



Delft University of Technology

## Local subspace identification of distributed homogeneous systems with general interconnection patterns

Yu, Chengpu; Verhaegen, M

**DOI**

[10.1016/j.ifacol.2015.12.192](https://doi.org/10.1016/j.ifacol.2015.12.192)

**Publication date**

2015

**Document Version**

Accepted author manuscript

**Published in**

IFAC-PapersOnline - 17th IFAC Symposium on System Identification

**Citation (APA)**

Yu, C., & Verhaegen, M. (2015). Local subspace identification of distributed homogeneous systems with general interconnection patterns. In Y. Zhao, E.-W. Bai, & J.-F. Zhang (Eds.), *IFAC-PapersOnline - 17th IFAC Symposium on System Identification* (pp. 585-589). (IFAC-PapersOnline; Vol. 48). IFAC. <https://doi.org/10.1016/j.ifacol.2015.12.192>

**Important note**

To cite this publication, please use the final published version (if applicable).

Please check the document version above.

**Copyright**

Other than for strictly personal use, it is not permitted to download, forward or distribute the text or part of it, without the consent of the author(s) and/or copyright holder(s), unless the work is under an open content license such as Creative Commons.

**Takedown policy**

Please contact us and provide details if you believe this document breaches copyrights.

We will remove access to the work immediately and investigate your claim.

# Local Subspace Identification of Distributed Homogeneous Systems With General Interconnection Patterns

Chengpu Yu \* Michel Verhaegen \*

\* Delft Center for Systems and Control, Delft University, Delft  
2628CD, Netherlands ([c.yu-4@tudelft.nl](mailto:c.yu-4@tudelft.nl), [m.verhaegen@tudelft.nl](mailto:m.verhaegen@tudelft.nl))

**Abstract:** This paper studies the local identification of large-scale homogeneous systems with general network topologies. The considered local system identification problem involves unmeasurable signals between neighboring subsystems. Compared with our previous work in Yu et al. (2014) which solves the local identification of 1D homogeneous systems, the main challenge of this work is how to deal with the general network topology. To overcome this problem, we first decompose the interested local system into separate subsystems using some state, input and output transform, namely the spatially lifted local system has block diagonal system matrices. We subsequently estimate the Markov parameters of the local system by solving a nuclear norm regularized optimization problem. To realize the state-space system model from the estimated Markov parameters, another nuclear norm regularized optimization problem is provided by taking into account of the inherent dependence of a redundant parameter vector. Finally, the overall identification procedure is summarized.

**Keywords:** Subspace identification, nuclear norm, networked systems

## 1. INTRODUCTION

Nowadays, the research on distributed system identification has attracted considerable attention. For a large-scale networked system, it is usually impossible to collect all the system input and output data, thus developing system identification methods that can identify local dynamics using local system input and output measurements becomes essential. In addition, the interconnected signals between neighboring subsystems are generally unobservable, such as the dynamics governed by PDEs, which poses an extra challenge to the local identification problem. This paper contributes to the local identification problem with unmeasurable interconnection signals.

In the literature, a number of identification algorithms for distributed systems have been reported. By parameterizing the system dynamics in terms of transfer functions, an instrumental variable technique is adopted in Ali et al. (2011) to identify distributed identical subsystems and a prediction method for closed-loop identification is implemented in Hof et al. (2013) for the identification of local modules in the network. In the above parameterized methods, the interconnected signals between neighboring subsystems are measurable, thus limiting their applicabilities. In Rice and Verhaegen (2011), the state-space represented dynamical systems are parameterized by exploiting the SSS (Sequential, Semi-Separability) of the system matrices, and the associated identification is dealt with by solving a non-linear (non-convex) optimization problem. One common feature of the above mentioned

prediction error identification methods (PEM) is the non-convex nature in general of the numerical solution.

Compared to the PEM, the subspace approaches can reliably obtain identification results using classic algebraic computations such as QR and SVD decompositions, see Verhaegen and Verdult (2007). When the concerned distributed and decomposable system has a circulant interconnection pattern, it is shown in Massioni and Verhaegen (2008) that the whole system can be decomposed into separate subsystems by some state, input and output transform. The overall system identification can then be performed by parallel identification of the individual subsystems. As an extension, the distributed identification under general network topologies is studied in Massioni et al. (2009). In this work, the associated state-space realization is accomplished by solving a Bilinear Matrix Inequality (BMI) problem; thus it is hard to ensure the global optimality of the solution. Since the above identification approaches require some global state, input and output transform, they cannot be scaled to the identification of large-scale systems.

When the interconnection signals are unmeasurable and only the local system input and output measurements are available, a subspace identification method is proposed in Haber and Verhaegen (2014) which approximates the unobservable neighboring states using a linear combination of inputs and outputs of a local neighborhood of subsystems, and the identification performance relies on the selection of that neighborhood. To avoid the neighborhood selection, a nuclear norm optimization based approach is presented in Matni and Rantzer (2014) by exploiting the low-order local dynamics and high-order global dynamics.

\* The work of C. Yu and M. Verhaegen is sponsored by the European Research Council, Advanced Grant Agreement No. 339681.

In this work, the neighboring states are considered to be energy-bounded disturbances and it only identifies impulse response elements without state-space realization. In our previous work Yu et al. (2014), the local system identification is handled by solving a nuclear norm regularized optimization problem which is formed by exploiting the structure and low rank properties of the terms in the data equation. Since this solution does not require any prior knowledge of the energy upper bound of the unmeasurable neighboring states, it can perform well under either weakly or strongly coupled networked systems.

In Yu et al. (2014), the local identification of 1D distributed homogeneous systems is considered and the corresponding solution relies on the block tri-diagonal properties of the system matrices. As a consequence, it cannot be straightforwardly applied to the identification of networked systems with general network topologies. Inspired by identification algorithm in Yu et al. (2014) and the properties of decomposable systems in Massioni et al. (2009), we propose a nuclear norm optimization based subspace identification in this paper. The spatially lifted local system is transformed into another state-space model with block diagonal system matrices by some state, input and output transform. Following the local system identification procedures shown in Yu et al. (2014), the estimation of Markov parameters and system matrices is carried out by solving a nuclear norm regularized optimization problem.

The paper is organized as follows. Section 2 describes the local identification problem of large-scale homogeneous systems. Section 3 proposes a subspace identification method which estimates the Markov parameters first, followed by the system realization. Section 4 summarizes the whole system identification approach, followed by the conclusions in Section 5.

## 2. PROBLEM FORMULATION

The considered networked system consists of a large number of identical subsystems, with the  $i$ -th subsystem  $\Sigma_i$  having the following dynamics:

$$\begin{aligned} x_i(k+1) &= A_a x_i(k) + A_b \sum_{j \in \mathcal{N}_i} x_j(k) + B u_i(k) \\ y_i(k) &= C x_i(k) + w_i(k), \end{aligned} \quad (1)$$

where  $x_i(k) \in \mathbb{R}^{n \times 1}$ ,  $u_i(k) \in \mathbb{R}^{m \times 1}$ ,  $w_i(k) \in \mathbb{R}^{p \times 1}$  and  $y_i(k) \in \mathbb{R}^{p \times 1}$  are the state, input, measurement noise and output of the  $i$ -th subsystem,  $\mathcal{N}_i$  denotes the set of the neighboring subsystems of the  $i$ -th subsystem.

In the networked system description in (1), we assume that  $C$  is a flat matrix, namely  $p \leq n$ . Otherwise, if  $C$  has a full column rank, the associated state can be represented in terms of the system output, and the local system identification boils down to the identification of an errors-in-variables (EIV) model which can be solved by many classic methods, see Chou and Verhaegen (1997); Verhaegen and Verdult (2007). In addition, we assume that the associated network topology is bidirectional, which is common in state-space represented systems governed by PDEs.

The problem of interest is to identify the system matrices  $C, A_a, A_b, B$  up to a similarity transform given the local

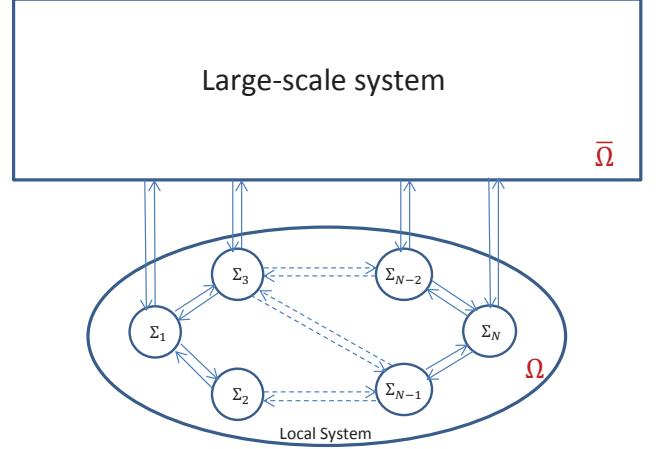


Fig. 1. Diagram of a local system in a large-scale network.

system input and output data and the local interconnection pattern, where the local system refers to the system contained in the ellipse in Fig. 1.

## 3. IDENTIFICATION METHOD

Denote by  $\Omega = \{\Sigma_1, \Sigma_2, \dots, \Sigma_N\}$  the set of subsystems in the local system and  $\bar{\Omega}$  the set of subsystems outside the local system, as shown in Fig. 1. Let  $\mathcal{N}_\Omega$  be the set of neighboring subsystems of  $\Omega$ , namely the subsystems in  $\mathcal{N}_\Omega$  are directly connected to the local system. It follows that  $\mathcal{N}_\Omega \subset \bar{\Omega}$ . The spatially lifted state-space form of the local networked system, which consists of subsystems  $\{\Sigma_i\}_{i=1}^N$  as shown in Fig. 1, can be written as

$$\begin{aligned} x(k+1) &= (I \otimes A_a + P \otimes A_b) x(k) + (R \otimes A_b) v(k) \\ &\quad + (I \otimes B) u(k) \\ y(k) &= (I \otimes C) x(k) + w(k), \end{aligned} \quad (2)$$

where  $x(k)$ ,  $u(k)$ ,  $y(k)$  and  $w(k)$  are the spatially lifted state, input, output and measurement noise, respectively. For example, for the local system illustrated in Fig. 1,  $x(k)$  is defined as  $x(k) = [x_1^T(k) \ x_2^T(k) \ \dots \ x_N^T(k)]^T$ .  $v(k)$  is an external signal for the local system which is stacked by states  $\{x_i(k)\}_{i \in \mathcal{N}_\Omega}$ . The matrix  $P$  is the pattern matrix of the local system while  $R$  is a pattern matrix describing the interconnection pattern between the subsystems in  $\Omega$  and  $\bar{\Omega}$ . It is noteworthy that the neighboring state vector  $v(k)$  in the above system equation is unavailable.

Since the system matrices in (2) have no sparse or banded structures, the identification problem seems to be challenging. However, we can observe that the local system model (2) without the unknown system input term is a decomposable system model (see Massioni et al. (2009)); hence, it can be transformed into another state-space model with block diagonal system matrices by some state, input and output transform.

*Lemma 1.* Let  $P = U \Lambda U^T$  with  $U$  an orthogonal matrix and  $\Lambda$  a real diagonal matrix. The decomposable system in (2) can be equivalently transformed into:

$$\begin{aligned}\hat{x}(k+1) &= \mathbf{A}\hat{x}(k) + \underbrace{(U^T R \otimes A_b)}_{\mathbf{R}} v(k) + \underbrace{(I \otimes B)}_{\mathbf{B}} \hat{u}(k) \\ \hat{y}(k+1) &= \underbrace{(I \otimes C)}_{\mathbf{C}} \hat{x}(k) + \hat{w}(k),\end{aligned}\tag{3}$$

where  $\hat{x}(k) = (U^T \otimes I)x(k)$ ,  $\hat{u}(k) = (U^T \otimes I)u(k)$ ,  $\hat{w}(k) = (U^T \otimes I)w(k)$  and  $\hat{y}(k) = (U^T \otimes I)y(k)$ . The system matrix  $\mathbf{A}$  is block diagonal and has the following forms:  $\mathbf{A} = I \otimes A_a + \Lambda \otimes A_b$ .

The above lemma can be easily derived following the results in Massioni et al. (2009).

### 3.1 Estimation of the Markov parameters

In (3), the system matrices  $\mathbf{A}, \mathbf{B}, \mathbf{C}$  are block diagonal except the matrix  $\mathbf{R}$ . The associated data equation of (3) can be written as

$$\hat{Y}_{s,r} = \mathcal{O}_s \hat{X}_r + \mathbf{T}_{u,s} \hat{U}_{s,r} + \mathbf{T}_{v,s} \hat{V}_{s,r} + \hat{W}_{s,r}, \tag{4}$$

$$\text{where } \hat{Y}_{s,r} = \begin{bmatrix} \hat{y}(1) & \hat{y}(2) & \cdots & \hat{y}(r) \\ \hat{y}(2) & \hat{y}(3) & \ddots & \hat{y}(r+1) \\ \vdots & \ddots & \ddots & \vdots \\ \hat{y}(s) & \hat{y}(s+1) & \cdots & \hat{y}(T) \end{bmatrix} \text{ with the sub-}$$

scripts  $s, r$  representing the numbers of vertical and horizontal blocks, respectively.  $\hat{U}_{s,r}$  and  $\hat{W}_{s,r}$  have the same

$$\text{structure as } \hat{Y}_{s,r}. \mathcal{O}_s = \begin{bmatrix} \mathbf{C} \\ \mathbf{CA} \\ \vdots \\ \mathbf{CA}^{s-1} \end{bmatrix} \text{ is the extended ob-}$$

servability matrix.  $\hat{X}_r = [\hat{x}(1) \cdots \hat{x}(r)]$  is a concatenat-

$$\text{ed state sequence. } \mathbf{T}_{u,s} = \begin{bmatrix} 0 & & & \\ \mathbf{CB} & 0 & & \\ \vdots & \ddots & \ddots & \\ \mathbf{CA}^{s-2}\mathbf{B} & \cdots & \mathbf{CB} & 0 \end{bmatrix} \text{ and}$$

$$\mathbf{T}_{v,s} = \begin{bmatrix} 0 & & & \\ \mathbf{CR} & 0 & & \\ \vdots & \ddots & \ddots & \\ \mathbf{CA}^{s-2}\mathbf{R} & \cdots & \mathbf{CR} & 0 \end{bmatrix}.$$

It is worth noting that the term  $\mathcal{O}_s \hat{X}_r$  in (4) has low rank and the term  $\mathbf{T}_{v,s} \hat{V}_{s,r}$  is totally unknown. Next, we shall show that the sum  $\mathcal{O}_s \hat{X}_r + \mathbf{T}_{v,s} \hat{V}_{s,r}$  has low rank under some mild conditions.

*Lemma 2.* Denote by  $|\Omega|$  the number of subsystems in  $\Omega$ , and  $|\mathcal{N}_\Omega|$  for the set  $\mathcal{N}_\Omega$ . The rank of the sum  $\mathcal{O}_s \hat{X}_r + \mathbf{T}_{v,s} \hat{V}_{s,r}$  satisfies that

$$\text{rank}(\mathcal{O}_s \hat{X}_r + \mathbf{T}_{v,s} \hat{V}_{s,r}) \leq |\Omega|n + |\mathcal{N}_\Omega|(s-1)n.$$

From the above lemma, we can find that the sum  $\mathcal{O}_s \hat{X}_r + \mathbf{T}_{v,s} \hat{V}_{s,r}$  has a lower rank with relation to  $\hat{Y}_{s,r}$  if  $|\Omega| \gg |\mathcal{N}_\Omega|$ , namely there are much more subsystems inside the local system than its neighboring subsystems in  $\mathcal{N}_\Omega$ .

In the sequel, we denote  $N = |\Omega|$ . By combining the N2SID method in Verhaegen and Hansson (2014) and the low rank

property of the sum  $\mathcal{O}_s \hat{X}_r + \mathbf{T}_{v,s} \hat{V}_{s,r}$ , we can derive the following nuclear norm regularized optimization problem

$$\min_{\tilde{Y}_{s,r} \in \mathcal{H}_{s,r}, \mathbf{T}_{u,s} \in \mathcal{T}_{u,s}} \sum_{k=1}^T \|\hat{y}(k) - \tilde{y}(k)\|_F^2 + \alpha \|\tilde{Y}_{s,r} - \mathbf{T}_{u,s} \hat{U}_{s,r}\|_*, \tag{5}$$

where  $\alpha$  is a regularization parameter.  $\mathcal{H}_{s,r}$  and  $\mathcal{T}_{u,s}$  are the sets of block Hankel and Toeplitz matrices having the same structures of  $\hat{Y}_{s,r}$  and  $\mathbf{T}_{u,s}$ , respectively.  $\tilde{Y}_{s,r}$  is a Hankel matrix constructed by  $\{\tilde{y}(k)\}_{k=1}^T$ , which has the same structure as  $\hat{Y}_{s,r}$ .

By exploring the structure of  $\mathbf{T}_{u,s}$  in (4), we can see that its block entries are further block diagonal matrices. More specially, the block entry  $\mathbf{CA}^i \mathbf{B}$  can be explicitly written as

$$\mathbf{CA}^i \mathbf{B} = \begin{bmatrix} C(A_a + \lambda_1 A_b)^i B \\ \vdots \\ C(A_a + \lambda_N A_b)^i B \end{bmatrix}.$$

When solving the optimization problem in (5), the above finer structures of  $\mathbf{T}_{u,s}$  are imposed as constraints. Fortunately, adding finer structure constraints does not affect the convexity of the optimization problem. Thus, the Markov parameters  $C(A_a + \lambda_i A_b)^j B$  for  $1 \leq i \leq N, 0 \leq j \leq s-2$  can be reliably obtained by solving (5).

### 3.2 Determine system matrices

After obtaining the Markov parameters, the realization of state-space model in (1) will be investigated. To cope with this problem, Massioni et al. (2009) proposes a new method which is to solve a BMI problem. Due to the lack of convexity, the obtained solution is likely to have local optima. In this subsection, we shall develop a convex solution for the realization problem.

For notational simplicity, we shall demonstrate the system realization approach using Markov parameters up to the fourth moment, i.e.  $\{C(A_a + \lambda_i A_b)^j B\}_{i=1, j=0}^{N, 4}$ . For any fixed  $j$ , we can see that  $C(A_a + \lambda_i A_b)^j B$  can be expressed by a linear combination of the parameters in the set  $\underline{CA}_1 A_2 \cdots A_j B$  with  $A_l \in \{A_a, A_b\}$  for  $1 \leq l \leq j$ .

$$\text{Let } \phi = \begin{bmatrix} CB \\ CA_a B \\ CA_b B \\ CA_a^2 B \\ CA_a A_b B \\ CA_b A_a B \\ CA_a^2 B \\ \vdots \\ CA_b^4 B \end{bmatrix}$$

be the parameter vector. Stacking

all the estimated Markov parameters together yields an

$$\text{augmented vector } \psi = \begin{bmatrix} CB \\ C(A_a + \lambda_1 A_b)B \\ \vdots \\ C(A_a + \lambda_N A_b)B \\ C(A_a + \lambda_1 A_b)^2 B \\ \vdots \\ C(A_a + \lambda_N A_b)^4 B \end{bmatrix}.$$

We can then find a matrix  $H$  such that

$$H\phi = \psi. \quad (6)$$

In the above equation,  $H$  and  $\psi$  are available, while  $\phi$  is to be estimated. Although  $H$  might be a tall matrix, equation (6) is generally under-determined.

*Lemma 3.* In equation (6), the coefficient matrix  $H$  has a rank satisfying:

$$\text{rank}(H) \leq 15p, \quad (7)$$

where the equality holds when the pattern matrix  $P$  has more than 5 different eigenvalues.

**Proof.** In the proof, we first consider the Markov parameters  $\{C(A_a + \lambda_i A_b)^j B\}$  up to the second moment. The corresponding linear estimation equation can be written as

$$\begin{aligned} & \left[ \begin{array}{c|c} I & \\ \hline I & \lambda_1 I \\ I & \lambda_2 I \\ \vdots & \dots \\ I & \lambda_N I \end{array} \right] \left[ \begin{array}{c} CB \\ CA_a B \\ CA_b B \\ CA_a^2 B \\ CA_a A_b B \\ CA_b A_a B \\ CA_b^2 B \end{array} \right] \\ & H \left[ \begin{array}{c} CB \\ C(A_a + \lambda_1 A_b)B \\ C(A_a + \lambda_2 A_b)B \\ \dots \\ C(A_a + \lambda_N A_b)B \\ C(A_a + \lambda_1 A_b)^2 B \\ C(A_a + \lambda_2 A_b)^2 B \\ \dots \\ C(A_a + \lambda_N A_b)^2 B \end{array} \right] \\ & = \left[ \begin{array}{c} CB \\ C(A_a + \lambda_1 A_b)B \\ C(A_a + \lambda_2 A_b)B \\ \dots \\ C(A_a + \lambda_N A_b)B \\ C(A_a + \lambda_1 A_b)^2 B \\ C(A_a + \lambda_2 A_b)^2 B \\ \dots \\ C(A_a + \lambda_N A_b)^2 B \end{array} \right], \end{aligned}$$

where  $I$  has size  $p \times p$ . From the structure of  $H$  we can find that: when  $\{\lambda_i\}_{i=1}^N$  contains at least 3 different elements, it has that  $\text{rank}(H) = (1 + 2 + 3)p = 6p$ . By induction, the coefficient matrix  $H$ , corresponding to the linear estimation using the Markov parameters up to the fourth moment, has the following rank property:  $\text{rank}(H) \leq (1 + 2 + 3 + 4 + 5)p = 15p$ , where the equality holds when there are more than 5 different elements in the set  $\{\lambda_i\}_{i=1}^N$ , namely  $P$  has more than 5 different eigenvalues.

From the above lemma, we can see that the ill condition of the linear estimation problem in (6) cannot be resolved by including more subsystems in the local system. As long as the pattern matrix  $P$  has more than 5 different eigenvalues, the matrix  $H$  can reach its maximum rank.

By taking account of the displacement structure of the parameter vector  $\phi$ , we can find that the following matrix constructed by the components of  $\phi$  is of low rank

$$\begin{aligned} \Gamma(\phi) &= \begin{bmatrix} CB & CA_a B & CA_b B & \dots & CA_b^2 B \\ CA_a B & CA_a^2 B & CA_a A_b B & \dots & CA_a A_b^2 B \\ CA_b B & CA_b A_a B & CA_b^2 B & \dots & CA_b^3 B \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ CA_b^2 B & CA_b^2 A_a B & CA_b^3 B & \dots & CA_b^4 B \end{bmatrix} \\ &= \begin{bmatrix} C \\ CA_a \\ CA_b \\ \vdots \\ CA_b^2 \end{bmatrix} \begin{bmatrix} B & A_a B & A_b B & \dots & A_b^2 B \end{bmatrix}. \end{aligned} \quad (8)$$

By combining the under-determined equation (6) and the low rank property of the matrix  $\Gamma(\phi)$ , we can derive the following nuclear norm regularized optimization problem

$$\min_{\phi} \|H\phi - \psi\|_F^2 + \beta \|\Gamma(\phi)\|_*, \quad (9)$$

where  $\beta$  is a regularization parameter to trade off the least-square term and the low-rank term.

Solving the optimization problem (9) yields the estimates of  $\phi$  and  $\Gamma(\phi)$ . Taking the SVD decomposition of  $\Gamma(\phi)$  yields that

$$\Gamma_\phi = [U_s \ U_n] \begin{bmatrix} \Sigma_s & \\ & \Sigma_n \end{bmatrix} \begin{bmatrix} V_s^T \\ V_n^T \end{bmatrix}, \quad (10)$$

where  $U_s \in \mathbb{R}^{7p \times n}$  and  $V_s \in \mathbb{R}^{7m \times n}$  are partial orthogonal matrices,  $\Sigma_s \in \mathbb{R}^{n \times n}$  and  $\Sigma_n$  are diagonal matrices with the nonzero entries of  $\Sigma_s$  being larger than those of  $\Sigma_n$ .

Let  $\underline{Q} = U_s$  and  $\underline{C} = \Sigma_s V_s^T$ . According to the structure of  $\Gamma(\phi)$ , we can establish that

$$\underline{Q} = \begin{bmatrix} C \\ CA_a \\ CA_b \\ CA_a^2 \\ CA_a A_b \\ CA_b A_a \\ CA_b^2 \end{bmatrix} \Pi \quad (11)$$

and

$$\underline{C} = \Pi^{-1} \begin{bmatrix} B & A_a B & A_b B & A_a^2 B & A_a A_b B & A_b A_a B & A_b^2 B \end{bmatrix} \quad (12)$$

where  $\Pi \in \mathbb{R}^{n \times n}$  is a nonsingular ambiguity matrix.

Then the estimates of  $C$  and  $B$  can be obtained as follows

$$\begin{aligned} \hat{C} &= \underline{Q}(1:p,:), \\ \hat{B} &= \underline{C}(:,1:m). \end{aligned} \quad (13)$$

In addition, the estimates of  $A_a$  and  $A_b$  can be estimated as

$$\begin{aligned} \hat{A}_a &= \begin{bmatrix} \underline{Q}(1:p,:) \\ \underline{Q}(p+1:2p,:) \\ \underline{Q}(2p+1:3p,:) \end{bmatrix}^\dagger \begin{bmatrix} \underline{Q}(p+1:2p,:) \\ \underline{Q}(3p+1:4p,:) \\ \underline{Q}(5p+1:6p,:) \end{bmatrix}, \\ \hat{A}_b &= \begin{bmatrix} \underline{Q}(1:p,:) \\ \underline{Q}(p+1:2p,:) \\ \underline{Q}(2p+1:3p,:) \end{bmatrix}^\dagger \begin{bmatrix} \underline{Q}(2p+1:3p,:) \\ \underline{Q}(4p+1:5p,:) \\ \underline{Q}(6p+1:7p,:) \end{bmatrix}. \end{aligned} \quad (14)$$

One inherent condition for the above system realization

is that  $\begin{bmatrix} C \\ CA_a \\ CA_b \end{bmatrix}$  has a full column rank. If we estimate the individual system matrices by adopting the Markov parameters  $C(A_a + \lambda_i A_b)^j B$  up to a higher moment, this inherent condition can be relaxed.

#### 4. SUMMARY OF THE IDENTIFICATION ALGORITHM

The developed local system identification method for large-scale homogeneous systems can be executed in three steps: (a) take a state, input and output transform according to the SVD decomposition of the local pattern matrix; (b) estimate Markov parameters under local system input and output data; (c) realize the state-space system model of a single subsystem. To ease the reference, the identification algorithm is summarized in **Algorithm 1**.

<b>Algorithm 1:</b> Local system identification of large-scale systems	
1)	Take the state, input and output transform according to (3);
2)	Estimate the Markov parameters $\{C(A_a + \lambda_i A_b)^j B\}_{i=1,j=0}^{N,s-2}$ by solving the nuclear-norm optimization problem in (5);
3)	Obtain the estimates of $\phi$ and $\Gamma(\phi)$ by solving the optimization problem in (9);
4)	Compute the SVD decomposition of $\Gamma(\phi)$ shown in (10);
5)	Determine $B$ and $C$ as shown in (13);
	Estimate $A_a$ and $A_b$ according to (14).

Since there are no specific constraints of the network topology, the proposed local system identification method possesses a wide range of applications. Specifically, the two dimensional homogeneous system is special case of distributed systems with general network topologies, so its associated identification problem can be tackled by the above presented method. In addition, for the developed identification method, it only requires the local system to be homogeneous; hence, it can be applied to the large-scale systems with distributed clusters where the cluster dynamics may be different from each other.

The developed identification algorithm is realized by solving two nuclear norm regularized optimization problems in (5) and (9). It can deal with the identification using input and output data with short lengths. For the regularization parameters  $\lambda$  and  $\beta$ , they can be empirically chosen using the cross-validation method described in Ljung (1999); Verhaegen and Verdult (2007).

#### 5. CONCLUSION

This paper has presented a subspace algorithm for the local identification of large-scale homogeneous systems with general network topologies. The crucial step in dealing with the general topology is to transform the original spatially lifted state-space system model into an equivalent one with block diagonal system matrices by taking some state, input and output transform. By taking account of the finer structures and low rank properties of the terms in the data equation, the associated Markov parameters using only local input and output data have been reliably obtained by solving a nuclear norm regularized optimization problem. One condition for the above operation is that the neighboring subsystems should be much less than the local subsystems. Further, a convex solution has been provided for the realization of the state-space model.

In this paper, the local interconnection pattern is assumed to be known. In our future work, how to detect the connections among local subsystems and further carry out local subspace identification will be investigated.

#### REFERENCES

- Ali, M., Ali, A., Chughtai, S., and Werner, H. (2011). Consistent identification of spatially interconnected systems. In *American Control Conference (ACC), 2011*, 3583–3588.
- Chou, C. and Verhaegen, M. (1997). Subspace algorithms for the identification of multivariable dynamic errors-in-variables models. *Automatica*, 33(10), 1857 – 1869.
- Haber, A. and Verhaegen, M. (2014). Subspace identification of large-scale interconnected systems. *Automatic Control, IEEE Transactions on*, 59(10), 2754–2759.
- Hof, P.M.V.D., Dankers, A., Heuberger, P.S., and Bombois, X. (2013). Identification of dynamic models in complex networks with prediction error methodsbasic methods for consistent module estimates. *Automatica*, 49(10), 2994 – 3006.
- Ljung, L. (1999). *System Identification: Theory for the user (Second Edition)*. Prentice Hall PTR.
- Massioni, P., Fraanje, R., and Verhaegen, M. (2009). Adaptive optics application of distributed control design for decomposable systems. In *Proceedings of the 48th IEEE Conference on Decision and Control*, 7113–7118.
- Massioni, P. and Verhaegen, M. (2008). Subspace identification of circulant systems. *Automatica*, 44(11), 2825 – 2833.
- Matni, N. and Rantzer, A. (2014). Low-rank and low-order decompositions for local system identification. *arXiv:1403.7175*.
- Rice, J. and Verhaegen, M. (2011). Distributed control in multiple dimensions: A structure preserving computational technique. *Automatic Control, IEEE Transactions on*, 56(3), 516–530.
- Verhaegen, M. and Hansson, A. (2014). Nuclear norm subspace identification (n2sid) for short data batches. *CoRR abs/1401.4273*.
- Verhaegen, M. and Verdult, V. (2007). *Filtering and System Identification: A Least Squares Approach*. Cambridge University Press.
- Yu, C., Verhaegen, M., , and Hansson, A. (2014). Subspace identification of local 1d homogeneous systems. *Submitted to 17th IFAC Symposium on System Identification, Beijing*.