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DOI
10.1007/978-3-319-99867-1_91

Publication date
2019

Document Version
Final published version

Published in
New Trends in Urban Drainage Modelling - UDM 2018

Numerical and CFD-Based Modelling of Concentrated Domestic Slurry in Turbulent Flow Through Circular Pipes

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Abstract. The concentration of domestic slurry has two advantages, it promotes resource recovery (nutrients and biomass) and saves water. But the design of a relevant sewerage requires a clear understanding of the frictional losses incurred during the transport of the slurry. This abstracts describes numerical & CFD-based methods to estimate losses while the concentrated slurry flows through circular pipes in a fully-turbulent flow. To model turbulent flows through circular pipes, one can rely on either the Newtonian *Moody Charts* appropriate for engineering applications or a computational fluid dynamics (CFD)-based analysis, made possible through the Newtonian *universal law of the wall*. However, our studies reveal that concentrated domestic slurry behaves like a non-Newtonian fluid, of the Herschel-Bulkley type. Therefore, the analysis of such a slurry would require modifications to both, existing engineering models and CFD methods. This abstract summarises a modified law of the wall suitable for Herschel-Bulkley fluids, which has been validated against experiments on concentrated domestic slurry. It further details possible non-Newtonian numerical engineering models that could be modified to assess frictional losses incurred by Herschel-Bulkley fluids. The latter will be a quicker and perhaps reliable alternative to computationally expensive CFD-analyses.

Keywords: Concentrated domestic slurry · Urban drainage
Herschel-Bulkley fluid · Non-Newtonian flow
Computational fluid dynamics · Turbulence

1 Introduction

Unlike a Newtonian fluid, the viscosity of a non-Newtonian fluid not only depends on the temperature and pressure but also on the **flow itself**. The simple constitutive relation of a Newtonian fluid reads,

$$\tau = \mu \dot{\Upsilon},$$

G. Mannina (Ed.): UDM 2018, GREEN, pp. 528–532, 2019.
https://doi.org/10.1007/978-3-319-99867-1_91

where τ is the shear stress, μ is the molecular viscosity and $\dot{\Upsilon}$ is the shear rate. In contrast, the constitutive relation of a non-Newtonian fluid of the Herschel-Bulkley type reads (Herschel and Bulkley 1923),

$$\tau = \left(\frac{\tau_y}{|\dot{\Upsilon}|} + m\left(\dot{\Upsilon}^{n-1}\right)\right)\dot{\Upsilon},$$

where, m is the consistency index, n is the behaviour index and τ_y is the yield stress. If $n = 0$, the fluid is a Bingham plastic and if $\tau_y = 0$, the fluid is a power-law fluid (Chhabra and Richardson 1999); both of which are non-Newtonian fluids.

Heywood and Cheng 1984 and Skelland 1967 summarise a range of numerical methods that could be used to predict frictional losses experienced by the above-mentioned non-Newtonian fluids in turbulent flows. Each method has its range of accuracy and limitations. Based on a range of experiments, Slatter 1995 proposed a simple numerical model suitable for Herschel-Bulkley fluids. Thomas and Wilson 1987 also proposed a numerical model but based on the theoretical nature of turbulent dissipation.

Malin 1998 on the other hand, illustrates a computational fluid dynamics (CFD)-based analysis of turbulent flows of Herschel-Bulkley fluids. Malin 1998 uses the κ-ε Reynolds-Averaged Navier-Stokes (RANS) model while replacing the molecular viscosity with an apparent viscosity calculated using the constitutive relations mentioned above.

On using the above-mentioned numerical methods, we noticed a discrepancy between the numerical predictions and the experimental observations on the flow of concentrated domestic slurry, as regards to the pressure losses in circular horizontal pipes. Further, using both κ-ε and the Reynolds Stress Model (RSM) with non-Newtonian viscosity alone, led to poor predictions of the pressure losses.

2 Materials and Methods

Concerning CFD, it is known that the *universal law of the wall* (Launder and Spalding 1974) is used to model the presence of a wall boundary (the pipe's wall in this case) for enabling accurate yet affordable simulations of wall-bounded flows. This law was developed and has been validated for **Newtonian fluids alone**.

The law of the wall for Newtonian fluids reads,

$$\frac{u}{\left(\frac{\tau_w}{\rho}\right)^{1/2}} = \frac{1}{K} ln\left(E\frac{\rho y \tau_y}{\mu}\right),$$

where K and E are constants, ρ is the density of the fluid, y is distance from a wall boundary and u is the velocity parallel to the wall at a distance y from it. Using this law, one can determine the wall shear stress τ_w through an appropriate CFD simulation.

However, this law when used with a Herschel-Bulkley fluid will lead to erroneous predictions of the wall shear stress and hence, the pressure losses.

Instead, Mehta et al. 2018, proposed a modified law of the wall that was derived using an approach similar to Launder and Spalding 1974 and relevant numerical methods summarised in Skelland 1967. The aim was to incorporate the effects of the yield stress and the fluid behaviour index in a single equation that could approximate the near-wall behaviour of Herschel-Bulkley fluids and enable the prediction of wall shear stress through a CFD simulation using the κ-ε and the RSM RANS models.

The modified law of the wall reads,

$$\frac{u}{\left(\frac{\tau_w-\tau_y}{\rho}\right)^{1/2}} = \frac{1}{nK} ln\left(E\frac{\rho}{m}y^n\left(\frac{\tau_w-\tau_y}{\rho}\right)^{1-\frac{n}{2}}\right),$$

The above equation is referred to as ψ_1. On the other hand, there are semi-empirical relations that relate the wall shear stress and the flow properties, such as the one by Tomita (see Skelland 1967) for **power-law** fluids,

$$\sqrt{\frac{1}{f_P}} = 4\log\left(Re_P\sqrt{f_P}\right) - 0.40,$$

where Re_P and f_P are defined as follows.

$$f_P = \frac{\tau_w}{\rho V^2}\cdot\frac{8(2n+1)}{3(3n+1)}$$

$$Re_P = \frac{D^n V^{2-n}\rho}{m}\cdot\frac{6\left(\frac{3n+1}{n}\right)^{1-n}}{2^n\left(\frac{2n+1}{n}\right)}$$

V is the average flow velocity through a pipe of diameter D. Various other approaches to model the pressure losses incurred by power law fluids have been proposed. However, Tomita (see Skelland 1967) also derived an expression similar to the above, for Bingham plastic fluids. Given its completeness, we shall follow Tomita's approach to derive an expression for Herschel-Bulkley fluids, which combined the effect of Bingham plastic and power-law fluids.

The study will also include a thorough comparison of most existing numerical models, the CFD-based analysis using the modified law of the wall and experimental data, to ascertain which numerical (or CFD) method is most suited to our purpose. The next section summaries the results obtained through CFD using the modified law of the wall.

3 Results and Discussion

Figure 1 contains **two y-axes**. The left y-axis represents τ_w while the right y-axis represents the Reynolds number Re_R (as proposed by Rudman et al. 2004). The x-axis represents the average flow velocity. The Reynolds number is shown with the crosses at each data point, whereas the experimental wall shear stress is shown with grey horizontal lines. Additionally, at each data point, the green error bar represents a 5% deviation and the magenta error bar, a 15% deviation from the experimental data. Although we present only one result here, the modified law of the wall was used to simulate a range of test-cases from literature and those performed by the co-authors. The details can be found in Mehta et al. 2018.

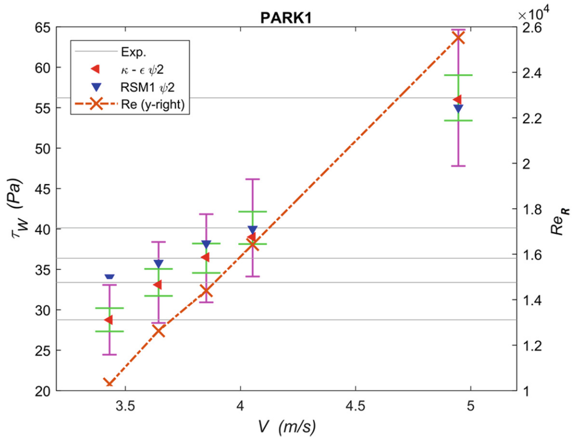

Fig. 1. $m = 0.0894$, $n = 0.73$, $\tau_y = 9.3$ Pa and $D = 51$ mm. The accuracy of the modified law of the wall combined with κ-ε and the RSM, is clearly visible.

4 Conclusions

The reported observations suggest that the proposed wall functions when combined with the standard κ-ε or RSM, provide reliable numerical predictions of the wall shear and hence, the pressure loss experienced by a Herschel-Bulkley fluid in turbulent flow through a circular horizontal pipe.

We are performing modifications of a few numerical models that have been proposed for power-law fluids and Bingham plastic fluids, the combined effect of which is seen in a Herschel-Bulkley fluid. We intend to present a comparison between the numerical methods that are currently being developing and the CFD analyses reported in the abstract, to ascertain if simple engineering models could replace CFD analyses for the turbulent flow of Herschel-Bulkley fluids.

Acknowledgement. This abstract is the result of the research carried out at the Delft University of Technology, Delft and funded by NWO-domain TTW, Foundation Deltares, Stowa, Foundation RIONED, Waternet, Waterboard Zuiderzeeland, Grontmij and XYLEM. We acknowledge the support of the funding bodies and their contributions to this research.

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