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HATT, Hydraulic and Astronomic Tidal Training
User manual

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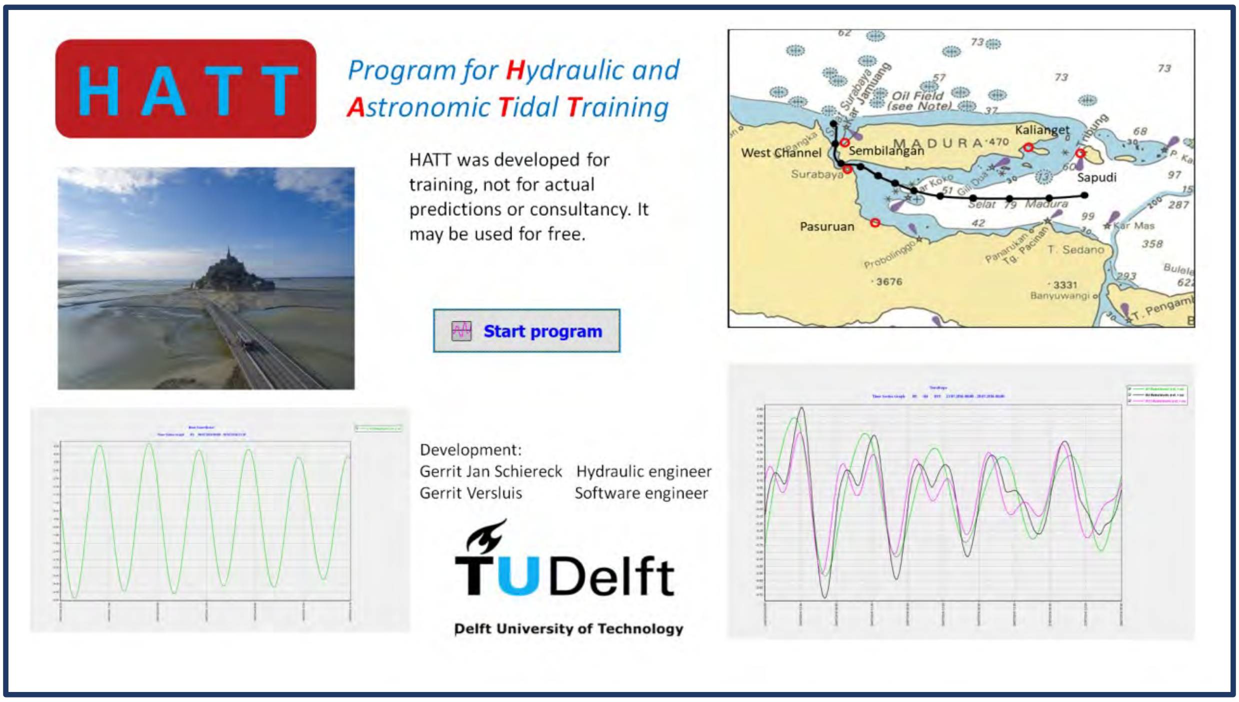
HATT

Hydraulic and Astronomic Tidal Training



Front screen HATT

MANUAL

- ***Instructions for use to exercise with tides in ports, bays and rivers***
- ***Theoretical backgrounds***

HATT, Hydraulic and Astronomic Tidal Training

User manual

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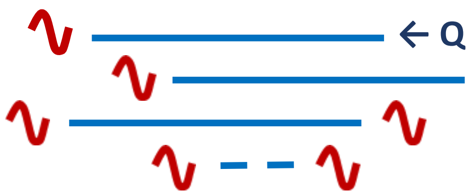
CONTENT

Note: When graphs in this manual are not easily legible (e.g. numbers too small), you can always use HATT to read them on your screen.

1. Introduction

HATT is a computer program that calculates astronomic tides in one or two points and water levels, discharges and velocities in a string of nodes and branches connected to one or both points. This gives the possibility of representing:

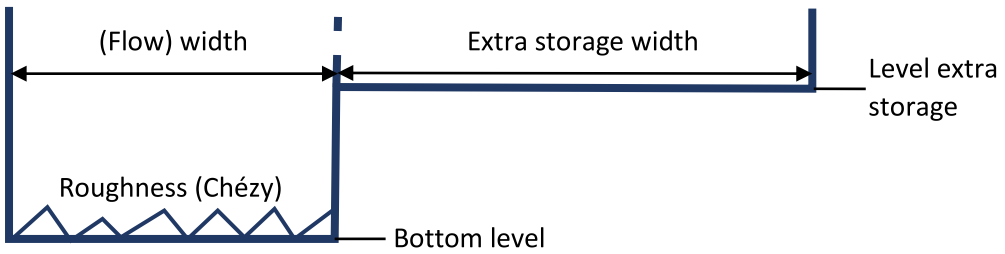
1. Tidal river with discharge upstream
2. Tidal bay with dead end
3. Tidal channels in coastal areas
4. (Seasonal) influence of secondary tidal constituents in 1 or 2 locations



The astronomic input for the boundary points consists of the 4 main constituents (M2, S2, K1, O1). Three constituents (N2, K2, P1) can be added, using the relations from equilibrium theory, see Annex A. The hydraulic input consists of the bathymetry of the channels in the string (Depth, Flow Width, Length, Roughness and Storage Width), see Annex B. In the channels (maximum 50 in total), discharges and velocities are calculated, while at the nodes between the channels the water levels are calculated.

Node 1 is always a tidal boundary condition (tide A). In the last node (number of nodes = number of branches + 1), a river discharge (Q River, which also can be 0, in case of a bay) **or** a second tidal boundary (tide B) is given. ***The maximum number of branches is 50.***

The branch data are as follows:



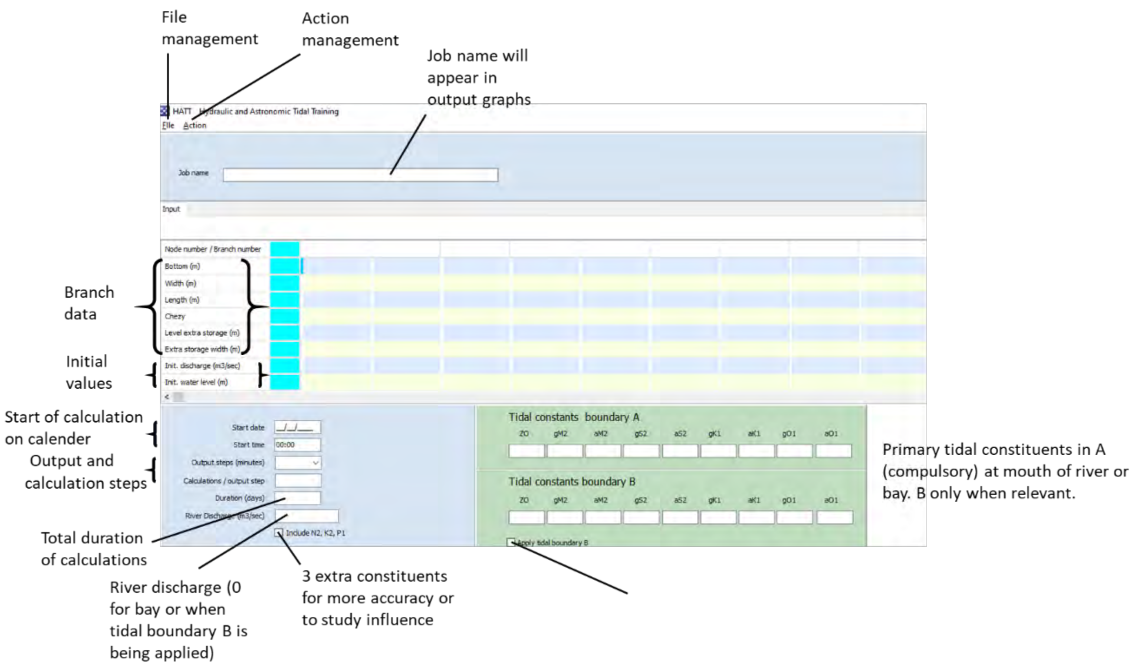
All dimensions are in m; the Chezy roughness value is in √m/s. Levels are given with regard to the same reference level (e.g. Mean Sea Level or some ordnance datum).

The program is started by clicking the Start program button in the middle of the cover screen.

For every possible use of HATT as mentioned above (1-5), an example will be given in this manual. The first one (Tidal river with discharge) will be elaborated more in detail, serving as a general example.

2. ***Input***

Opening screen HATT



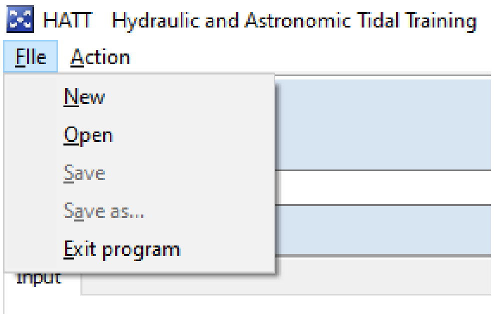
Start calculation: dd/mm/yyyy hh/mm Example: 24/05/1992 00/00

Output and calculation step: Output can be obtained every 5 to 120 minutes (choice menu). The calculation step can be chosen as any number of calculations per output step. Example: output every 30 minutes and 6 calculations per output step leads to a calculation step of 300 s. Note: This step should not be larger than the limits given in Annex C

File management (left upper corner)

New: Create a new file
Open: Open an existing file

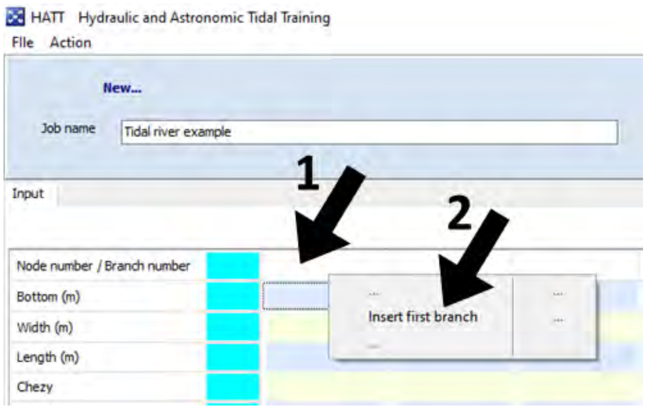
With a new file, input data have to be given on the opening screen. A job name is given (here: Tidal river example)



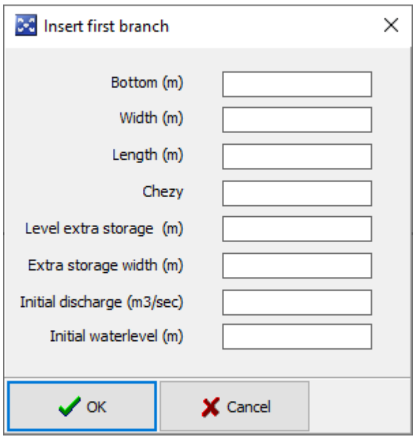
Branch input

- 1. Left click white cell next to Node number/Branch number
- **Insert first branch** button pops up
- 2. Left click button

A window for branch data pops up

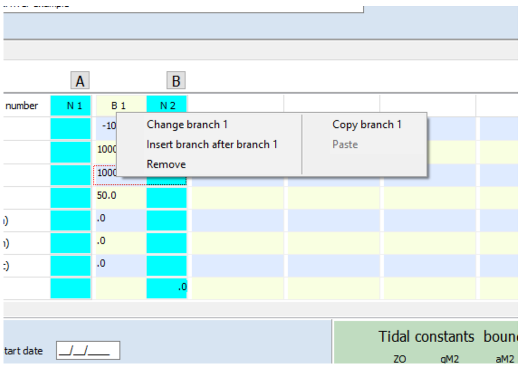


Fill in the values. Bottom, width, length and Chezy are mandatory. *Note: The bottom means 'bottom position', not depth! So , when the bottom is 10 m below 0-level, the input is -10. The 0-level can be chosen by the user.*
Default values for Level extra storage and Extra storage are 0. So, no need to fill in any value when there is no extra storage.



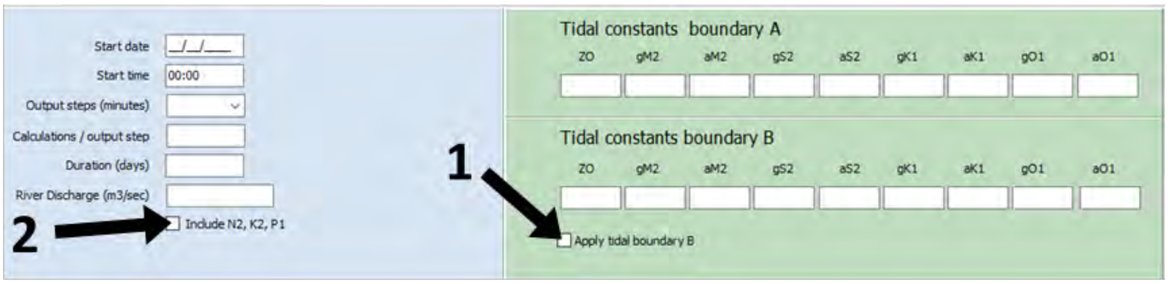
Default values for Initial discharge and water level are also 0. Well chosen values can help to speed up the computation but usually these values are not known, hence the first day or so, is necessary to reach a situation in which all values are correctly related to each other. See first example.

After clicking OK, the data will be incorporated into the overall table and N(ode)1, B(ranch)1 and N(ode)2 appear on top. For the second branch, click the white cell on top of the 2nd column and repeat the procedure. When you want to copy the data of the first branch, click B1, click Copy Branch 1 and Paste in the next column. The other options in this screen are self-evident. **Insert branch after branch 1** has the same effect as clicking the white cell on top of the 2nd column.



Repeat these steps until all branch data have been filled in.

Tidal input



M2, S2, K1 and O1 are the 4 primary constituents as given in e.g. the Admiralty Tide Tables (ATT). Z_0 is the zero level used in the computations. g, the phase angle, is in degrees (0-360) and a, the amplitude, is in m. Use of a second tidal boundary is only effective when ticking the box below the constituents table, see arrow 1.

Note: *In ATT Z_0 is the chart datum (LLW, an extreme low water to avoid grounding of ships when making a tidal prediction). But in technical work Z_0 is e.g. Mean Sea Level (MSL), hence $Z_0 = 0$.*

Additional to the primary constituents, the program can add 3 more: N2, K2 and P1. They are related to M2, S2 and K1 respectively, according to equilibrium theory (N2/M2 = 0.192, K2/S2 = 0.272, P1/K1 = 0.331). f you want to include these, tick the box at arrow 2. See also Case 4 and Annex A on astronomy.

Node 1 is always a tidal boundary. At the last branch there are three possibilities:

- River discharge > 0, representing a river with upstream discharge input
- River discharge = 0, representing a bay without river inflow
- Tidal boundary B, representing tidal channels with tide at both sides

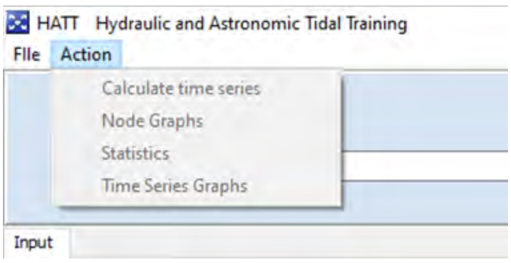
Note: Tidal boundary B is applied when the 'Apply tidal boundary B' box is ticked, also when a river discharge is given. In that case it will be overwritten.

River discharge input

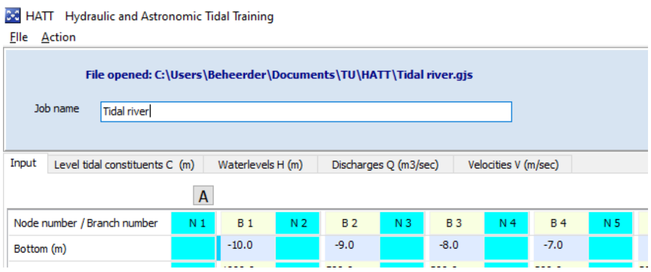
The inflow at the upstream end of the river (upstream of last branch) in m3/s. Note the flow is positive from upstream to tidal boundary A.

Action management

First, with **Calculate time series** the program will produce results for water levels, discharges and velocities throughout all branches. These results are stored in a table. From this table, graphical results can be derived.
Node Graphs are cross-sections of this table at a certain time level, giving water levels along the nodes.
Time Series Graphs are cross-sections at a certain location, giving water levels as a function of time.
With **Statistics**, an overview of minimum, average and maximum values is presented for all water levels, discharges and velocities.



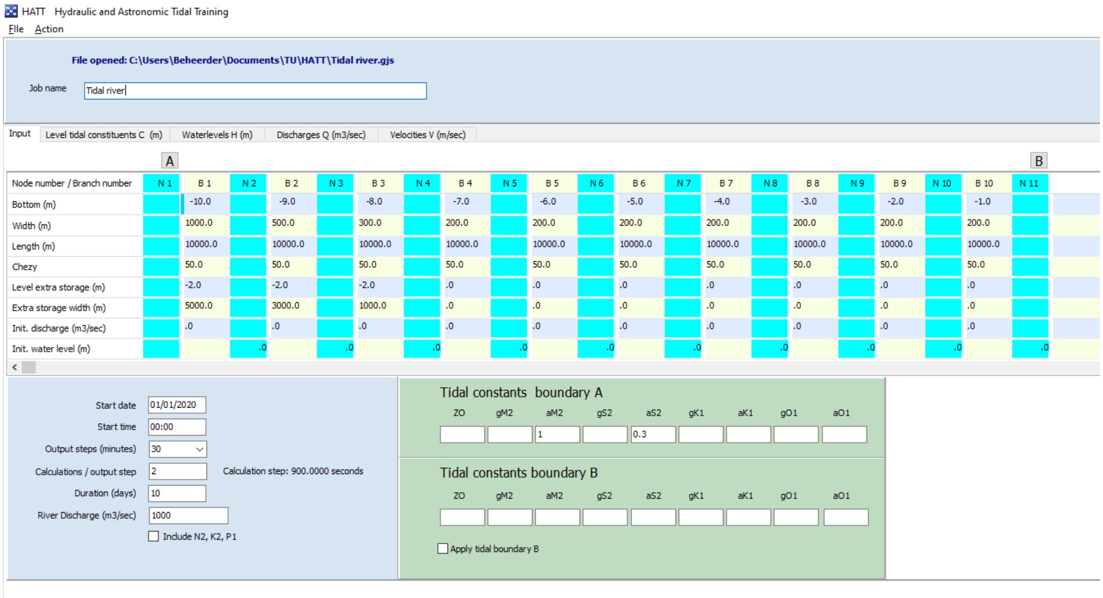
Note: Every time you open a HATT file, you have to recalculate ('Calculate time series') to see results. The program does not save the calculations. The calculation process takes very little time. Once finished, above the branch data 4 headings will appear: Level tidal constituents C, Water levels H, Discharges Q, Velocities V. Clicking these, gives access to tables with calculated values. Also, when you change something in the input (branch dimensions, tidal data, river discharge, calculations have to be done again.



3. ***Output***

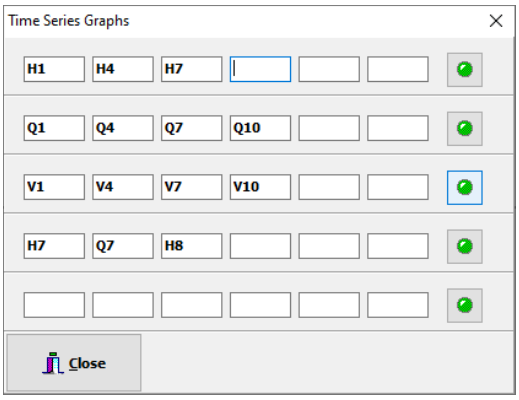
The output is described in detail with the example cases

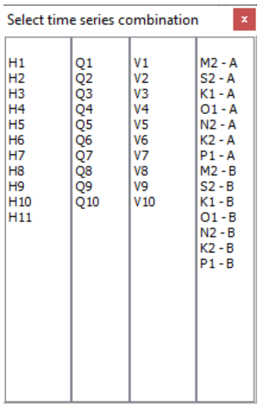
Case 1: Tidal river with discharge upstream



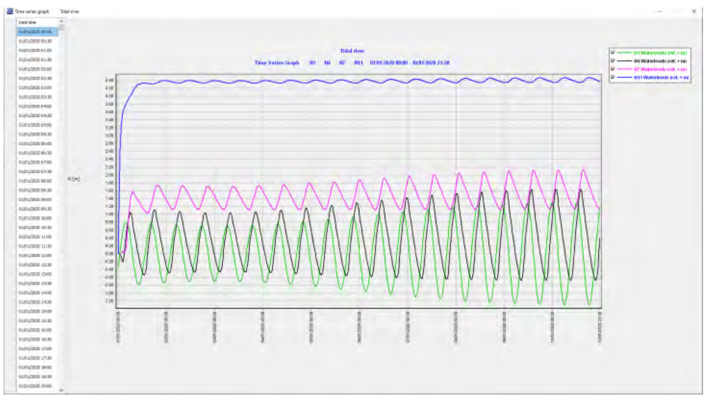
A river stretch with a total length of 100 km, 10 branches with varying width and depth. In the mouth extra storage to e.g. represent side branches, harbors or swamps. A river discharge of 1000 m3/s and a tidal boundary with only M2 and S2, without any value for phase angle g (default = 0). This is simply done to show a spring-neap cycle, not for any specific date. The total duration is 10 days.

To show the results we start with: Action-Time Series Graphs. A window pops up in which the desired graphs are defined by double clicking in the boxes and choose from the menu. Here water levels (H), discharges (Q) and velocities (V) in 4 nodes and branches, equally divided along the river.

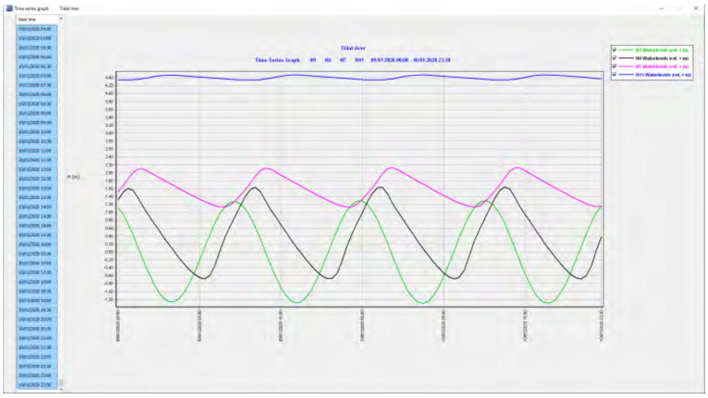




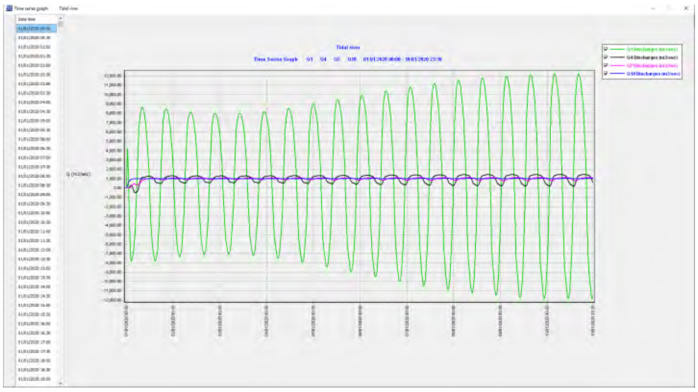
Water levels:
The initial period, in which the assumption that all initial values are 0 is corrected, is clearly visible. The water in node 11 has to rise more than 4 m. Here, it takes less than one day but it can be advantageous to use better estimations. The spring-neap cycle (with only M2 and S2) is clearly visible. The tide upstream is small



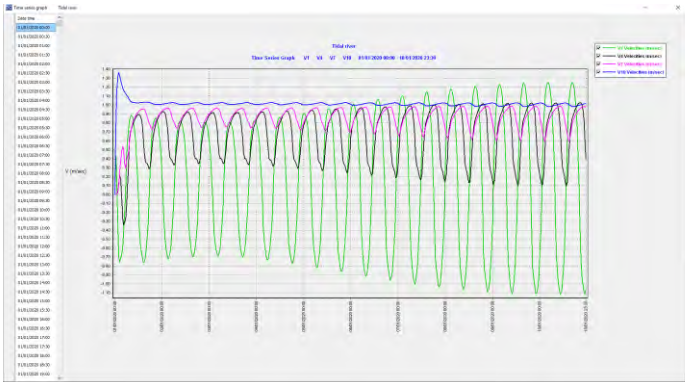
The results will be presented for the computation period (here 10 days). If you want more detail, select a period in the column with values left of the graph and choose with a right mouse click: **Show range.** Now only the last two days are being represented.



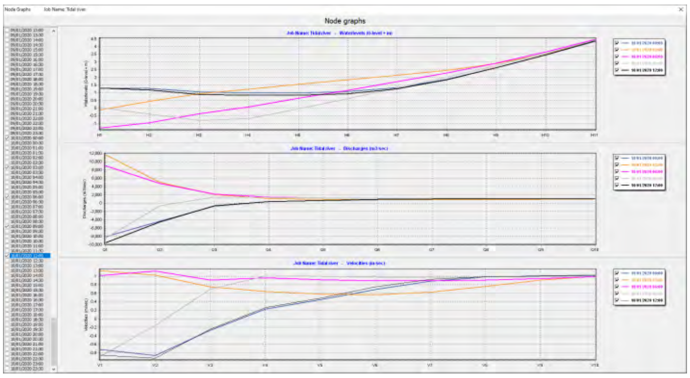
This graph shows the discharge in branch 1, 4, 7 and 10. The average is 1000 m3/s, as is the river input. In the river mouth, the variation in the discharge is large, due to the extra storage near the coast, which has to be filled and emptied by the tide. In the branches more upstream, the discharge is nearly constant.



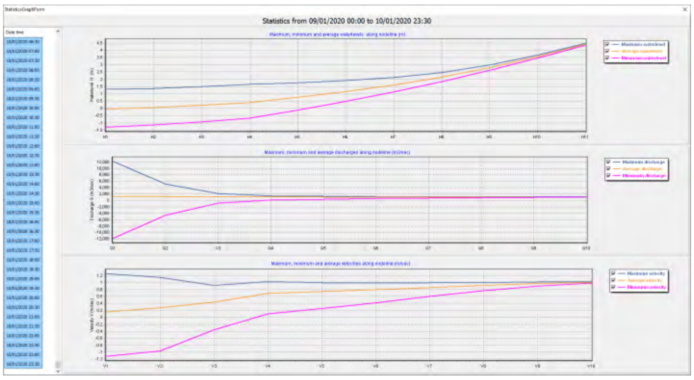
And the velocities for the same branches



Node graphs can be presented for every moment in the calculation results. To do so, tick the box (not the date and time) in the column on the left. Here this is done for the last day of the computations for every 3 hours, representing almost a full tidal cycle. Note: Node graphs within the initial period are not so useful from a hydraulic point of view, unless you are interested in the numerical process.

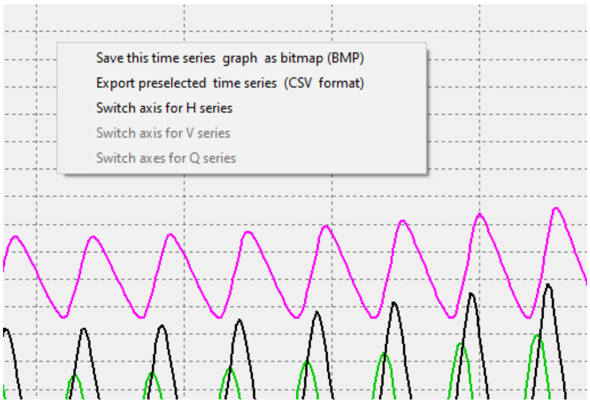


With “Statistics” the minimum, average and maximum water levels, discharges and velocities along the river are given. Default is the presentation of the whole calculation period, but with the same procedure as with the time series you can select the period you want (here the last 2 days).

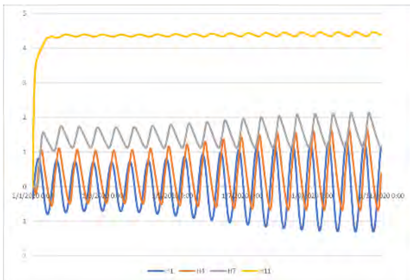


The graphs can be processed further in two ways:

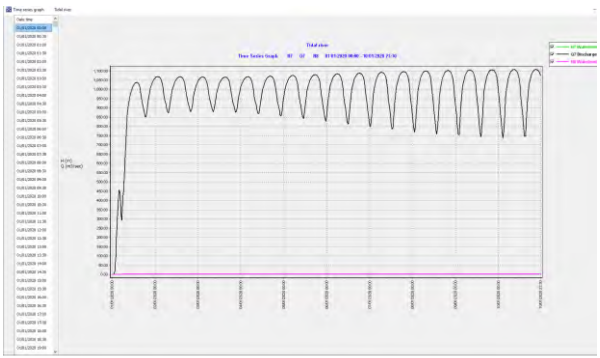
- It can be exported as a bitmap picture to incorporate in documents
- It can be converted into a .txt-file (Comma Separated format), which can be read into a spreadsheet program.

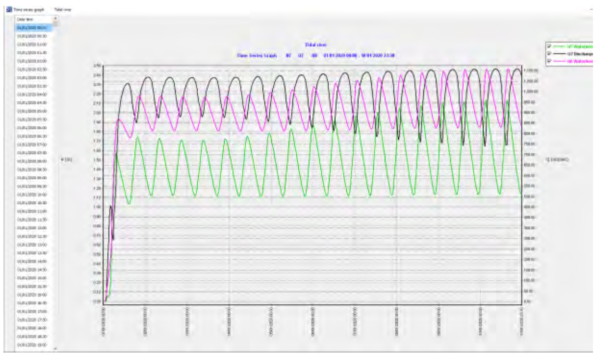


Note: Date and time is not converted into Excel format correctly but the date/time column can easily be replaced manually with Excel formatted data. For the graph, a scatter diagram should be selected, where lines can be chosen instead of dots. Here the Excel result of the time series graph as presented in HATT format above.



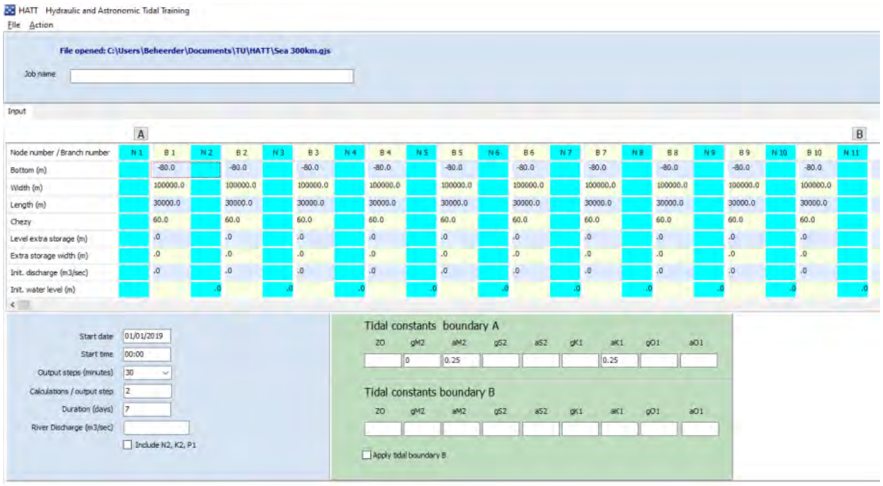
Switch axes is useful when two parameters with very different values are being presented in one graph. Here the water levels in node 7 and 8, plus the discharge in branch 7 is given. On the left hand side, the default graph is given with one vertical scale, where the discharge has much larger numbers. Switch axis (here for Q) presents the discharge numbers on the axis at the right hand side, making all parameters readable.





Case 2: Tidal bay with dead end

The opening screen for this example shows 10 identical branches 80 m deep, 100 km wide and 30 km long, hence a 300 km long bay, actually more a sea. Boundary condition is a tide with only M2 and K1, each with an amplitude of 0.25 m. A weak, mixed tide (f = 1), often encountered in oceans.



For one day at the end of the computation, the statistics show a steady increase from the boundary at the ocean, with a tidal range of less than 1 m, to a range of almost 5 m at the end. Discharges and velocities show the opposite: large values at the entrance and (almost) 0 at the end of the sea. This is also visible in the time graph with the ocean boundary and the tide at the end.

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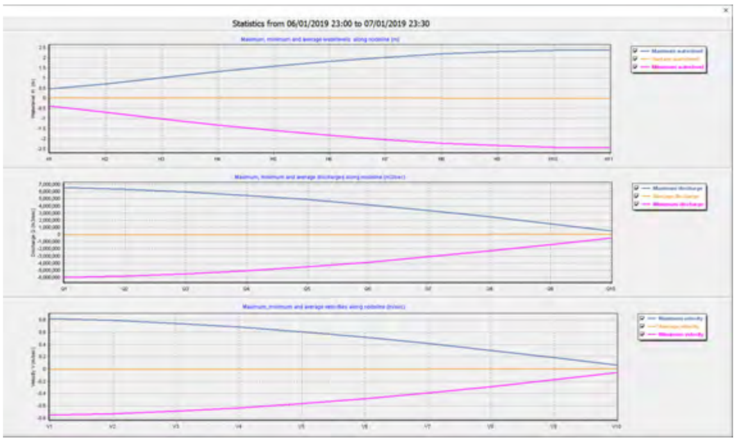
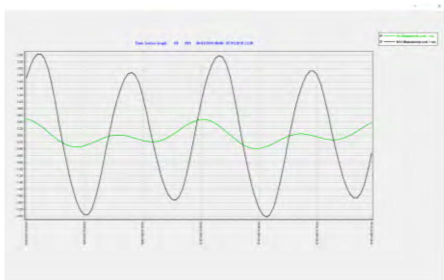
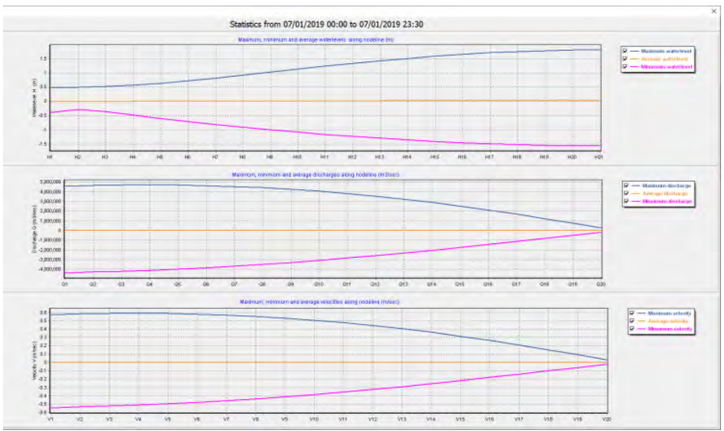
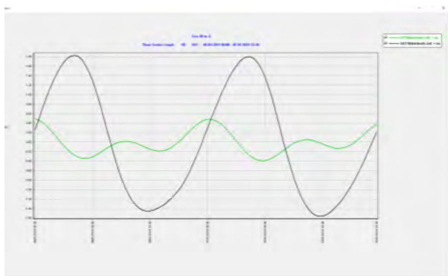




300 km long sea
Green: tide at ocean boundary
Black: tide at end of sea

When we now double the length of the sea (20 branches with length = 30 km), with the same tide as boundary condition, we get a similar picture, now with a tidal range at the end of more than 3 m:

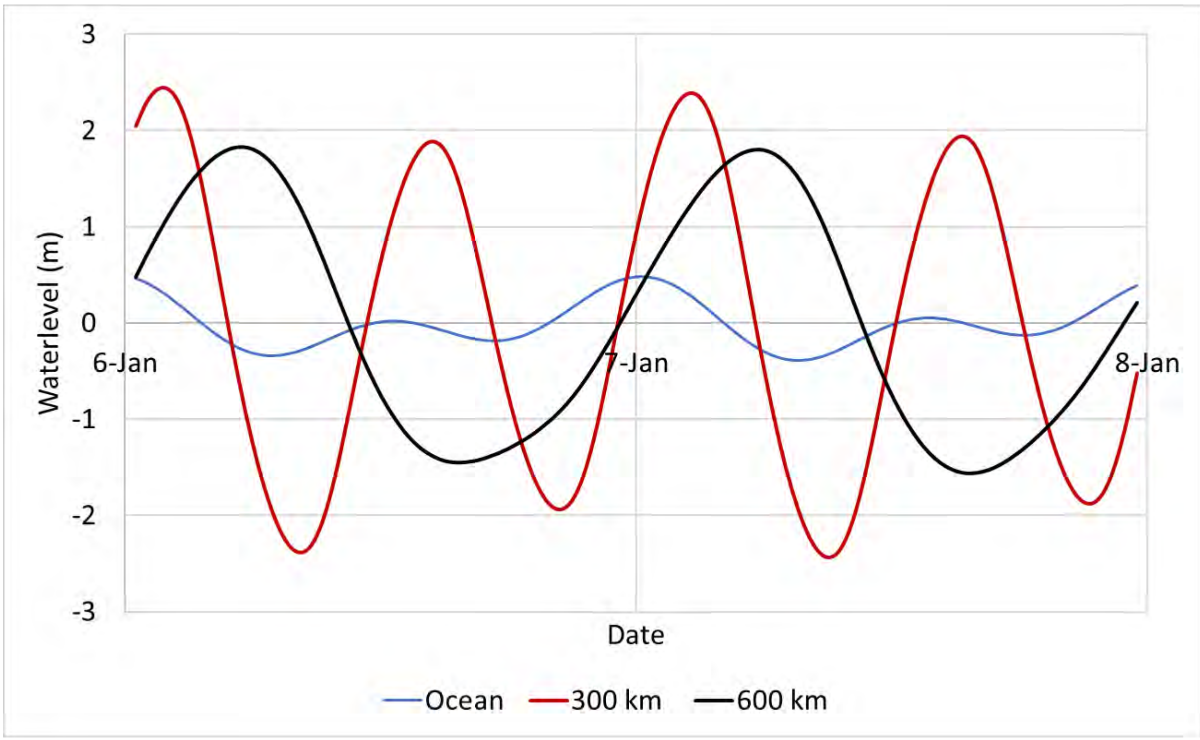




600 km long sea
Green: tide at ocean boundary
Black: tide at end of sea

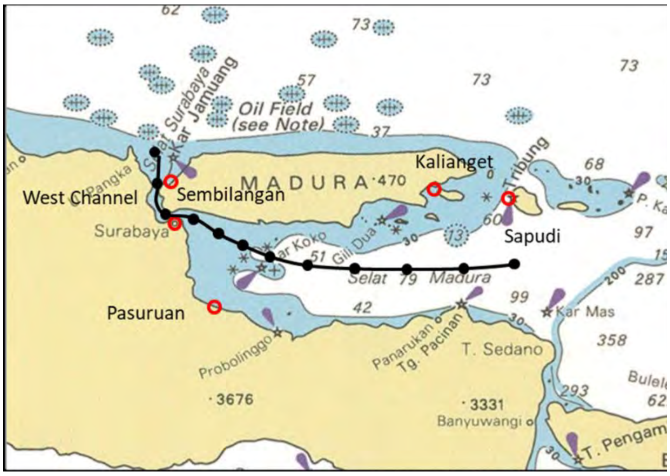
Similar but with one big difference: With the 300 km sea we have two tidal curves each day and with the 600 km sea only one! The explanation can be found in Annex 2. With a depth of 80 m, the celerity of a tidal wave = $\sqrt{(g*D)}$ = 28 m/s. For M2, this leads to a wave length of 28 * 12.4206 * 3600 = 1251 km and for K1: 28 * 23.9345 * 3600 = 2413 km. That means that 300 km is near ¼ of the wave length of M2 and 600 km near ¼ wave length of K1.

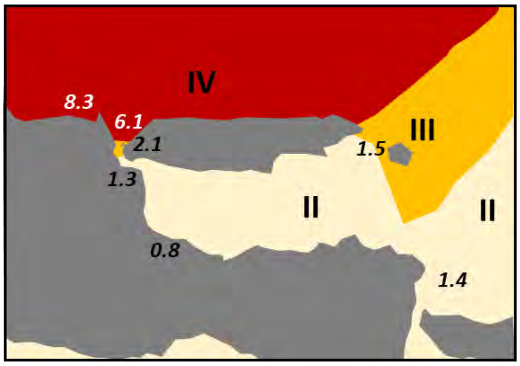
Exporting both results to Excel and combining them, gives the following picture:



The same weak, mixed tide as ocean boundary condition for two seas with different length, results in strong tides at the end of the sea, purely semi-diurnal for the 300 km sea and purely diurnal for the 600 km sea. This shows how the difference in tidal character is mainly governed by the bathymetry of seas and oceans. In various locations in the world, but specifically in Southeast Asia, this phenomenon is frequently encountered.

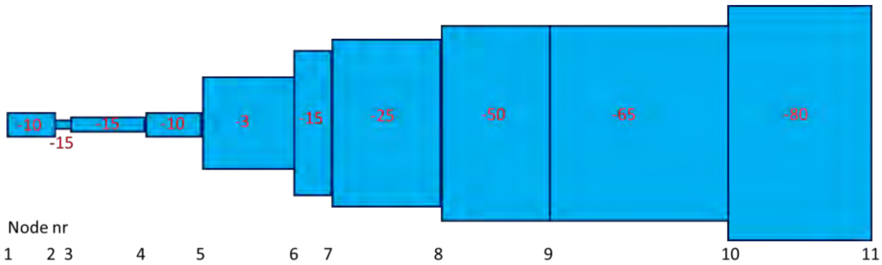
Case 3 Tidal channels in coastal areas





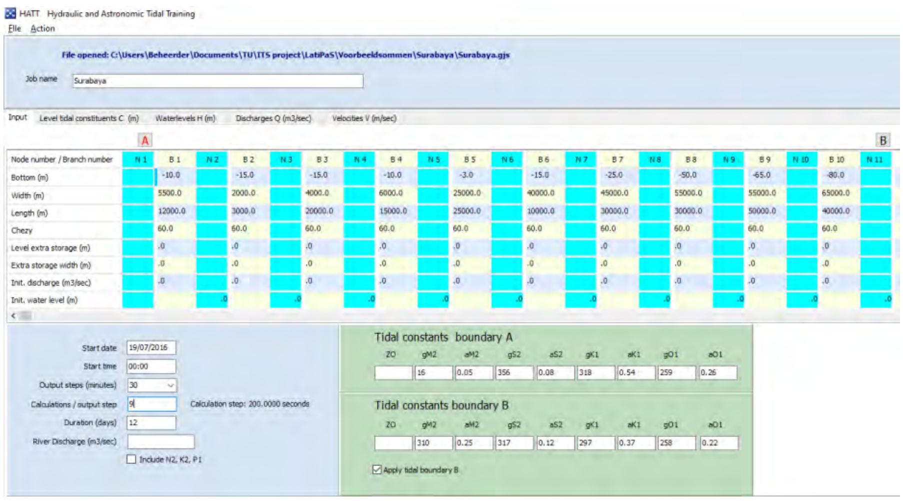
To illustrate the use of HATT with two tidal boundaries, we take the situation around Surabaya. North of Java and Madura is the Java Sea where, in that part, the tide is purely diurnal (type IV, see Annex A). In the Strait of Madura, between Java and Madura, the tide is mixed, mainly semi-diurnal (type II), while at the east side of the Strait, the tide is mixed, mainly diurnal (type III). So, in between two diurnal zones, there is an area with semi-diurnal characteristics.

Length, width and depth used in schematization, see also opening screen below

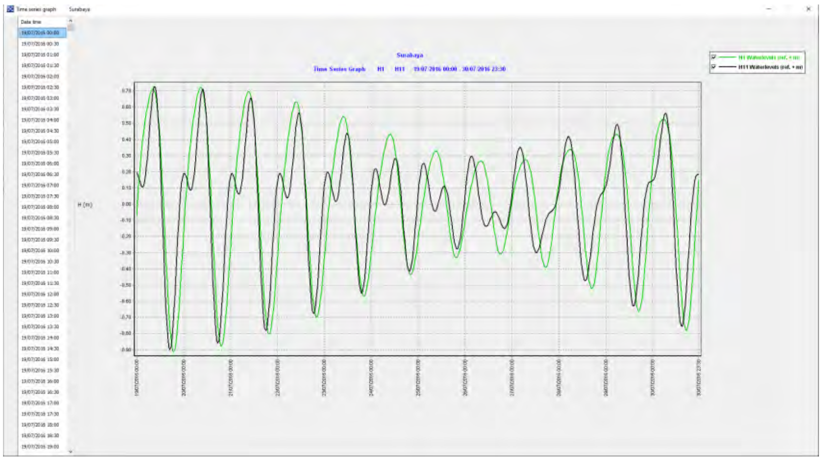


The opening screen of this case.

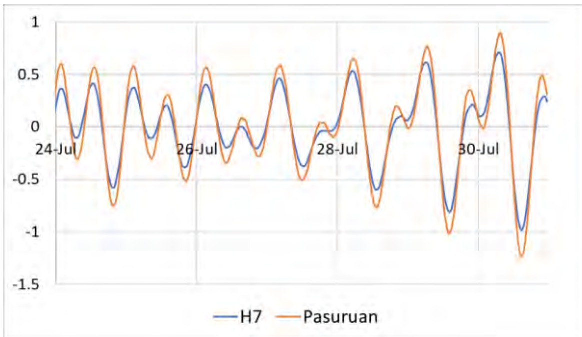
The tidal constituents come from the Admiralty Tide Tables

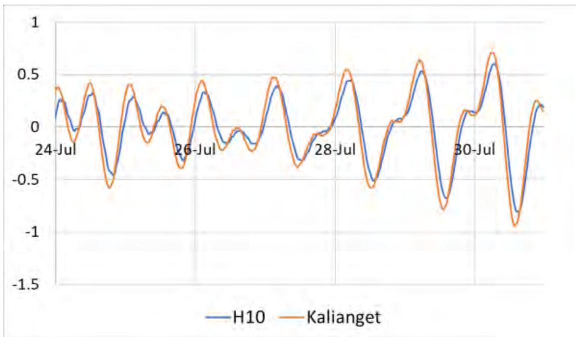


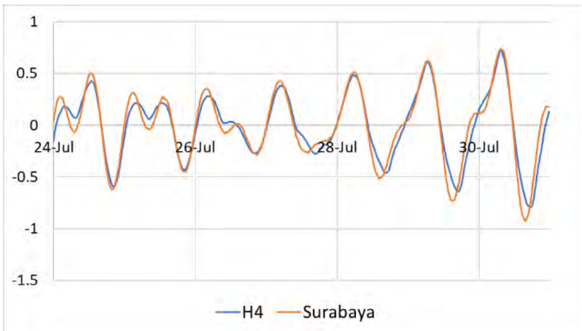
These lines show the boundary conditions. The green line is BC A in node 1 in the Java Sea (Karang Jamuang) and the black line is BC B in node 11 at Sapudi at the east side of Strait Madura

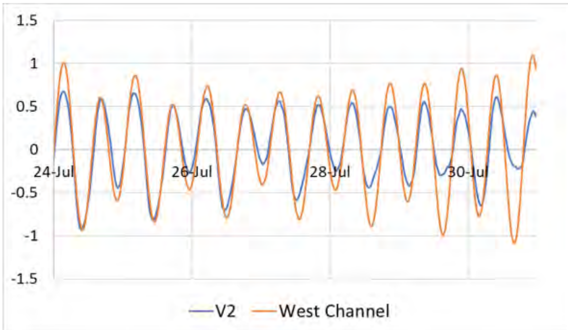


Below are some comparisons between the **hydraulic** computations with HATT, solving the equations of Annex B between two boundary conditions (blue line) and the **astronomic** computations, coming from the motion of Moon and Earth, using the local tidal constituents from ATT (orange line).



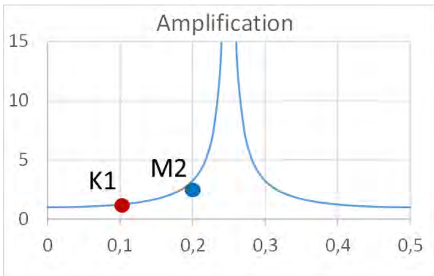




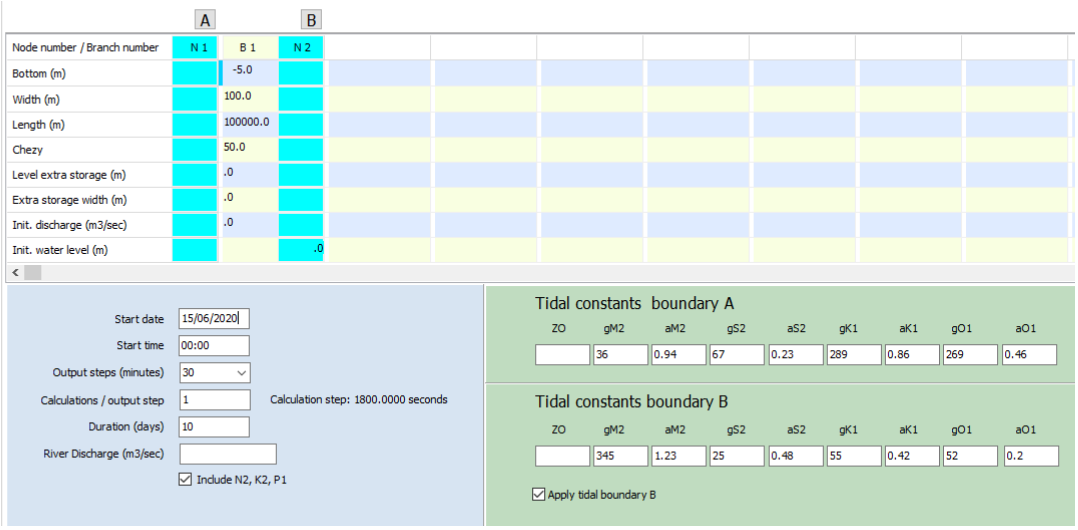


It appears that, despite the simplified 1-dimensional approach with rectangular cross-sections, HATT is doing reasonably well. Good enough for studying phenomena.

The physical explanation of the phenomena around Surabaya comes from the behavior of waves in closed channels. Amplification of the semi-diurnal constituent M2 is larger than for the diurnal K1 in Strait Surabaya. Hence, the character of the tide going from East to West becomes more semi-diurnal.

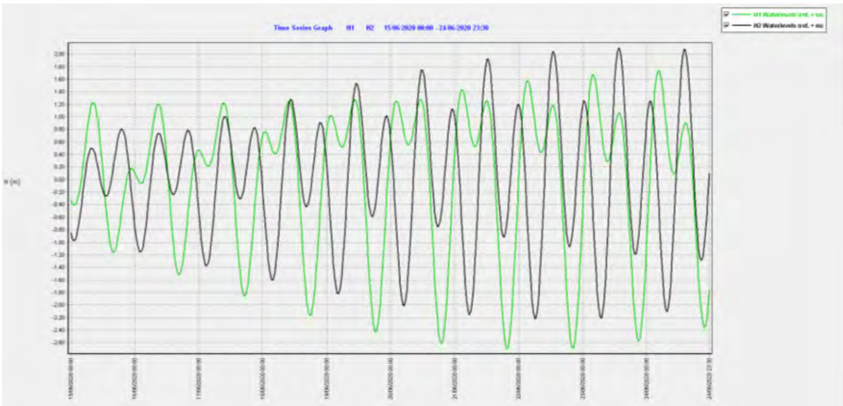


Case 4 (Seasonal) influence of secondary tidal constituents in 1 or 2 stations

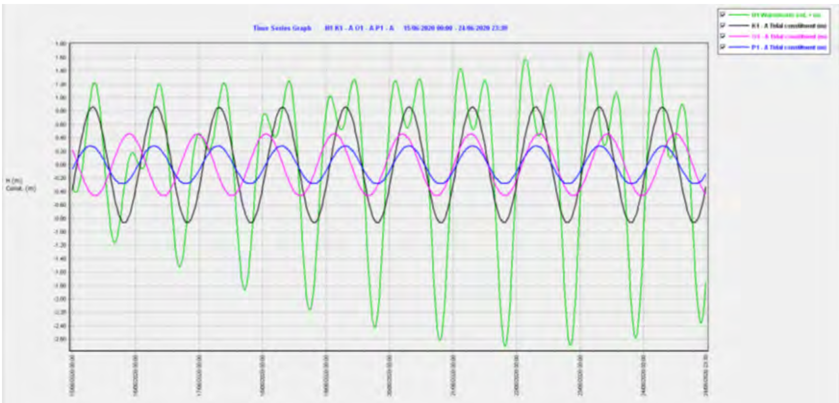


To see the seasonal effects of tidal constituents, we take two different tides, one mixed, mainly diurnal (Vancouver, f = 1.13, see Annex A, tide A in node 1) and one mixed, mainly semi-diurnal (Bombay, f = 0.36, tide B in node 2). In between we assume a very long branch (100 km). This branch has no meaning (HATT needs at least one branch); the length is chosen to avoid any problem with numerical instability. The locations are in different time zones (Vancouver +8, Bombay -5.30). Here, there is no need to adapt them to one time zone. We are not comparing the two curves, we are looking into the seasonal effects of the tidal constituents for each station.

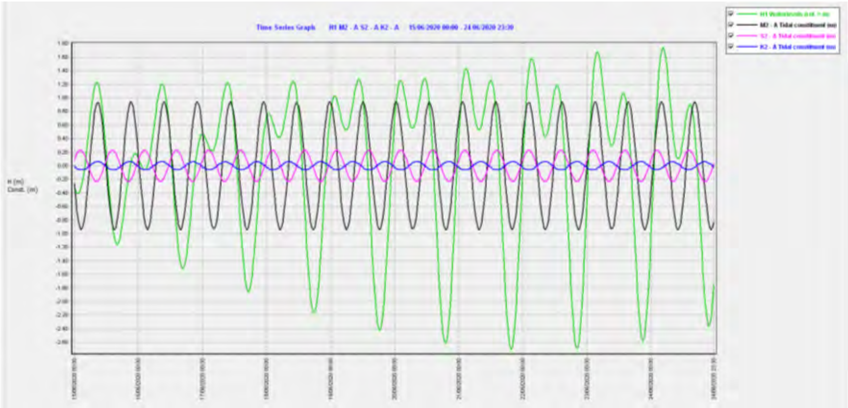
First we look at the situation around June 21 (Sun at the Tropic of cancer, Northern summer solstice). Vancouver (green) is diurnal, with semi-diurnal features around HW. Bombay (black) is semi-diurnal with large differences in HW and LW (“daily inequality”).



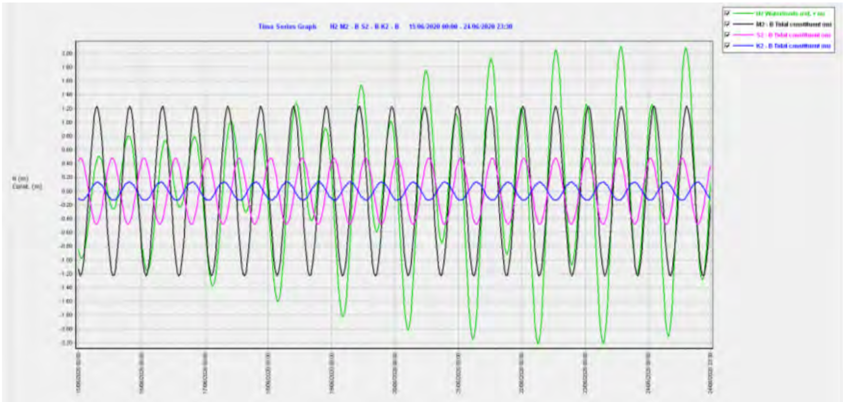
Vancouver:
For diurnal tides, spring occurs when K1 (black) and O1 (purple) coincide. K1 and P1 (blue) coincide the whole period (maximum declination of sun). This also increases the diurnal character in Vancouver. Spring tides have a range ≈ 4.4 m, neaps ≈ 2.2 m.



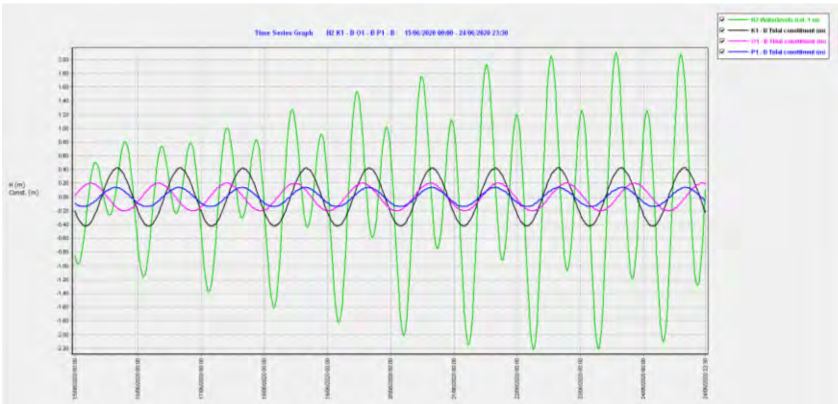
Vancouver:
S2 (purple) and K2 (blue) are out of phase in June, so the semi-diurnal component is weak, making Vancouver more diurnal. When M2 (black) and S2 coincide, the semi-diurnal dip around HW is strongest.



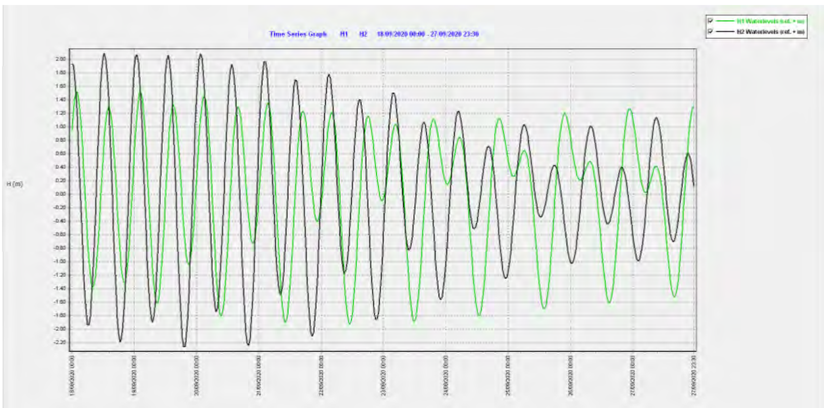
Bombay:
Springtide with semi-diurnal tides occurs when M2 (black) and S2 (purple) coincide. S2 and K2 (blue) are out of phase in June. Range during spring i≈ 4.2 m and 1.7 m with neap.



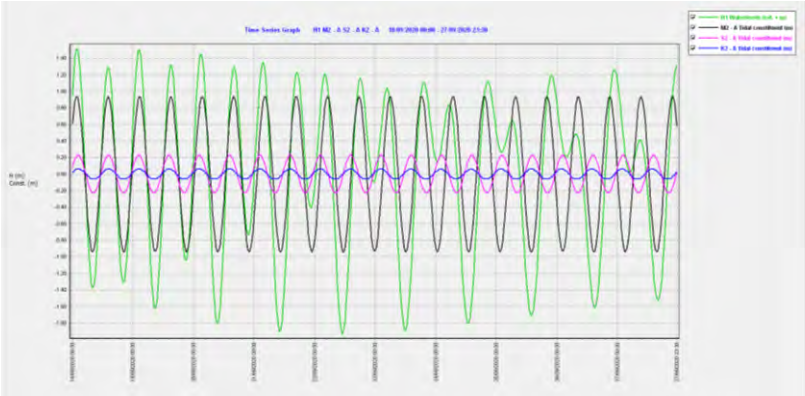
Bombay:
K1 (black) and P1 (blue) coincide in June, giving a large daily inequality. This is even stronger when K1 and O1 (purple) coincide.



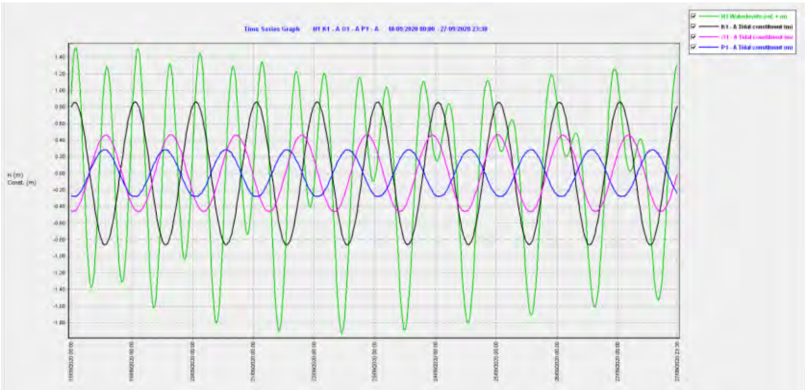
Next, we look at the tides in September (Sun at equator, equinox). Vancouver (green) now shows more semi-diurnal characteristics and Bombay (black) less daily inequality than in June.



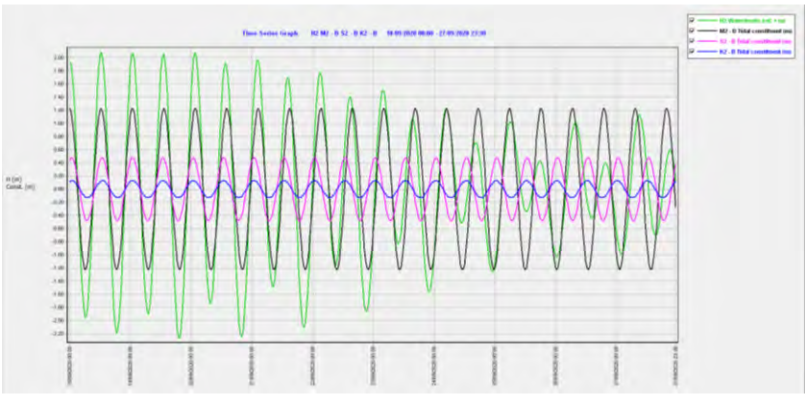
Vancouver:
S2 (purple) and K2 (blue) coincide in September, so the semi-diurnal component is strong, making Vancouver more semi-diurnal. During neap, when M2 (black) and S2 are out of phase, the character is more diurnal.



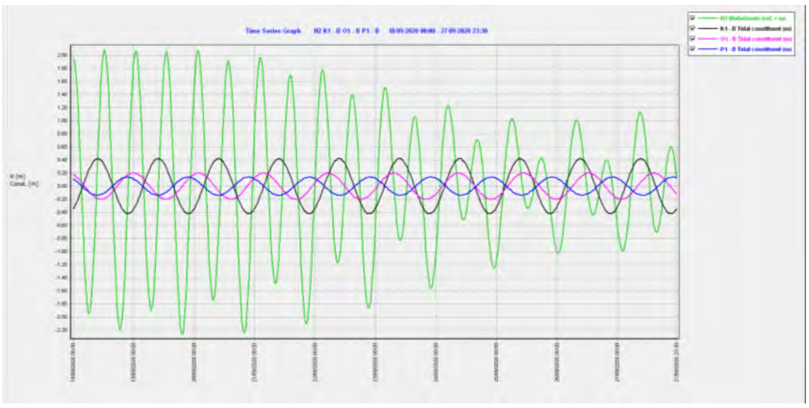
Vancouver:
In September, K1 (black) and P1 (blue) are out of phase, giving lower spring tides (range now ≈ 3 m).



Bombay:
S2 (purple) and K2 (blue) coincide in September giving strong semi-diurnal tides.



Bombay:
K1 (black) and P1 (blue) are out of phase in September, giving a much smaller daily inequality than in June. When K1 and O1 (purple) coincide, it is again considerable.



Annex A Astronomy

Tidal constituents

The attraction forces between Moon, Earth and Sun are represented by tidal constituents. These constituents, for a certain location, come from harmonic analysis of observed water levels at that location. The tidal movement can be described in detail with many constituents, for which long registration periods are necessary. Here, only the 7 most important constituents are being used, 4 primary and 3 secondary constituents. The 4 primary constituents can be found in tide tables, such as the Admiralty Tide Tables (ATT) or on the internet, the 3 secondary constituents used in HATT are derived from equilibrium theory, using a fixed relation with the primary constituents.

These 7 constituents are (1 indicates one High Water and 1 Low Water per day, diurnal tides and 2 indicates 2 HW and 2 LW per day, semi-diurnal tides:

Consti-tuent	***Origin***	***Period (hr)***	***Speed (°/hr)***	***a (amplitude, m)***	***g (phase angle, °)***
M2	**Principal Moon**	**12.4206**	**28.9841**	**From Tide table**	
S2	**Principal Sun**	**12.0000**	**30.0000**	**From Tide table**	
K1	**Declination Moon/Sun**	**23.9345**	**15.0407**	**From Tide table**	
O1	**Declination Moon**	**25.8193**	**13.9430**	**From Tide table**	
N2	Elliptic Moon	12.6583	28.4397	0.194 * a M2	= g M2
K2	Declination Moon/Sun	11.9672	30.0821	0.272 * a S2	= g S2
P1	Declination Sun	24.0659	14.9589	0.331 * a K1	= g K1

The character of the tide is given by ***f = (aK1 + aO1) / (aM2 + aS2)*** with the following division

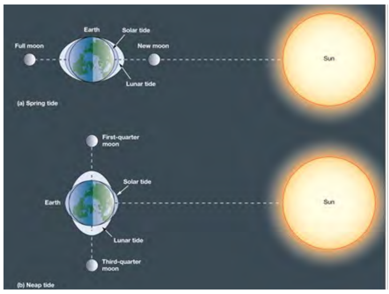
I	$f < 0.25$	Purely semi-diurnal
II	$0.25 < f < 1.5$	Mixed, mainly semi-diurnal
III	$1.5 < f < 3$	Mixed, mainly diurnal
IV	$f > 3$	Purely diurnal

Beats

The constituents show so-called beats, given by: $T_{beat} = T1*T2/|T1-T2|$, causing fluctuations in the strength of the tides. The most important fluctuations are:

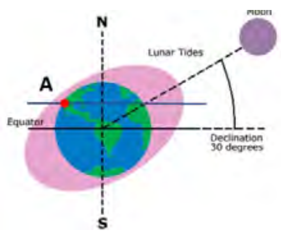
Semi-diurnal neap-spring cycle, related to the phases of the Moon, given by the beat M2-S2: T_{beat} M2-S2 = 12.4206*12.0000 / 0.4206 = 354.368 hr = 14.77 days.

After Full moon and New Moon, when the attraction forces of Moon and Sun reinforce each other, springtide occurs. After First and Third Quarter, when the forces work in different direction, neap tide occurs.

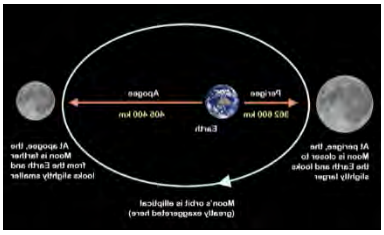


Diurnal neap-spring cycle, related to the declination of the Moon, given by the beat K1-O1: T_{beat} K1-O1 = 23.9345*25.8193/|23.9345-25.8193| = 327.8714 hr = 13.66 days.

After maximum North or South declination of the Moon, diurnal springtides occur, and with 0 declination diurnal neap tides.

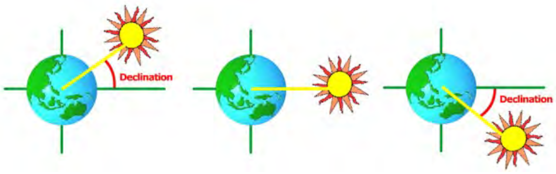


The distance of the Moon to the Earth is not constant but is maximum at Perigee and minimum at Apogee, due to the elliptic orbit of the Moon.



N2 is a correction of the principal attraction force, indicated by M2. Together they have a beat of: 12.4206*12.6583/|12.4206-12.6583| = 661.437 = 27.56 days.

Finally, S2-K2 and K1-P1 both show a beat of 182.62 days (= half year). S2 and K2 reinforce each other in March and September (so-called equinoctial tides, when the principal forces are not weakened by the Sun's declination) while K1-P1 do so in June and December (Northern summer and winter solstice), when declination, causing diurnal tides, is maximum



June March/Sept. December

These beats can be made explicitly visible in HATT by presenting the constituents involved in the Time series graph

Time zones

Each of the 4 main constituents (M2, S2, K1, O1) is given in the ATT with a phase angle (g) and an amplitude (a). a is simply half the wave height of the constituent, g is the phase angle to come from the astronomic situation to the local clock-time for each station. When the tidal prediction is made for one station or for more stations in the same time zone, the results are for the local time.

When two stations from different time zones have to be compared to the same clock time, the phase angle, g, needs a correction, taking into account the frequency or speed of each constituent. The correction for a different time zone (TZ) then is:

$$g_{\text{New TZ}} = g_{\text{ATT TZ}} + (\text{ATT TZ} - \text{New TZ}) * \text{Speed}$$

Example

Station A is in Time zone -7 (West Indonesia), station B in zone -9 (East Indonesia). The choice is to express both for Time zone -7 (same as A). The correction for B then becomes (-9 + 7) = -2 times the speed of each constituent, resulting in:

$g_{\text{ATT TZ}}$ **B**	**Correction**	$g_{\text{New TZ}}$ **B**
gM2: 156	-2 * 29 = -58	98
gS2: 93	-2 * 30 = -60	33
gK1: 103	-2 * 15 = -30	73
gO1: 12	-2 * 14 = -28	344

Note 1: The phase angles are given in degrees, so there is no need to use decimals in the speed numbers.
Note 2: g is always between 0 and 360

With these new phase angles, the High and Low waters in B will now occur 2 hours earlier!

Annex B Hydraulics

Basic equations

The conservation of momentum ("equation of motion") for long (tidal) waves in 2 dimensions reads:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + g\frac{\partial h}{\partial x} - fv + \frac{g}{d\,C^2}u(u^2 + v^2)^{0.5} + \frac{\tau_{sx}}{\varrho} = 0$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + g\frac{\partial h}{\partial y} - fu + \frac{g}{d\,C^2}v(u^2 + v^2)^{0.5} + \frac{\tau_{sy}}{\varrho} = 0$$

Local inertia   Convective inertia   Gravity   Coriolis   Friction   Wind stress

And the conservation of mass ("continuity equation") where the first term represents the change in water level and the other two the net flow.

$$\frac{\partial h}{\partial t} + \frac{\partial du}{\partial x} + \frac{\partial dv}{\partial y} = 0$$

In HATT this is reduced to 1 dimension with:

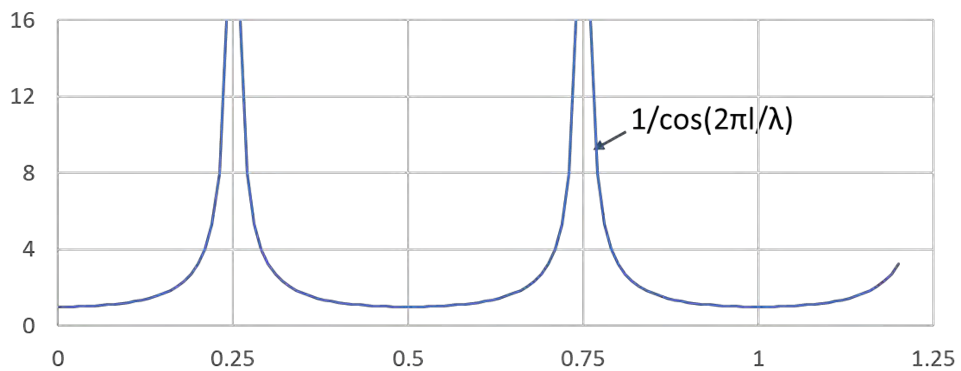
$$\frac{\partial u}{\partial t} + \frac{g\partial h}{\partial x} + \frac{g\,u2}{d\,C2} = 0 \quad \text{and} \quad \frac{\partial h}{\partial t} + \frac{\partial du}{\partial x} = 0$$

Friction

With the first two terms a frictionless wave can be described: $\boldsymbol{h = a\cos(\sigma t - kx)}$, representing the water level in a progressive wave, with: σ = 2π/T (T is wave period) and k = 2π/λ (λ is wave length). The velocity is given by: ***u = (a/d) c*** $\boldsymbol{\cos(\sigma t - kx)}$, in which **c =** √(gd), the so-called wave celerity (h and u are in phase). The velocity is small compared to the celerity: umax = (a/d) c. With a = 1 m this becomes u = 1 m/s for a water depth of 10 m and 0.1 m/s for a depth of 1000 m.

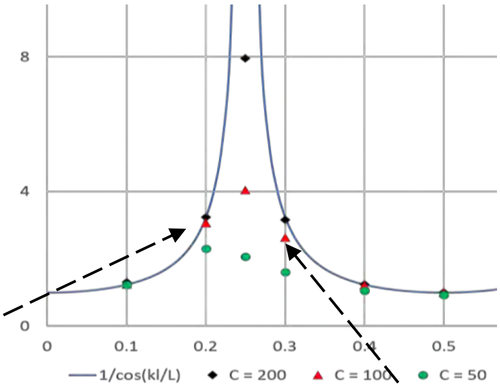
When this wave comes to a wall, it reflects, resulting in a standing wave with: ***h = 2a . cos σt . cos kx*** and ***u = (2a/d) . c . sin σt . sin kx*** (h and u now out of phase). For an open bay, with a length of l m and x = 0 at the end of the bay, this leads to:

$$\frac{ampl\ (x=0)}{ampl\ (x=l)} = \frac{2a\cos k.0}{2a\cos k.l} = \frac{1}{\cos 2\pi l/\lambda}$$

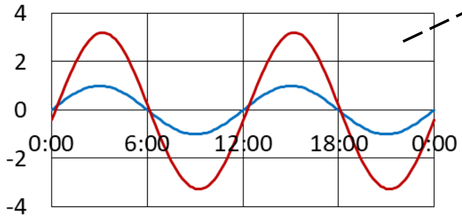


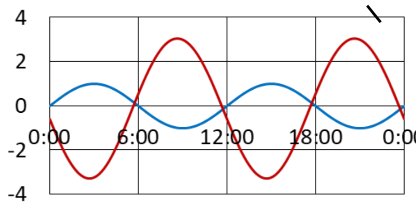
This indicates a (theoretically) infinitely high amplification (resonance) of the wave height when l = 0.25 λ, l = 0.75 λ, etc, while with l = 0.5 λ, l = λ, etc, the amplitude at the end is equal to the amplitude at the mouth.

With all three terms in HATT, the results can look like:



For estuaries with a depth of 35 m, the resonance is studied with different roughness values. With an extremely high Chezy value (200 √m/s), the results are very much in line with the analytical solution, with more realistic values, the results deviate.
For l/λ= 0.2, see below, the water levels at the end and at the entrance of the bay are in phase, while for l/λ= 0.3, these lines are 180° out of phase, since there is a node now present in the longer bay.



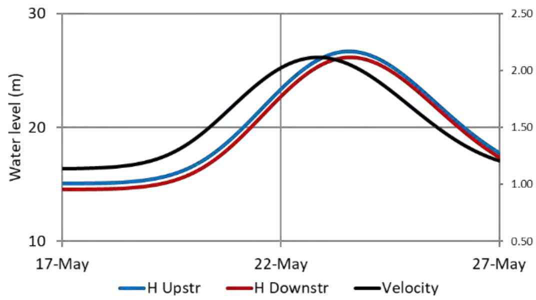


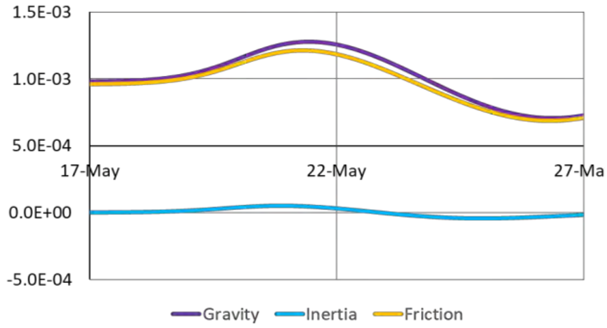
Inertia

With the last two terms in the momentum equation, the flow in a river or canal is described:

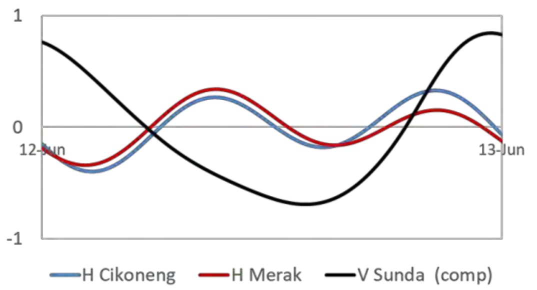
$\frac{g\partial h}{\partial x} + \frac{g\,u2}{d\,C2} = 0$ With the sign of u taken positive in the downstream direction, the gradient of the water level becomes negative, giving: $\frac{u2}{d\,C2} = \frac{\partial h}{\partial x}$, leading to ***u = C√(dI)***, the Chezy equation.

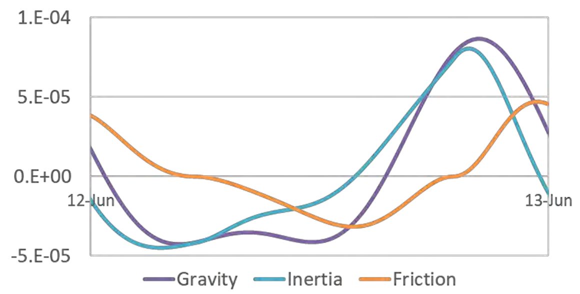
Of the three terms in the momentum equation in HATT, inertia can often be neglected in river flow. As an example here a river with a discharge of 3000 m^3/s and a flood wave of 18000 m^3/s with a duration of 10 days is given. The inertia term is small compared to gravity and friction.



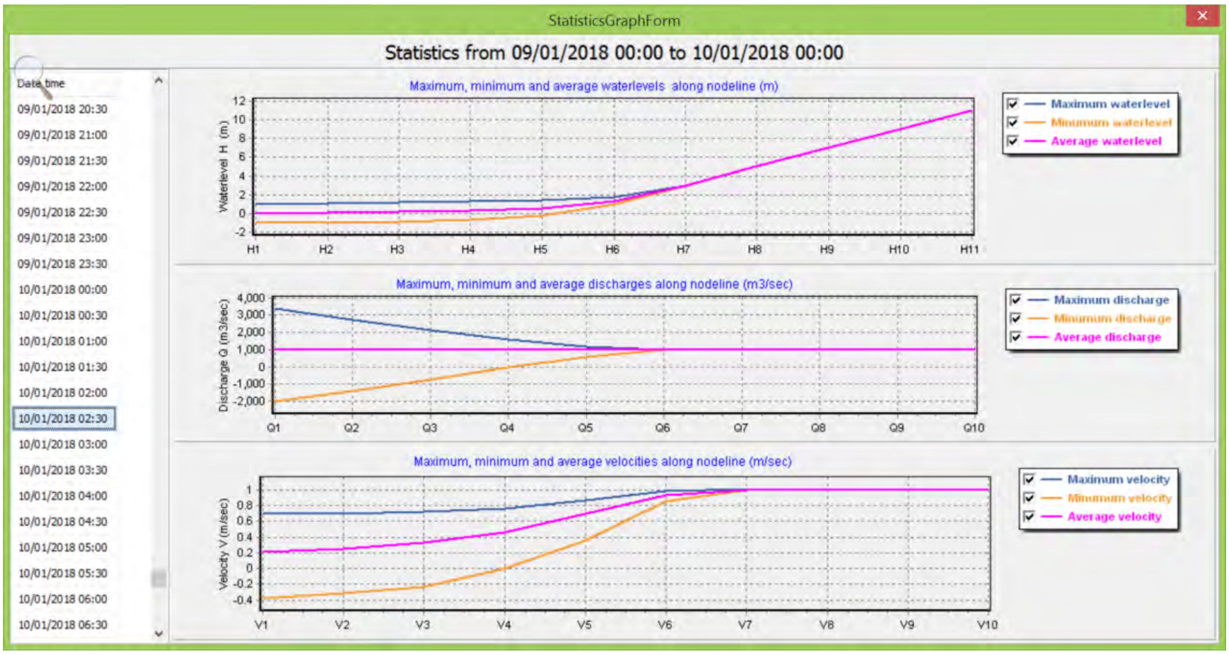


In tidal flow, all three terms are equally important, see this example for Strait Sunda:



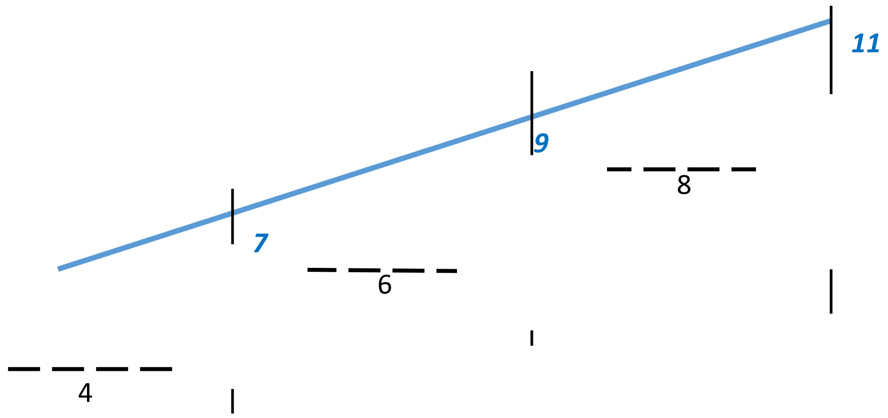


HATT is meant to study tidal flows, river flow at the upstream boundary can only be a constant value. The picture below shows the results (maximum, minimum and average values for water levels, discharges and velocities) in a river with a slope of 2.10^{-4}, a constant discharge of 1000 m^3/s and a C-value of 50 $\sqrt{m}/s$. The average discharge is 1000 m^3/s everywhere, since that amount of water is flowing to the sea through the whole river.



The tidal boundary at sea is a simple M20tide with an amplitude of 1 m.In the upstream part, there is no more tidal movement, the velocity (and discharge) have a constant value (u = 1 m/s). The water levels and the branch bottoms are:

Node Branch Node Branch Node



With a bottom level of + 8 m and an average water level of (+ 9 + 11) / 2 = + 10m, the water depth in the most upstream branch then is 2 m. According to the Chezy-equation, the velocity then should be: $\boldsymbol{u = C \sqrt{(dI)}}$, giving: $u = 50 * \sqrt{(2 * 2 * 10^{-4})} =$ 1 m/s, which is indeed the case in the result graph.

Note: The branch bottoms in HATT have only one value. This could be interpreted as a horizontal bottom but it seems better to see it as just the depth in the middle of the branch.

Annex C Numerical method

The basic equation in HATT are rewritten with the discharge ***Q*** instead of the velocity ***u*** to describe the flow, which results for the momentum equation into:

$$\frac{\partial Q}{\partial t} + gA\frac{\partial h}{\partial x} + g\frac{Q|Q|}{C^2AR} = 0 \quad (1)$$

With this equation, the flow in the branches is computed, while for the computation of the water level in the nodes, the mass balance equation is being used:

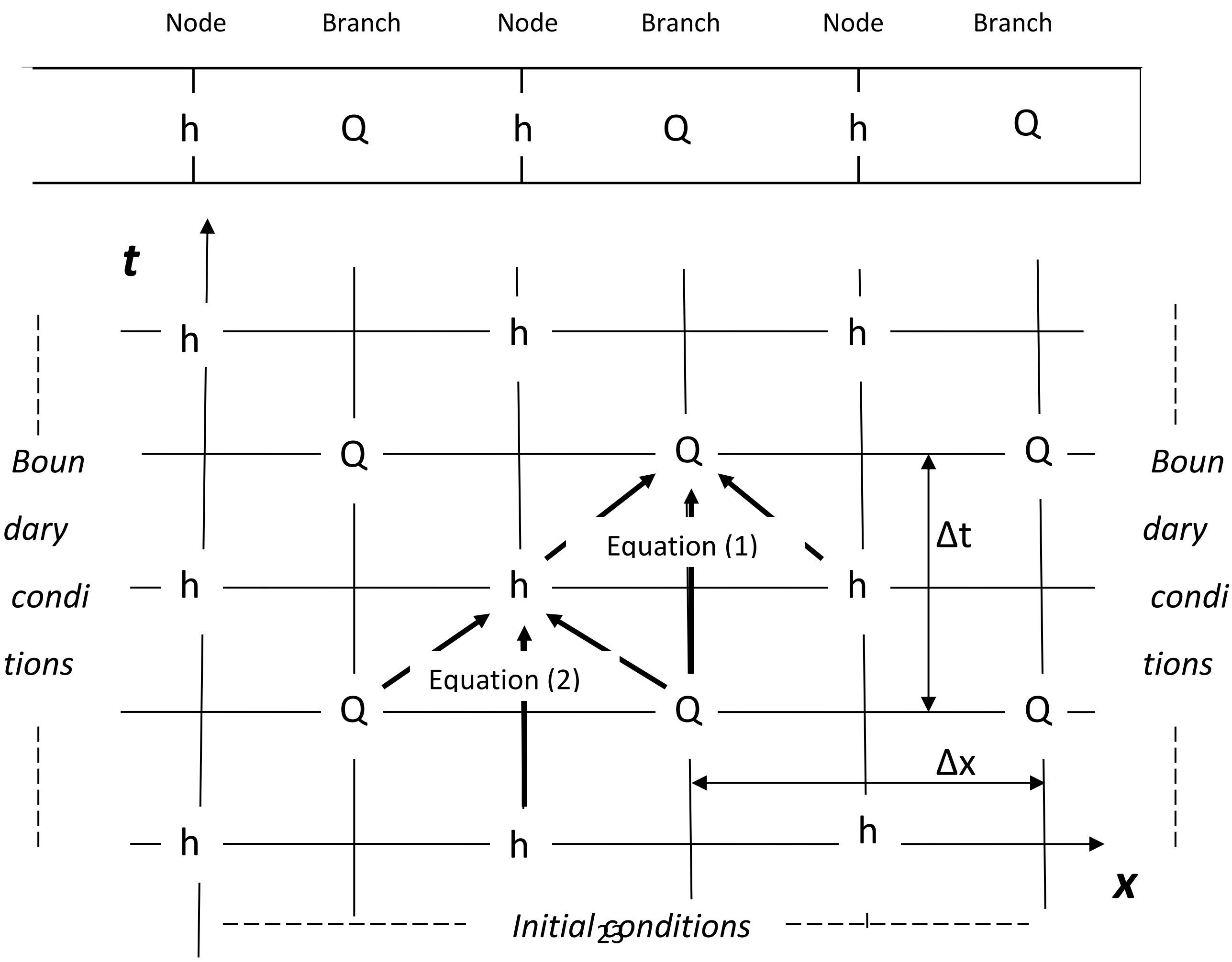
$$\frac{\partial h}{\partial t} = \frac{\sum Q}{S} \quad (2)$$

The momentum equation is linearized by using the absolute value of the discharge at the old time level in the friction term, leading to finite difference equations to solve the basic equations:

$$\frac{Q_{new} - Q_{old}}{\Delta t} + \frac{gQ_{new}|Q_{old}|}{C^2AR} = gA\frac{h_1 - h_2}{\Delta x} \qquad (1n)$$

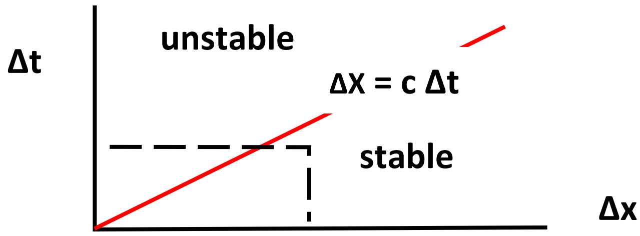
$$h_{new} - h_{old} = \frac{\sum Q\Delta t}{S} \qquad (2n)$$

The differential equations are solved numerically with a so-called “leap-frog” scheme, on a staggered grid in space and in time:



This is an explicit scheme, meaning that only information of the old time level is used. That means limitations to length and/or time steps: ***Δt < Δx/c***, for all branches. Δt is the calculation time step (DTCalc), Δx the branch length and c the wave celerity. c = √gd, in which g = acceleration of gravity and d = depth.

$$\Delta t < \frac{\Delta x}{c} = \frac{\Delta x}{\sqrt{gd}} \qquad (4)$$



In words one could say: to remain stable, the calculation should not leap forward in time or space beyond the wave.