Applied Mathematics on Urban Space

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Abstract The aim of this chapter is to explain what Space Syntax is. Firstly, the types of spatial elements used in Space Syntax is discussed, secondly, the various mathematical formulas of various space syntax methods are elaborated, and finally Space Syntax' contribution to theory building on built environments are discussed.

1 Introduction

Research and methodological development on built environments are located on the intersection between natural, human and sociological sciences. During the Greek and Roman period, as well as in the Renaissance period, mathematics was applied in the geometry of buildings as well as in urban planning. During the twentieth century, various approaches from the social and human sciences were applied for understanding how cities work in relation to society. Due to large social and economic changes during the Industrial Revolution, the role of health, population growth and economic development had to be treated together with uncontrolled urban growth. There exist several writings on how we understand cities and how we should design them. However, concrete methods on analysing urban form and urban space are lacking in most of these writings.

In the past three decades the space syntax method, developed by Bill Hillier and his colleagues at the University College London, has been applied to urban studies. This method consists of calculating configurative spatial relationships in built environments.

According to Hillier, space syntax is four things. First, space syntax is operating with a concise definition of urban space. Second, it is a family of techniques for analysing cities as the networks of space formed by the placing, grouping and orientation of buildings. Third, it is a set of techniques for observing how these networks

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of space relate to functional patterns such as movement, land use, area differentiation, migration patterns and even social well-being and malaise. Fourth, based on the empirical results from the two first things, space syntax has made it possible to make a set of theories about how urban space networks relate in general to the social, economic and cognitive factors which shape them and are affected by them. The techniques have been applied to a large number of cities in different parts of the world. In this way, a substantial database now exists of cities which have been studied at some level using space syntax (Hillier et al. 2007).

What space syntax calculates is the two primary all-to-all (all street segments to all others) relations. On the one hand, it measures the *to-movement*, or accessibility, potential, of each street segment with respect to all others. On the other hand, it measures the *through-movement* potential of each street segment with respect to all pairs of others. Each of these two types of relational pattern can be weighted by three different definitions of distance. The metric distance measures the city's street and road net as a system of shortest paths, while the topological distance, calculate the city's street and road net as a system of fewest turns paths. Finally, the geometrical distance gives a picture of the city's street and road net as a system of least angle change paths. Each type of relation can be calculated at different radii from each street segment, defining radius again either in terms of shortest, fewest turns or least angle paths (Hillier and Iida 2005, pp. 557–558).

Space syntax is under constant development. Its contribution to theories on built environments and methodology develop at the intersection of natural, social and technical sciences. So far, research projects range from anthropology or cognitive sciences to applied mathematics and informatics and touch upon philosophical issues. The evolution of space syntax asks for communication not just between various cultural contexts, but likewise between different scientific domains. Space syntax research is placed in the overlap between the applied mathematics from the natural sciences, cognition and orientation issues from the human sciences and human behaviour (social and economic related activities) from the social sciences.

2 Definition of Urban Space

Hillier distinguishes between *extrinsic* and *intrinsic* properties of space. Extrinsic ones determine the way in which spatial units relate to one another. Here settlements are regarded as sets of spaces defined by surrounding physical objects. Volumes, textures and size are not taken into consideration. Here, spaces are shape-free. It is just their inter-relational aspect or structure that is taken into consideration. Each space has one or more functions either in terms of occupation or with regard to movement (Hillier 1999, p. 1). Extrinsic properties of space determine both built form and its possible function.

Intrinsic properties of space relate to visible aspects of things we can see, i.e. shape, size, volumes and texture of physical objects or built mass. They present themselves mostly through geometrical properties. They account for the articulation of social meaning via a built form (Hillier 1999, p. 1). We have many words for describing the extrinsic properties of space. Words like "a narrow street, a large square, a massive building etc" make it possible to describe the artefacts of a city.

Describing the extrinsic properties of space, thus, requires to represent parts of the complex urban reality in models. Therefore, spatial modelling of built environments simplifies reality. In the case of space syntax, the fewest and longest sight lines axial map is seen as a representation of publicly accessible spaces. It models or represents the required correspondence between world and model. Naturally, the model is simpler than the thing modelled.

3 The Method of Calculations

Recent software development has made it possible to calculate and combine geometrical relationships (in terms of angular weighting between axial lines) and metric relationships with topological spatial relationships.

A global axial integration analysis implies to calculate how spatial integrated a street axe is in terms of the total number of direction changes to all others in a town or city. The fewer changes of direction to all other streets, the higher global integration values. Conversely, streets with many direction changes to all others tend to have low global integration values. Hence, they are spatially segregated.

Figure 1 left shows an axial map of a simple settlement, named town X, consisting of the main street with some side streets and some smaller back streets. The right upper corner in Fig. 1 shows an integration analysis of the settlement carried out by the computer program Depthmap. Various integration values are represented by colours. The red axes are the most integrated ones, while the blue ones are the most segregated ones. As shown in the lower part in Fig. 1, the justified graph shows how the system can be experienced from the most integrated main street and the segregated back street. In the case of the fewest line axial map, the lines are represented as nodes and the intersection of lines as connections between the nodes. The colours of the circles are the same as in the axial integration analyses. The justified graph shows how a whole system is connected to one another in an abstract manner.

The more integrated a street is, the shorter topological distance it has to all other streets. Other way around, the more segregated a street is, the longer topological distance to all other streets. When comparison the justified graphs of the back and the main street, the graph's structure looks more like a "tree" in the back street case, while it looks more like a "bush" in the case of the main street. The higher number of spaces in a shorter topological distance from space, the more likely this space will have a high integration value. The "bush" shaped justified graph is thus topological shallow. If most of the spaces are located many topological steps away, the more likely this space implies low integration values. In this case, it is a highly segregated space, and the "tree" shaped justified graph is topologically deep.

Since town X has a simple spatial system consisting of a few streets, it can be calculated manually. The computer programs Axwoman (Jiang et al. 2000) and Depthmap

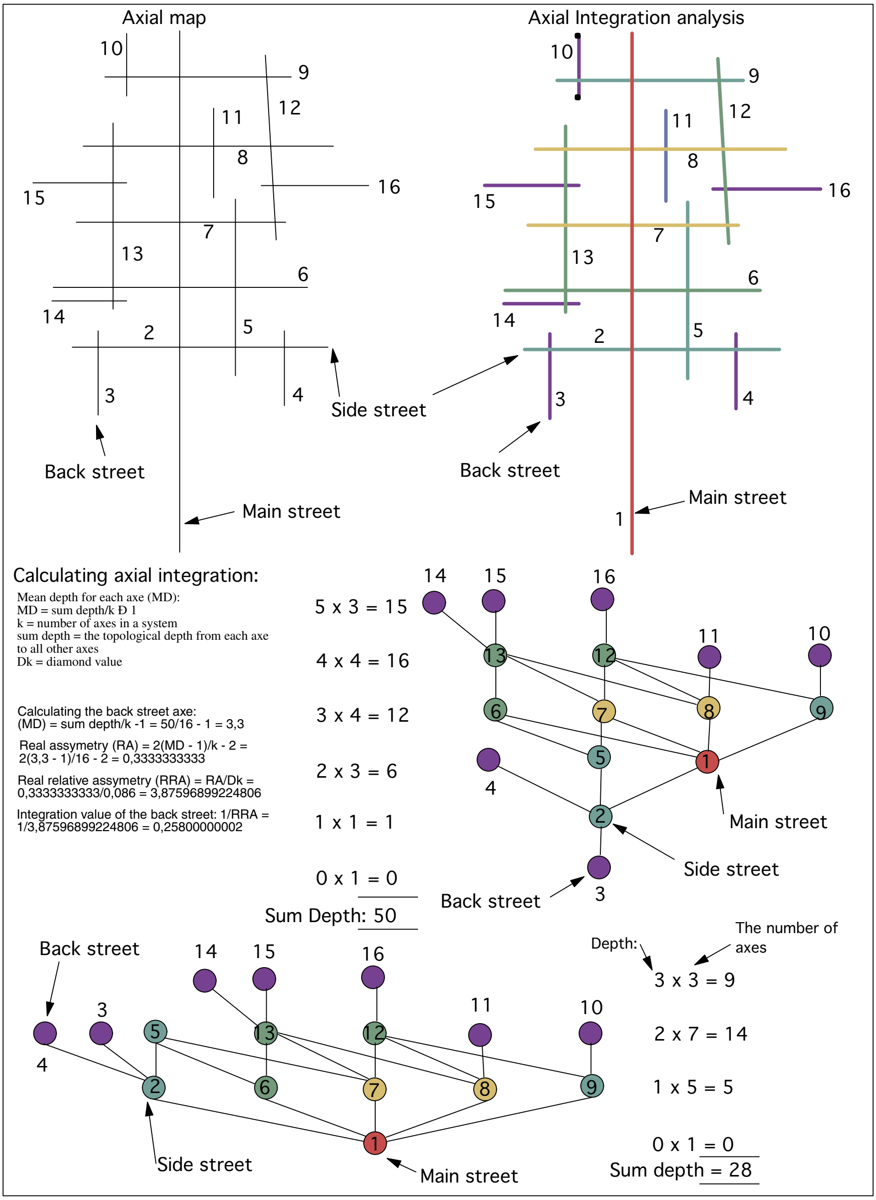

Fig. 1 A simple example on calculating spatial relationships

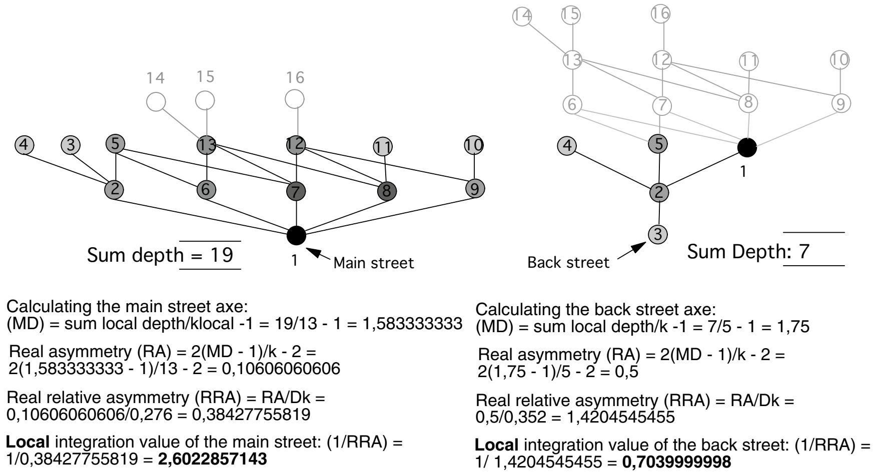

Fig. 2 The principle of calculating local integration

(Turner 2007) are able to calculate large cities with thousands of axes inter-related with one another.

As research shows, commercial activities take place in the most global integrated streets (Hillier 1996, p. 175; Hillier et al. 1993, pp. 31, 36 and 61; van Nes 2002, pp. 287–303). Dwelling areas are mostly located in the segregated areas (Hillier 1996, pp. 175–179; Hillier and Hanson 1984, p. 140).

A local axial integration analysis contributes to reducing the edge effect from global integration analyses. Figure 2 illustrates what local integration with a radius like 3 is like in town X. It calculates the value of the axe in a topological radius like 3 from each street. When calculating the value of the main street, the axes number 14, 15 and 16 are not taken into account. In the case of the back street, only 5 axes are included in the local integration analysis. The system in the left shows a local integration analysis of every street in town X.

Like global axial integration, local axial integration can measure the spatial impacts of the whole city before and after urban interventions. As research has shown, the flow rates of pedestrians through cities correlate with local integration values while vehicle flow rates correspond with global integration ones (Hillier et al. 1998, pp. 59, 84). Moreover, local axial integration gives an indication of local shopping areas in a city.

Figure 3 shows a global and local integration analysis of Haarlem. The locally most integrated streets, coloured in red, are where the most vital pedestrian-friendly shopping streets are. Along these streets, there is a mixture of individual shops, chain stores and cafés. Along the globally integrated streets, Haarlem's main city centre is located with a high number of shops, large chain stores, the municipality hall and the cinema.

Fig. 3 Global (left) and local (right) axial integration of Haarlem

Applying global and local integration analyses on Dutch cities tend to show weak results. Axial analyses count each change of direction as one topological step, even though the angle is close to 180°. In this way, many centres in cities with curved streets tend to get a broken up street net consisting of many short axial lines. As follows, the local axial integration values will get low in these kinds of areas, which does not always correspond with the location pattern of shops.

The angular analysis is essentially an extension of visibility graph analysis and axial analysis (Turner 2001, p. 30.1). What the angular analysis adds to the various integration analyses is that each axial line is weighted by the angle of their connections to other axial lines. As shown in Fig. 4, two axes which are almost 180° has a shallow angle of incidence while two axes with almost 90° have a sharp angle of incidence (Dalton 2001, p. 26.7). For making angular analyses, the axial map is broken up into segments. For example, a long axis crossing several other axes consist of several segments. Now it is possible to make integration analyses of the segments as well as taking the segments' angular relationship into account in the spatial analyses.

As research has shown, streets' angular relationships play a role in the way people orientate themselves through built environments. People tend to conserve linearity through their routes, with minimal angular deviation (Conroy Dalton 2001, p. 47.8). By changing direction, people tend to choose an angle close to 90° or to 180°. Urban blocks with rare angles, like those of 30 and 60°, tend to make people get lost.

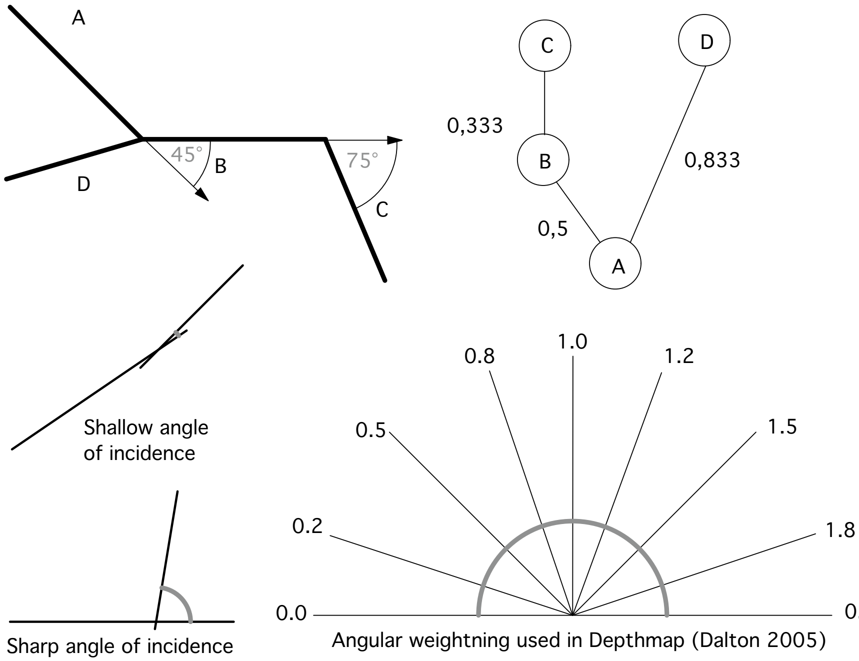

Fig. 4 Angular weighting of axial lines

Moreover, people tend to choose the longest street with the shortest angle towards their aimed direction. With other words, people choose the straightest possible routes in order to avoid the complexity for way-finding through urban street grids (Conroy Dalton 2001, p. 47.11).

In order to calculate the angular segment analyses of a street segment, the total angular turn from one segment to another segment is calculated. Values are given on different angles. For example, an angle with 45° has the value like 0,5, and angle with 90° has the value like 1, and an angle with 180° has a value like 2 (Turner 2005, p. 148). Consider four street segments connected to one another in different angles, as illustrated in Fig. 4. The depth from segment A to B is 0.5, since one makes a turn of 45°. The depth from segment A to C is 0.833 due to that one has to make a turn of 45° first to segment B and then a turn of 30° to segment B. The values are shown in the justified graph on the upper right in Fig. 4.

When calculating the angular mean depth, or the local angular integration, the case shown in Fig. 4 is used as an example. The angular mean depth from street segment A, is calculated as follows:

$$angular\ mean\ depth\ of\ (A) = \frac{(B)0.5 + (C)0.833 + (D)0.833}{(3)} = \mathbf{0.722} \quad (1)$$

The angular mean depth analysis highlights the main routes network through cities and regions. Moreover, the edge effect from the axial analysis is reduced. The least angle analysis seems to be the best predictor of movement, followed closely by the fewest turns (the results from the global and local integration analyses). Metric distance comes far behind the two first ones (Hillier et al. 2007).

The software Depthmap is able to make a segment map from an axial map. This map is the base for the angular analysis. At every node, the angular relationship between the connected lines can now be taken into account. When trying out various radii, the following results are obtained: the lower the radius, the more the main routes in the small local centres of a city are highlighted, the higher the radius, the more the main routes through the city are highlighted. As shown in Fig. 5, a local angular integration analysis highlights the most vital shopping streets in Haarlem as well as the streets where most of the larger companies and municipality hall are located.

One of the criticisms of space syntax has been that it does not take into account metrical properties in analyses of the mobility network (Ratti 2004, p. 501). As

Fig. 5 Angular analyses of Haarlem centre with R = 3

mentioned earlier, metrical distances show the least correlation between pedestrian and car traffic flow rates and the spatial analyses in comparison with the geometrical and topological distances. However, there is a difference between metrical distance and metrical radii.

Figure 6 shows some examples of how various types of radii affect various types of street networks. Town Y has on the left side of its main street an orthogonal street network and on the right side an organic street structure. When applying a two-step analysis from the main route axis, almost all streets in the strict orthogonal grid can be reached within two direction changes. Conversely, the local catchment area in the organic street network is rather poor. Likewise, when applying the two-step analysis to a street segment (below left in the figure), the local catchment area reduces slightly for both types of street networks. When applying the metrical radius from a segment, the strict orthogonal street network as well as the organic street network has more or less the same catchment area. The next step is to show what metrical radiuses add to the analyses of topological and geometrical distances.

Figure 7 shows the most used space syntax analyses of town Y. In the global axial and segment analyses, the main street is the highest integrated street. When splitting the axial map into segments, and running a segment integration analyses, the middle part of the main route is the most integrated on both global and local levels. When analysing the angular choice on global and local levels, the main route shows and a side street shows the largest through movement potentials. The angular segment integration analysis measures the "to-movement" potentials whereas the angular choice analysis measures the "through-movement" potentials on various scale levels. The first one highlights the potentials for urban centres, whereas the latter for the potentials for movement flow between various urban areas.

In order to demonstrate what a metrical radius implies for urban centrality when combining it with topological and geometrical distances, an example of a new and an old town is used. The Dutch new town Zoetermeer is 50 years old and has around 123.500 inhabitants. Originally, it was a small village until the late 1960s with 7000 inhabitants. The new town was constructed with the intention to catch up the population growth in the Hague City. The car traffic routes and pedestrian and bicycle routes are separated. Figure 8 shows angular choice segment analyses with a low and a high metrical radius for Zoetermeer's mobility network. All routes are included in the analyses. As can be seen in the figure on the left, the streets where the children's playgrounds are located are highlighted in red. Conversely, as shown in the figure on the right, the main routes between the various local areas are highlighted in red. These routes are trafficked by only vehicles. Zoetermeer's main mega shopping mall is located in the middle, where the density of the integrated main routes is the highest. The degree of orientability is low for visitors in Zoetermeer, due to its labyrinthly structured local street network and that the main route network is located far outside the various local residential areas.

When applying the same analyses to the old Dutch town Haarlem, a different structure can be seen. Haarlem was founded in 1245, and today it has around 155.500 inhabitants. Figure 9 shows an analysis of Haarlem's street and road network with a low radius. The town's local shopping streets are highlighted in red. Likewise, all

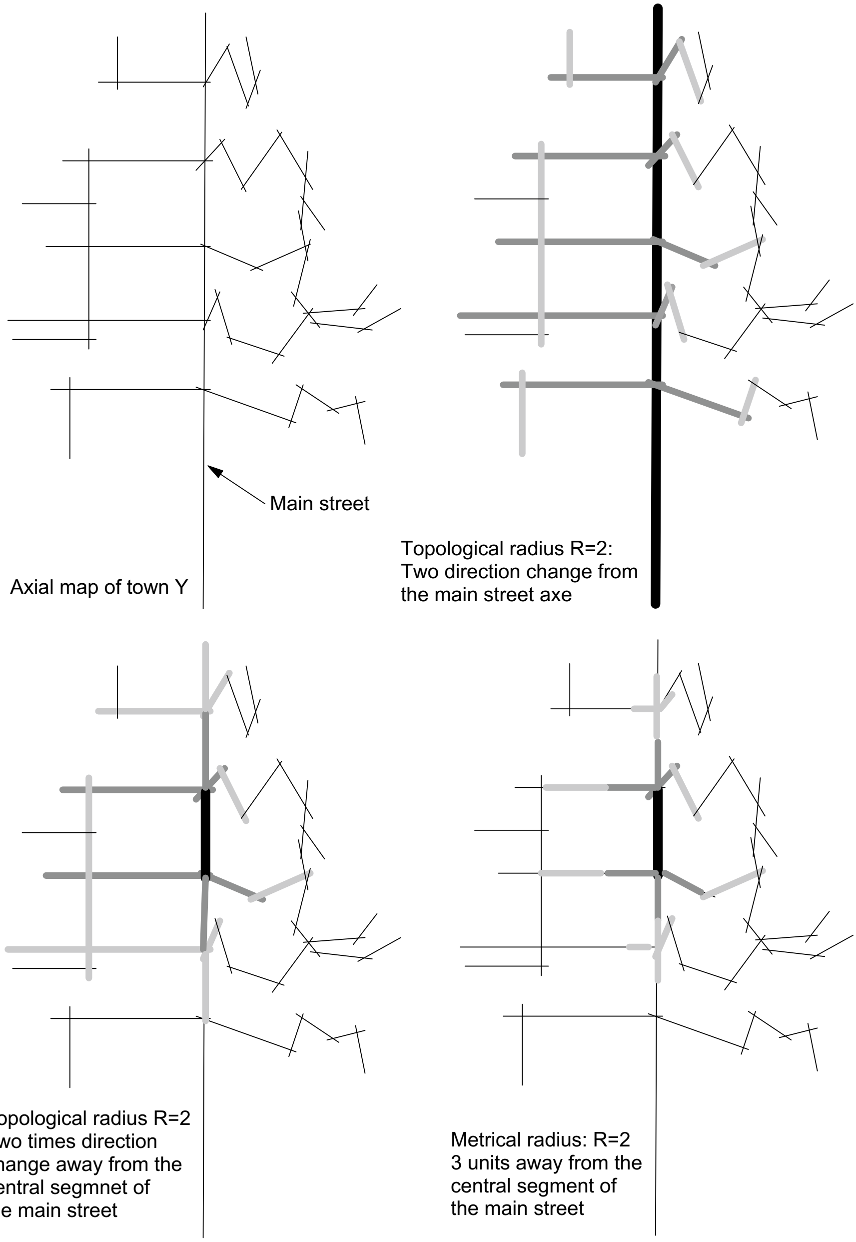

Fig. 6 The difference between a topological radius and a metrical radius

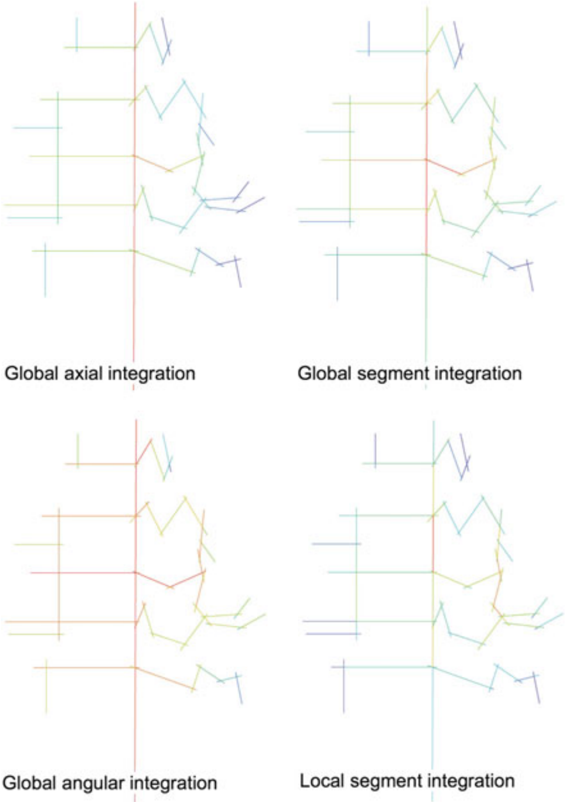

Fig. 7 Various integration analyses of Town Y

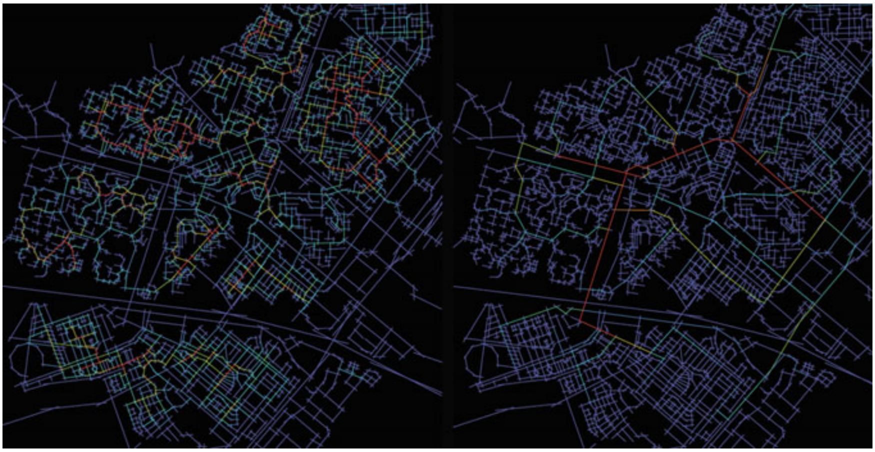

Fig. 8 Angular choice analyses with a low (left) and high (right) metrical radius for Zoetermeer

Fig. 9 Angular choice analyses with a low (left) and high (right) metrical radius for Haarlem

its central shopping streets are highlighted. When applying a high radius, as shown in Fig. 9, the main routes through the urban areas are highlighted in red. These main routes cross the local centres and the main centre highlighted in the small radius analysis.

As presumed, an optimal location for shops tends to be in streets that are accessible within a short metrical radius to a neighbourhood's dwellers as well as to a place that catches the through travellers. Therefore, the variation of shops in local shopping areas tends to be higher in old towns than in new towns. Even though the density of inhabitants can be high in a post-war neighbourhood, these areas generally offer a local supermarket. The supermarket is mostly used by the dwellers of the neighbourhood. In pre-war neighbourhoods, the variation of shops tends to be much higher than in post-war areas, due to their strategic location in the mobility network. These streets can easily be reached from the main route network.

A criticism often directed at new towns tends to be that they are "sleeping towns" with a lack of street life (van Casteren 2008). The use of the metrical radii can shed some light on this discussion. The main route network in post-war neighbourhoods is separated from the local centres whereas it is integrated with the local centres in pre-war neighbourhoods.

All cities are made up of a very large number of short streets and a very small number of long streets and roads. This can be seen on all scale levels in which gives the city street networks a clearly fractal structure. The foreground network is largely made up of longer streets or roads whose ends are linked by highly obtuse, nearly straight connections. The longer the line, the more likely it is to end with a nearly straight connection. The main routes through cities on all scale levels tend to consist of a set of longer lines connected to each other with almost 180° angles. The main routes net through urban areas it the armature for linking the city's edges to its centre. The natural interface of co-presence through movement from centres to edges is made efficient and possible, and affects the location of economic activities. The foreground network is the general component of the city (Hillier et al. 2007, pp. 2–4).

Conversely, the background network is largely made up of shorter streets, which tend to intersect and end at a near right angle. The shorter the street, the more likely it is to end at a right angle. Most silent dwelling streets tend to be metrically short. The background network reproduces the cultural pattern, and is the conservative component of the city. Various cultures have different local radius measures (Hillier et al. 2007, pp. 2–4).

In many ways, the generative spatial laws are the spatial parameter for human cognition. What space syntax measures are various degrees of inter-visibility i.e. how many people can see each other, and various degree of accessibility i.e. how a city's edges can reach its centre. The latter one concern the through movement pattern of "betweenness". When built environments grow in a natural way, extensions occur through the way avoiding blocking longer streets or roads (Hillier and Iida 2005, pp. 557–557).

Experimenting with mathematical formulas contributes to new discoveries (Hacking 1991). In many ways, space syntax is developed through "trial and error" research. New phenomena are created through experiments and tested out on a wide range of built environments. The challenges have been to give names on these various discovered phenomena and to present it to an audience outside the research community. At least experiments contribute to new development or improvement of existing methods for analysing urban space and to provide understandings on how built environments works.

All these spatial measurements can be correlated with a variety of other numerical data expressing social activities such as flows of human movement through the street net, land use pattern, land values, and distribution of crime. Thus, spatial and social factors can be correlated with one another. Hence, it is possible to study one urban area in correlation with its whole city.

So far, in urban research where space syntax has been applied, the following results were obtained. First of all, the degree of spatial integration is a strong predictor of pedestrian and traffic flow rates (Hillier et al. 1998, pp. 80–84). Pedestrian flows tend to follow various local integration values, while vehicle movement tends to follow the global integration values. As research has shown, applying the space syntax method on the configuration of the street net is better able to explain how the spatial set up of built environments influences the flow of traffic than the modelling techniques of the road engineers (Penn 2003, p. 33). Moreover, correlations between various degrees of integration and distribution of various types of crime have been found in research (Hillier and Sahbaz 2005; López and van Nes 2007; Hillier and Shu 2000). Moreover, a strong correlation was also found between streets' degree of integration and land values (Desyllas 2000), building density (Van Nes et al. 2012), land use diversity (Ye et al. 2014), degree of ethnic mixture (van Nes and Aghabeick 2015), degree of gender mixture (van Nes and Rooij 2015) and location pattern of shops (van Nes 2002).

4 Space Syntax' Contribution to Theory Building and Understanding on How to Build Environments Works

Building systematic theories on the built environment is still in a beginning phase. The reason is that most writings on built environment tend to have a *normative* approach (Hillier and Hanson 1984, p. 5), where most authors describe *how* to make a good city. What is lacking is a description of what a good city *is* or how a city *function* spatially and in relation to society. Conversely, writings in the field of urban sociology lack concise definitions of space or understandings on the physical framework on where various social interactions take place.

The use of space syntax has contributed to an understanding of the spatial structure of the city as an object shaped by a society on the one hand and on the other hand how it can generate or affect certain socio-economical processes in a society. To some extent, space syntax is able to predict some types of economic processes as an effect on urban interventions. Likewise, space syntax provides understandings on the spatial possibilities for certain social activities such as crime, social segregation and anti-social behaviour. It is all about how spatial integration and segregation conditions social integration and segregation.

The biggest challenge at this moment is to build descriptive theories on how cities work. It has to be done from three different perspectives. The first approach is on the relation between society and space. Here the focus is to get understandings on how activities in a society influence the shape, pattern and structure of a built environment. Research in the field of social anthropology and archaeology belongs under this approach. The aim is to gain an understanding of various cultures based on their built form. A hermeneutic approach is used here, and therefore clear explanations or theory building between cause (the society) and effect (the built form) from the natural science tradition cannot be done (von Wright 1971).

The second approach focuses only on the spatial relationships of the built environment. In line with the positivistic tradition, it is obvious that Hillier's theories on spatial laws or combinatorics (Hillier 1996, Chap. 8) have a strong link between cause and effect:

The principle of centrality: A central placed object increases the topological depth more than one placed at the edge.

The principle of extension: Partitioning a longer line increases the topological depth than a short one.

The principle of contiguity: Contiguous blocks increase topological depth more than separate ones.

The principle of compactness: Straight lines increase topological depth more than "curled" lines.

Intentions and human rationalities are not taken into account here.

The third approach is on the relationship between space and society. This approach has both a hermeneutic and positivistic approach. It is about how built form affect activities in society. Here again, human rationality has to be taken into account. Where the human intentions are unambiguous, it is possible to predict the effects on society as an effect on spatial changes of built form. Marked rationality is an unambigious rationality, where it is about profit maximising. Therefore, various space syntax researches have contributed to the *theory of the natural movement economic process*. The spatial structure of the street network influences the movement rates through an urban street net and where economic activities take place. Attractors, such as shops, retail and large firms tend to locate themselves along the most integrated streets (Hillier et al. 1993, pp. 31 and 61).

Figure 10 shows the relationship between configuration, attraction (the location of shops) and movement. It explains how a built environment function independent on planning processes regard the location pattern of economic activities, human movement through the urban network and the configuration of the street grid. Movement and attractors influence each other. The more people in a street, the more it attracts shops to locate along these streets. The more shops locating along a street, the more they attract people into this street. It gives a multiple effect process. After all, movement and attractors do not influence the configuration of the street net.

Likewise, the recently proposed *theory of the natural urban transformation process* (Ye et al. 2014) is rooted in marked rationality. The more integrated or accessible the street network is, the higher density of the buildings and the higher degree of

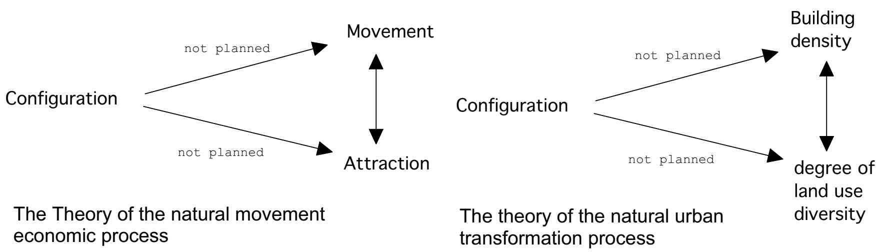

Fig. 10 The Theory of the natural movement economic process and the Theory of the natural urban transformation process

land use diversity. The spatial structure of the street network is the underlying driver for urban transformation processes.

Other kinds of human rationality, such as the occurrence of crime, anti-social behaviour, the location of various ethnical groups in cities and the occurrence of fear in built environments cannot be predicted. Therefore, there exist no theories on for example the relationship between space and anti-social behaviour.

Political forces and organisatoric constraints can overrun the spatial forces in built environments. A strong planning system on different levels and organisatoric constraints can block a natural location of economic activities at strategic optimal locations. Likewise, ethnic conflicts can contribute to that people avoid central integrated areas. Therefore one has to be aware of a country's planning system, political forces or ethnic conflicts on the one hand, and on the other hand the generative power of the street and road net.

In essence, subsequent considerations distinguish between a theory able to offer an *explanation* of phenomena and a theory proposing an *understanding* thereof. As concluded, the theories on spatial combinatorics, the natural movement economic process, and natural urban transformation process can offer an explanation of changes in a built environment in terms of cause and effect, while research related to social rationality, archaeology or historical research, space and crime or anti-social behaviour, cognitive aspects aims at an understanding of the culture or meaning associated with the causes at issue. Moreover, research concerning how activities in society affects urban space requires a hermeneutic approach, whereas research concerning how a spatial layout can affect activities in society requires both a positivistic as well as a hermeneutic approach. Seemingly, human behaviour as an effect on spatial structure depends on the type of rationality of human intentions and behaviour the research is focusing on. Marked rationality can use positivistic explanation models, whereas other kinds of rationality rely on hermeneutic ones.

What does space syntax add to studies on built environment? At least it offers concise spatial tools to measure spatial changes in the built environment, independent on context related situations where cultural aspect must be taken into account. In this way, space syntax is able finding some spatial evidence on presumptions and observations. Even though Norberg-Schulz criticises a quantitative approach in studies on built environment (Norberg-Schulz 1967, p. 202), a space syntax approach can at least provide some exact evidence on how some spatial components of built environments create lively or quiet places based on applying some simple mathematic calculations on spatial relationships.

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