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## Extended abstract

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# POCKETS OF TURBULENCE IN PLANE COUETTE FLOW

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*Abstract*
We track the onset of chaos in plane Couette flow. The formation of a chaotic saddle through a boundary crisis is discussed. We show that inside the chaotic saddle, new saddle-node bifurcations create stable pockets of turbulence, which subsequently undergo boundary crisis. This leads to a non-monotonic variation of characteristic lifetimes with Reynolds number.

## THE ONSET OF CHAOS

Studies of the transition to turbulence in linearly stable shear flows, such as pipe flow and plane Couette flow, have revealed a route to turbulence that passes through the formation of a chaotic saddle [1, 2]. We here present the analysis of the state-space structures and the bifurcations that lead to the creation of this saddle. The chaotic saddle supports long transients preceding sustained turbulence.

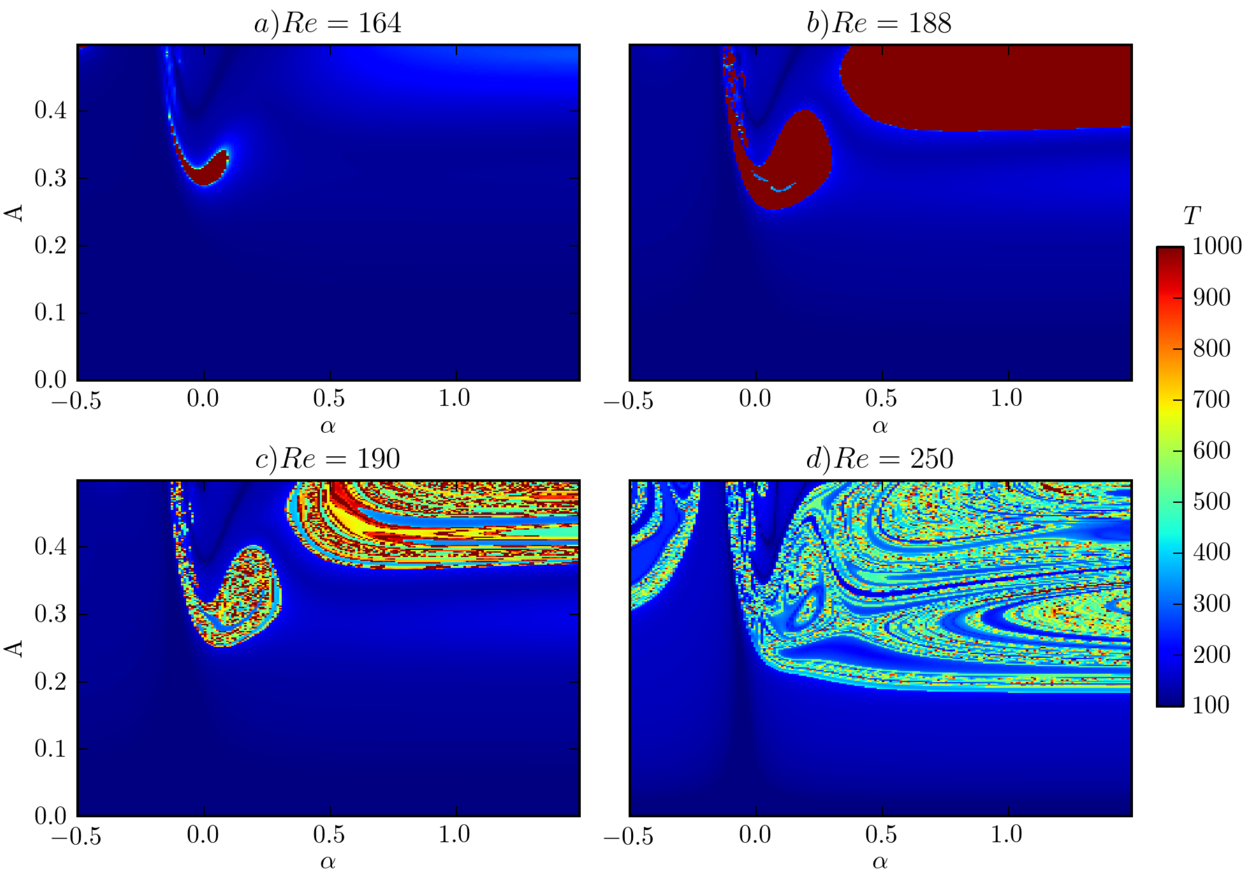


**Figure 1.** Two-dimensional slices of the state-space of transitional plane Couette flow. Coordinates are chosen such that the x-axis is an interpolation between the lower- and upper-branch Nagata-Busse-Clever states. The y-axis is the amplitude of the interpolated state. The structure of state-space is shown by the lifetimes $T$ of initial conditions. (a) Just after the first saddle-node bifurcation. (b) Just before the boundary crisis. (c) Just after the boundary crisis. (d) At higher Reynolds number.

We study plane Couette flow in a small periodic domain of size $2\pi \times 2 \times \pi$ in the downstream, wall-normal and spanwise directions with imposed shift-and-reflect symmetry. In this cell, the Nagata state [4] is created as the lower-branch of a saddle-node bifurcation at Reynolds number $Re = 163.8$. It has exactly one unstable direction, while the corresponding upper-branch solution is stable. The latter one is hence an attractor that coexists with the laminar attractor. Figure 1(a) illustrates the situation by showing the lifetimes of initial conditions in a two-dimensional slice of the state-space at $Re = 164$. The small dark-red region corresponds to the basin of attraction of the upper-branch, the blue region to states that decay to the laminar state. The fixed-point undergoes a series of bifurcations that result in a chaotic attractor [3]. Its basin of attraction grows with increasing Re, figure 1(b).

At $Re = 188.7$, the chaotic attractor has expanded so much that it collides with the lower-branch state, leading to a boundary crisis. At this point, the basin of attraction opens up and the chaotic attractor turns into a chaotic saddle. By comparing the situation before (figure 1(b) at $Re = 180$) and after (figure 1(c) at $Re = 190$) the crisis, we still see the chaotic saddle as a region of rapidly varying lifetimes.

As Reynolds number increases further, the chaotic saddle grows to fill more and more parts of the state-space, figure 1(d) at $Re = 250$.

## CREATING MORE CHAOS

Inside the chaotic saddle the distribution of lifetimes follows an exponential scaling, with a characteristic lifetime $\tau$. Figure 2(a) shows $\tau$ as a function of Reynolds number, with each value of $\tau$ calculated from 50000 trajectories. Just above the crisis point in Re, the lifetime decays quickly. For higher Re it varies irregularly, before increasing more steadily for $Re$ above 300.

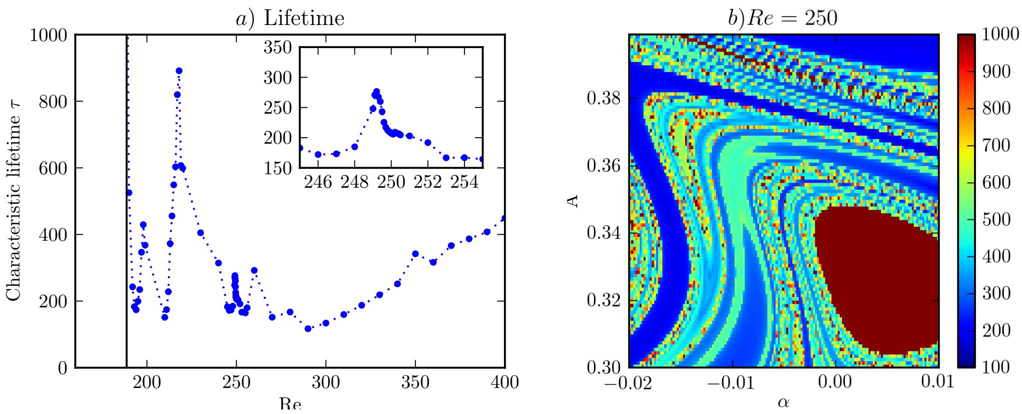


**Figure 2.** Non-monotonic lifetimes. (a) The characteristic lifetime as a function of Reynolds number varies non-monotonically. (b) A "pocket of turbulence", associated with a saddle-node bifurcation of periodic orbits, immersed in the chaotic saddle.

We are able to associate the non-monotic variation of lifetimes to the creation of "pockets of turbulence": in a small region in state-space a stable orbit and a hyperbolic point are created in a saddle-node bifurcation; this pocket grows, opens up and merges with other pockets, leading to an increase in lifetimes.

The crisis-bifurcation discussed above gives a first example of such a pocket. As a second example we study the peak around $Re = 250$, magnified in the inlet of figure 2(a). This peak can be related to a pair of periodic orbits which are created in a saddle-node bifurcation at $Re = 249.01$. In this bifurcation, the upper-branch orbit is an attractor, the lower-branch orbit has one unstable direction. The situation is hence similar to the one described above for the Nagata-solutions, only that now the saddle node bifurcation occurs inside the chaotic saddle. We see this chaotic pocket as the dark-red region in figure 2(b), where states never decay to the laminar state, immersed in the colorful saddle. The attractor is destroyed in a new crisis at $Re = 250.13$, leaving behind a larger chaotic saddle than before.

The creation of invariant solutions in saddle-node bifurcations and the destruction in crisis bifurcations provides one mechanism underlying the generally observed increase of turbulent lifetimes with Re. Note however that lifetimes can vary non-monotonically.

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