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Free flow speed estimation: a probabilistic, latent approach. Impact of speed limit changes and road characteristics

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ABSTRACT

The estimation of the free flow speed (FFS) distribution is important for capacity analysis, determination of the level-of-service, and setting speed limits. Subjective time headway thresholds have been commonly used to identify vehicles travelling under free flow speed conditions i.e., vehicles whose speeds are not influenced by the vehicle in front. Since, the headway a driver operates under the free flow state is subjective and varies from driver to driver, such approaches can introduce biases in the FFS estimation. Therefore, in this paper a parametric probabilistic latent approach is proposed based on discrete choice utility theory to estimate the FFS distribution on urban roads and simultaneously the probability that drivers perceive their state as constrained by the vehicle in front. This methodology is used to estimate the impacts of road characteristics and Posted Speed Limit (PSL) changes on the FFS distribution using an extensive dataset of speed observations from urban roads with varying characteristics. The results show that the simultaneous estimation of the free flow speed distribution and the state the driver is in (e.g., free or constrained) is feasible. The analysis indicates that the FFS is influenced by several road characteristics such as land use, on-street parking and the presence of sidewalks. The PSL change impacts not only the distribution of the free flow vehicles but also the speed distribution of the constrained vehicles. The constrained probabilities vary depending on the PSL change with higher probabilities for lower speed limits.

Keywords: *free flow speed distribution, urban roads, road characteristics, posted speed limits, probability to be constrained, maximum likelihood estimation.*

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1. INTRODUCTION

Knowledge of the free flow speed (FFS) distribution is important for several reasons: (i) for design purposes the 85th percentile of the FFS distribution is used to establish speed limits (Deardoff et al., 2011; Elvik, 2010); (ii) in simulation studies it is an important parameter for unimpeded vehicles (Caliper, 2008; PTV AG, 2015); (iii) in 'before-after' traffic safety studies it is used to estimate the impact of a specific countermeasure on road safety (Silvano and Bang, 2015); and (iv) in traffic performance studies it is used to establish the "*ideal*" traffic conditions *(base free flow speed)* to be adjusted by road-traffic factors (HCM, 2010; Bang and Carlsson, 1995). From a design point of view, it is also important to identify the factors which influence the FFS distribution, such as road characteristics and traffic regulations, and quantify their influence. Normally, studies on speed models investigate the impact of road characteristics on the mean free flow speed or speed percentile (e.g., 85th percentile speed). Road characteristics influencing the FFS distribution include land use (e.g., suburbs, city center, shopping areas), road function (e.g., local, main, collector, and arterial roads), number of lanes per direction, carriageway width, lane width, presence of medians, presence and width of shoulder, presence of on-street parking, presence of sidewalks, number of pedestrians, access point density, intersection density, link length, etc. (Ericsson, 2001; Aronsson and Bang, 2007; Hansen et al., 2007; Wang et al., 2006; Silvano and Bang, 2015). In addition, road characteristics can be modified or introduced to impact the FFS distribution. For instance, on urban roads, introducing on-street parking and sidewalks for pedestrians impact the FFS of the vehicles using the facility (Trivector AB, 2009; Silvano and Bang, 2015).

Traffic regulations, such as Posted Speed Limits (PSL), impact the FFS distribution as well. However, in the literature, the inclusion of the impact of PSL on speed models is not clear. Some authors argue that PSL is highly correlated with road characteristics and difficult to capture. Recently, a study by Himes et al., (2013), who conducted extensive statistical analysis, recommends that PSL should be included in speed models as exogenous variable due to its significant association with the mean speed. To estimate the impact of PSL on the FFS, field experiments are normally carried out by changing the existing PSL, typically in steps of 10 km/h. According to Elvik (2010) a change of ±10 km/h in the PSL results in a change of ± 2.5 km/h on the mean speed. However, drivers ignoring the PSL are an increasing concern because it is becoming a normal driving behavior leading to reduced traffic safety (Mannering, 2009).

The FFS estimation is not straightforward. It should be based on observations from vehicles whose speeds are not influenced by the vehicle in front, or under free flow conditions. At free flow conditions, the speed of a driver is only influenced by the road environment and driver preferences. However, the state under which a driver operates (i.e. free flow or constrained) is not directly observed, i.e., the driver state is latent. For the estimation of the FFS, cut-off headway values were typically used in previous studies (Vogel, 2002; Silvano & Bang, 2015). However, since the headway a driver operates under the free flow state is subjective and varies from driver to driver, such approaches can introduce biases in the FFS estimation. Therefore, a new research methodology needs to be developed which releases this limitation. Furthermore, the impacts of road geometric characteristics and speed limit changes on the FFS have not been thoroughly investigated, particularly in urban roads where road geometry varies greatly.

The main goal of this paper is to propose a methodology for the estimation of the FFS distribution which can: a) evaluate the impact of road geometric characteristics and traffic regulations on the FFS; and b) simultaneously estimate the probability that a driver perceives his/her state as constrained or not by the vehicle in front, by using a latent formulation. The methodology used for the analysis is based on discrete choice utility theory.

The remainder of the paper is organized as follows: Section 2 presents previous studies on the identification of the FFS distribution based on time headways. Section 3 describes the study methodology and model formulation. Section 4 presents the application of the methodology with the results of the impact of speed limits and road characteristics. Finally, Section 5 concludes the study and presents future research directions.

2. BACKGROUND

Normally, studies aiming to estimate the FFS distribution collect speed data under low flow conditions (<1000 veh/h/ln) with large time headways between two successive vehicles in order to guarantee that vehicles are not constrained (Dawson and Chimini, 1968; Branston, 1979; Dixon et al., 1999; HCM, 2010; Wang et al., 2014). The HCM (2010) is a widely used methodology to estimate the FFS for freeways and urban segments. Level of Service (LOS) 'A' is associated to free flow conditions with vehicles being unimpeded to maneuver in the traffic stream. For urban segments, the methodology to estimate the LOS is data intensive including factors such as flow rate, number of lanes, segment length, speed limits, access point density, capacity, etc. In the HCM (2010) the FFS represents the mean speed of through vehicles at low-volume conditions. The methodology includes the estimation of the *base free flow* speed which is then adjusted for several factors such as cross section with or without curb presence, access point density and adjustment for signal spacing. As a result, with the help of previous tabulated values, the mean FFS is estimated in order to compute the LOS. According to the HCM (2010), the adjusted *free flow* speed is given by:

$$S_{f0} = S_0 + f_{CS} + f_A \tag{1}$$

S_{f0} is the adjusted FFS, S_0 is the *base free flow* speed, f_{CS} is the adjustment for cross section (with or without curb presence), and f_A is the adjustment for access point density.

Several studies have been dedicated to estimate the FFS distribution based on speeds of vehicles with time headways larger than a certain threshold, determined from studies in the literature (e.g. Silvano and Bang, 2015). Branston (1979) assumed that the FFS distribution was normally distributed and used data from motorway M4 in London. Two criteria were used for a vehicle to be in the FFS distribution: (i) time headway larger than some headway threshold 'T' and (ii) relative speed (speed leader minus speed follower) larger than some standard deviation criterion (τ). The author tested different values for 'T' (3.5 to 5.5 seconds) and τ (2, 2.5, and 3 standard deviations) to fit a theoretical distribution to the data. The author found that the free flow speed distribution was fairly insensitive to the values chosen for 'T' and τ varying only 2% in the results. Botma (1999) discussed the use of several methods to estimate the FFS distribution. The author introduced the notion of censored observations i.e., vehicles which are constrained with a certain probability due to different driver and vehicle characteristics.

Hoogendoorn (2005a) proposes to estimate the FFS distribution with a non-parametric approach using time headway and speed data. The author validates the approach by generating synthetic data as follows: i) Constrained headway $N{\sim}(\mu_x = 2\ sec, \sigma_x = 0.5\ sec)$; ii) Free flow headway exponentially distributed ($\lambda = 0.167\ veh/sec$); iii) Free flow speed $N{\sim}(\mu_{v^0} = 80\ km/h, \sigma_{v^0} = 12\ km/h)$. Thus actual headways h_i and speeds v_i were classified as follows:

$$h_i = max\{x_i, \mu_i\} \tag{2}$$

$$v_i = \begin{cases} v_i^0 & x_i < \mu_i \\ min\{v_i^0, v_{i-1}\} & x_i \geq \mu_i \end{cases} \tag{3}$$

Where, x_i is the constrained headway, μ_i is the free flow headway, v_i^0 is the free speed, v_{i-1} is the speed of the leader. At free flow conditions, the speed of the vehicle equals to its desired speed; otherwise, the speed of the vehicle is the minimum between its free speed and the speed of the lead vehicle. Later on, Hoogendoorn (2005b) applied the non-parametric approach to estimate the free flow speed distribution with data from a multilane motorway. Gaps and relative speeds were used to establish the separation criteria between the free flow and constrained vehicles. The probability of being constrained is given by:

$$\theta(d_i, \Delta \mathrm{v}_i) = \theta_1(d_i) \cdot \theta_2(\Delta v_i) \tag{4}$$

$\theta_1(d_i)$ is the probability to be constrained at a distance d_i. $\theta_2(\Delta v_i)$ is the probability of a vehicle to be constrained with relative speed Δv_i. (Relative speed between the leader and the subject vehicle, $v_{i-1} - v_i$). The author found that the mean FFS for all vehicles and for different vehicle classes is higher in the outer lane (left lane) compared to the right lane. Moreover, the author found that the mean FFS is lower in the morning compared to the noon and evening mean FFS speeds. Traffic congestion might be a possible explanation according to the author since congestion makes drivers less willing to speed up. The approach was sequential first estimating the regime and then the FFS distribution.

The literature on the analysis of time headway observations is quite extensive. Early studies developed statistical models to fit some probability density function to the time headway data. For instance, Dawson and Chimini (1968) developed a probability model for single-lane traffic flow on two-lane, two-way roads. Flow rates were stratified from 150 to 1050 veh/h with steps of 100 veh/h in each level. The authors established the headway threshold where the logarithmic curve became a straight line. Researchers has modeled time headways as a random variable with different distributions for the free flow vehicles and constrained vehicles. For instance, Buckley (1968) developed a Semi-Poisson model to estimate the proportion of drivers with constrained operation. According to the author, constrained vehicles drive at average speeds which are equal to the speed of the leader with a preferred tracking headway. Cowan (1974) proposed to treat the headways with two random components as well: i) a "tracking" component (followers) and ii) a "free" component (leaders). The author proposed several models. In the first model, a negative exponential distribution model was introduced only for the *free* component. A second model assumed a shifted negative exponential distribution to account for the fact that there is a minimum headway which cannot be "zero". In a third model, there is a proportion of vehicles "tracking" their predecessors (θ) at some headway τ and a proportion of vehicles travelling freely $(1-\theta)$ with headways greater than τ. In a fourth model, a distribution is introduced for the "tracking" proportion. The author estimated the parameters of the fourth model with data from 1324 successive headways observed for two hours and found a mean tracking headway τ of 1.70 seconds and the proportion of vehicles tracking their predecessor θ equal to 0.35. Branston (1976) evaluated headway models previously proposed by Miller (1961) and Buckley (1968) with data from the M4 motorway in London and a two-way road in Indiana. He found a mean following headway of 1.3 and 1.6 seconds in the M4 for the fast and slow lane respectively. A mean following headway of 2-seconds was found for the road in Indiana.

Luttinen (1996) proposed a four-step methodology to analyze time headway data that includes: i) the probability density function; ii) the hazard function; iii) the coefficient of variation; and iv) the kurtosis vs. squared skewness plot. The data used in the study were collected on low-speed roads (e.g., 50, 60, and 70 km/h) by inductive loops on two-lane two-way roads in Finland. The author argues that there are three driving regimes: free flow speed (leaders), car-following (constrained) and in-transition drivers (i.e., changing state from being a leader to becoming a follower). During the in-transition state, vehicles begin to adjust their speed to the leader's speed when they are 9 – 8 seconds from the vehicle in front. He found that at headways greater than 10 seconds the mean relative speed, ($\Delta V = V_{leader} - V_{follower}$), is negative thus concluding that the vehicle behind is free from the influence of the lead vehicle. Moreover, the author points out that the best headway model is the semi-Poisson model proposed by Buckley (1968) with the negative exponential distribution to model the proportion of free vehicles and the gamma distribution to account for the proportion of constrained vehicles.

A different approach to estimate the threshold between the free flow and constrained regime on urban roads with 50 km/hr posted speed limit was carried out by Vogel (2002). The data were collected in a 4-leg intersection for 24 hours during 6 consecutive days. Four measurement stations were placed at each approach at a distance of 116 m from the intersection. The data were collected by means of pneumatic tubes and speeds and time headways were derived. The data were classified in groups of one second from 1 to 12 s headway and the author estimated correlation factors among the groups. For the 1-s group, the R-value was 0.8 which decreased gradually as the headway increased and for headways larger than 6 s the R-value leveled out approaching to zero thus the author concluded that all vehicles with headways larger than 6 s were in the free flow regime.

In other studies, headways (time and space headways) have also been investigated using Markov process models (Chen et al., 2010; Wang, et al., 2008). Chen et al., (2010) defined three Markov processes classified by speed intervals: 0-5 m/s, 5-10 m/s, and 10-15 m/s. Afterwards, the authors defined 10 possible states based on observed headways ranging from 0 to 9 s in 1-s increments. Observations were considered free at headways larger than 10 s. The authors claim that the model can generate headway distributions similar to observed headways. In Wang et al., (2008) the space headway was investigated by using a Markov-gap Cellular Automata (CA) model. The aim was to reproduce by simulation the empirical headway distribution. The authors concluded that the model was able to capture the variations of the space headway between two consecutives vehicles.

In conclusion, time headway thresholds are likely to vary due to traffic, and road conditions, and driver characteristics. For instance, a threshold of 3 s was used for 2-lane highways in the HCM (2000) and a threshold of 6 s was found on urban roads by Vogel (2002). Factors such as flow rate, operating speeds, speed limits, weather conditions, and road geometric characteristics may influence time headway thresholds (Ayres et al., 2001). Moreover, the headway threshold is likely to vary among drivers due to different driving experiences, gender, age, etc., and may differ for the same driver as well, e.g. depending on the driver's trip purpose and departure time (Brackstone et al., 2009). Vehicle characteristics may also influence the headway threshold due to factors such as vehicle age, power engine, maintenance, etc. More importantly, a model that can evaluate the impact of road characteristics and changes in traffic regulations (e.g., speed limit changes), on the FFS is missing in the literature. Therefore, the estimation of the impact of road characteristics and traffic regulation changes on the FFS distribution, while simultaneously estimating the probability to be constrained by the vehicle in front, addresses important drawbacks of previous studies.

3. METHODOLOGY

Road geometric characteristics impact the FFS in urban roads. Factors increasing the FFS include wider lanes, higher number of lanes, presence of shoulder and median. On the other hand, factors such as the presence of sidewalks, on-street parking, presence of vulnerable road users (e.g., cyclists, pedestrians, mopeds), and vehicle composition reduce the FFS (Silvano and Bang, 2015; Bang and Carlsson, 1995; Aronsson and Bang, 2007; Wang et al., 2006;

Brundell-Freij and Ericsson, 2005; Ericsson, 2001; Hansen et al., 2007; Wang et al., 2014; Balakrishnan and Sivanandan, 2015). Therefore, the mean FFS is dependent on the geometric layout factors of urban roads which may imply as well that the headway threshold, which distinguishes free flow vehicles from constrained vehicles, varies as well. This section introduces a probabilistic approach, to estimate the FFS distribution in urban roads while taking into account the potential impact of road geometric characteristics and traffic regulation.

From a behavioral perspective, traffic flow is composed of two states: (i) free flow drivers and (ii) constrained drivers (Luttinen, 1996; Michael et al., 2010; Vogel, 2002). Free flow moving drivers can attain their desired speed. Drivers in the constrained state are expected to drive on average at speeds which are equal to the speed of the vehicle in front (Buckley, 1968). Therefore, under constrained conditions the speed of a vehicle follows closely the speed of the lead vehicle while in the free flow state drivers most likely follow their desired speed. For estimation purposes, it is assumed that the available data include point measurements of individual vehicle speeds on a road, along with headways, and characteristics of the road and surrounding environment. The state, e.g., constrained or free flow is not observed. Hence, the underlying state the driver is in, is latent and the estimation methodology explicitly recognizes this. Figure 1 illustrates the model structure showing the driving states as latent.

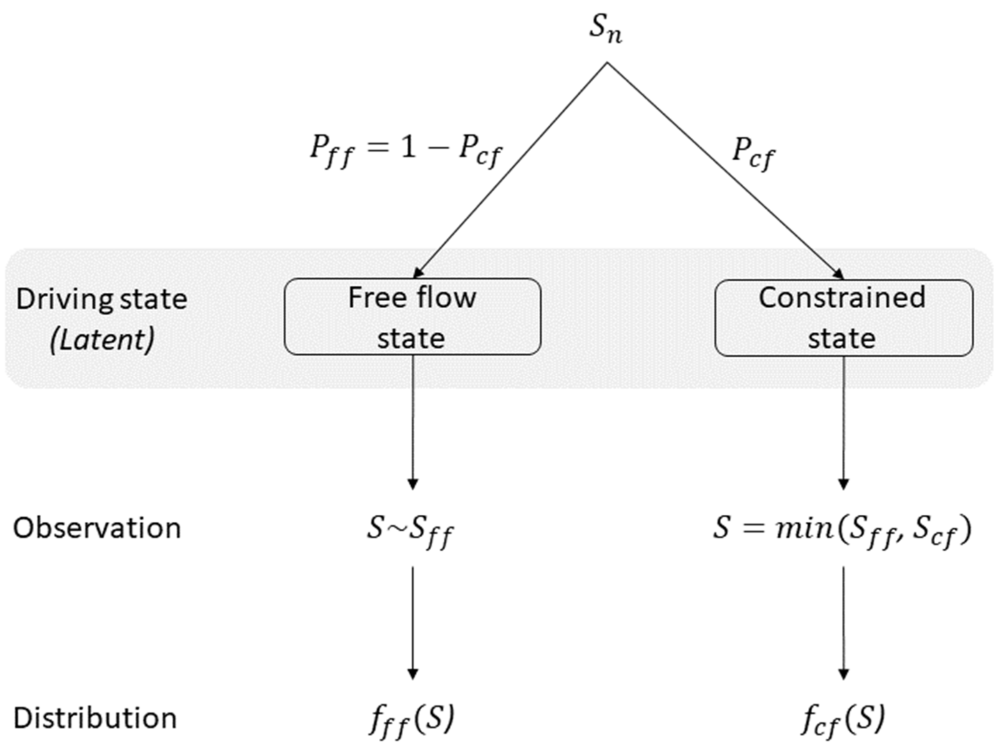


Figure 1 Model structure

S_n is the speed of a vehicle. S_{ff} is the free flow speed of the driver. S_{cf} is the speed of the vehicle when it is influenced by the speed of the vehicle in front. f_{ff} is the free flow speed distribution. f_{cf} is the constrained speed distribution. P_{ff} is the probability to be free and P_{cf} is the probability to be constrained.

3.1 The probability to be constrained

Time headway thresholds, often deterministic (fixed threshold), have been used extensively in the literature to determine the traffic state. If the time headway is less than the specified threshold, the driver is assumed to be in the constrained state; otherwise, the appropriate free flow state applies. However, the selection of the threshold is not a trivial task. For example, Luttinen (1996) conducted extensive studies on time headways on low-speed rural and urban roads (e.g., 50 km/h and 70 km/h) and found that at headways larger than 10 s the mean relative speed is negative ($v_{i-1} - v_i < 0$) pointing out that the subject vehicle is free from the influence of the leader. In another study, Vogel (2002) found that the speed correlation between the subject vehicle and its leader levels out (approaches to zero) at headways larger than 6 s on urban roads. The author also pointed out that at lower speeds, constrained vehicles need longer headways to reduce the influence of the vehicle in front. On the other hand, the constrained state takes place at shorter time headways. For example, on freeways, the car-following regime is assumed at headways shorter than 4 s (Bando et al., 1995; Ahmed, 1999; Toledo, 2003).

In this paper, the state the driver is in (constrained or not) is treated as a latent state, and the model assumes that a driver makes decisions based on how they perceive their individual state. Thus, the model captures how drivers perceive their driving state, as being influenced by the vehicle in front or not. The probability of a driver to be in the constrained state, P_{cf}, is modeled as a binary discrete choice problem with linear utility functions. In the literature, discrete choice models have been frequently used to model drivers' decisions and behavior. For example, Farah and Toledo (2010) estimated the probability of a driver to overtake on two-lane rural roads using binary choice model, Koutsopoulos and Farah (2012) modeled drivers' decisions to accelerate, decelerate, or do-nothing under constrained conditions as a discrete choice problem, Singh and Li (2012) investigated lane changing decisions as a discrete choice problem, and Silvano et al., (2016) modeled drivers' decisions to yield to cyclists using a similar framework. In this study, the probability of the driver to be in the constrained or free flow state is also modelled as a binary discrete choice problem. It is assumed that the utility, V_{cf}, of being in the constrained state, is a function of explanatory variables (e.g., time headway, relative speed). The specification of the utility depends on the available data. The utility of the free flow state, V_{ff}, is set to 0. Thus, the probability that driver n is in the constrained state, assuming a Gumbel distribution for the error terms, can be given by:

$$P_{cf} = \frac{e^{V_{cf}}}{e^{V_{cf}} + e^{V_{ff}}} = \frac{e^{V_{cf}}}{1 + e^{V_{cf}}} \tag{5}$$

$$P_{ff} = 1 - P_{cf} \tag{6}$$

V_{cf} is the systematic utility of the constrained state:

$$V_{cf} = \beta_0 + \sum_k \beta_k \cdot X_k \tag{7}$$

X_k are explanatory variables (depending on available data) and β_k are the corresponding parameters to be estimated.

3.2 Free flow speed distribution

A vehicle whose speed is not influenced by the vehicle in front is considered to travel at the desired speed. Hence, the speed represents an observation from the FFS distribution, $f_{ff}(S)$. For estimation purposes and consistent with the literature (Branston, 1979; Dixon et al., 1999), we assume that the FFS follows a normal distribution, $f_{ff}(S) \sim N(\mu_{ff}, \sigma_{ff})$;

$$f_{ff}(S) = \frac{1}{\sigma_{ff}} \phi \left(\frac{S - \mu_{ff}}{\sigma_{ff}} \right) \tag{8}$$

S is the free flow speed of a vehicle. μ_{ff} and σ_{ff} are the mean and standard deviation of the FFS distribution, respectively.

The mean free flow speed, μ_{ff}, is influenced by, among other factors, road geometric characteristics, as previously stated. Thus, the mean FFS expression can be formulated as a function of those influencing factors:

$$\mu_{ff} = \alpha_0 + \sum \alpha_i \cdot X_i \tag{9}$$

α_0 is a constant, and α_i and X_i are parameters and explanatory variables respectively, affecting the mean FFS. The exact specification depends on the nature of the available data.

3.3 Constrained speed distribution

When vehicles are close to each other, the interactions among vehicles results in drivers not being able to attain their desired speeds. In the constrained state (e.g. the driver feels constrained by the vehicle in front), a vehicle has a speed that is either its desired speed or is constrained by the speed of the vehicle in front (Hoogendoorn, 2005a), i.e.:

$$S = \min\{S_{ff},\ S_{cf}\} \tag{10}$$

and,

$$f(\ S_{cf}) = \frac{1}{\sigma_{cf}} \phi \left(\frac{S_{cf} - S_{front}}{\sigma_{cf}} \right) \tag{11}$$

S_{ff} is the free flow speed of the driver with a probability density function given by Eq. (8). S_{cf} is the speed of the vehicle when it is influenced by the speed of the vehicle in front, S_{front}, and it is assumed to follow a normal distribution around S_{front}, $S_{cf} \sim N(S_{front}, \sigma_{cf})$.

Eq. (10) states that if the speed of the vehicle in front is larger than the driver's free flow (desired) speed, the driver travels at its desired speed; otherwise it follows the speed of the leader (since the state is constrained). Equation (11) indicates that if the lead vehicle's speed is the constraining factor, then it is assumed that the speed of the subject vehicle is normally distributed around the lead vehicle's speed, S_{front}, and σ_{cf} is the standard deviation of the constrained speed distribution.

For two random variables $X_1 \sim N(\mu_1, \sigma_1)$ and $X_2 \sim N(\mu_2, \sigma_2)$ and correlation coefficient, ρ, the random variable $Y = \min\{X_1,\ X_2\}$ has a distribution given by (Nadarajad and Kotz, 2008):

$$f(y) = f_1(y) + f_2(y),$$

where;

$$f_1(y) = \frac{1}{\sigma_1}\phi\left(\frac{y-\mu_1}{\sigma_1}\right)\Phi\left(\frac{\rho(y-\mu_1)}{\sigma_1\sqrt{1-\rho^2}} - \frac{y-\mu_2}{\sigma_2\sqrt{1-\rho^2}}\right) \quad (12)$$

$$f_2(y) = \frac{1}{\sigma_2}\phi\left(\frac{y-\mu_2}{\sigma_2}\right)\Phi\left(\frac{\rho(y-\mu_2)}{\sigma_2\sqrt{1-\rho^2}} - \frac{y-\mu_1}{\sigma_1\sqrt{1-\rho^2}}\right) \quad (13)$$

Hence, the pdf of the speed S under constrained conditions, assuming that S_{ff} and S_{cf} are not correlated, is given by:

$$f_{cf}(S) = \frac{1}{\sigma_{cf}}\phi\left(\frac{S-S_{front}}{\sigma_{cf}}\right)\Phi\left(-\frac{S-S_{ff}}{\sigma_{ff}}\right) + \frac{1}{\sigma_{ff}}\phi\left(\frac{S-\mu_{ff}}{\sigma_{ff}}\right)\Phi\left(-\frac{S-S_{front}}{\sigma_{cf}}\right)$$

$$= \frac{1}{\sigma_{cf}}\phi\left(\frac{S-S_{front}}{\sigma_{cf}}\right)\left(1-\Phi\left(\frac{S-\mu_{ff}}{\sigma_{ff}}\right)\right) + \frac{1}{\sigma_{ff}}\phi\left(\frac{S-\mu_{ff}}{\sigma_{ff}}\right)\left(1-\Phi\left(\frac{S-S_{front}}{\sigma_{cf}}\right)\right) \quad (14)$$

3.4. Maximum likelihood formulation

The parameters of the model are estimated jointly using the maximum likelihood approach. Based on the framework illustrated in Figure 1, the probability density function of a speed observation S_n is given by:

$$P(S_n) = (1-P_{cf}) \cdot f_{ff}(S_n) + P_{cf} \cdot f(S_n)$$

$$= (1-P_{cf}) \cdot \frac{1}{\sigma_{ff}}\phi\left(\frac{S-\mu_{ff}}{\sigma_{ff}}\right) +$$

$$P_{cf} \cdot \left[\frac{1}{\sigma_{cf}}\phi\left(\frac{S-S_{front}}{\sigma_{cf}}\right)\left(1-\Phi\left(\frac{S-\mu_{ff}}{\sigma_{ff}}\right)\right) + \frac{1}{\sigma_{ff}}\phi\left(\frac{S-\mu_{ff}}{\sigma_{ff}}\right)\left(1-\Phi\left(\frac{S-S_{front}}{\sigma_{cf}}\right)\right)\right] \quad (15)$$

The likelihood function for all vehicles 1,…, N (assuming independence) is given by:

$$\mathcal{L} = P(S_1, S_2, \dots S_n) = \prod_{n=1}^{N} P(S_n) \quad (16)$$

The log-likelihood function is then defined by:

$$\mathcal{LL} = \sum_{n=1}^{N} logP(S_n) \quad (17)$$

4. APPLICATION

The methodology discussed in the previous section is applied using an extensive dataset from more than 30 locations in Sweden.

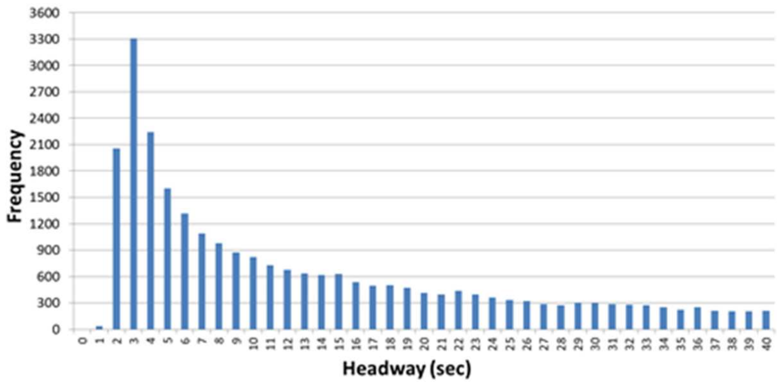
4.1 Data

A new speed limit scheme was evaluated in Sweden on urban roads (Hyden et al., 2008; Bang and Silvano, 2012). Depending on road characteristics, the speed limit on urban roads with existing speed limit of 50 km/h was changed. The selection of sites was undertaken by the Swedish Transport Administration and Municipal traffic authorities. The before-change data collection started in September 2009 and was concluded in June 2010. The after-change data collection started in September 2010 and was concluded in June 2011. The collected data correspond to only one-day before the speed limit change and one-day after the speed limit change. The measurement location at each site was placed midblock on a sufficiently long link, with right of way at minor intersections and avoiding nearby traffic signals, roundabouts, and crosswalks. The data were collected by means of pneumatic tubes and data loggers, on average over 7 hours per site during daytime. The sites represent a broad range of urban road characteristics (summarized in Table 1). The text files from the data loggers were interpreted with the Axel Passage Interpreter (Archer, 2003) to obtain passage times, speeds, flow, vehicle types and travel direction. The data set used in this study is related to 32 of the sites that were considered in the study by Bang and Silvano (2012), from 8 urban areas across Sweden with 50 km/h speed limits. The dataset was composed of about 31,000 speed and time headway observations with 50 km/h PSL and about 36,000 observations with 40 km/h PSL.

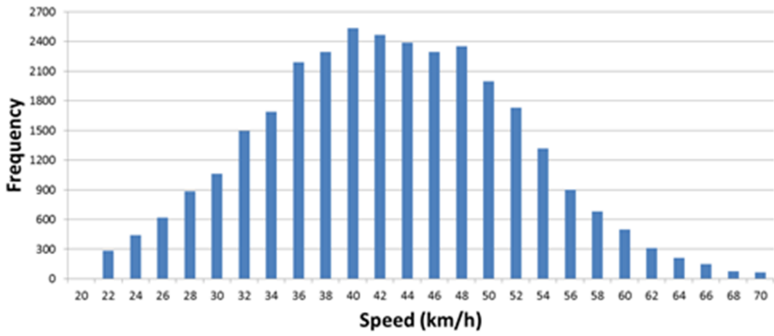
TABLE 1 Road Characteristics

Variable	**Definition**	**Number of sites per category**
Posted Speed Limit	PSL	Before: 50 km/h = 32 sites After: 40 km/h = 32 sites
Land Use	City Center = 0 Suburb = 1	Center = 6 (Presence of commercial development) Suburb = 26 (Homebased neighborhoods)
Parking-only	No = 0 Yes = 1	No = 30 Yes = 2 (Presence of parked vehicles only)
Sidewalk-only	No = 0 Yes = 1	No = 13 Yes = 19 (Presence of sidewalk only)
Parking + Sidewalk	No = 0 Yes = 1	No = 26 Yes = 6 (Presence of parked vehicles + sidewalk)
Speed (km/h)	All observations	Before: mean=41.97; std.=9.33 After: mean=40.41; std.=8.45
Headways (s)	All observations	Before: mean=23.09; median=11.66; std.=33.44 After: mean=23.37; median=12.67; std.=48.80
Flow rate (veh/h/ln)	All observations	Before: mean=142; std.=72.4; max=305; min=18 After: mean=158; std.=72.5; max=364; min=48

Figure 2 presents the headway and speed distributions for all observations with 50 km/h PSL. Similarly, Figure 3 depicts the headway and speed histograms for all observations after the speed limit reduction to 40 km/h. It is important to point out that the data collection aimed at capturing free flow vehicles from as many sites as possible with varying geometric conditions. The speed measurements took place under light to moderate traffic, which is appropriate for free flow estimation. As a result, the range of the speed data collected is from 20 to 70 km/h (See Figures 2b and 3b)).

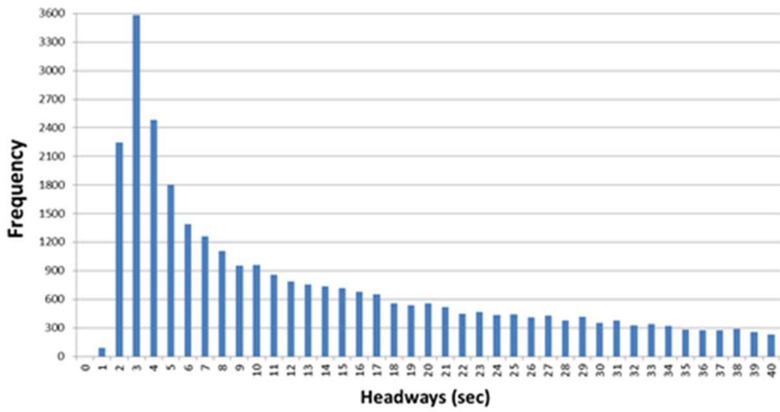


(a)

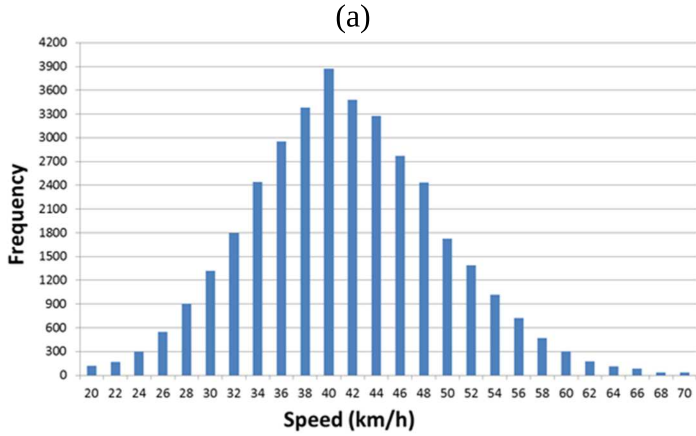


(b)

Figure 2 Speed and headway histograms (all observations - 50 km/h PSL)



(a)



(b)

Figure 3 Speed and headway histograms (all observations - 40 km/h PSL)

4.2 Results

The Gauss econometric package (Aptech System, 1994) was used for the simultaneous estimation of the model parameters. The analysis is based on the speed and headway observations dataset composed of the 32 sites before and after the speed limit reduction from 50 km/h to 40 km/h.

A. Before speed limit change (50 km/h PSL)

Table 2 summarizes the results of the simultaneous model estimation. According to the results, the main variables that impact the free flow speed are land use, the presence of parking, and the presence of sidewalks. The probability of the constrained state is mainly impacted by the time headway and the relative speed between successive vehicles, but only when the space headway is less than 60 m. For longer space headways, the impact was not significant. All the parameters are significant at the 95% confidence level and with the expected sign. The constant of the mean free flow speed, α_0, is 46.02 km/h, and the standard deviation, σ_{ff}, 7.76 km/h.

TABLE 2 Parameter estimates before speed limit change (PSL=50 km/h)

Description			**Estimate**	***t*-statistic**
Free flow speed distribution				
Free Flow Constant (km/h)		α_0	46.02	227.42
Land Use	(Centre=0; Suburb=1)	α_1	5.56	36.38
Parking-only	(Yes=1; No=0)	α_2	-8.58	-17.85
Sidewalk-only	(Yes=1; No=0)	α_3	-5.52	-39.21
Parking + Sidewalk	(Yes=1; No=0)	α_4	-9.35	-47.19
Free flow Standard deviation		σ_{ff}	7.76	7.73
Constrained speed distribution				
Constrained Standard deviation		σ_{cf}	4.51	4.47
Probability to be constrained				
Constant		β_0	4.7129	28.81
Time Headway (s)		β_1	-0.6934	-22.87
Relative Speed (m/s) ($S_n - S_{n-1}$) (for Space Headway < 60m)		β_2	-1.3361	-3.48
Number of sites and observations				
No. Observations			31019	
No. Sites			32	
Mean *LL(β)* per observation			-3.35754	

Variable *Relative Speed* $(S_n - S_{n-1})$ *for Space Headway < 60m* is an interaction variable defined as:

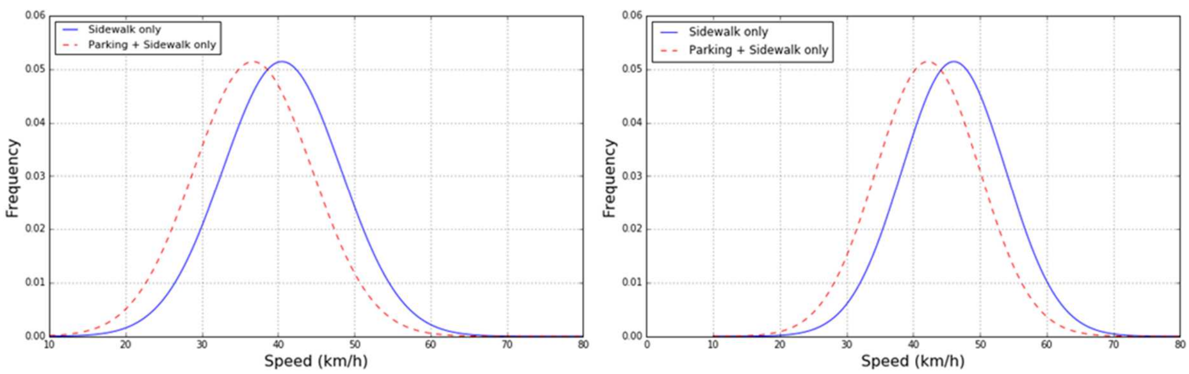
$$(S_n - S_{n-1}) \cdot (Space_{headway} < 60m) \tag{18}$$

Where $Space_{headway}$ is a dummy variable,

$$Space_{headway} = \begin{cases} 1 & if\ space\ headway < 60m \\ 0 & otherwise \end{cases} \tag{19}$$

Free flow speed distribution and impact of street characteristics

Figure 4 shows the impact of the presence of parking and sidewalk classified by land use on the free flow speed distribution before the speed limit change. Figure 4(a) shows the results on city center roads, the solid line shows the impact of sidewalk-only on the free flow speed, while the dashed line shows the impact of the interaction of parking and sidewalk presence. Figure 4(b) shows the speed results in suburb areas which are higher compared to city center roads. City center roads are normally characterized by the presence of sidewalks, parking, pedestrians, cyclists and shops. The mean free flow speed on a city center road with the presence of sidewalk-only, is 40.5 km/h and the impact of parking and sidewalk combined results in the lowest mean speed of 36.7 km/h. On the other hand, roads located in suburb areas show on average, higher mean free flow speed compared to city center areas. For example, a road in suburb area with the presence of parking-only has a mean speed of 43.0 km/h. The same road with the presence of sidewalk-only has a mean speed equal to 46.1 km/h, and a road with the presence of parking and sidewalk combined show a mean free flow speed equal to 42.2 km/h, while a road located in a suburb area without parking and no sidewalk has a mean free flow speed of 51.6 km/h. Parking has a strong impact, reducing the mean free flow speed by 8.5 km/h. The presence of sidewalk reduces the mean speed as well by 5.5 km/h. The effect of parking and sidewalk combined results in the highest impact reducing the mean free flow speed by 9.35 km/h yet the impact is lower compared to adding parking and sidewalk impacts separately (-14.1 km/h) and this is consistent with our a-priori expectations.



(a) City center areas (b) Suburb areas

Figure 4 Impact of road characteristics (50 km/hr PSL)

Table 3 summarizes the free flow speed for typical urban roads with different road characteristics based on the model estimation results of Table 2.

Table 3 Mean free flow speed for different facility types with PSL 50 km/hr

Land Use	Center	Center	Suburb	Suburb	Suburb	Suburb
Parking-only	No	No	Yes	No	No	No
Sidewalk-only	Yes	No	No	Yes	No	No
Parking + Sidewalk	No	Yes	No	No	Yes	No
Average free flow speed (km/h)	*40.5*	*36.7*	*43.0*	*46.1*	*42.2*	*51.6*

These results are in line with previous research on the impact of road characteristics on the mean free flow speed (Ericsson, 2001; Aronsson and Bang, 2007; Hansen et al., 2007; Wang et al., 2006; Silvano and Bang, 2015). The results clearly indicate that on-street parking, sidewalk presence, and their interaction reduces the mean free flow speed in city centers and suburban areas on urban roads.

Probability to be constrained

According to the results in Table 2, the time headway parameter, β_1, is negative, indicating that shorter time headways result in drivers with higher probabilities of perceiving their state as constrained by the vehicle in front. According to the model specification the relative speed, ΔS, with an estimated value of -1.3361 m/s only impacts the interaction between the subject vehicle and the vehicle in front if the distance between them is less than 60 m (this distance was determined based on separate estimations of the model with different values for the distance threshold).

Figure 5 illustrates the impact of the time headway and relative speed on the probability to be constrained in two cases: (i) the relative speed is zero, reflecting a subject vehicle following at a speed equal to the speed of the vehicle in front and (ii) the relative speed is $-0.55\ m/s$ ($-2\ km/h$) reflecting the case where the subject vehicle is traveling faster by 2 km/h than the vehicle in front. According to the results, in the first case ($\Delta S = 0$), the probability to be in the constrained state is 63% at 6 s time headways. The results also show a critical time headway of 6.8 s, where the probability that a driver perceives the situation as constrained is 50%. The probability of the constrained state is high at headways shorter than 4 sec (>88%) which is a threshold used for car-following regimes in various studies (Bando et al., 1995; Ahmed, 1999; Toledo, 2003), whereas, the probability is under 10% for headways larger than 10 sec. For $\Delta S = -2\ km/h$, the probability to be constrained increases rapidly, since the subject driver travels at a higher speed than the vehicle in front. The probability of the constrained state is high (>88%) up to 5 s headways.

The estimated probabilities show how drivers perceive the traffic situation, as being constrained or not. At short headways, it is most likely that drivers feel constrained (high probability), whereas, at large headways, it is very unlikely that they feel constrained (low probability). However, there is still a proportion of drivers who may feel free at short headways or feel constrained at large headways, as shown by the resulting probabilities.

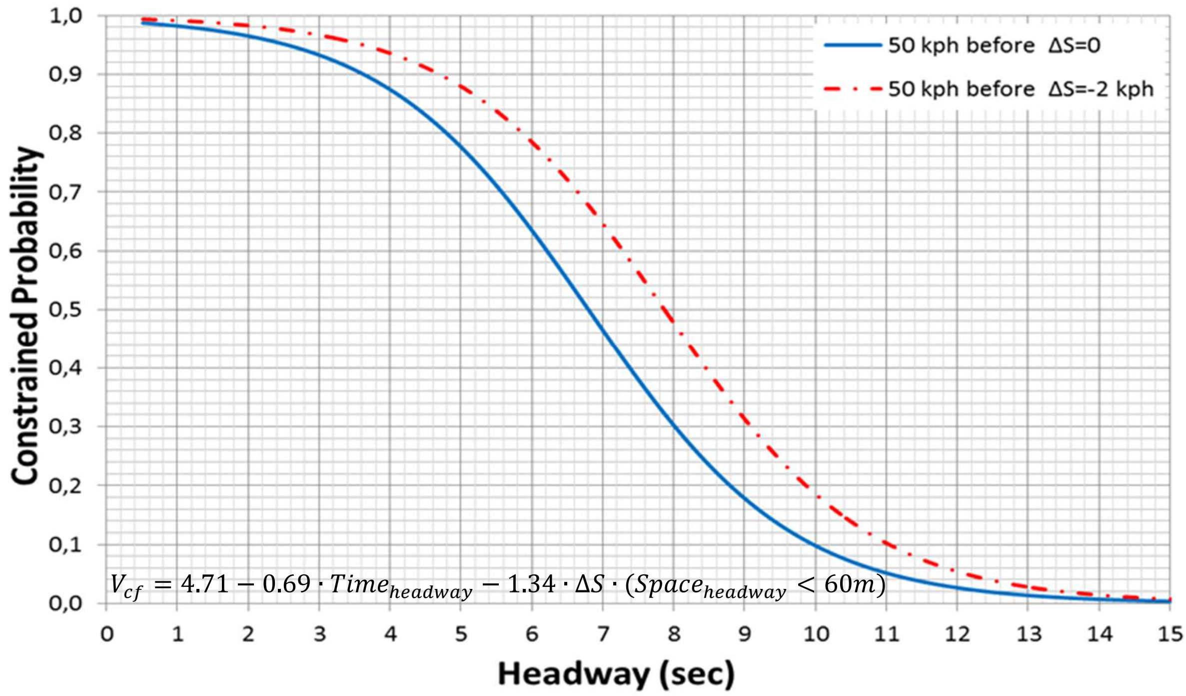


Figure 5 Probability to be constrained (50 km/h PSL)

B. After speed limit change (40 km/h PSL)

The estimation results are summarized in Table 4. In the after case, the constant of the mean free flow speed, α_0, is 45.08 km/h which is reduced by 0.94 km/h compared to the before case in which the speed limit was 50 km/h. The standard deviation, σ_{ff}, is 8.25 km/h which is increased by 0.49 km/h. In general, the results are in line with previous research on the impact of road characteristics on the mean free flow speed (Ericsson, 2001; Aronsson and Bang, 2007; Hansen et al., 2007; Wang et al., 2006; Silvano and Bang, 2015). The impact of the road characteristics is lower compared to the case of PSL of 50 km/h, as expected. However, the impact of parking only is nearly the same with a very small increase of 0.21 km/h in the after case. The highest reduction of the speed limit change is for land use which has been reduced by 4.1 km/h. The presence of sidewalk only has been reduced changing from -5.52 km/h to -3.05 km/h which is a reduction of 2.47 km/h. Similarly, the impact of the interaction of parking and sidewalk has decreased from -9.35 km/h to -6.16 km/h showing a reduction of 3.16 km/h.

TABLE 4 Parameter estimates after speed limit change (40 km/hr PSL)

Description			**Estimate**	***t-statistic***
Free flow speed distribution				
Free Flow Constant (km/hr)		α_0	45.08	238.63
Land Use	(Centre = 0; Suburb=1)	α_1	1.46	12.10
Parking-only	(Yes = 1; No = 0)	α_2	-8.79	-26.64
Sidewalk-only	(Yes = 1; No = 0)	α_3	-3.05	-20.08
Parking + Sidewalk	(Yes = 1; No = 0)	α_4	-6.16	-32.18
Free flow Standard deviation		σ_{ff}	8.25	8.22
Constrained speed distribution				

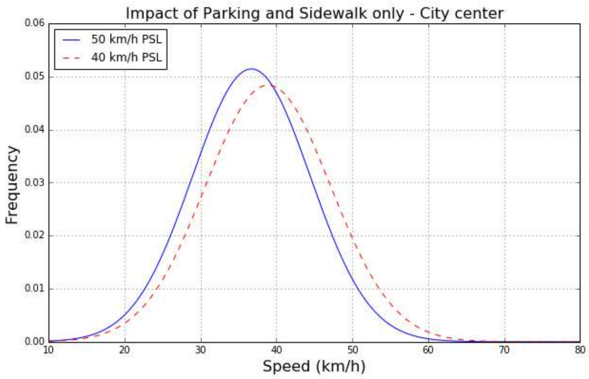
Constrained Standard deviation	σ_{cf}	4.36	4.32
Probability to be constrained			
Constant	β_0	5.3292	28.71
Time Headway (s)	β_1	-0.7503	-23.20
Relative Speed (m/s) $(S_n - S_{n-1})$ (for Space Headway < 60m)	β_2	-1.2692	-4.04
Number of sites and observations			
No. Observations		36058	
No. Sites		32	
Mean *LL(β)* per observation		-3.34007	

Table 5 shows the estimated mean free flow speed for typical streets. The results show that the speed limit change was not an effective measure to reduce speed levels on roads with already low speeds located in city center areas. Silvano and Bang (2015) also found a similar pattern on roads with already low current speeds indicating that the speed limit reduction can result in undesirable impacts increasing the mean speed. According to the results, the mean free flow speed on a road with parking and sidewalk is 38.9 km/h, which is an increase of 2.2 km/h compared to the before case (PSL of 50 km/h). Also, a road with sidewalk-only results in a mean free flow speed of 42.0 km/h which is an increase of 1.5 km/h compared to the higher PSL of 50 km/h. On the other hand, the speed limit reduction did effectively reduce the mean free flow speed on roads located in suburb areas. For instance, a road with parking-only has mean speed equal to 38.1 km/h which is a reduction of nearly 5 km/h compared to the previous speed limit. Similarly, a road with sidewalk-only results in a mean speed of 43.5 km/h showing a reduction of 2.6 km/h. The impact of the interaction of parking and sidewalk in suburb areas results in a mean speed of 40.4 km/h indicating a reduction of 1.8 km/h from the previous speed limit. Furthermore, a road without parking and no sidewalk has mean free flow speed of 46.5 km/h showing a reduction of 5.1 km/h on the mean speed for PSL equal to 50 km/h.

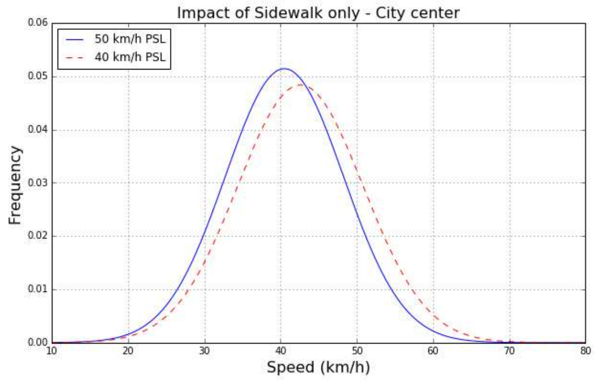
Table 5 Mean free flow speed for different facility types with PSL at 40 km/h

Land Use	Center	Center	Suburb	Suburb	Suburb	Suburb
Parking-only	No	No	Yes	No	No	No
Sidewalk-only	Yes	No	No	Yes	No	No
Parking + Sidewalk	No	Yes	No	No	Yes	No
Average free flow Speed (km/h)	*42.0*	*38.9*	*38.1*	*43.5*	*40.4*	*46.5*

Figure 6 shows the impact of road characteristics and speed limit changes on the free flow speed. For instance, Figure 6(a) shows the impact of parking and sidewalk in city center areas. Similarly, Figure 6(b) shows the impact of sidewalk-only in city center roads. Figure 6(c) shows the impact of sidewalk-only on driving speeds in suburban roads, Figure 6(d) depicts the impact of parking-only, and Figure 6(e) shows the impact of no parking and no sidewalk presence in suburban areas. Figure 6(f) plots the impact of the interaction of parking and sidewalk.



(a)



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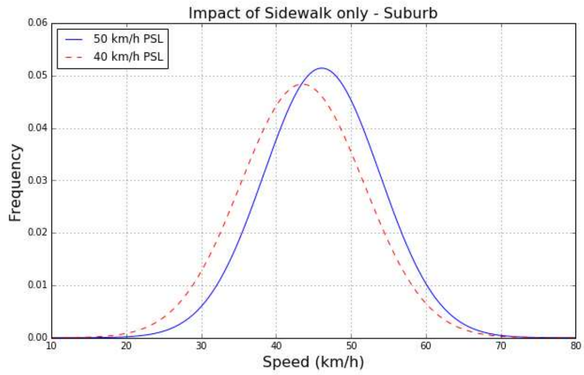

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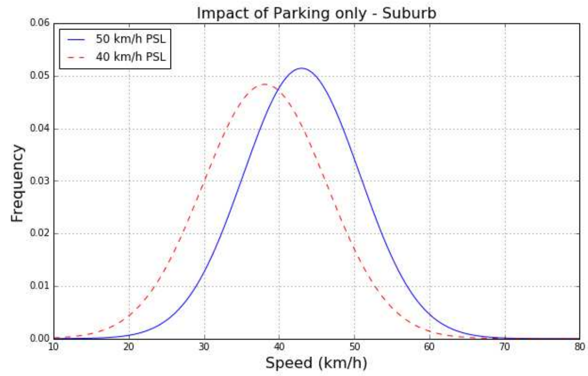

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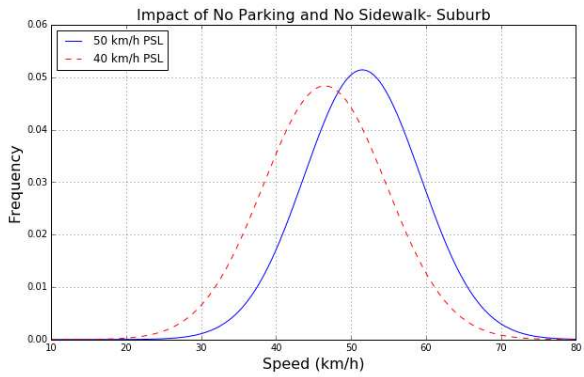

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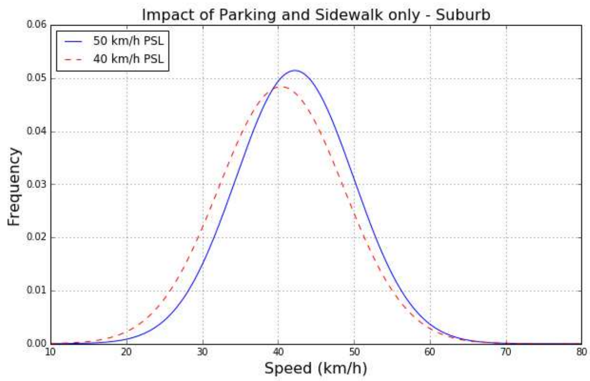

(f)

Figure 6 Impact of speed limit reduction **(**parking, sidewalk, and land use)

The probability that the driver perceives the state as constrained was estimated as well and compared to the different speed limit cases under investigation (i.e., 50 km/h and 40 km/h) and for the two relative speed cases ($\Delta S = 0\ km/h$, and $\Delta S = -2km/h$). According to the results, comparing Table 2 and Table 4, the time headway parameter, β_1, is not very sensitive to the speed limit changes with an estimated value of -0.6934 and -0.7503 for the before and after speed limit change respectively. On the other hand, the constant of the utility function, β_0, increases from a value of 4.7129 for 50 km/h PSL to a value of 5.3292 for speed limit of 40 km/h resulting in higher probabilities of perceiving the state as constrained for the lower speed limit.

Figure 7 compares the probability of the constrained state as a function of the time headway for the two PSL cases and the two relative speed values (0 km/h and –2 km/h). The results for relative speeds equal to zero show that the probability of the constrained state is 63% at 6 s time headways with 50 km/h speed limit. However, at 40 km/h, the constrained probability becomes 70% for the same time headway. In the case of the 40 km/h PSL the critical time headway, where the constrained probability is 50%, is 7.1 s (compared to 6.8 s when the PSL is 50 km/h). The impact of the speed limit change is small but as expected, increasing the constrained probability. Similar observations apply for the case were the relative speed is $\Delta S = -2km/h$.

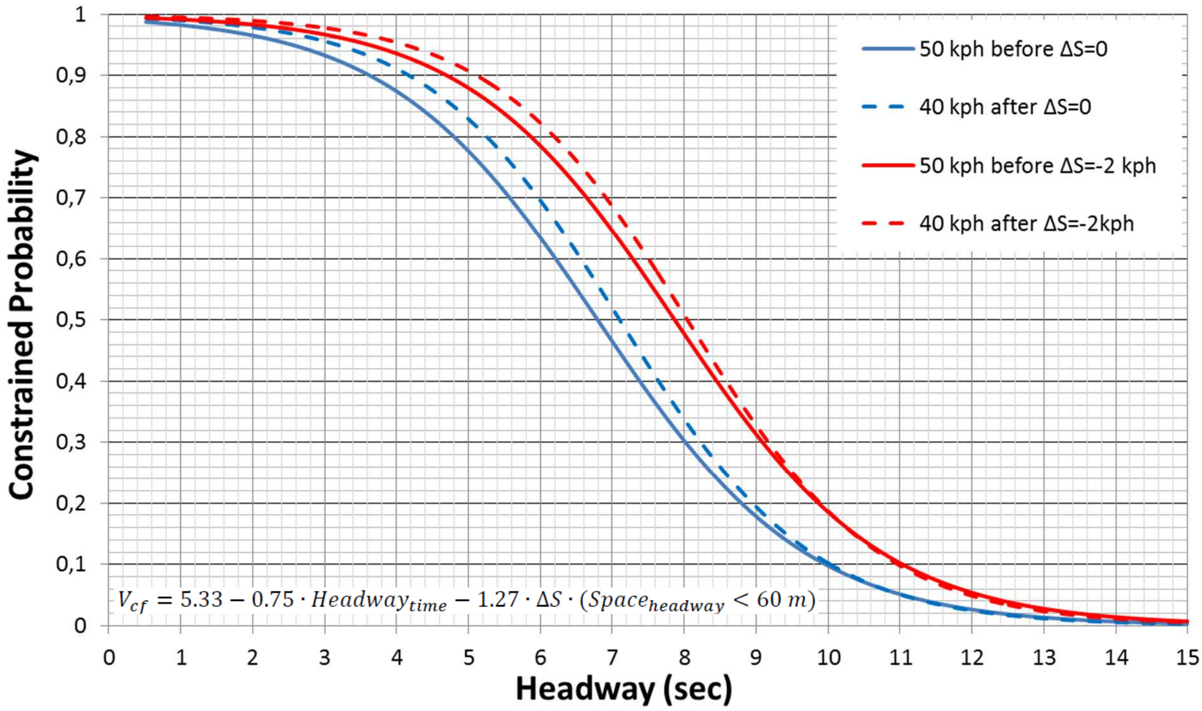

Figure 7 Probability to be constrained and speed limit changes

As discussed in the literature review section of the paper, Vogel (2002) found a deterministic fixed threshold of 6 sec on urban roads to discriminate free flow vehicles from constrained vehicles using data from a single intersection (i.e., site-specific results). Luttinen (1996) suggests that vehicles are free from the influence of the leader at time headways larger than 10 seconds, mainly on rural roads. Furthermore, the author suggests that there is a third group of drivers (called in-transition) from being a leader and becoming a follower pointing out that from 9 to 7 seconds drivers adjust their speed to the speed of the vehicle in front. The results in Figure 7 for $(\Delta S = 0)$ show that at headways larger than 10 seconds the constrained probability is lower than 10% whereas, at headways shorter than 10 seconds the probability increases rapidly with drivers being potentially in-transition state up to 7 seconds. At headways shorter than 4 seconds, the constrained probability is high, as expected.

Buckley (1968) states that a driver in the car-following regime drives at a speed equal on average to the speed of the vehicle in front. This was also assumed in the model proposed in this paper for the case where the subject vehicle was constrained by the vehicle in front. This means that the relative speed $\Delta S = (S_n - S_{n-1})$ follows a distribution with 0 mean. The results in Table 2 and Table 4 show the variations of the estimated standard deviations of the car-following regimes on the speed limit cases. The CF standard deviation before the PSL change is 4.51 km/h which reduces to 4.36 km/h when the speed limit is 40 km/h. The speed limit reduction thus impacts the variability of the relative speeds between the subject and the leading vehicle, as shown in Figure 8 which compares the two distributions.

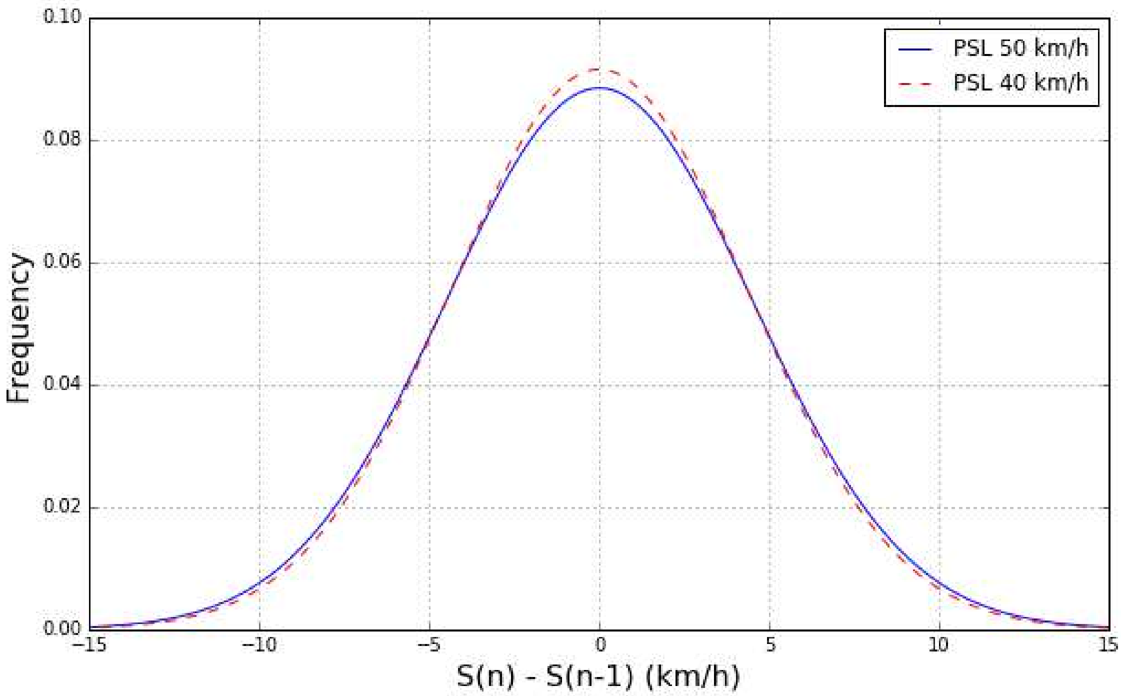

Figure 8 Constrained relative speed distribution

Comparison with fixed threshold results

Table 6 shows the results of the free flow speed estimated by regression models using different threshold values compared to the proposed latent approach. The difference in the estimated values (latent approach vs regression at a predefined threshold) is significant (e.g., Headway > 5 sec, t-stat=5.32). The proposed latent approach shows a higher mean free flow speed (> 1 km/h) compared to the regression models. The latent approach considers all observations in the estimation.

Table 6 Comparison of latent approach vs. regression approach at different thresholds

PSL 50 km/h	Headway threshold					No threshold
	Headway > 4 sec	Headway > 5 sec	Headway > 6 sec	Headway > 7 sec	Headway > 10 sec	Latent approach
Free flow speed (km/h) (land use = 1)	50.27	50.28	50.24	50.25	50.16	51.58

5. CONCLUSION

The estimation of the free flow speed distribution is important for all types of transportation facilities such as freeways, rural, and urban roads since it captures the influence of various factors such as road geometry, traffic regulations, driver preferences, etc. Time headways have been widely used to distinguish between free flow and constrained vehicles but the cut-off point (critical headway threshold) is likely to vary based on the type of the transportation facility and the traffic conditions. Therefore, the main objective and contribution of this study is to propose a probabilistic approach to estimate simultaneously the free flow speed distribution and the probability that drivers perceive their state as constrained, overcoming the limitation of arbitrary cut-off point used in previous studies. At the same time, the proposed methodology is able to incorporate the impact of road geometric characteristics and speed limit changes on the mean of the free flow speed, using an extensive dataset from more than 30 urban sites in Sweden with varying characteristics.

The results show that the mean free flow speed is strongly influenced by several road characteristics such as land use, parking, and the presence of sidewalks. The reduction in the posted speed limit decreases the impact of sidewalk by 2.5 km/h and the impact of the interaction of parking and sidewalk by 3.2 km/h. Land use (center or suburb) shows the highest impact on speed reduction of 4.1 km/h. On the other hand, the impact of parking only is nearly unchanged with the reduction of the posted speed limit. Furthermore, the results show that the speed limit may not be an effective intervention to reduce speed levels on roads with already low speeds (e.g., city center roads), where other factors such as parking intensity, pedestrian and bicyclist flows also impact behavior. The speed limit reduction effectively decreases the mean speed on roads with higher actual speeds (e.g., suburb areas).

The paper has shown that the proposed modeling framework is capable of simultaneously estimating the impact of changes in the road environment e.g., speed limit reduction, on the FFS distribution and the estimation of the constrained probability with reasonable results. The model can also evaluate the impact of road geometric characteristics on the FFS as it is a generic model (i.e., not site-specific) including 32 sites with varying characteristics.

The estimated probabilities show how drivers are most likely to perceive their driving situation, given her/his desired speed. For example, the constrained probability is high (>88%) at headways shorter than 4 s while at headways larger than 10 s the probability is low (<10%). The critical headway, where the constrained probability is 50% is around 7 s. Moreover, the relative speed between the subject and leading vehicle has a strong impact on the constrained probability, as expected.

Future research can benefit from richer datasets that include a variety of traffic conditions as well as more detailed information about parking activity intensity, mid-block crossing pedestrian flows, and cyclist flows, so that their impact on the free flow distribution can be captured more accurately. It will be interesting to estimate the model under mixed conditions, where, for some period of time, because of an incident, conditions have become very congested and evaluate the ability of the model to put low weight in such observations while estimating the free flow speed distribution.

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