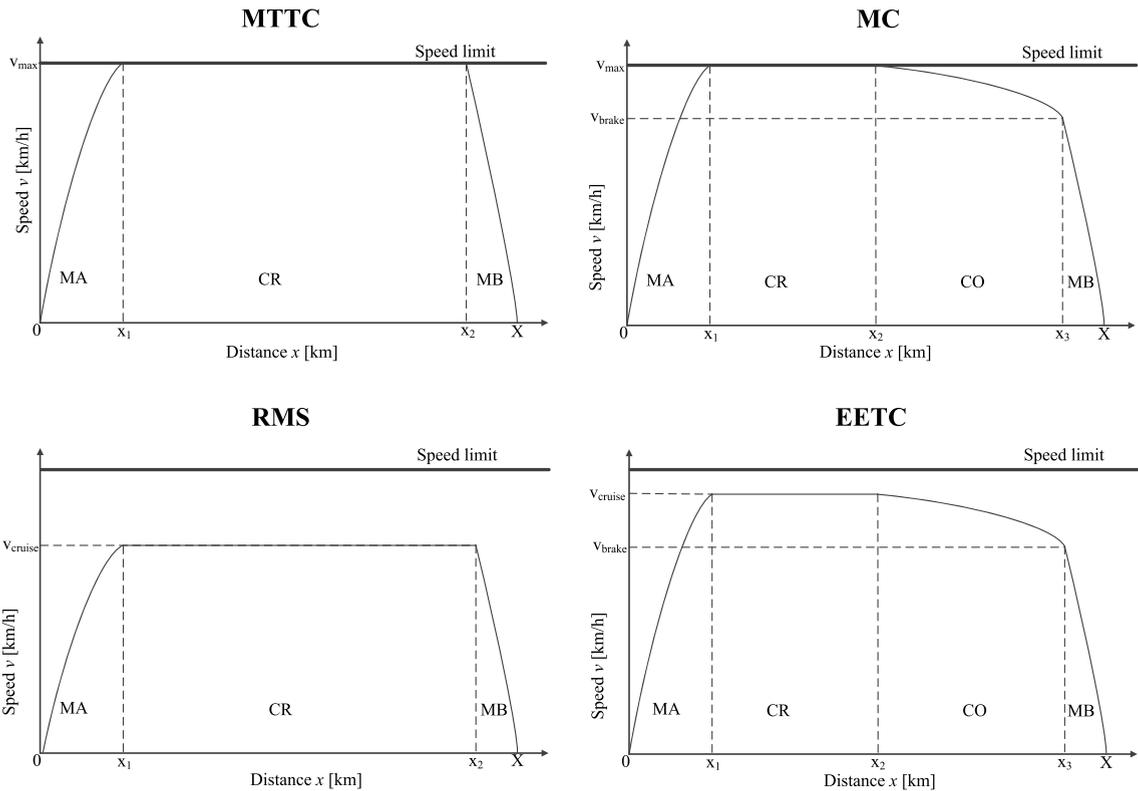


Fig. 2. The different driving strategies: MTTC (top left), MC (top right), RMS (bottom left), and EETC (bottom right). (MA = maximum acceleration, CR = cruising, CO = coasting and MB = maximum (service) braking, and the switching points x_1 , x_2 , and x_3 between driving regimes).

3. Driving strategies

In this section we first describe the four driving strategies used in this study (see Section 3.1), and then we give a more elaborate description of the main model in Section 3.2, followed by an explanation on how we solved the optimal train control problem in Section 3.3.

3.1. Four driving strategies

There are several driving strategies that are applied by train drivers. In most of these driving strategies the following *driving regimes* play an important role (see Scheepmaker and Goverde (2015)):

- (Maximum) acceleration;
- Cruising (maintaining a certain speed level);
- Coasting (turning off traction);
- (Maximum) braking.

In this study we compared the following four, in practice and theory most commonly used driving strategies: Minimum Time Train Control (MTTC), Maximum Coasting (MC), Reduced Maximum Speed (RMS), and Energy-Efficient Train Control (EETC). We have chosen MTTC as a reference driving strategy to compare the other eco-driving strategies that aim to arrive exactly on-time at the next station (i.e. not too late or not too early). The MC and RMS driving strategy are commonly applied in practice, while the EETC driving strategy is studied a lot in scientific research and gives the theoretical energy optimal solution. We explain the different driving strategies below:

1. The Minimum Time Train Control (MTTC), also known as the technical minimum running time strategy. This is not an eco-driving strategy. In this strategy the driver drives as fast as is technically possible, using the maximum permitted speed on every part of the track (see Fig. 2). This driving strategy is usually in practice applied to make up for lost time (due to delays). In addition, this driving strategy is used as a reference to compare the other driving strategies in our paper, as this strategy is unambiguously defined.
2. The Maximum Coasting Strategy (MC), see Fig. 2. This strategy aims at reducing total traction energy, by coasting as much as possible. In this strategy the driver drives as fast as possible and turns off the traction as soon as possible during the train trip, so that the train can coast to the nearest stop without using energy. This driving strategy uses the running time supplements in the timetable to arrive exactly on-time at the next station. The MC driving strategy is applied in practice by train drivers at the Netherlands Railways NS, where they use the UZI method (In Dutch: *Universeel Zuinig rijden Idee*) (Velthuisen and Ruijsendaal, 2011; Scheepmaker and Goverde, 2015; Luijt et al., 2017).
3. The Reduced Maximum Speed Strategy (RMS). The driver chooses and maintains a maximum speed in order to arrive exactly on time. This strategy focuses on optimizing a fixed cruising speed (see Fig. 2). This driving strategy is applied by the Swiss Federal Railways SBB, where the information about the cruising speed is presented in the train driver information system LEA (digital timetable) (Graffagnino et al., 2019).
4. The Energy-Efficient Train Control (EETC) strategy. This strategy focuses on determining and applying the optimal cruising speed and the optimal coasting points for each individual train trip (see Fig. 2). This strategy combines cruising and coasting and aims to minimize total traction energy consumption given the amount of running time in the timetable. The results of the EETC model are the theoretical optimum. Most scientific research on energy-efficient train driving is focused on the topic of EETC, e.g. Howlett (2000), Khmel'nitsky (2000), Liu and Golovitcher (2003), and Albrecht et al. (2016a, b). For an overview on the topic of EETC we refer to the review paper of Scheepmaker et al. (2017).

All strategies apply (maximum) acceleration and (maximum) braking. The main difference is the cruising speed, the use of coasting and the speed at the beginning of the braking phase.

3.2. Optimal train control

In this section the optimal train control problem is briefly explained and the optimal control structure are derived by using *Pontryagin's Maximum Principle (PMP)* (Pontryagin et al., 1962; Lewis et al., 2012; Ross, 2015). Derivations with distance as independent variable can be found in for example Albrecht et al. (2016a, b). In this paper, we use time as independent variable, because this leads to more stable results for the pseudospectral method compared to distance as independent variable (Goverde et al., 2019). We consider both *mechanical braking (MeB)* and *regenerative braking (RB)*. During mechanical braking the train uses braking disks or pads in order to convert kinetic energy into heat while during regenerative braking the kinetic energy is converted into electricity by using the engine of the train as a generator, which can be used by the train itself, stored in batteries or fed back to the catenary system (González-Gil et al., 2013).

We start with the minimum time train control problem in Section 3.2.1. Afterwards, we consider the energy-efficient train control problem with regenerative braking only in Section 3.2.2 and mechanical braking only in Section 3.2.3. The maximum coasting problem is explained in Section 3.2.4 and the reduced maximum speed driving strategy is discussed in Section 3.2.5.

3.2.1. Minimum time train control problem

This section briefly discusses the minimum time train control problem (MTTC). For a more detailed derivation of this problem we refer to [Albrecht et al. \(2016a, b\)](#). The aim of the MTTC problem is to minimize total running time of the train J [s] with t_f [s] defined as the final time and t_0 [s] as the initial time by

$$\text{Minimize } J = t_f - t_0, \quad (5)$$

subject to the constraints and the endpoint conditions

$$\dot{x}(t) = v(t) \quad (6)$$

$$\dot{v}(t) = f(t) + b_r(t) + b_m(t) - r(v) - g(x) \quad (7)$$

$$f(t)v(t) \leq p_{f,max} \quad (8)$$

$$-b_r(t)v(t) \leq p_{b,max} \quad (9)$$

$$0 \leq v(t) \leq v_{max}(x) \quad (10)$$

$$0 \leq f(t) \leq f_{max} \quad (11)$$

$$-b_{r,min} \leq b_r(t) \leq 0 \quad (12)$$

$$-b_{m,min} \leq b_m(t) \leq 0 \quad (13)$$

$$t_0 = 0, x(t_0) = 0, x(t_f) = X, v(t_0) = 0, v(t_f) = 0, \quad (14)$$

where time t [s] is the independent variable, distance x [m] and speed v [m/s] are the state variables with their derivatives to time $\dot{x} = dx/dt$ and $\dot{v} = dv/dt$. Furthermore, the control variables are the mass-specific traction force or acceleration f [m/s²], mass-specific regenerative braking force b_r [m/s²], and mass-specific mechanical braking force b_m [m/s²], which are computed by dividing the total force F [N] over the total rotating mass, e.g. $F(t)/(\rho m)$ with the rotating mass factor ρ [-] and the train mass m [kg]. The mass-specific traction control is bounded between the minimum of the maximum mass-specific traction force and the maximum mass-specific power divided by the speed, i.e. $f(v) \in [0, \min(f_{max}, p_{f,max}/v)]$. The braking forces are bounded between zero and the maximum braking force, i.e. $b_r(v) \in [\min(-b_{r,min}, -p_{b,max}/v), 0]$ for regenerative braking and $b_m(v) \in [-b_{m,min}, 0]$ for mechanical braking. The train cannot apply both traction and braking at the same time, i.e., $f \cdot (b_r + b_m) = 0$. The total resistance W [N] is determined by the train resistance and the line resistance, i.e. $W = R + G$. The train resistance R [N] is given by Davis

$$R(v) = r_0 + r_1 v + r_2 v^2, \quad (15)$$

with the non-negative coefficients r_1 , r_2 , and r_3 ([Davis, 1926](#)). The mass-specific train resistance $r(v)$ [m/s²] is computed by dividing the train resistance force by the total rotating mass, i.e. $r(v) = R(v)/(\rho m)$. The line resistance is mainly determined by the gradient force $G(x)$ [N], which are positive if the gradient is uphill and negative if the gradient force is downhill. The mass-specific line resistance is $g(x) = G(x)/(\rho m)$ [m/s²]. We do not include curves and tunnel resistance.

The Hamiltonian H [-] is defined by

$$H(x, v, \lambda_1, \lambda_2, f, b_r, b_m, t) = \lambda_2 f + \lambda_2 b_r + \lambda_2 b_m + \lambda_1 v - \lambda_2 r(v) - \lambda_2 g(x), \quad (16)$$

with the costate variables $\lambda_1(t)$ [s/m] and $\lambda_2(t)$ [s²/m]. The augmented Hamiltonian \bar{H} [-] is defined by

$$\begin{aligned} \bar{H}(x, v, \lambda_1, \lambda_2, \mu, f, b_r, b_m, t) = & H + \mu_1 (f_{max} - f) + \mu_2 (b_r + b_{r,min}) + \mu_3 (b_m + b_{m,min}) + \mu_4 (p_{f,max} - f v) \\ & + \mu_5 (p_{b,max} + b_r v) + \mu_6 (v_{max} - v), \end{aligned} \quad (17)$$

with the nonnegative Lagrange multipliers μ_1 [s²/m], μ_2 [s²/m], μ_3 [s²/m], μ_4 [s³/m²], μ_5 [s³/m²], and μ_6 [s/m]. The costates λ_1 and λ_2 satisfy the following differential equations

$$\dot{\lambda}_1(t) = -\partial \bar{H} / \partial x = \lambda_2 g'(x) \quad (18)$$

$$\dot{\lambda}_2(t) = -\partial \bar{H} / \partial v = -\lambda_1 + \lambda_2 r'(v) + \mu_4 f - \mu_5 b_r + \mu_6. \quad (19)$$

Note that (18) indicates that λ_1 is constant where the gradient is constant.

Pontryagin's Maximum Principle gives the optimal controls by maximizing the Hamiltonian ([Pontryagin et al., 1962](#)). Moreover, by applying the *Karush-Kuhn-Tucker (KKT)* conditions on the augmented Hamiltonian, we are able to derive the optimal control structure from the necessary optimality conditions:

$$\left(\widehat{f}(t), \widehat{b}_r(t), \widehat{b}_m(t) \right) = \begin{cases} (f_{\max}(v(t)), 0, 0) & \text{if } \lambda_2(t) > 0 \quad (\text{MA}) \\ (r(v_{\max}) + g(x), 0, 0) \text{ and } r(v_{\max}) + g(x) \leq 0 & \text{if } \lambda_2(t) = 0 \quad (\text{CR1}) \\ (0, r(v_{\max}) + g(x) + b_m, r(v_{\max}) + g(x) + b_r) \text{ and } r(v_{\max}) + g(x) > 0 & \text{if } \lambda_2(t) = 0 \quad (\text{CR2}) \\ (0, -b_{r,\min}(v(t)), -b_{m,\min}(v(t))) & \text{if } \lambda_2(t) < 0 \quad (\text{MB}) \end{cases} \quad (20)$$

The results indicate that the MTTC driving strategy consists of three driving regimes, which depends on the value and sign of costate λ_2 . The first driving regime is maximum acceleration (MA) in which the train applies the minimum between the maximum specific traction force (11) and maximum specific power divided by speed (8). Second, during cruising at the speed limit the control is equal to the total train resistance in order to remain at the speed limit (i.e. depending on the sign of the total train resistance using traction (CR1) or braking (CR2)). Note that it is not possible to apply traction and braking at the same time. Finally, the train applies maximum braking force during the maximum braking (MB) regime. Note that there is no preference for regenerative or mechanical braking in this scenario, since the aim is to minimize total travel time and not the energy consumption.

3.2.2. Energy-efficient train control problem with regenerative braking only

The second optimal control problem we consider is the energy-efficient train control (EETC) problem with regenerative braking (RB) only. We included the effect of regenerative braking by the term $(b_r(t) + b_m(t))\eta$, where η [-] considers the total efficiency of both the traction η_t [-] and the catenary η_c [-], so $\eta = \eta_t\eta_c$. EETC with mechanical braking is achieved by replacing b_r by b_m and setting $\eta = 0$, which is discussed in Section 3.2.3. Like Albrecht et al. (2016a, b) we assume that the train only applies regenerative braking and no mechanical braking. We briefly provide the EETC problem formulation and we refer to Albrecht et al. (2016a, b) for more details about the general EETC problem derivation. The aim of the optimal train control is to minimize total mass-specific energy consumption J [m^2/s^2] between two consecutive stops, while arriving on time given the amount of time within the timetable T [s]:

$$J = \min \int_{t_0}^{t_f} (f(t) + \eta b_r(t))v(t)dt, \quad (21)$$

subject to the constraints (6–12) and the endpoint conditions

$$t_0 = 0, t_f = T, x(t_0) = 0, x(t_f) = X, v(t_0) = 0, v(t_f) = 0. \quad (22)$$

We define the Hamiltonian H [m^2/s^3] by

$$H(x, v, \lambda_1, \lambda_2, f, b_r, b_m, t) = (\lambda_2 - v)f + (\lambda_2 - \eta v)b_r + \lambda_1 v - \lambda_2 r(v) - \lambda_2 g(x), \quad (23)$$

with the costates $\lambda_1(t)$ [m^2/s^2] and $\lambda_2(t)$ [m/s]. The augmented Hamiltonian \bar{H} [m^2/s^3] is given by

$$\begin{aligned} \bar{H}(x, v, \lambda_1, \lambda_2, \mu, f, b_r, b_m, t) = & H + \mu_1(f_{\max} - f) + \mu_2(b_r + b_{r,\min}) + \mu_4(p_{f,\max} - f v) \\ & + \mu_5(p_{b,\max} + b_r v) + \mu_6(v_{\max} - v). \end{aligned} \quad (24)$$

where the nonnegative Lagrange multipliers are defined by μ_1 [m/s], μ_2 [m/s], μ_4 [-], μ_5 [-], and μ_6 [m/s^2]. The differential equations of the costates λ_1 and λ_2 state

$$\dot{\lambda}_1(t) = \lambda_2 g'(x) \quad (25)$$

$$\dot{\lambda}_2(t) = f + \eta b_r - \lambda_1 + \lambda_2 r'(v) + \mu_4 f - \mu_5 b_r + \mu_6. \quad (26)$$

Again, by using the PMP and KKT conditions we can derive the optimal control structure consisting of:

$$\left(\widehat{f}(t), \widehat{b}_r(t) \right) = \begin{cases} (f_{\max}(v(t)), 0) & \text{if } \lambda_2(t) > v(t) \quad (\text{MA}) \\ (r(v_f) + g(x), 0) & \text{if } \lambda_2(t) = v_f \quad (\text{CR1}) \\ (0, 0) & \text{if } \eta v(t) < \lambda_2(t) < v(t) \quad (\text{CO}) \\ (0, r(v_r) + g(x)) & \text{if } \lambda_2(t) = \eta v_r \quad (\text{CR2}) \\ (0, -b_{r,\min}(v(t))) & \text{if } \lambda_2(t) < \eta v(t) \quad (\text{MB}). \end{cases} \quad (27)$$

The optimal control now depends on the costate λ_2 in relation to speed v . The driving regimes MA and MB are similar to (20). The CO driving regime corresponds to coasting, where there is zero control. The driving regime CR1 indicates cruising by partial traction at cruising speed v_f , where a balance exists between the traction force and the total train resistance. The CR2 driving regime is cruising by partial regenerative braking at cruising speed v_r and can only be maintained during downhill gradients. MB indicates maximum braking by applying regenerative braking.

3.2.3. Energy-efficient train control problem with mechanical braking only

In this section we briefly consider mechanical braking (MeB) only for the EETC problem. More details can be found in Albrecht et al. (2016a, b). Basically, the optimal control problem is the same as in Section 3.2.2 replacing b_r (regenerative braking) by b_m (mechanical braking) and setting $\eta = 0$, thus, mechanical braking does not generate energy. This leads to the following objective function (J [m^2/s^2]):

$$J = \min \int_{t_0}^{t_f} f(t)v(t)dt, \quad (28)$$

subject to constraints (6, 8, 10, 11, 13), endpoint conditions (22), and constraint

$$\dot{v}(t) = f(t) + b_m(t) - r(v) - g(x). \quad (29)$$

The Hamiltonian H [m^2/s^3] is defined as

$$H(x, v, \lambda_1, \lambda_2, f, b_m, t) = (\lambda_2 - v)f + \lambda_2 b_m + \lambda_1 v - \lambda_2 r(v) - \lambda_2 g(x). \quad (30)$$

The augmented Hamiltonian \bar{H} [m^2/s^3] is given by

$$\bar{H}(x, v, \lambda_1, \lambda_2, \mu, f, b_m, t) = H + \mu_1(f_{\max} - f) + \mu_3(b_m + b_{m,\min}) + \mu_4(p_{f,\max} - fv) + \mu_6(v_{\max} - v), \quad (31)$$

where the nonnegative Lagrange multipliers are defined by μ_1 [m/s], μ_3 [m/s], μ_4 [-], and μ_6 [m/s^2]. The costates λ_1 [m/s^2] and λ_2 [m/s] satisfy the following differential equations:

$$\dot{\lambda}_1(t) = \lambda_2 g'(x) \quad (32)$$

$$\dot{\lambda}_2(t) = f - \lambda_1 + \lambda_2 r'(v) + \mu_4 f + \mu_6. \quad (33)$$

After maximizing the Hamiltonian and applying the KKT conditions we can derive the optimal control structure:

$$\widehat{f}(t), \widehat{b}_m(t) = \begin{cases} (f_{\max}(v(t)), 0) & \text{if } \lambda_2(t) > v(t) & \text{(MA)} \\ (r(v_f) + g(x), 0) & \text{if } \lambda_2(t) = v_f & \text{(CR1)} \\ (0, 0) & \text{if } 0 < \lambda_2(t) < v(t) & \text{(CO)} \\ (0, r(v_m) + g(x)) & \text{if } \lambda_2(t) = 0 & \text{(CR3)} \\ (0, -b_{m,\min}(v(t))) & \text{if } \lambda_2(t) < 0 & \text{(MB)}. \end{cases} \quad (34)$$

Compared to (27) the only difference is that the cruising by partial regenerative braking CR2 is replaced by cruising by partial mechanical braking CR3 at cruising speed v_m and that the maximum braking (MB) is now applied by mechanical braking instead of regenerative braking.

3.2.4. Maximal coasting

The third driving strategy is maximal coasting (MC). The optimal control problem is equal to (21) for RB and to (28) for MeB, subject to the constraints (6–12) for RB, (6, 8, 10, 11, 13) for MeB, and endpoint conditions (22). In addition, we included a constraint that forced the train to apply cruising at the speed limit by:

$$0 \leq v(t) - v_s(t) \leq v_{\max}, \quad (35)$$

where $v_s(t)$ is defined as a restricted speed that forces the train to maintain the speed limit during cruising. We also check the MTTC driving strategy to see where the train starts to cruise at the speed limit and we consider this the first distance where the train should apply cruising at the speed limit. For the distance where the model suggests to start cruising below the speed limit (such as suggested by the EETC model) we avoid this by setting $v_s(t) = v_{\max}$ given (10), and for all other distance $v_s(t) = 0$. The process to determine the optimal start of the coasting phase by changing $v_s(t)$ is done iteratively. The resulting optimal driving regimes include MA, (CR at speed limit), CO and MB, which are equal to the driving regimes given by (27) for RB and (34) for MeB. The only difference is the driving regime cruising CR1, CR2, and CR3, where the speed is equal to the speed limit v_{\max} similar to CR1 and CR2 in (20).

3.2.5. Reduced maximum speed

The fourth driving strategy considered is the reduced maximum speed (RMS) below the speed limit, which is a cruising driving strategy without coasting. Again the optimal control problem is equal to (21) for RB and (28) for MeB subject to the constraints (6–11) for RB and (6, 8, 10, 11, 13) for MeB and endpoint conditions (22). In addition, we included an extra path constraint that avoids the train to apply coasting by:

$$f(t) - r(v) - g(x) \geq 0. \quad (36)$$

The RMS driving strategy leads to the same driving regimes MA, MB, CR1, CR2, and CR3 as given by (27) for RB and (34). The only difference is that the driving regime coasting (CO) is not included.

3.3. Pseudospectral optimal train control

We use the *Radau Pseudospectral Method (RPM)* in order to solve the optimal control problem (Wang and Goverde, 2016a). This is a direct solution method, which discretizes the optimal control state and control variables using collocation points (Betts, 2010). Next, the discretized optimal control problem is rewritten to a nonlinear programming (NLP) problem, which is solved using efficient algorithms. For more details on the translation of the optimal train control problem into a NLP problem, we refer to Wang and Goverde

(2016a).

The main focus in the literature on pseudospectral optimal train control is on multiple-phase, in which the optimal control problem is divided into subproblems or phases with constant values for speed limit and gradient. The total optimal control problem is then solved for each phase and the phases are connected using linking functions (Wang et al., 2013, 2014; Wang and Goverde, 2016a, b, 2017, 2019). Single-phase pseudospectral methods decrease the computation time and can be applied when the number of discontinuities in the constraints is limited, because the quality of the model results depends strongly on the amount and location of the collocation points (determined by the algorithm) (Scheepmaker et al., 2019). Therefore, some constraints may be violated between the collocation points such as speed restrictions or gradients, since the discretization points are not exactly at the locations where the speed limit or gradient changes. In this paper we consider the single-phase model using the software MATLAB with the toolbox of GPOPS version 4.1 (Rao et al., 2011) to develop the model which we call PROMO (PseudospectRal Optimal train control Model) based on Scheepmaker and Goverde (2016) and Scheepmaker et al. (2019). We use a single core for the computations applied on a 2.1 GHz processor with 8 GB RAM.

4. Description case study

In this Section we describe the key characteristics and conditions that formed the basis of our test scenarios. The test scenarios per rolling stock type (see Section 4.1) with the different characteristics are summarized in Section 5. We first constructed a reference scenario by using what we considered to be standard values for each train type. Then we varied one of the key characteristics or conditions to create a new scenario. Some of these characteristics and conditions are based on the actual situation at NS. One example is the NS timetable, which includes a running time supplement of at least 5%. We measured the KPIs for different driving strategies on a single train without conflicts. We start this section with a description of the fixed characteristics within our study (see Section 4.1). We then describe track (see Section 4.2), rolling stock (see Section 4.3), and timetable related characteristics (see Section 4.4). In most scenarios we assume that there are no other trains preceding our test train. However in Section 4.5 we add a train sequencing condition.

4.1. Fixed characteristics

The following standard characteristics were used in our study: for all our scenarios we used a single train without conflicts. The ATP in most of our test scenarios uses a braking curve, for example ETCS L2. There is only one scenario with a conventional ATP (NS '54/ATB). We only considered electrically powered rolling stock (i.e., trains that obtain their power from the catenary system using a pantograph). The catenary voltage amounted to 1.5 kV DC with a maximum of 4 kA per train (Dutch situation). We assume the efficiency of the catenary to be 80% in the case the train applies regenerative braking and its energy is fed back to the catenary system (Scheepmaker and Goverde, 2016). All trains in our model started at the same location at the departure station (at the end of the platform length), and stopped at the end of a platform length of the station of arrival. The timetable was conflict-free in fractions of seconds. The maximum permitted speed (linespeed) on the track was 140 km/h. We used a fixed train length (fixed number of carriages) per rolling stock type. We used three types of rolling stock in our model: an Intercity (IC), a Sprinter (SPR), and a freight train (FT). Details of the rolling stock characteristics can be found in Table 3. A brief overview of the rolling stock is given below:

Table 3

Standard rolling stock characteristics (NS, 2018). Note that the train resistance equation between brackets for FT is only valid for 0% train load scenario.

Rolling Stock Characteristic	IC	SPR	FT
Rolling StockType	VIRM VI	SLT VI	BR186 (Traxx e-loc)
Number of coaches/wagons	6	6	28 (class Fals)
Train length	162 m	101 m	515 m
Total train weight	391,000 kg	198,000 kg	2,400,000 kg
Empty train weight	375,418 kg	189,600 kg	786,000 kg
Train load weight	15,582 kg (35%)	8,400 kg (35%)	1,614,000 kg (100%)
Train resistance equation: $r_0 + r_1 v + r_2 v^2$			
r_0	2,711.3 N	1,375.8 N	30,658 (10,949) N
r_1	43.43 N/m	37.48 N/m	796.97 (284.49) Ns/m
r_2	7.82 N ² /m ²	6.75 N ² /m ²	141.05 (48.81) Ns ² /m ²
Maximum traction force	213.9 kN	170 kN	236.22 kN
Maximum power	2,157 kW	1,755 kW	5,600 kW
Standard and ATB braking deceleration	0.5 m/s ²	0.5 m/s ²	0.5 m/s ²
Traction efficiency	87.5%	87%	87.7%
Catenary efficiency	80%	80%	80%
Maximum regenerative braking force	142.5 kN	150 kN	–
Maximum speed	160 km/h	160 km/h	95 km/h

- For the IC we used the most commonly used type of rolling stock at NS, which is the VIRM (In Dutch: *Verlengd InterRegio Materieel*). We used a VIRM type VI, which has six coaches.
- The SPR was a SLT VI (Sprinter Light Train) consisting of six coaches. This type of rolling stock is used on short distances.
- For the FT we chose the BR186 (In German: *BauReihe* 186). This e-loc is often used for freight transport in the Netherlands. This train consists of a locomotive and 28 freight wagons (type Falns used to transport coal).

The standard track length for both the IC, and the FT was 50 km in most test scenarios.

For the SPR we used a track length of 5 km. We used tracks without curves and an amount of cross wind of 10 km/h (this is considered to be the average amount of cross wind in the Netherlands). For the IC and the SPR we chose the set of train resistance coefficients of the tactical Dutch timetable design system, Donna. Our FT train resistance coefficients are based on the data of the strategic timetable design system DONS.

4.2. Track related characteristics

Most of our scenarios are based on track lengths between 5 and 50 km, that are common in the Netherlands. For the purpose of a sensitivity analysis we wanted to study the effects of longer track lengths on the four driving strategies. Therefore, we introduced an additional scenario with a track length of 150 km for both IC and FT.

In order to study the effect of track gradients on the driving strategies, we developed a test scenario in which we introduced one track gradient with a height difference of 20 m over a distance of 3000 m the middle of the track, equal to $20/3000 = 6.67\%$ (see Fig. 4). This was only for IC and FT under the assumption that a track gradient on a short track (SPR) would not add to this study. In this scenario the trains first encounter a downward, and then an upward gradient. We reasoned that this would be more demanding with respect to driver anticipation than a situation with first an upward and then a downward gradient.

4.3. Rolling stock related characteristics

We used a load factor of 0% (zero load), 35% (the default) and of 100%. The latter refers to the weight of the train itself (zero load) including full loading. In passenger trains full loading includes the weight of all the (folding) seats occupied by passengers, the weight of one train driver, and one train guard (with an average weight per person of 70 kg (BS-EN-15663, 2017)) plus the rotating mass. For the freight trains the maximum load was the train weight (zero load) plus the maximum cargo weight.

In our model Regenerative Braking (RB) is standard for SPR and IC. Since not all the NS rolling stock is equipped with regenerative brakes we defined a separate test scenario for Mechanical Braking (MeB). In our paper we assume that the SPR train uses 100% regenerative braking and the IC train uses 72.89% regenerative braking (the rest is mechanical braking) with a maximum deceleration rate of 0.5 m/s^2 (NS, 2018). The FTs in the Netherlands only use mechanical brakes. That is why all the FT test scenarios in our model use MeB.

We tested the effectiveness of driving strategies under limited speed restrictions (SR). Besides the standard situation, in which there are no speed restrictions, we introduced an SR in the middle of the route in which the line speed of 140 km/h was lowered to 100 km/h (for IC and SPR), and 80 km/h (for FT) respectively.

4.4. Timetable related characteristics

We compared the runs of our four driving strategies with different running time supplements. At NS the advised running time supplement for SPR and IC is at least 5% rounded up to whole minutes (ProRail, 2016). In practice this results in an average running time supplement of 10%. Note that in this study we did not round up the running time supplement. For IC and SPR in this study the standard running time supplement was 10%. We also measured the effects of a running time supplement of 5%, and 15%, and compared the results to a reference value without running time supplement (a situation in which MTTC is applied).

For FT we used a standard running time supplement of 4.3%, based on the difference between the minimum running time with a maximum speed of 90 km/h and of 95 km/h. During timetable design in the Netherlands freight trains are modelled with 0% running time supplements for the heaviest composition (maximum train load) and the slowest train allowed (maximum speed of 90 km/h) (ProRail, 2016). The running time supplements for freight trains are then determined in practice by lighter freight trains as well as freight trains with a higher maximum speed. For freight trains we used alternative running time supplements of 7.5% and 10%.



Fig. 3. Track characteristics IC/FT entering a station. Total track length = 50 km.

4.5. Train sequencing

In a busy, tightly planned network, IC and FT trains cannot run early without encountering the effects of preceding trains. In order to study the effect of driving strategies under this situation we introduced a condition in which a preceding train occupies the entry track of our approaching simulated train. We chose the shortest platform clearing times where the planned timetable was conflict-free for a train entering the station at linespeed. We studied this condition only for IC in both the standard ETCS L2 and the conventional Dutch NS '54/ATB situation.

In these scenarios the speed and time at the signal before the entry signal (km 47) is used as input (see Fig. 3). We assumed that the driver does not change the driving strategy up to this signal. Braking was taken at 0.5 m/s^2 without reaction times or speed dependence. In this way, braking curve intervention in ETCS L2 was avoided, and at the same time this conformed to minimal braking for NS '54/ATB. Acceleration was likewise taken at 0.5 m/s^2 .

5. Results

In this section we summarize our findings per group of test scenarios as defined in Table 4. Section 5.1 contains an elaborate description of our findings in the reference scenarios. In the following sections we will predominantly highlight the differences with the reference scenarios. We start with track related scenarios (see Section 5.2), followed by load factor scenarios (in Section 5.3), braking characteristics (in Section 5.4), speed restrictions in Section 5.5, running time supplements in Section 5.6, and finally platform clearing scenarios in Section 5.7. The values per evaluation criterion based on the KPIs for each of the different test scenarios are presented in Table 5 (for SPR), 6 and 9 (for IC), and 7 (for FT). The columns in Tables 5–7 indicate the different KPIs and the evaluation criteria to measure the KPIs. Under 'Timeliness' we find the deviation in seconds from the planned arrival time. For MTTC this is usually a negative value, since the train arrives ahead of time. The columns under 'Energy consumption' display the total energy consumption per driving strategy, and in addition the percentage extra energy saved by a particular driving strategy compared to MTTC. The 'Environment' column displays the average noise made during the train trip, and under 'Environment-Maintenance' the percentage of wear (KPI Maintenance) generated by a driving strategy, compared to MTTC in the reference scenario (which is set at 100%) is displayed. This is equal to the amount of brake grindings produced, which is another evaluation criterion for the KPI Environment. The rows contain the driving strategies per scenario as defined in Table 4.

The results of the workload analysis are presented separately in Table 8 for the scenarios 0 ('Reference'), 1.2 ('Gradient'), 4.1 ('Speed restriction'), 6.1 ('Platform clearing ETCS L2'), and 6.2 ('Platform clearing NS '54/ATB'). Finally, Table 9 is focused on the platform clearance scenarios, in which the test train is hindered by a previous train at the arrival platform (scenarios 6.1 and 6.2). This table has basically the same structure as Tables 5–7, but it also includes the KPI safety for NS '54/ATB, and the KPI brand image (see Section 5.7).

5.1. Reference scenarios

On short distances (SPR) the energy consumption of EETC and MC is identical (see Fig. 5 and Table 5). These driving strategies lead to the highest energy savings i.e. 40.4% compared to the non-eco-driving strategy MTTC. On this distance EETC will not reach the line speed, nor will it apply a cruising regime (CR), but it will switch from maximum acceleration (MA) immediately to coasting (CO). Note that the small oscillations in cruising speeds observed in the speed profiles are due to numerical approximations of the real cruise speed.

In comparison RMS leads to 36.8% more energy savings than MTTC, which is less than EETC and MC. On longer distances (IC) we see that EETC attains an optimal cruising speed of 136 km/h. This amounts to 24.8% more energy savings than MTTC (see Fig. 5 and Table 6). There is a mere 0.01% difference in energy savings between EETC and MC, which makes MC nearly as energy-efficient. RMS has the highest energy consumption of the eco-driving strategies (20.7% less than MTTC). Similar to IC we find that for FT (see Fig. 5 and Table 7) both EETC and MC lead to the highest energy savings with 12.7% (for EETC) and 11.7% (for MC) more than MTTC. RMS has with energy savings of 9.3% more than MTTC again the highest energy consumption of the eco-driving strategies.

The results of the workload analysis of the reference scenario (see Table 8) show that on short distances (SPR) MC is the least demanding (TIS = 14.0) and that the workload of MTTC is only slightly higher (TIS = 15.0). On longer distances (50 km) the driver will experience the lowest workload when applying MTTC (TIS = 15.0). EETC causes the highest workload on both short and long distances (TIS = 22.2), while MC (long distance variant) and RMS score equally high (TIS = 18.0). The workload of the latter two is considerably higher than that of MTTC, but at the same time considerably lower than the workload of EETC.

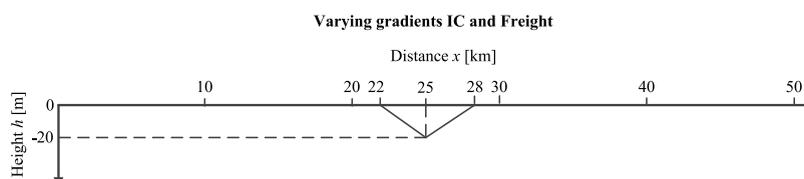


Fig. 4. Track gradient IC and FT.

Table 4

Test scenarios per rolling stock type (SPR = Sprinter, IC = Intercity, FT = Freight train, SR = Speed Restriction, MeB = Mechanical Braking, RB = Regenerative Braking, NA = Not Applicable, Run. time suppl. = Running time supplement, L2 = ETCS L2, ATB = NS '54/ATB, Plat. clear. = platform is clear on arrival of test train at the signal preceding the entry signal).

Scenarios train type SPR	Track length	Gradient	Load factor	Braking (MeB/RB)	SR	Run. time suppl.	Plat. clear.	ATP
0: Reference	5 km	0 m	35%	RB	No	10%	Yes	L2
2.1: Load factor (zero load)	5 km	0 m	0%	RB	No	10%	Yes	L2
2.2: Load factor (fully loaded)	5 km	0 m	100%	RB	No	10%	Yes	L2
3.1: Braking (MeB)	5 km	0 m	35%	MeB	No	10%	Yes	L2
4.2: SR (100 km/h)	5 km	0 m	35%	RB	100 km/h	10%	Yes	L2
5.1: Run. time suppl. (5%)	5 km	0 m	35%	RB	No	5%	Yes	L2
5.2: Run. time suppl. (15%)	5 km	0 m	35%	RB	No	15%	Yes	L2
Scenarios train type IC								
0: Reference	50 km	0 m	35%	RB	No	10%	Yes	L2
1.1: Track length (150 km)	150 km	0 m	35%	RB	No	10%	Yes	L2
1.2: Gradient (20 m)	50 km	20 m	35%	RB	No	10%	Yes	L2
2.1: Load factor (zero load)	50 km	0 m	0%	RB	No	10%	Yes	L2
2.2: Load factor (fully loaded)	50 km	0 m	100%	RB	No	10%	Yes	L2
3.1: Braking (MeB)	50 km	0 m	35%	MeB	No	10%	Yes	L2
4.2: SR (100 km/h)	50 km	0 m	35%	RB	100 km/h	10%	Yes	L2
5.1: Run. time suppl. (5%)	50 km	0 m	35%	RB	No	5%	Yes	L2
5.2: Run. time suppl. (15%)	50 km	0 m	35%	RB	No	15%	Yes	L2
6.1: Platform clear (L2)	50 km	0 m	35%	RB	No	10%	No	L2
6.2: Platform clear (ATB)	50 km	0 m	35%	RB	No	10%	No	ATB
Scenarios train type FT								
0: Reference	50 km	0 m	100%	MeB	No	4.3%	Yes	L2
1.1: Track length (150 km)	150 km	0 m	100%	MeB	No	2.5%	Yes	L2
1.2: Gradient (20 m)	50 km	20 m	100%	MeB	No	4.3%	Yes	L2
2.1: Load factor (zero load)	50 km	0 m	0%	MeB	No	4.3%	Yes	L2
4.1: SR (80 km/h)	50 km	0 m	100%	MeB	80 km/h	4.3%	Yes	L2
5.1: Run. time suppl. (7.5%)	50 km	0 m	100%	MeB	No	7.5%	Yes	L2
5.2: Run. time suppl. (10%)	50 km	0 m	100%	MeB	No	10%	Yes	L2

The results of the pass-by noise measurements show that on short runs (SPR) the average pass-by noise produced during a run of MTTC is about 0.9 dB higher than that of the eco-driving strategies (see Table 5). On long runs for IC (see Table 6) the difference is less (between 0.63 and 0.7 dB higher) and for FT runs (see Table 7) it is even smaller (between 0.3 and 0.39 dB). Since the SPR brakes 100% regenerative there is no wear on short distances. For the IC we see that (see Table 6, under Environment-Maintenance) MC produces the least percentage of wear (26%), EETC results in more wear (35%), and of the three eco-driving strategies RMS has the highest percentage of wear (66%). For FT (see Table 7, under Environment-Maintenance) the driving strategies relate to each other in the same way as for IC (MC 25%, EETC 35%), although the scores of RMS on wear are almost as high as MTTC (i.e. 90%).

Note that in general the computation times of our model are quite long as can be seen in Table 5, Table 6, and Table 7. In the literature, initialization methods exist and allow a strong decrease of the computation time, see e.g. Lejeune et al. (2016). However, we are not considering a real-time Driver Advisory System, but we do an offline comparison between different driving strategies. Therefore, we accept longer computation times and we did not include initialization methods to decrease the computation speeds. We just provided the computation times to indicate the performance of our model.

5.2. Track related scenarios

On long tracks (150 km) we see that (in contrast to the results of the reference scenario) RMS saves more energy than MC (see Fig. 6 and Table 6). For IC RMS attains 16.7% more energy savings than MTTC, while MC generates extra energy savings of 13.8% compared to MTTC. EETC still saves the most energy with additional savings of 18.6%. For FT (see Fig. 6 and Table 7) we see a comparable result on energy consumption (extra energy savings compared to MTTC amount to 5.7% for EETC, 5.0% for RMS, and 4.6% for MC). We used a running time supplement of 2.5% for FT, because a larger running time supplement could not be accommodated for by MC.

We see that a gradient affects all driving strategies (see Fig. 7). For IC MTTC applies a braking regime when driving downhill to prevent the train from exceeding the line speed. The energy profile shows that energy is regenerated during this regime. When driving downhill MC applies an extra coasting, and a short braking regime. Similar to MC, EETC applies a coasting regime when driving downhill. In line with Howlett (2016) EETC applies the same cruising speed (130.2 km/h) both before and after the gradient. In Fig. 7 it seems as if RMS also applies a coasting regime on the downhill slope, however the data shows that at this point RMS applies an extremely low amount of traction, since (36) restricts RMS from applying a coasting regime. The ranking of the driving strategies, with respect to energy consumption, is the same as in the reference scenario.

For FT we see that the uphill slope is so steep that none of the driving strategies can maintain speed. In this case the optimal cruising speed of EETC before the gradient (95 km/h) differs from the cruising speed after the gradient (90.3 km/h). EETC applies a coasting regime on the uphill slope. This results in a speed profile similar to that of MTTC. The effect of the gradient is the most distinct in the speed curve of MC. As a result of the relatively large percentage of running time supplement (4.3%) MC uses the first coasting phase (on

Table 5

Results SPR for each test scenario per evaluation criterion (SPR = Sprinter, PMx = particular matter (brake grindings), SR = Speed Restriction, MeB = Mechanical Braking, RB = Regenerative Braking, P1 = percentage of savings compared to MTTC, P2 = pass-by noise compared to MTTC.

Results SPR	Timeliness	Energy consumption		Environment	Environment-maintenance	Computation time (s)
	Deviation arrival time (s)	(kWh)	P1 (%)	P2 (dB)	PMx = Wear (%)	
Scenario 0. Reference (35% load)						
MTTC-RB	-20.24	40.94	0.0%	0.00	0%	74.40
EETC-RB	0	24.40	40.4%	0.90	0%	10.61
MC-RB	0	24.40	40.4%	0.90	0%	10.61
RMS-RB	0	25.88	36.8%	0.89	0%	13.98
Scenario 2.1. Load factor (zero load)						
MTTC-RB	-21.83	40.18	0.0%	0.00	0%	111.72
EETC-RB	0	23.46	41.6%	0.92	0%	9.74
MC-RB	0	23.46	41.6%	0.92	0%	9.74
RMS-RB	0	24.96	37.9%	0.91	0%	12.42
Scenario 2.2. Load factor (fully loaded)						
MTTC-RB	-17.28	42.64	0.0%	0.00	0%	60.18
EETC-RB	0	26.26	38.4%	0.87	0%	10.44
MC-RB	0	26.26	38.4%	0.87	0%	10.44
RMS-RB	0	27.83	34.7%	0.86	0%	12.19
Scenario 3.1. Braking (MeB)						
MTTC-MeB	-20.24	64.52	0.0%	0.00	100%	74.83
EETC-MeB	0	35.89	44.4%	0.90	49%	10.66
MC-MeB	0	35.89	44.4%	0.90	49%	10.66
RMS-MeB	0	39.36	39.0%	0.89	57%	13.02
Scenario 4.2. SR						
MTTC-RB	-23.35	40.15	0.0%	0.00	0%	104.41
EETC-RB	0	17.17	57.2%	0.83	0%	12.44
MC-RB	0	17.17	57.2%	0.83	0%	12.44
RMS-RB	0	18.19	54.7%	0.82	0%	18.76
Scenario 5.1. Running time supplement (5%)						
MTTC-RB	-10.12	40.94	0.0%	0.00	0%	74.40
EETC-RB	0	29.22	28.6%	0.48	0%	10.88
MC-RB	0	29.22	28.6%	0.48	0%	10.88
RMS-RB	0	30.95	24.4%	0.47	0%	14.86
Scenario 5.2. Running time supplement (15%)						
MTTC-RB	-30.36	40.94	0.0%	0.00	0%	74.40
EETC-RB	0	21.17	48.3%	1.28	0%	11.29
MC-RB	0	21.17	48.3%	1.28	0%	11.29
RMS-RB	0	22.54	44.9%	1.28	0%	14.23

the downhill slope) optimally by coasting maximally to a speed of 50 km/h. This ensures that the (final) coasting curve is almost equal to that of EETC (from 90.3 km/h onwards). RMS maintains the same optimal cruising speed of 90 km/h, both before and after the gradient. On energy savings the driving strategies score similar to the IC runs.

The workload of all the driving strategies increases considerably compared to the reference scenario on a track with a gradient (see Table 8). For the driving strategies MTTC (TIS = 24.0), RMS (TIS = 26.0), and MC long distance variant (TIS = 27.0) the TIS is eight or nine points higher than in the reference scenario. EETC, which already had the highest TIS score, scores on this scenario (TIS = 36.4), i. e. 14.2 points higher than in the reference scenario.

In this scenario, wear of MC is considerably higher than in the reference scenario (see Table 6 for IC, and Table 7 for FT), making it similar to that of EETC. For IC we see that there is a small increase in wear of RMS compared to the reference scenario (from 66% to 68%). However, for FT runs wear of RMS, which was already high in the reference scenario, increases to 99%, making it almost equally high as wear of MTTC.

5.3. Load factor

From the load factor scenarios (see Figs. 8 and 9) we learn that the higher the load, the longer the (maximum) acceleration regime. This results in a higher cruising speed for RMS and EETC, and a longer coasting regime for MC and EETC. For MTTC a longer acceleration regime results in higher energy consumption (see e.g. Table 6, with a zero load MTTC consumes 326.3 kWh, a 35% load (reference scenario) results in an energy use of 330.9 kWh, and a fully loaded IC consumes 338.7 kWh).

The lighter the train, the more running time is available for eco-driving. For all eco-driving strategies we see that the lighter the train, the more extra savings compared to MTTC can be obtained (e.g. see Table 5 driving strategy RMS, train type SPR: a zero load results in 37.9%, a load of 35% in 36.8%, and a fully loaded train in 34.7% extra savings compared to MTTC). Note that for MC (IC), a train with zero load will not be hindered by the speed restriction of 80 km/h at the end of the track, and can maintain the coasting regime longer, see Fig. 8. The coasting capacity with a zero load is low, so the train will already have reached a speed lower than 80 km/h at that point. As a result, the extra energy savings (compared to MTTC) for MC with a zero load is relatively high compared to

Table 6

Results IC for each test scenario per evaluation criterion (IC = Intercity, PMx = particular matter (brake grindings), SR = Speed Restriction, MeB = Mechanical Braking, RB = Regenerative Braking, P1 = percentage of savings compared to MTTC, P2 = pass-by noise compared to MTTC.

Results IC	Timeliness	Energy consumption		Environment	Environment-maintenance	Computation time (s)
	Deviation arrival time (s)	(kWh)	P1 (%)	P2 (dB)	PMx = Wear (%)	
Scenario 0. Reference (35% load)						
MTTC-RB	-141.35	330.9	0.0%	0.00	100%	140.55
EETC-RB	0.00	248.9	24.8%	0.70	35%	34.90
MC-RB	0.00	249.0	24.8%	0.63	26%	36.21
RMS-RB	0.00	262.5	20.7%	0.72	66%	22.59
Scenario 1.1. Track length (150 km)						
MTTC-RB	-398.54	863.3	0.0%	0.00	101%	18.10
EETC-RB	0.00	702.7	18.6%	0.68	35%	32.10
MC-RB	0.00	743.9	13.8%	0.52	15%	179.66
RMS-RB	0.00	719.0	16.7%	0.68	68%	16.94
Scenario 1.2. Gradient (20 m)						
MTTC-RB	-141.35	339.0	0.0%	0.00	108%	32.45
EETC-RB	0.00	249.2	26.5%	0.69	35%	67.03
MC-RB	0.00	253.1	25.3%	0.63	34%	40.90
RMS-RB	0.00	263.0	22.4%	0.72	68%	67.70
Scenario 2.1. Load factor (zero load)						
MTTC-RB	-142.81	326.3	0.0%	0.00	96%	23.39
EETC-RB	0.00	245.7	24.7%	0.70	33%	38.54
MC-RB	0.00	247.5	24.2%	0.63	24%	51.88
RMS-RB	0.00	258.8	20.7%	0.72	63%	69.26
Scenario 2.2. Load factor (fully loaded)						
MTTC-RB	-138.63	338.7	0.0%	0.00	106%	22.67
EETC-RB	0.00	251.6	25.7%	0.68	38%	30.01
MC-RB	0.00	253.7	25.1%	0.62	30%	54.39
RMS-RB	0.00	269.5	20.4%	0.70	71%	60.29
Scenario 3.1. Braking (MeB)						
MTTC-MeB	-141.35	373.5	0.0%	0.00	181%	149.17
EETC-MeB	0.00	259.9	30.4%	0.65	51%	29.43
MC-MeB	0.00	260.0	30.4%	0.63	47%	82.14
RMS-MeB	0.00	290.7	22.2%	0.72	119%	27.98
Scenario 4.2. SR						
MTTC-RB	-144.76	361.1	0.0%	0.00	100%	26.38
EETC-RB	0.00	247.8	31.4%	0.71	39%	117.11
MC-RB	0.00	248.4	31.2%	0.68	34%	36.02
RMS-RB	0.00	269.3	25.4%	0.73	84%	27.42
Scenario 5.1. Running time supplement (5%)						
MTTC-RB	-70.68	330.9	0.0%	0.00	100%	140.55
EETC-RB	0.00	270.2	18.3%	0.35	40%	28.21
MC-RB	0.00	270.7	18.2%	0.33	40%	49.54
RMS-RB	0.00	286.9	13.3%	0.37	73%	24.95
Scenario 5.2. Running time supplement (15%)						
MTTC-RB	-212.03	330.9	0.0%	0.00	66%	140.55
EETC-RB	0.00	227.9	31.1%	1.03	31%	54.84
MC-RB	0.00	241.2	27.1%	0.87	24%	129.32
RMS-RB	0.00	241.0	27.2%	1.05	58%	25.42

scenarios with a higher load (see Table 6 under Energy consumption: 24.2% for zero load, 24.8% for a load of 35%, and 25.1% for a fully loaded train).

5.4. Braking characteristics

We find that when mechanical braking is applied (see Fig. 10), the difference in energy savings between MTTC and the eco-driving strategies becomes larger. For SPR, under the driving strategy EETC or MC, the extra savings compared to MTTC amount to 44.4%, as opposed to 40.4% when regenerative braking is applied. For RMS this results in extra savings compared to MTTC of 39% with mechanical braking, and 36.8% with regenerative braking. For IC we see similar results. For IC we see a steep rise (by a factor 1.45 for EETC, and 1.8 for the other driving strategies) in the percentage of wear compared to the reference scenario (see Table 6 under PMx = Wear (%)).

5.5. Speed restrictions

In the results of the speed restriction scenario (see Fig. 11), we see that the difference in energy savings between MTTC and the eco-driving strategies is larger than in the reference scenario. We see e.g. that for SPR EETC and MC save 57.2% more energy compared to

Table 7

Results FT for each test scenario per evaluation criterion (FT = Freight Train, SR = Speed Restriction, MeB = Mechanical Braking, P1 = percentage of savings compared to MTTC, P2 = pass-by noise compared to MTTC).

Results FT	Timeliness	Energy consumption		Environment	Environment-Maintenance	Computation time (s)
	Deviation arrival time (s)	(kWh)	P1 (%)	P2 (dB)	PMx = Wear (%)	
Scenario 0. Reference (running time supplement 4.3% and fully loaded)						
MTTC-MeB	-89.94	3127	0.0%	0.00	100%	29.97
EETC-MeB	0	2730	12.7%	0.36	35%	36.60
MC-MeB	0	2760	11.7%	0.30	25%	33.49
RMS-MeB	0	2837	9.3%	0.39	90%	29.82
Scenario 1.1. Track length (150 km)						
MTTC-MeB	-147.36	8939	0.0%	0.00	100%	14.20
EETC-MeB	0	8428	5.7%	0.33	37%	25.19
MC-MeB	0	8531	4.6%	0.32	18%	28.39
RMS-MeB	0	8491	5.0%	0.36	92%	29.00
Scenario 1.2. Gradient (20 m)						
MTTC-MeB	-84.40	3211	0.0%	0.00	104%	349.23
EETC-MeB	0	2824	12.1%	0.33	33%	122.10
MC-MeB	0	2858	11.0%	0.31	33%	34.35
RMS-MeB	0	2929	8.8%	0.36	99%	418.55
Scenario 2.1. Load factor (zero load)						
MTTC-MeB	-217.25	997.74	0.0%	0.00	35%	24.99
EETC-MeB	0	773.91	22.4%	0.89	10%	36.91
MC-MeB	0	830.44	16.8%	0.64	3%	87.88
RMS-MeB	0	799.25	19.9%	0.92	25%	25.94
Scenario 4.2. SR (80 km/h)						
MTTC-MeB	-81.17	3191	0.0%	0.00	122%	296.52
EETC-MeB	0	2737	14.2%	0.33	35%	114.42
MC-MeB	0	2755	13.7%	0.31	35%	73.42
RMS-MeB	0	2857	10.5%	0.36	99%	35.81
Scenario 5.1. Slack time (7.5%)						
MTTC-MeB	-157.90	3127	0.0%	0.00	100%	29.97
EETC-MeB	0	2560	18.1%	0.64	32%	65.69
MC-MeB	0	2679	14.3%	0.50	15%	39.55
RMS-MeB	0	2636	15.7%	0.67	81%	44.39
Scenario 5.2. Slack time (10%)						
MTTC-MeB	-210.53	3127	0.0%	0.00	100%	29.97
EETC-MeB	0	2439	22.0%	0.85	30%	33.00
MC-MeB	0	2640	15.6%	0.64	10%	93.08
RMS-MeB	0	2516	19.5%	0.89	77%	34.32

Table 8

Results workload (short = short distance variant (SPR); long = long distance variant (IC/FT) (SPR = Sprinter, IC = Intercity, FT = Freight Train). Note that the determination of the total interference scores per driving strategy for the reference scenario is described in detail in [Table 2](#).

Scenarios	Total interference score per driving strategy				
	MTTC	RMS	MC short	MC long	EETC
0. Reference - golden run	15.0	18.0	14.0	18.0	22.2
1.2. Gradient	24.0	26.0	-	27.0	36.4
4.1. Speed restriction	28.0	31.0	24.0	32.0	42.4
6.1. Platform clearing (ETCS L2)	31.0	26.0	-	18.0	22.2
6.2. Platform clearing (NS '54/ATB)	31.0	26.0	-	22.0	26.2

MTTC (see [Table 5](#)), while in the reference scenario this amounted to 40.4%. This does not change the ranking of the eco-driving strategies with respect to the total energy consumption, i.e. in this scenario EETC still saves the most energy, followed by MC, and then by RMS.

For both FT and IC we see that in this scenario wear increases for most driving strategies. Only for the driving strategy EETC (for FT) wear remains the same as in the reference scenario, see [Table 7](#). For FT the wear of MC becomes 10 percentage points higher than in the reference scenario. Especially for IC we see a substantial increase in the wear of RMS from 66% in the reference scenario to 84% in the speed restriction scenario (see [Table 6](#)). For FT wear of RMS was already 90% in the reference scenario. In this scenario it becomes similar to MTTC (i.e. 99%), see [Table 7](#).

The workload of EETC becomes almost twice as high as in the reference scenario (see [Table 8](#)) with a TIS of 42.4. On short distances, MC has the lowest TIS of 24.0. This is considerably less than MTTC (TIS = 28.0). On longer distances the score of RMS (TIS = 31.0) and MC (TIS = 32.0) with respect to workload is similar. In this scenario, as in scenario 1.2. Gradient (See [Table 8](#)), we see that the difference in workload between on the one hand RMS and MC, and on the other hand EETC increases.

Table 9

Results IC for the platform clearing scenarios (IC = Intercity, PMx = particular matter (brake grindings), MeB = Mechanical Braking, P1 = speed at 47 km (Signal at danger), P2 = distance between the end of the first braking regime and the beginning of the second braking regime, P3 = time left between the end of the first braking regime and the beginning of the second braking regime, P4 = percentage of savings compared to MTTC, P5 = pass-by noise compared to MTTC).

Results platform clearing scenarios (IC)	Safety			Timeliness	Energy consumption		Environment	Environment-Maintenance	Brand Image
	P1 (km/h)	P2 (km)	P3 (s)	Deviation arrival time (s)	(kWh)	P4 (%)	P5 (dB)	PMx = Wear (%)	Unplanned stop
6.1. Platform Clearing (ETCS L2)									
MTTC-RB	140	–	–	–46.88	361.8	0.0%	0.00	135%	Yes
EETC-RB	98	–	–	0.00	248.9	31.2%	0.69	35%	No
MC-RB	85	–	–	0.00	249.0	31.2%	0.63	26%	No
RMS-RB	125	–	–	0.00	262.5	27.4%	0.72	66%	No
6.2. Platform Clearing (NS '54/ATB)									
MTTC-RB	140	–238	–21	–28.37	338.2	0.0%	0.00	109%	Yes
EETC-RB	98	668	60	0.00	248.9	26.4%	0.54	35%	No
MC-RB	85	879	79	17.20	249.0	26.4%	0.47	26%	No
RMS-RB	125	129	12	0.00	262.5	22.4%	0.56	66%	No

5.6. Running time supplement

The results of the test scenarios with varying running times show that in general the larger the percentage of running time supplement, the lower the optimal cruising speed, and the longer the coasting regime. In addition, the more the running time supplement, the sooner the train starts a coasting regime. On short distances (SPR) EETC and MC refrain from applying a cruising regime. As stated before, extra savings compared to MTTC runs can be attained by applying an eco-driving strategy. However, these additional energy savings are not directly proportional to the running time supplement. The Pareto curve (see Fig. 12) shows that the relative energy savings become smaller when the running time is longer i.e. for EETC and MC (SPR) we see that in comparison to MTTC, a 5% running time supplement leads to 28.6% extra energy savings, while a 10% running time supplement leads to additional savings of 40.4%, and a 15% running time supplement leads to 48.3% (see Table 5).

On longer distances (IC), EETC starts coasting at more or less the same point for every variation in running time supplement, while MC starts coasting at an earlier point as the percentage of the running time supplement increases. When there is a relatively small percentage of running time supplement (5%) additional energy savings of MC (18.2%) are comparable to those of EETC (18.3%) (see Table 6). However, when there is a relatively large percentage of running time supplement (15%) RMS saves slightly more energy than MC (see Table 6). In this situation RMS has 27.2% and MC 27.1% additional savings compared to MTTC, while EETC remains the most energy-efficient driving strategy. We see similar results for FT (see Table 7). However, energy savings of RMS become significantly higher than those of MC with a running time supplement of either 7.5% or 10%.

We find that wear diminishes as the percentage of running time supplement increases, and that (similar to energy consumption) the rate with which it diminishes is not directly proportional to the percentage of running time supplement (see Tables 6 and 7 under PMx = Wear (%)), e.g. on long distances (IC) the percentage of wear for RMS with a running time supplement of 5% is 73%, with a running time supplement of 10% it is 66%, and with a running time supplement of 15% it is 58%. Similar to the reference scenario MC produces the least percentage of wear, while of the eco-driving strategies RMS scores the highest.

5.7. Platform clearing scenarios

In this section we will look at the effects of driving strategies in a more dense network in which the platform clearing times of a previous train at the platform can influence the arrival of the test train (see Fig. 3). The energy consumption of the eco-driving strategies is the same as in the reference scenario. Only MTTC consumes more energy. Therefore, the additional savings of the eco-driving strategies compared to MTTC increase, with approximately 6.5 under ETCS L2 (scenario 6.1), and 1.5 percentage points under NS '54/ATB (scenario 6.2), see Table 9.

Under ETCS L2 we see that the workload of an EETC or MC driver is the same as in the reference scenario (see Table 8). MTTC has in both platform clearing scenarios the highest workload, which includes making an unplanned stop at the entry signal, and informing passengers (the latter task is mandatory within NS proceedings). The workload of RMS also increases significantly compared to the reference scenario (from a TIS of 18 to a TIS of 26), making RMS the eco-driving strategy with the highest workload in scenario ETCS L2, and a workload similar to EETC under NS '54/ATB. Although, the workload of MC increases in the NS '54/ATB scenario, in relative terms the difference with EETC remains the same, making MC the driving strategy with the lowest workload in this scenario.

Note that only for MTTC an additional braking and acceleration regime is necessary. This can be seen in the results for wear. For the eco-driving strategies this evaluation criterion is the same as in the reference scenario. For MTTC we see an increase in wear of 35 percentage points under the ETCS L2 scenario, and of 9 percentage points under the NS '54/ATB scenario (see Table 9).

Safety is measured in scenario 6.2 (see Table 9). The results of this scenario show that for MTTC the distance, and time remaining between the first and final braking regime is negative, i.e. –238 m and –21 s, respectively (see Table 9). Negative values mean that the

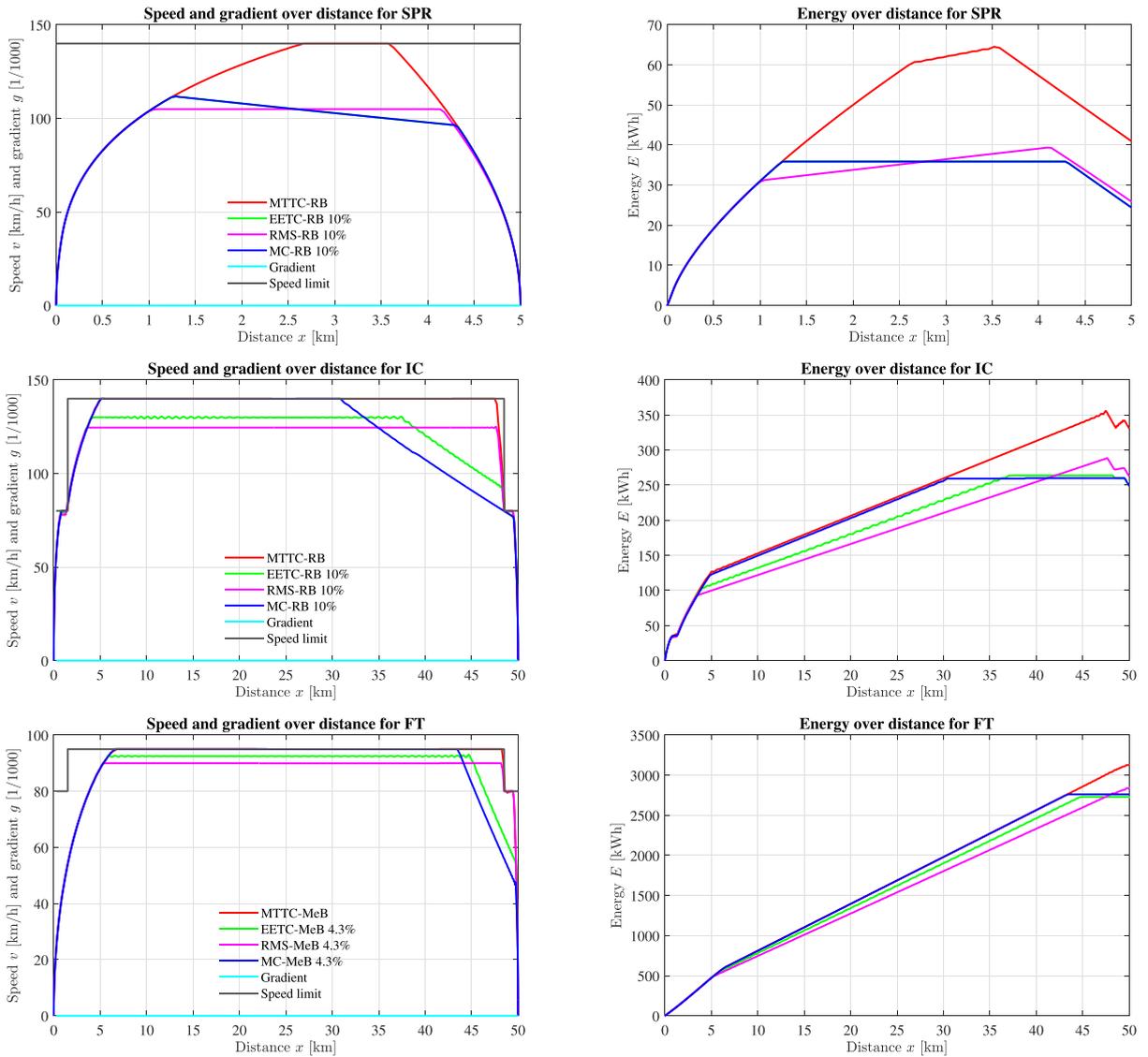


Fig. 5. Results reference scenario. Note that for SPR the driving strategy EETC is identical to MC and therefore not visible in this figure.

driver (in order to avoid a SPAD) will have to brake harder than the minimal required braking deceleration of 0.43 m/s^2 , making the risk of a SPAD for this driving strategy high. For RMS, there is a distance of 129 m left to be covered within 12 s. This is a large enough margin for the driver to stop the train in front of the signal at danger. For MC and EETC a distance of more than 600 m remains, to be covered within 60–80 s. The margin here is large.

In both these scenarios MTTC experiences hinder from the previous train, that is still at the platform as the MTTC train passes the signal at warning. As a consequence, the train arrives less early than in the reference scenario (-46.88 s under ETCS L2, and -28.37 s under NS '54/ATB, see Table 9), but still not timely. Under ETCS L2 all eco-driving strategies manage to arrive timely, but under NS '54/ATB only RMS and EETC are timely. Although, the MC train arrives at the signal at warning a number of seconds earlier than under RMS, it passes this signal with a significantly lower speed (85 km/h). The MC train is then obliged by the ATP to maintain a speed of 40 km/h over a longer period of time than under RMS. This causes the MC train to arrive 17.2 s later than the planned arrival time. Since MTTC causes an unplanned stop in both these scenarios this driving strategy will have a negative impact on the brand image for passengers.

6. Discussion

In Section 5 we have seen that driving strategies affect the train operator's KPIs in various ways, and that the circumstances of a TOC or FOC determine which driving strategy will be most beneficial for its operation. There is no one size fits all solution, when it

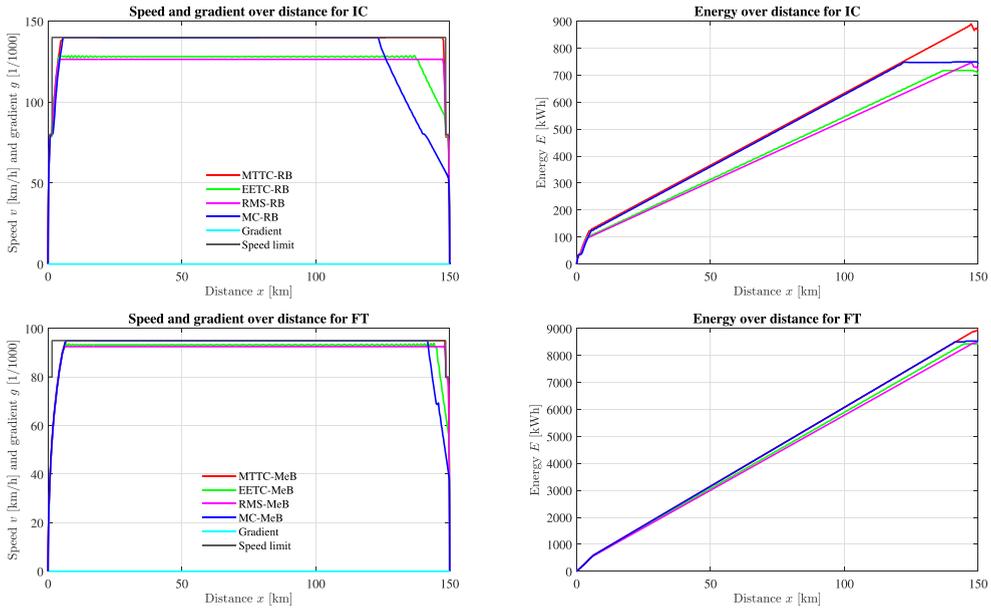


Fig. 6. Results for scenario with a track length of 150 km for IC and FT.

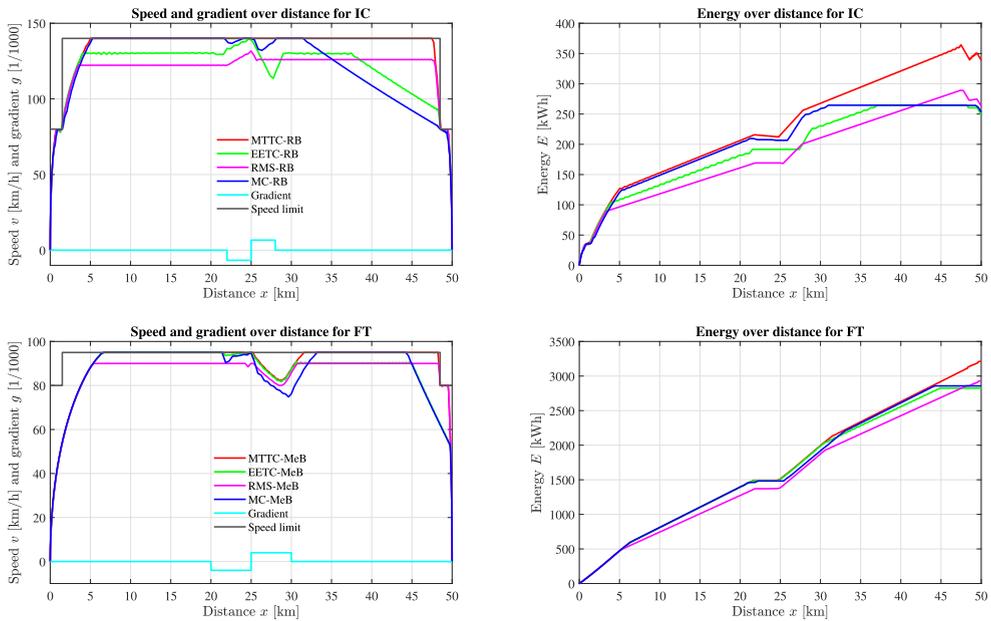


Fig. 7. Results for varying gradient scenario for IC and FT.

comes to selecting an appropriate driving strategy for a TOC or FOC. We see for instance, that it depends on the level of automation, whether or not the minimal energy solution, EETC is an appropriate choice.

In this section we discuss the findings in Section 5 in the light of previous studies, conventional wisdom, and the known literature on the subject of train driving strategies. Based on our results in Section 5 we first draw some general conclusions as to which driving strategy is, under which circumstance the most beneficial for the KPIs in our toolbox (see Section 6.1). We then conclude this section in Section 6.2 by discussing the implications of our main results on a number of issues concerning train operation. Some of these issues could be studied further on basis of our results, and could subsequently lead to additional insights into the effects of driving strategies on KPIs relevant to train operation.

Note that, although in this study all seven KPIs in the toolbox are important, the KPI energy consumption is often mentioned first, and often receives more attention than the other KPIs, because most of our driving strategies are eco-driving strategies, and the focus of

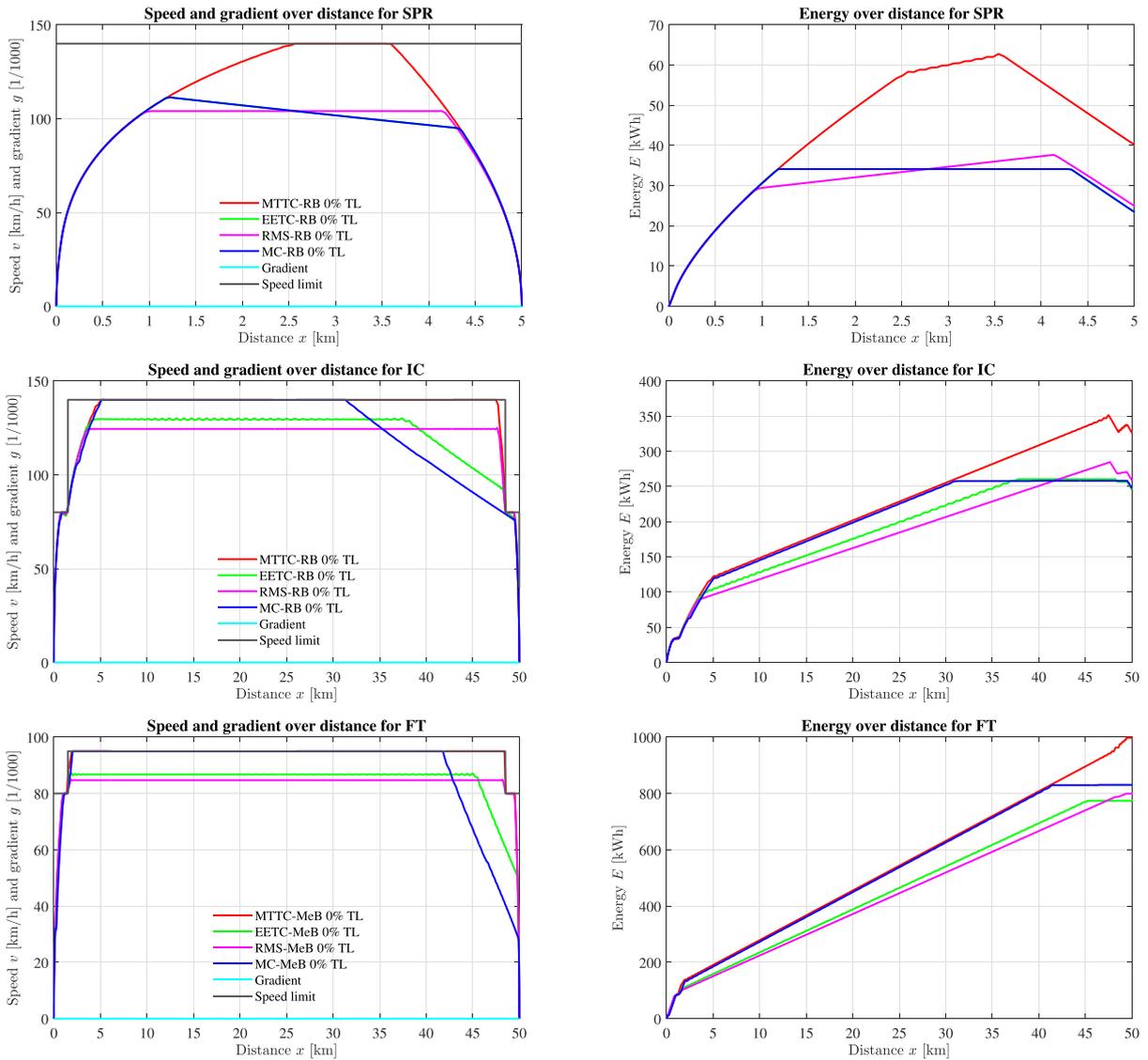


Fig. 8. Results zero load scenario.

research up until now has predominantly been on this KPI.

6.1. Choosing an appropriate driving strategy

For fully automated trains the importance of the KPIs energy consumption, environment and cost of maintenance (wear) for a TOC or FOC determine the choice for a driving strategy. The KPI workload is not an issue in this situation and therefore EETC could be an appropriate choice. However for semi-automatic train operation, it is important to consider a fall back scenario (in case there is a manual take-over by the driver) in which either RMS or MC is applied, so that there is a reasonable workload for the driver.

On short (metro-like) distances MC (like EETC), has the lowest energy consumption. This is counter to the findings in previous research (see Li and Lo (2014)), where for highly regenerating rolling stock, RMS is presented as favorable to MC with respect to energy consumption. For TOCs with many metro-like distances, a high-density network, low running time margins (less than 15%), and manual train control, MC seems to be the most appropriate driving strategy. Under these circumstances, MC has in addition to the lowest energy consumption also the lowest workload. Moreover, it scores best on environmental pollution, and cost of maintenance.

Only when there is a relatively large percentage of running time supplement (15% or more), or a relatively long track, RMS manages to save more energy than MC. A TOC or FOC which only operates on long hauls (more than 50 km) without running the risk of crossing busy tracks, or hinder from other trains, and with predominantly manual operation would be best served with RMS as its preferred driving strategy. However, consideration should be given to the KPI cost of maintenance because RMS causes a relatively

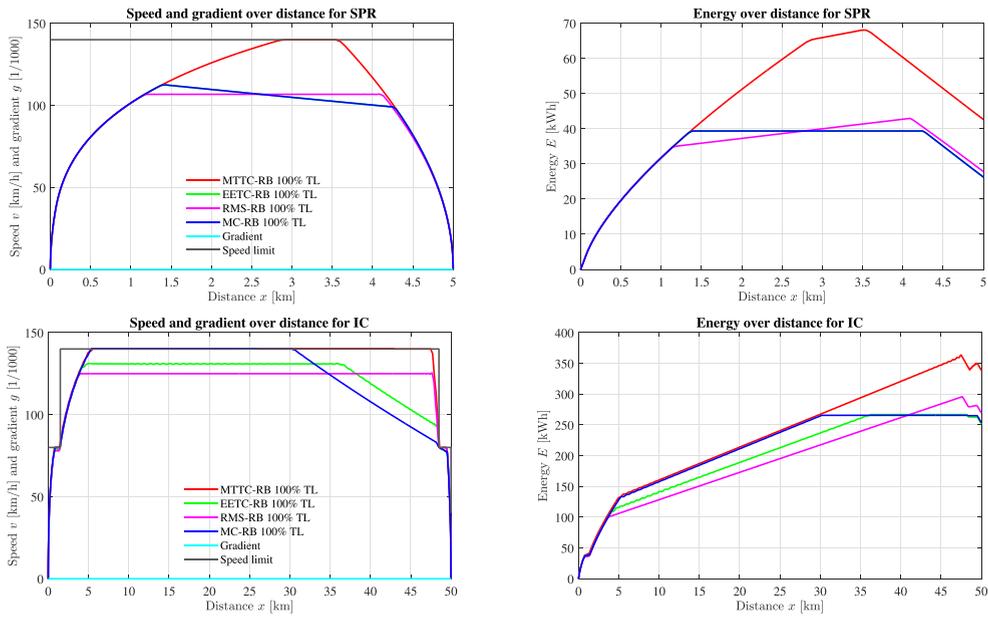


Fig. 9. Results fully loaded scenario for SPR and IC.

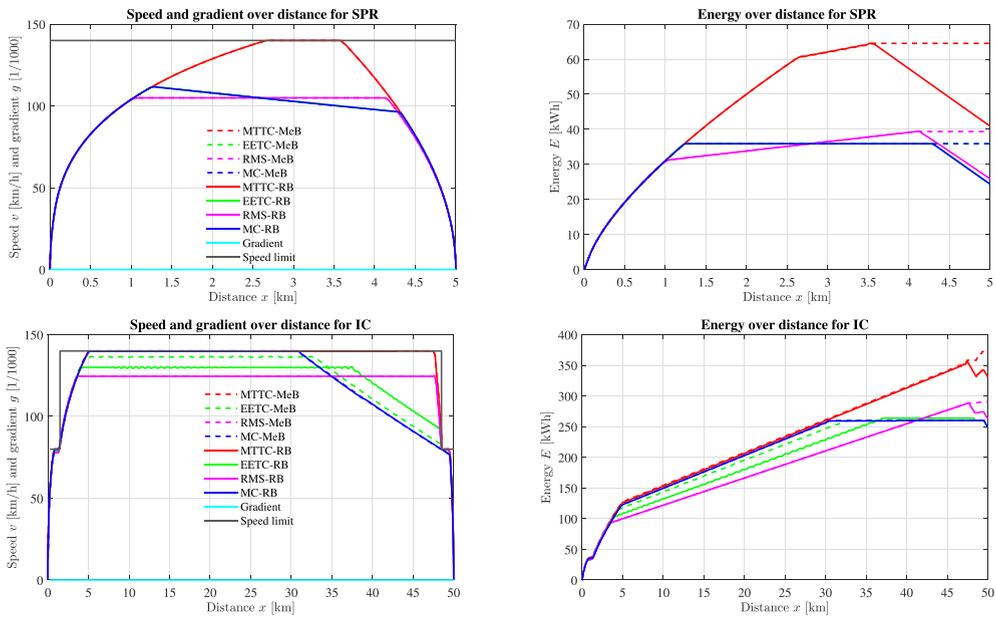


Fig. 10. Results mechanical versus regenerative braking for SPR and IC.

large amount of wear, and also to the KPI environment because of the relative large amount of brake grindings generated by this driving strategy.

6.2. Train operating issues

It was suggested by Van Dongen and Schuit (1989) that the reduction in catenary losses by reducing acceleration would change the optimum in the driving strategy. However, our findings show that increased traction at the beginning of a train run does not necessarily lead to higher energy consumption. On the contrary, although MC uses the maximum amount of energy in the acceleration regime (unlike for instance RMS), we find that the total use of energy during an MC run is usually significantly lower than that of RMS.

In some energy saving programs there was a strong focus on regenerative braking. In this study, we certainly see that regenerative

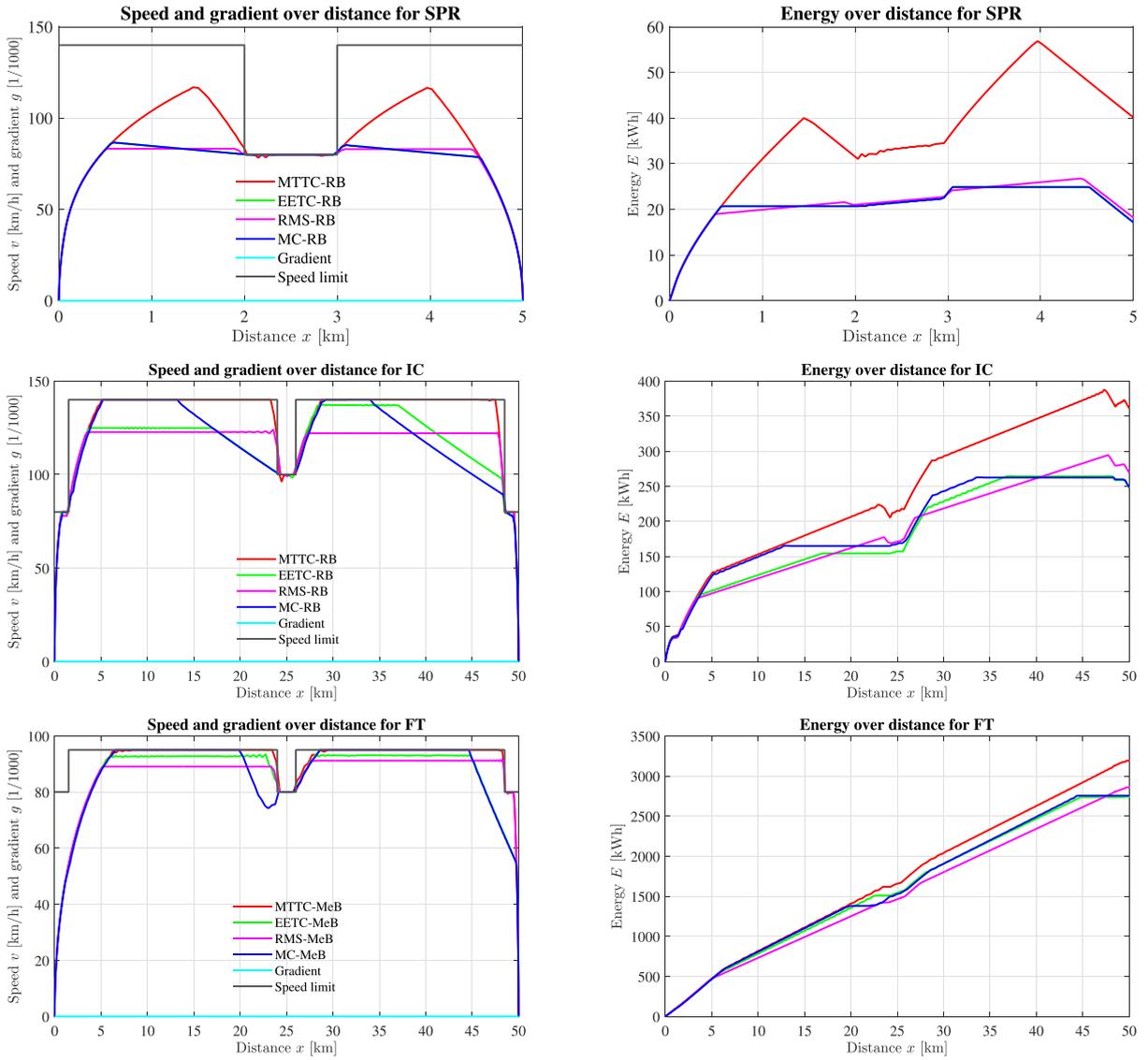


Fig. 11. Results speed restriction scenario.

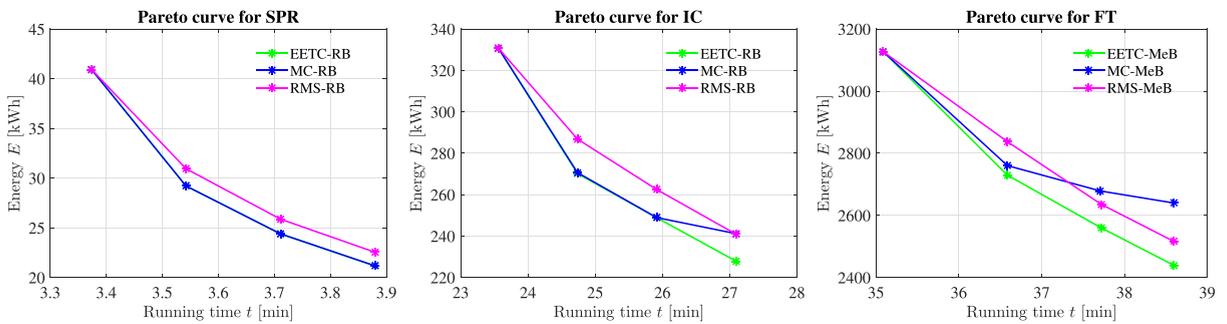


Fig. 12. Pareto curves varying running time (left SPR, middle IC, and right FT).

braking makes additional energy savings possible. However, a strict focus on regenerative braking as the only energy saving measurement is a less optimal approach, because according to our findings, an eco-driving strategy with mechanical braking saves more energy than MTTC, even if MTTC uses regenerative braking. Therefore, energy saving programs should first and foremost focus on choosing, and implementing an appropriate eco-driving strategy (see Luijt et al. (2017)), and only then add regenerative braking as an extra measurement to save more energy.

Although there are eco-driving (training) programs without electronic devices that have brought about substantial energy savings (see e.g. Luijt et al. (2017)), the implementation of a DAS could offer TOCs and FOCs the opportunity to operate more precisely, as calculations can be performed more accurately. A DAS can alleviate tasks of the driver (KPI workload), by calculating the start of the coasting regime and/or the cruising speed. This could require less computation effort of drivers. However, the KPI workload of the driver could be influenced negatively if a DAS gives several updates during a run. This can cause distraction and can increase the workload, depending on the specific implementation. An additional study into the effect of human aspects of each specific DAS implementation should be considered.

Pass-by noise could at certain locations along a track be a critical factor, that limits capacity (i.e. number of trains allowed to pass this location). Our study does not offer a direct solution to this problem (since we compared the average pass-by noise per driving strategy over the full track length), but it can help in a comprehensive weighing of the different KPIs. An analysis on pass-by noise on a specific location might lead to a trade-off between optimal energy savings, and noise reduction, leading possibly to an optimized speed profile with for instance an additional coasting regime at this location.

An interesting addition to our study would be a comparison of driving strategies on a higher line speed (e.g. 160 km/h). In previous field tests in the Netherlands (see Weltevreden (2015)), we saw that a higher line speed with the same timetable led to an increase in energy consumption, because drivers would arrive ahead of time. It would be interesting to study this scenario for our eco-driving strategies, and find out whether the possibility of longer traction in the acceleration regime, and as a consequence a longer coasting regime, could lead to reduced energy consumption under these circumstances.

Under a conventional ATP, all eco-driving strategies are safer compared to MTTC (i.e. have a low risk of a SPAD). This means that under eco-driving a conventional ATP system offers better protection than under MTTC. The fact that ATP protection under eco-driving is better, does not necessarily reduce the number of SPADs, because drivers who apply eco-driving in the vast majority of their train trips might not be sufficiently aware of this risk, when applying MTTC to make up for lost time. Further study on this subject and in addition on other human factors such as the risk of distraction in this case, could lead to additional insights.

For the four driving strategies studied, the seven KPIs are shown to yield different optimal solutions. None of these four driving strategies scores optimal on all. When applying these results, other driving strategies can be developed that optimize on a (weighed) mix of these KPIs. Such an optimum depends on the specific situation, as the relative importance of (sometimes location-specific impacts) varies. This study establishes the seven KPIs and evaluation methods and establishes a toolbox rather than defining an optimum for a specific situation. In future research TOCs and FOCs can optimize specific preferences with the help of this toolbox.

7. Conclusions

In this study we defined a toolbox of SMART KPIs for train operation, that are influenced by the train driving strategy. Those KPIs are safety, timeliness, energy consumption, workload of the driver, environmental pollution, cost of maintenance and brand image (of a TOC). We chose four of the most used train driving strategies to study their effect on the KPIs in the toolbox. The general conclusion is that any eco-driving strategy has advantages over MTTC on most of these KPIs. In this section we summarize our findings per KPI, and then conclude with suggestions for further research.

All eco-driving strategies score similarly high on safety (under a conventional ATP), and on timeliness. All these strategies make it possible to approach an entry signal at danger without having to brake excessively, so that (in contrast to MTTC) there is a relatively low risk of a SPAD. Furthermore, in most scenarios eco-driving strategies manage to arrive exactly on the second of the planned time, while MTTC usually arrives ahead of time.

There is a significant difference between the eco-driving strategies with respect to energy consumption. EETC has by definition the lowest energy consumption, and MTTC the highest. Energy savings of MC are equally or nearly as high as of EETC. Only on long tracks (150 km) RMS saves slightly more energy than MC. We can conclude that MC is comparable to EETC in energy consumption, while (counter to our expectations) RMS is not. So coasting is an essential regime for energy savings.

On sparsely used routes MTTC has a low workload compared to the eco-driving strategies. However, on a busy train network where MTTC runs the risk of unplanned stops it has the highest workload, while MC has the lowest workload. When trains do not experience hinder of previous trains clearing the platform, MC has the lowest workload, but only on short distances. On longer distances (50 km) MC and RMS are comparable in workload, while EETC scores significantly higher. In some scenarios the workload of EETC is twice as high as in the reference scenario. In order to ensure both timeliness and energy consumption the support of some sort of electronic device for the driver (DAS) is indispensable. From our findings it is apparent that it will be much harder (near impossible) for a driver to apply EETC than either RMS or MC without the use of a DAS.

Compared to MTTC any eco-driving strategy will diminish the average pass-by noise with more or less the same amount. MC causes the lowest wear on the braking blocks, and therewith produces the least amount of brake grindings per train trip. This makes MC the most environmental friendly strategy with the least maintenance costs. Under EETC wear is usually somewhat higher than under MC. However, RMS causes significantly more wear. In some situations it is almost equal to that of MTTC. MTTC is by definition the least environmental friendly, and has the highest costs in maintenance.

On a busy train network such as in the Netherlands, MTTC runs a higher risk of an unplanned stops near the arrival station, which

will have a negative effect on the brand image for passengers. In contrast, none of the eco-driving strategies run the risk of causing an unplanned stop.

Further research on additional characteristics and conditions will increase the insight into the effect of driving strategies on the KPIs presented in this paper. A few examples are the effects of driving strategies on slippery tracks, the maintenance condition of the bearings, improved catenary capacity for either powering or accepting (regenerated) energy (e.g. 3 kV), a higher line speed (160 km/h), a larger variation in gradients (e.g. for TOCs and FOCs with tracks on mountainous terrain), and different degrees of delays of preceding trains. Furthermore, a more profound research on the KPI cost of maintenance by studying the effect of driving strategies on wear of wheels, rails, and the catenary system could yield additional insights.

Acknowledgements

We would like to thank NS, and especially the department for Energy and Environment, for making this research possible. We would like to express our gratitude to Richard van der Weide for discussing the measurements for the KPIs workload, and comfort with us, to Vera Verstappen for helping us in determining the measurements for the KPI driver workload and to Ard Kuijpers for his advice on how to determine the evaluation criterion pass-by noise. We would also like to thank Mark van Hagen for assessing the evaluation criterion for brand image, and Michael Bastiani for assessing our approach to the KPI safety. Special thanks to NS expert train drivers Martijn de Jong and Wietze Schut who verified our hierarchical task analysis.

References

- Albrecht, A.R., Howlett, P.G., Pudney, P.J., Vu, X., Zhou, P., 2016a. The key principles of optimal train control - part 1: formulation of the model, strategies of optimal type, evolutionary lines, location of optimal switching points. *Transp. Res. Part B Methodol.* 94, 482–508. <https://doi.org/10.1016/j.trb.2015.07.023>.
- Albrecht, A.R., Howlett, P.G., Pudney, P.J., Vu, X., Zhou, P., 2016b. The key principles of optimal train control - part 2: existence of an optimal strategy, the local energy minimization principle, uniqueness, computational techniques. *Transp. Res. Part B Methodol.* 94, 509–538. <https://doi.org/10.1016/j.trb.2015.07.024>.
- Betts, J.T., 2010. *Practical Methods for Optimal Control and Estimation Using Nonlinear Programming*. Advances in Design and Control. SIAM, Philadelphia, PA, USA. <https://doi.org/10.1137/1.9780898718577>.
- BS-EN-15663, 2017. *Railway Applications - Vehicle Reference Masses*. Technical Report. BSI Group.
- Clements, M., Minson, R., Moiso, L., Moll, J., Moore, J., Scott, A., 2011. *Eco-driving: Understanding the Approaches, Benefits and Risks*. Technical Report T839. Rail Safety and Standards Board.
- Davis, W.J., 1926. The tractive resistance of electric locomotives and cars. *Gen. Electr. Rev.* 29.
- Doran, G.T., 1981. There's a S.M.A.R.T. way to write managements' goals and objectives. *Manag. Rev.* 70, 35–36.
- Era, 2016. *Railway Safety in the European Union 2016*. Biennial Report. <https://doi.org/10.2821/129870>. <https://www.era.europa.eu/Document%2dRegister/Documents/RailwaySafetyPerformance2016finalE.pdf>.
- González-Gil, A., Palacin, R., Batty, P., 2013. Sustainable urban rail systems: strategies and technologies for optimal management of regenerative braking energy. *Energy Convers. Manag.* 75, 374–388. <https://doi.org/10.1016/j.enconman.2013.06.039>.
- Goverde, R.M.P., Scheepmaker, G.M., Wang, P., 2019. Pseudospectral Optimal Train Control. *European Journal of Operational Research* (submitted for publication).
- Graffagnino, T., Schäfer, R., Tuchschnid, M., Weibel, M., 2019. Energy savings with enhanced static timetable information for train drivers. In: *Rail Norrköping Conference*.
- Horrey, W.J., Wickens, C.D., 2004. Driving and side task performance: the effects of display clutter, separation, and modality. *Hum. Factors: J. Human Factors Ergon. Soc.* 46, 611–624. <https://doi.org/10.1518/hfes.46.4.611.56805>.
- Howlett, P.G., 2000. The optimal control of a train. *Ann. Oper. Res.* 98, 65–87. <https://doi.org/10.1023/a:1019235819716>.
- Howlett, P.G., 2016. A new look at the rate of change of energy consumption with respect to journey time on an optimal train journey. *Transp. Res. Part B Methodol.* 94, 387–408. <https://doi.org/10.1016/j.trb.2016.10.004>.
- Howlett, P.G., Pudney, P.J., 1995. *Energy-efficient Train Control*. Springer, London, UK. <https://doi.org/10.1007/978-1-4471-3084-0>.
- Khmelnitsky, E., 2000. On an optimal control problem of train operation. *Autom. Contr. IEEE Trans.* 45, 1257–1266. <https://doi.org/10.1109/9.867018>.
- Lejeune, A., Chevrier, R., Vandanjon, P., Rodriguez, J., 2016. Towards eco-aware timetabling: evolutionary approach and cascading initialisation strategy for the bi-objective optimisation of train running times. *IET Intell. Transp. Syst.* 10, 483–494. <https://doi.org/10.1049/iet-its.2014.0309>.
- Lewis, F.L., Vrabe, D.L., Syrmos, V.L., 2012. *Optimal Control*. John Wiley & Sons, Hoboken, NJ, USA.
- Li, X., Lo, H.K., 2014. Energy minimization in dynamic train scheduling and control for metro rail operations. *Transp. Res. Part B Methodol.* 70, 269–284. <https://doi.org/10.1016/j.trb.2014.09.009>.
- Liu, R.R., Golovitcher, I.M., 2003. Energy-efficient operation of rail vehicles. *Transp. Res. A Policy Pract.* 37, 917–932. <https://doi.org/10.1016/j.tra.2003.07.001>.
- Lu, S., Hillmansens, S., Ho, T., Roberts, C., 2013. Single-train trajectory optimization. *IEEE Trans. Intell. Transp. Syst.* 14, 743–750.
- Luijt, R.S., van den Berge, M.P.F., Willeboordse, H.Y., Hoogenraad, J.H., 2017. 5 years of Dutch eco-driving: managing behavioural change. *Transp. Res. A Policy Pract.* 98, 46–63. <https://doi.org/10.1016/j.tra.2017.01.019>.
- Ministerie van Infrastructuur en Milieu, 2012. *Reken- en meetvoorschrift geluid (in Dutch)*. report.
- NS, 2018. *TreinPlein*. <https://treinplein.net>.
- Olsson, N.O.E., Haugland, H., 2004. Influencing factors on train punctuality - results from some Norwegian studies. *Transp. Policy* 11, 387–397. <https://doi.org/10.1016/j.tranpol.2004.07.001>.
- Panou, K., Tzieropoulos, P., Emery, D., 2013. Railway driver advice systems: evaluation of methods, tools and systems. *J. Rail Transp. Plan. Manag.* 3, 150–162. <https://doi.org/10.1016/j.jrtpm.2013.10.005>.
- Pontryagin, L.S., Boltyanskii, V.G., Gamkrelidze, R.V., Mishchenko, E.F., 1962. *The Mathematical Theory of Optimal Processes*. Wiley, Hoboken, NY, USA.
- ProRail, 2016. *Network Statement 2018*.
- Rao, A.V., Benson, D., Darby, C., Mahon, B., Francolin, C., Patterson, M.A., Sanders, I., Huntington, G.T., 2011. *User's Manual for GPOPS Version 4.X: A MATLAB Software for Solving Multiple-phase Optimal Control Problems Using Hp-Adaptive Pseudospectral Methods* (Technical Report).
- Ross, I.M., 2015. *A Primer on Pontryagin's Principle in Optimal Control*. Collegiate Publishers, San Fransisco, CA, USA.
- RSSB, 2008. *Understanding Human Factors a Guide for the Railway Industry*. Technical Report. RSSB.
- Scheepmaker, G.M., Goverde, R.M.P., 2015. The interplay between energy-efficient train control and scheduled running time supplements. *J. Rail Transp. Plan. Manag.* 5, 225–239. <https://doi.org/10.1016/j.jrtpm.2015.10.003>.
- Scheepmaker, G.M., Goverde, R.M.P., 2016. Energy-efficient train control including regenerative braking with catenary efficiency. In: *IEEE International Conference on Intelligent Rail Transportation*, pp. 116–122.
- Scheepmaker, G.M., Goverde, R.M.P., Kroon, L.G., 2017. Review of energy-efficient train control and timetabling. *Eur. J. Oper. Res.* 257, 355–376. <https://doi.org/10.1016/j.ejor.2016.09.044>.

- Scheepmaker, G.M., Pudney, P.J., Albrecht, A.R., Goverde, R.M.P., Howlett, P.G., 2019. Optimal running time supplement distribution for energy-efficient train control. *J. Rail Transp. Plan. Manag.* (submitted for publication).
- Van Dongen, L.A.M., Schuit, J.H., 1989. Energy-efficient driving patterns in electric railway traction. In: *International Conference on Main Line Railway Electrification*. IEEE, York.
- Van Leeuwen, J.J.A., 2017. Trillingshinder - Gedifferentieerd rijden. Samenvatting onderzoek naar het effect van rijsnelheid van goederentreinen op trillingsopwekking in de bodem uitgevoerd op zes locaties in Nederland plus een analyse van andere oorzaken van trillingen (in Dutch). Report. DGMR.
- Velthuisen, F., Ruijsendaal, S., 2011. *De UZI Basis Methode* (In Dutch). Report, NS.
- Vernersson, T., Lundén, R., 2014. Wear of brake blocks for in-service conditions-Influence of the level of modelling. *Wear* 125–131. <https://doi.org/10.1016/j.wear.2013.12.008>.
- Wang, P., Goverde, R.M.P., 2016a. Multiple-phase train trajectory optimization with signalling and operational constraints. *Transp. Res. C Emerg. Technol.* 69, 255–275. <https://doi.org/10.1016/j.trc.2016.06.008>.
- Wang, P., Goverde, R.M.P., 2016b. Two-train trajectory optimization with a green wave policy. In: *Transportation Research Board Records*.
- Wang, P., Goverde, R.M.P., 2017. Multi-train trajectory optimization for energy efficiency and delay recovery on single-track railway lines. *Transp. Res. Part B Methodol.* 105, 340–361.
- Wang, P., Goverde, R.M.P., 2019. Multi-train trajectory optimization for energy-efficient timetabling. *Eur. J. Oper. Res.* 272, 621–635. <https://doi.org/10.1016/j.ejor.2018.06.034>.
- Wang, Y., De Schutter, B., den Boom, T.J.J., Ning, B., 2013. Optimal trajectory planning for trains – a pseudospectral method and a mixed integer programming approach. *Transp. Res. Part C* 29, 97–114.
- Wang, Y., De Schutter, B., den Boom, T.J.J., Ning, B., 2014. Optimal trajectory planning for trains under fixed and moving signaling systems using mixed integer linear programming. *Contr. Eng. Pract.* 22, 44–56.
- Weltevreden, M., 2015. Wat Is Het Verschil in Energieverbruik ERTMS versus ATB Ritten? (in Dutch) ERTMS, Kennisbank. http://www.ertmskennisbank.nl/resultaten-pilot/a1542_Antwoord-Vraag-81.
- Wickens, C.D., 2002. Multiple resources and performance prediction. *Theor. Issues Ergon. Sci.* 3, 159–177. <https://doi.org/10.1080/14639220210123806>.